

Credit Risk Analysis and Security Design*

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Abstract

We consider a stylized model of lending in which the lender analyzes the borrower's credit risk prior to the loan decision. Unless the lender obtains the full surplus from the project her credit policy will be too conservative, i.e., she will reject projects that would have obtained financing in a first-best world. The optimal contract in place minimizes this inefficiency. In a general setting with a continuous cash-flow distribution we show that the unique optimal contract is a standard debt contract. Among all possible contracts debt is the one that makes the lender the *least* conservative, thus providing her with optimal incentives to make (constrained) efficient credit decisions. Hence the common folk wisdom whereby giving banks equity makes them less conservative in their lending practice is generally not correct.

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1 Introduction

One of the fundamental questions in the security design literature is under what conditions simple security designs such as standard debt are optimal. This paper considers a stylized model that is representative of many real-world lending decisions. A lender publicly offers a contract, analyzes the borrower’s credit risk, and—based on the outcome of the analysis—either rejects or accepts the borrower. The credit risk analysis can be potentially complex and have multiple dimensions. For instance, a loan officer in charge of small business loans would typically base his recommendation both on “hard” information such as financial statements as well as his subjective beliefs as to whether the business owner is honest and the business is likely to do well in the future. The loan officer’s recommendation is assumed to come in the form of a one-dimensional score—or signal—where higher values indicate that the borrower (or the project for which he is seeking finance) is a better credit risk. Given that both the information entering the signal and the process of weighing and aggregating the information are partly subjective, it is natural to assume that the signal cannot be contracted upon.¹ At the end of the day it is thus exclusively at the lender’s discretion whether to grant the loan or not.

Our first observation is that—unless the lender obtains the entire surplus from the project—her credit policy will be too conservative.² That is, the cutoff signal chosen by the lender below which the borrower is rejected is too high relative to a first-best world where total surplus is maximized. This is not driven by the fact that the signal is a noisy indicator of the borrower’s credit risk. It holds equally if the signal is perfectly informative. Neither is it driven by the type of security in place. It holds for *any* security where the repayment is bounded from above by the project cash flow. Finally, the result is not driven by asymmetric information on the part of the borrower. While our results extend to the case where the borrower is privately informed about his (or his project’s) credit risk, our base model assumes that the borrower and lender have common priors at the time when the borrower applies for the loan.

The *optimal* security offered by the lender minimizes the inefficiency arising from excessive

¹One interpretation is that the signal is private information. But even if the signal is publicly observable, contracting upon it is pointless as long as the information entering the signal or the process of weighing and aggregating the information into a signal are not fully transparent or subjective in nature. The lender would then always report the signal that is optimal for her *ex post*. In the paper we also show that it is strictly suboptimal to offer a menu of contracts from which the lender can choose after observing the signal.

²If the lender obtains the entire surplus the (optimal) security is naturally straight equity.

project rejection. In a model with a continuous cash-flow distribution we show that the unique optimal security is standard debt. Debt maximizes the lender's payoff in low cash-flow states, thereby maximizing her return from low-type projects, i.e., projects with a high probability mass on low cash-flow outcomes. Having less to worry about low-type projects the lender is willing to apply a lower cutoff signal even though this increases the conditional likelihood of financing a low-type project. In other words, the cutoff signal implemented under a debt contract is lower than that implemented under any other security, and therefore closest to the cutoff signal chosen in a first-best world. Our argument that debt implements optimal lending decisions is both new and consistent with the widespread use of debt in situations where lenders perform credit risk analyses.

Various other rationales for the optimality of debt have been proposed in the literature. Debt minimizes ex-post verification costs (Townsend (1979), Gale and Hellwig (1985)) or the deadweight loss from inflicting non-pecuniary penalties upon borrowers (Diamond (1984)). If there is ex-post moral hazard on the part of the borrower debt provides the borrower with incentives to work hard (Innes (1990)) or repay funds (Bolton and Scharfstein (1990), Hart and Moore (1998)). Finally, debt is optimal if the issuer has private information either before (Nachman and Noe (1994), formalizing the intuition in Myers and Majluf (1984)) or after the security design (DeMarzo and Duffie (1999), Biais and Mariotti (2001)). In contrast to all these models the incentive problem in our model is with the lender, not the borrower. Also, as we have already emphasized, our argument for why debt is optimal is not related to private information on the part of the borrower. A more detailed account of the differences between our model and other models is provided in the paper.

Our results have implications for the policy debate surrounding the separation of banking and commerce in the U. S. There, banks are—with few exceptions—forbidden to hold significant equity stakes in nonfinancial firms.³ Berlin (2000) summarizes the research agenda as follows: “Do restrictions against U.S. banks holding equity make a difference for banks’ behavior? Are U.S. banks’ borrowers at a disadvantage because their lenders are too cautious when evaluating project risks and too harsh when a borrower experiences financial difficulties?” While in our model lenders are indeed too cautious, this is not because they hold debt, but because they do

³These restrictions derive from the Banking Act of 1933, also known as Glass-Steagall Act. The recent Gramm-Leach-Bliley Act of 1999 modestly expands banks’ powers to hold equity, but stops well short of permitting banks to hold mixed debt-equity claims as a normal lending practice.

not capture the full surplus from the project. In fact, we show that it is debt, not equity, that makes lenders the *least* cautious in their lending practice.⁴

In two extensions of the model we explore the potential role of borrowers' private wealth, which can be either used as collateral or to co-finance the investment. In either case the effect is that the lender's credit policy can be made more lenient. Pledging private wealth as collateral protects the lender against high-risk projects, while reducing her investment naturally limits her risk exposure. In both cases the lender will adjust the cutoff signal downwards. For a range of parameter values the cutoff signal now coincides with that chosen in a first-best world. The unique optimal contract is again standard debt: in addition to making the lender less conservative debt also minimizes the (costly) use of the borrower's private wealth.

Following Broecker (1990), a number of papers have examined the role of pool externalities arising from the fact that rejected borrowers worsen the average quality of loan applicants. The focus of this literature is clearly different from ours. In particular, it does not consider optimal security design. Manove, Padilla, and Pagano (2001) have a model in which high-type borrowers post collateral to avoid paying for testing costs. In our model testing costs are zero—hence there is no inefficiency from too little testing. Rather, the inefficiency is that positive NPV projects are not financed. Finally, in Garmaise (2001) investors receive a signal about project quality prior to bidding for the financing of a project in a first-price auction. Unlike our model, there is no underinvestment in his model as all projects are profitable.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 derives the lender's optimal credit policy and the optimal contract between the borrower and lender. It also contains a discussion of the main differences between our model and other models of debt. Section 4 reconsiders some of the main assumptions of the model and discusses their robustness. Section 5 extends the model to situations where the borrower has private wealth that can be either used as collateral or to co-finance the investment. Section 6 concludes. All proofs are in the Appendix.

⁴Existing research addressing the question of whether banks should hold equity focuses on different issues, such as the effect of bank equity holdings on borrowers' project choice (John, John, and Saunders (1994)), and banks' behavior during debt workouts and conflicts between the firm and nonequity stakeholders during financial distress (James (1995), Berlin, John, and Saunders (1996)).

2 The Model

An entrepreneur (“the borrower”) has a project requiring an investment k . The borrower has private wealth w , where $0 \leq w < k$. For the moment we shall set $w = 0$. In Section 5 we consider the case where $w > 0$. Financing is provided by a lender who can assess the borrower’s credit risk prior to the loan decision.

2.1 Project Technology

There are several possible project types denoted by $\theta \in \Theta$. For expositional convenience we assume that the type set Θ is countable. Our results can be extended to arbitrary type sets, except to the case where Θ is a singleton. Types are denoted by natural numbers $\theta \in \Theta := \{1, 2, \dots, \bar{\theta}\}$, where $\bar{\theta} := \infty$ if there is an infinite number of types.

A type- θ project generates a random, non-negative cash flow x drawn from the distribution function $G_\theta(x)$ with support $X := [\underline{x}, \bar{x}]$. The support is the same for all types. We assume that $\bar{x} > \underline{x} \geq 0$, where \bar{x} can be either finite or infinite. The corresponding density $g_\theta(x)$ is positive for all $x \in X$ and has finite expected value $\mu(\theta) > 0$.⁵

We next impose an ordering on the family of distribution functions, $G_\theta(x)$. We assume that high-type projects put strictly more probability mass on high cash flows than low-type projects in the sense of the Monotone Likelihood Ratio Property (MLRP).

Assumption 1. *For any pair $(\theta, \theta') \in \Theta$ with $\theta' > \theta$ the ratio $g_{\theta'}(x)/g_\theta(x)$ is strictly increasing in x for all $x \in X$.*

MLRP is a common assumption in contracting models. (See Milgrom (1981) for an economic interpretation of MLRP.) In Section 4.2 we show that our results also hold under a weaker assumption if we make an additional assumption about the contract space.

We assume that it is profitable to invest in projects with a sufficiently high θ . To provide a role for credit risk analysis, we moreover assume that projects with a sufficiently low θ are unprofitable.

⁵Continuity of the cash-flow distribution function is a standard assumption in the security design literature and implies that we can restrict attention to deterministic sharing rules. We assume absolute continuity to introduce the Monotone Likelihood Ratio Property below. All our results extend to the case where $g_\theta(x) = 0$ at the boundaries of X .

Assumption 2. $\mu(1) < k$, $\mu(\bar{\theta}) > k$ if $\bar{\theta}$ is finite, and $\mu(\theta) > k$ for all sufficiently high $\theta \in \Theta$ if $\bar{\theta} = \infty$.

Assumption 2 implies that $\underline{x} < k$. Hence the riskless cash flow is strictly less than the investment cost.

2.2 Credit Risk Analysis

As sketched in the Introduction, the credit risk analysis generates a noncontractible score—or signal— s . (One interpretation is that the lender privately observes s .) The signal has support $S := [0, 1]$ and is drawn from the distribution function $F_\theta(s)$ with continuous density $f_\theta(s)$, where $f_\theta(s) > 0$ for all $s \in (0, 1)$. The assumptions concerning the density function ensure that even a small change in the contract (i.e., security) will have an effect on the lender’s credit policy. Finally, while we believe the assumption that s is noisy is realistic, it is not critical for our results. Section 4.3 shows that the model can be easily rephrased as a model in which there is a continuum of project types and a fully informative signal.

There are at least two potential costs, or inefficiencies, associated with credit risk analysis: (i) making type-1 and type-2 errors, i.e., misclassifying bad projects as good and vice versa, and (ii) spending too little or too much time or effort on the analysis. As it is not the focus of the paper we shall abstract from the second problem by assuming that the analysis is costless. In practice, technological innovations such as credit scoring have drastically reduced the cost of reviewing loan applications. For instance, credit scoring has reduced the loan approval process to well under an hour (Mester (1997)), while the actual cost of a commercially available credit scoring model averages about \$1.5 to \$10 per loan (Muolo (1995)).

Unlike other security design models (e.g., Nachman and Noe (1994), DeMarzo and Duffie (1999), Biais and Mariotti (2001)), our results do not require that the borrower possesses private information. To underscore this point we assume that prior to the credit risk analysis the borrower and lender have common priors $h(\theta)$, where $h(\theta) > 0$ for all $\theta \in \Theta$. In Section 4.4 we show that our results naturally extend to the case where the borrower has private information about the project quality.

One implication of the fact that the borrower and lender have common priors is that the credit risk analysis generates new, valuable information that has been unavailable previously (rather than merely providing the lender with information the borrower already has). After

all, credit risk models are typically based on sophisticated statistical techniques and draw on the experience from thousands of historical loans. It is unlikely that the borrower’s personal estimate of the project’s credit risk is as accurate as that of an experienced loan officer. As we pointed out above, however, our results also hold if the borrower knows more than the lender at the time he applies for a loan.

Similar to $G_\theta(x)$, we impose an ordering on the family of signal-generating distributions, $F_\theta(s)$ by assuming that high-type projects put strictly more probability mass on high signals than low-type projects in the sense of MLRP.

Assumption 3. *For any pair $(\theta, \theta') \in \Theta$ with $\theta' > \theta$ the ratio $f_{\theta'}(s)/f_\theta(s)$ is strictly increasing in s for all $s \in S$.*

By Bayes’ rule, the lender has posterior beliefs that the borrower is of type θ with probability

$$h(\theta | s) := \frac{h(\theta)f_\theta(s)}{\sum_{\theta' \in \Theta} h(\theta')f_{\theta'}(s)}. \quad (1)$$

The conditional expected cash flow given the signal s is $\sum_{\theta \in \Theta} \mu(\theta)h(\theta | s)$. Assumption 3 then has the following natural interpretation: observing a high signal is good news in the sense that the conditional expected cash flow is increasing in s . (This follows directly from the fact that under Assumption 3 posterior beliefs satisfy MLRP.) In the Appendix we show that the reverse is also true, i.e., the conditional expected cash flow is increasing in s only if Assumption 3 holds. Finally, to rule out trivial cases where the optimal credit policy is independent of the signal we assume that investing is unprofitable if $s = 0$ but profitable after if $s = 1$, i.e., $\sum_{\theta \in \Theta} \mu(\theta)h(\theta | s) < k$ if $s = 0$ and $\sum_{\theta \in \Theta} \mu(\theta)h(\theta | s) > k$ if $s = 1$.⁶

2.3 Lending Process

The timing is as follows. The lender publicly announces a contract—or repayment scheme— $t(x)$ (the “security”). The borrower then decides whether to apply for the loan. If the borrower applies the lender assesses the borrower’s credit risk. Based on the resulting signal s the lender either accepts or rejects the borrower. Both the security $t(x)$ and the accept/reject-decision (“credit policy”) are chosen optimally.

⁶For instance, this is satisfied if at $s = 0$ it holds that $f_\theta(0) = 0$ for $\theta > 1$ and $f_\theta(0) > 0$ for $\theta = 1$ and at $s = 1$ it holds that $f_\theta(1) = 0$ for $\theta < \bar{\theta}$ and $f_\theta(1) > 0$ for $\bar{\theta} = 1$. (If $\bar{\theta} = \infty$ then $f_\theta(1) = 0$ must hold for all sufficiently low values of θ .)

We shall assume that the borrower receives a payment from the lender only if he has been approved. For instance, courts might not be able to distinguish between someone who was rejected and someone who has never applied. In this case a payment conditional upon rejection is not feasible. Second, such a payment might attract a large pool of fraudulent entrepreneurs, or “fly-by-night operators” (Rajan (1992), von Thadden (1995)), i.e., applicants without a reasonable investment opportunity who know for sure they will be rejected. The only reason why these applicants would apply is to cash in the payment. In this case a payment conditional upon rejection is feasible but not optimal.⁷ Note that this line of reasoning also rules out the possibility of buying the borrower’s project prior to assessing the credit risk.

While the contract cannot condition on s the lender could—in principle—signal her information by choosing a contract from a prespecified menu after observing s . We show in Section 4.1 that the unique optimal menu contains a single contract, implying that all borrowers who are accepted obtain the same conditions. Hence in our model loan uniformity arises endogenously as part of an optimal design. Alternatively—and related in spirit to DeMarzo and Duffie (1999)—one could ask the question what is the optimal “robust” security that works well for a variety of different borrowers. The reason for this could be that fine-tuning is costly or that discretion regarding loan conditions invites rent-seeking or collusion between the loan officer and the borrower. In this case loan uniformity is imposed by assumption. Whether uniformity is endogenously optimal or assumed is not important for our purpose.

In practice all of the above arguments are likely to matter. For instance, Saunders and Thomas (2001) note: “loan decisions made for many types of retail loans are reject or accept decisions. All borrowers who are accepted are often charged the same rate of interest and by implication the same risk premium. [...] In the terminology of finance, retail customers are more likely to be sorted or rationed by loan quantity restrictions rather than by price or interest rate differences.” The same authors also provide evidence suggesting that the cost of fine-tuning may outweigh the benefits: “a \$50,000 loan yielding three percent over cost of funds provides

⁷Specifically, suppose fraudulent entrepreneurs have projects that yield a certain payoff of zero. The lender finds out whether the entrepreneur is fraudulent only by performing the credit risk analysis. (The credit risk analysis includes an assessment of the entrepreneur’s books and business plan.) Stipulating a payment upon rejection would attract all fraudulent entrepreneurs in the population. By making the pool of fraudulent entrepreneurs sufficiently large we can then rule out the optimality of such payments. Note that if the payment was made to a third party, this party would have an incentive to collude with a fraudulent entrepreneur.

only \$1,500 p.a. gross revenues, before provisions of credit losses and allocation of overheads. This level of gross revenue can pay for very little of the loan officer’s analytical and monitoring time”. Finally, loan officers indeed seem to have limited discretion in adjusting loan terms: “relationship managers [...] had become largely a salesforce with limited discretion to depart from centralized criteria on loan decisions” and “the discretion of relationship managers within the guidelines for assessing risk, agreeing loans and setting terms under which he or she operates is normally very limited” (UK Competition Commission (2002)).

Since $w = 0$ the repayment scheme must satisfy $t(x) \leq x$. Note that this allows negative transfers from the lender to the borrower. We finally invoke the standard assumption that the repayment to the lender be nondecreasing in the realized cash flow.

Assumption 4: $t(x)$ is nondecreasing.

If Assumption 4 did not hold the borrower could make an arbitrage gain by borrowing money ex post to boost the cash flow. Since the loan is riskless any third party would be willing to provide the required funds. In Section 4.2 we show that by strengthening Assumption 4 one can relax Assumption 1 which requires that the cash-flow distribution satisfies MLRP.

While we assume that the lender makes the contract offer we explicitly want to leave open the possibility that the surplus is shared between the borrower and lender. We do this by assuming that in expectation the borrower must receive at least $\bar{V} \geq 0$. If $\bar{V} > 0$ the borrower captures a strictly positive fraction of the surplus. In practice, the division of surplus between the borrower and lender will depend on their relative bargaining powers, which in turn will depend on the competitiveness of the capital market.⁸

3 Optimal Credit Policy and Security Design

The analysis proceeds in two steps. We first derive the lender’s optimal credit policy. High-type projects generate a more favorable signal distribution than low-type projects. Conversely, a high signal generates a more favorable distribution of types, and hence project cash flows. Since the lender’s payoff is nondecreasing in the project cash flow we have that both the first- and second-best credit policy follow a simple cutoff rule: if the signal is above the cutoff signal the borrower is accepted, otherwise he is rejected. In a second step we derive the optimal contract

⁸See Inderst and Müller (2002b) for a model endogenizing \bar{V} as a function of capital market competition.

between the borrower and lender. We finally point out the differences between our model and related models of security design.

3.1 Optimal Credit Policy

As a benchmark let us begin with the first-best credit policy. The first-best credit policy is characterized by a unique cutoff signal s^{FB} with the following properties.

Lemma 1. *The first-best credit policy is to approve the loan if $s \geq s^{FB}$ and to reject the loan if $s \leq s^{FB}$, where $0 < s^{FB} < 1$. The first-best cutoff signal is unique and given by*

$$\sum_{\theta \in \Theta} \mu(\theta) h(\theta | s^{FB}) = k. \quad (2)$$

Lemma 1 follows from the fact that higher types both have a higher expected cash flow and are more likely to generate a higher signal. At $s = s^{FB}$ the project just breaks even, implying that acceptance and rejection are both optimal. For all $s > s^{FB}$ the expected project cash flow is strictly greater than the investment cost k .

As s is not contractible there is no simple way to implement the first-best credit policy. The lender's incentives to accept or reject a project will then depend on the contract in place. We now turn to the second-best credit policy, i.e., the credit policy implemented by the lender given the underlying contract $t(x)$. Denote the lender's (gross) payoff from a type- θ project by

$$U_{\theta}(t) := \int_{x \in X} t(x) g_{\theta}(x) dx.$$

As $t(x)$ is nondecreasing the lender's payoff is increasing in the borrower's type. Since high types are more likely to generate high signals than low types, the second-best credit policy involves (again) a unique cutoff signal. Denote this cutoff signal by $s^{SB}(t)$. Note that s^{SB} depends on $t(x)$. Unlike the first-best case it is now possible that $s^{SB}(t) = 1$. If $s = s^{SB}(t) = 1$ the lender is either indifferent between accepting and rejecting (if $\sum_{\theta \in \Theta} U_{\theta}(t) h(\theta | 1) = k$) or she strictly prefers to reject (if $\sum_{\theta \in \Theta} U_{\theta}(t) h(\theta | 1) < k$). To simplify the optimal policy rule we assume that in case of indifference the lender rejects.⁹

Lemma 2. *The lender's optimal—or second-best—credit policy is to approve the loan if $s > s^{SB}(t)$ and to reject it if $s \leq s^{SB}(t)$, where $0 < s^{SB}(t) \leq 1$. If $\sum_{\theta \in \Theta} U_{\theta}(t) h(\theta | 1) \leq k$ the*

⁹As indifference is a zero-probability event this is without loss of generality.

second-best cutoff signal is $s^{SB}(t) = 1$, while if $\sum_{\theta \in \Theta} U_{\theta}(t)h(\theta | 1) > k$ the second-best cutoff signal is unique and given by

$$\sum_{\theta \in \Theta} U_{\theta}(t)h(\theta | s^{SB}(t)) = k. \quad (3)$$

Accordingly, if the lender's expected payoff at $s = 1$ is less than or equal to k the optimal policy rule is to reject for all $s \in S$. By contrast, if the lender's expected payoff at $s = 1$ is greater than k there exists a threshold signal $s^{SB}(t) < 1$ at which she just breaks even, while for all signals $s > s^{SB}(t)$ her profit is strictly positive and increasing in s . The optimal policy rule is then to reject if $s \leq s^{SB}(t)$ and to accept if $s > s^{SB}(t)$. For expositional convenience we shall frequently omit the argument in $s^{SB}(t)$ and write s^{SB} , while keeping in mind that s^{SB} depends on $t(x)$. Finally, comparing (2) and (3) shows that the first- and second-best cutoff signals coincide if and only if $U_{\theta}(t) = \mu(\theta)$ for all θ , i.e., if and only if the lender obtains the entire cash flow.¹⁰

Is the lender too conservative or too lenient under the second-best credit policy? Suppose the lender does not obtain the entire cash flow. That is, suppose $t(x) < x$ for some x , implying that $U_{\theta}(t) < \mu(\theta)$. By (2) we have

$$\sum_{\theta \in \Theta} U_{\theta}(t)h(\theta | s^{FB}) < \sum_{\theta \in \Theta} \mu(\theta)h(\theta | s^{FB}) = k,$$

i.e., the lender no longer breaks even upon observing s^{FB} . Since the lender's expected payoff $\sum_{\theta \in \Theta} U_{\theta}(t)h(\theta | s)$ is strictly increasing in s this implies that the (second-best) optimal cutoff signal must lie strictly above s^{FB} . In other words, under the second-best credit policy the lender is too conservative relative to the first-best benchmark.

Lemma 3. *Unless the lender obtains the entire surplus the second-best cutoff signal lies strictly above the first-best benchmark, i.e., $s^{SB} > s^{FB}$.*

Interestingly, the argument that the lender is too conservative does not rest on Assumption 4 stating that repayment schemes be nondecreasing: the lender is too conservative under *any* contract $t(x)$ where $t(x) < x$ on a set of positive measure. While the second-best credit policy might no longer involve a single cutoff if $t(x)$ is decreasing over some range, it still holds that

¹⁰For this argument it is crucial that $t(x) \leq x$. If the borrower could make a repayment that is greater than x in some states (e.g., by liquidating collateral) the first- and second best credit policy might coincide even if the lender does not obtain the entire cash flow (on average). See Section 5.1 for details.

the set of signals for which the borrower is accepted is strictly smaller than the corresponding set in a first-best world.¹¹

EXAMPLE. As an illustration of Lemma 3 suppose there are two types, $\theta = 1, 2$ (low- and high-type projects, respectively). By Assumptions 1 and 4 we have that $U_1(t) < U_2(t)$ (unless $t(x) = U_1(t) = U_2(t) = 0$), i.e., the lender's utility under the low-type project is strictly lower than under the high-type project.

The lender's expected utility given the signal s is

$$U_1(t)h(1 | s) + U_2(t) [1 - h(1 | s)].$$

Hence the lender's expected utility is a weighted average of $U_1(t)$ and $U_2(t)$, where the "weight" associated with the lower utility, $h(1 | s)$, is decreasing in s . Unless $t(x) = x$ the lender's expected utility at $s = s^{FB}$ is strictly less than k . To make it again equal to k the lender must decrease the weight associated with the lower utility, $h(1 | s)$, and increase the weight associated with the higher utility, $1 - h(1 | s)$. That is, the lender must increase s , implying that $s^{SB} > s^{FB}$.

3.2 Optimal Security Design

The previous analysis suggests that there are two cases. If $\bar{V} = 0$ the lender can extract the entire cash flow, implying that the unique optimal contract is $t(x) = x$. The corresponding credit policy is first-best efficient.

By contrast, if $\bar{V} > 0$ we necessarily have that $t(x) < x$ for some x , implying that the optimal credit policy is inefficient. The optimal contract minimizes this inefficiency. Formally, the optimal contract maximizes the lender's expected payoff

$$U(t) := \sum_{\theta \in \Theta} h(\theta) [1 - F_\theta(s^{SB})] U_\theta(t)$$

subject to the constraint that in expectation the borrower receives at least \bar{V} :

$$V(t) := \sum_{\theta \in \Theta} h(\theta) [1 - F_\theta(s^{SB})] V_\theta(t) \geq \bar{V}, \quad (4)$$

¹¹This is easily established. Let $\Delta(x) := x - t(x)$ and $\Delta(\theta) = \int_{x \in X} \Delta(x) g_\theta(x) dx$. For $\bar{V} > 0$ it must hold that $\Delta(\theta) > 0$. The set of all signals for which the loan is approved, \bar{S} , contains all s satisfying $\sum_{\theta \in \Theta} h(\theta | s) [\mu(\theta) - \Delta(\theta)] \geq k$. If \bar{S} is not empty it follows from continuity of the posterior distribution in s that \bar{S} is compact, implying by the definition of s^{FB} that $\bar{S} \subset [s^{FB}, 1]$ and that $[s^{FB}, 1] \setminus \bar{S}$ has positive measure.

where s^{SB} is defined in Lemma 2, $V_\theta(t) := \int_{x \in X} [x - t(x)] g_\theta(x) dx$, and $t(x)$ satisfies Assumption 4. Clearly, if \bar{V} is too large the lender cannot break even. In all other cases a non-trivial contract under which the borrower is accepted with positive probability exists. In the following we assume that \bar{V} is sufficiently small in the sense just described.

In equilibrium the constraint (4) must bind, implying that any surplus in excess of \bar{V} accrues to the lender. As a claimant to the residual surplus the lender wants to minimize the efficiency loss due to excessive rejection.¹² Standard debt accomplishes this objective. Subject to the monotonicity constraint in Assumption 4 a debt contract maximizes the lender's payoff in low cash-flow states. Since low cash-flow states are more likely under low-type projects debt thus maximizes the lender's payoff from low-type projects. Having less to worry about low-type projects the lender implements a credit policy that is less conservative than under other security. Put differently, a debt contract minimizes the gap between the first- and second-best cutoff signal and therefore the efficiency loss from excessive rejection. And yet, as long as we have $\bar{V} > 0$ the second-best cutoff signal remains above the first-best benchmark.

Proposition 1. *If $\bar{V} = 0$ the unique optimal contract is $t(x) = x$. The lender captures the entire surplus and the optimal credit policy is first-best efficient.*

By contrast, if $\bar{V} > 0$ the unique optimal contract is a standard debt contract. That is, there exists a repayment $R > k$ such that the unique optimal contract is $t(x) = x$ for all $x \leq R$ and $t(x) = R$ for all $x > R$.

If $\bar{V} = 0$ the lender gets the entire project cash flow. The optimal contract $t(x) = x$ can be either interpreted as 100 percent equity or as standard debt with repayment level $R = \bar{x}$. The interesting case, however, is clearly where $\bar{V} > 0$, i.e., where the borrower captures some of the surplus generated by the project.

In the Appendix we show that the size of the repayment depends on \bar{V} . If \bar{V} is small the borrower captures only a small fraction of the expected surplus, and R will be high. Conversely, if \bar{V} is high the repayment R will be low. We conclude by showing that the greater the fraction of surplus captured by the borrower (i.e., the lower the repayment R), the more conservative is the lender's credit policy.

Corollary 1. *An increase in \bar{V} reduces R and raises the second-best cutoff signal s^{SB} , thereby*

¹²The same would hold if the borrower were to make the contract offer. The argument for why standard debt is optimal in our model is thus independent of who makes the contract offer.

pushing the second-best credit policy further away from the first-best optimum.

If \bar{V} is a function of credit market competition Corollary 1 has the following tentative implications. If—due to an increase in competition—borrowers capture a greater fraction of the surplus, lenders become more conservative and any given borrower will be rejected with higher probability. Several studies document a reduction in credit volume for small firms following an increase in credit market competition, e.g., Petersen and Rajan (1995) and Cetorelli and Gambera (2001). These studies also find that the adverse effect of an increase in competition is strongest for firms that are young, small, and relatively opaque. In other words, firms for which there is considerable quality uncertainty. These are precisely the kind of firms where credit risk analysis is most valuable.

3.3 Theories of Debt

Sections 3.1-3.2 provide a new, and intuitive argument for the optimality of debt based on the notion that credit decisions depend on the type of security in place. While the lender is generally too conservative (unless $\bar{V} = 0$), debt minimizes the associated inefficiency. Let us briefly point out the main differences between our model and other models of debt. Evidently, our argument is different from costly state-verification models (Townsend (1979), Gale and Hellwig (1985), and repeated lending models with non-verifiable cash flow (Bolton and Scharfstein (1990), Hart and Moore (1998), DeMarzo and Fishman (2000), Inderst and Müller (2002a)).

Nachman and Noe (1994), DeMarzo and Duffie (1999), and Biais and Mariotti (2001) consider private information on the part of the borrower either before or after the security design. While our model can be extended to incorporate such private information (Section 4.3), private information is not crucial—or necessary—for our results. To underscore this point we assume that the borrower and lender have symmetric information at the security design stage. After the security design, if anything, it is the lender who has private information (about the signal), not the borrower.

More fundamentally, and going back to Myers and Majluf (1984), the idea in private information models is that debt limits underpricing as it is relatively “information insensitive” to the borrower’s private information. If possible, the borrower would like to issue riskless debt, thereby fully eliminating the lemons problem.¹³ By contrast, in our model the problem is to

¹³Riskless debt may have other disadvantages, however. While there is no mispricing, issuing riskless debt

trade off errors of the first and second type. We showed in Section 3.1 that the first-best credit policy can be implemented only by giving the lender 100 percent equity. Hence the idea is not to shield the lender from cash-flow risk but—on the contrary—to expose her to cash-flow risk so that she would make efficient lending decisions. In a sense, our model is much closer to a moral hazard model than to a private information model. As the signal is noncontractible the security is designed to give the lender optimal incentives to implement an efficient credit policy, much like the optimal contract in a moral hazard model is designed to provide the agent with incentives to exert effort.

Like this model, Innes (1990) also explores the interaction between security design and moral hazard. In his model the incentive problem is with the borrower (i.e., the entrepreneur), while in our model it is with the lender. Yet there is a second, interesting difference between the kind of moral hazard examined here and that examined in Innes' model. For simplicity suppose there are two effort levels, $\theta = 1, 2$. Denote the agent's utility (including effort cost) under the contract t by $U_\theta(t)$. In Innes' model the agent's incentives depend solely on the utility difference $U_2(t) - U_1(t)$. Accordingly, a non-monotonic contract generating the same utility difference as 100 percent equity can attain the first best outcome (Innes (1990), Section 4). By contrast, in our model the lender's incentives to approve the loan depend both on the difference $U_2(t) - U_1(t)$ *and* on the absolute levels $U_1(t)$ and $U_2(t)$. Unless $U_1(t)$ and $U_2(t)$ assume their first-best values (i.e., unless the lender obtains the entire surplus) the first-best outcome cannot be attained—regardless of what value the difference $U_2(t) - U_1(t)$ takes.

4 General Discussion

4.1 Menu of Contracts

While the contract cannot directly condition on s there remains the possibility that the lender chooses a contract from a prespecified menu after observing s , hence potentially revealing the signal. We shall now investigate this possibility formally. By standard arguments we can restrict attention to direct mechanisms of the following kind. The lender offers a menu of contracts. After observing the signal the lender can reject or accept the applicant. If the applicant is

forces the issuer to retain a potentially large fraction of the project cash flow. See DeMarzo and Duffie (1999) for a model where the issuer trades off the retention cost of debt against the benefit of limiting mispricing.

accepted the lender is free to choose any contract from the menu.¹⁴ A menu is a collection $(t_i)_{i \in I}$ of contracts satisfying Assumption 4, where I is some index set and each contract is *ex post* optimal for the lender for some $s \in S$ given the other contracts in the menu.

In the Appendix we show that the unique optimal menu consists of a single contract. The intuition is roughly as follows. Suppose $\bar{V} > 0$.¹⁵ Clearly, the menu cannot contain $t(x) = x$, for the lender would then always choose this contract, implying that the borrower earns zero expected payoff. But if $t(x) = x$ is not contained in the menu our previous logic applies, suggesting that $s^{SB} > s^{FB}$. The optimal menu thus (again) minimizes the efficiency loss from excessive project rejection.

Denote the lowest signal at which a contract is implemented by s^{SB} , and denote the contract implemented at this signal by t^{SB} . At (t^{SB}, s^{SB}) the lender just breaks even. Suppose further that for higher signals $s_i > s^{SB}$ different contracts $t_i \neq t^{SB}$ are implemented. By revealed preference, at $s = s_i$ the lender's payoff from implementing the associated *ex post* optimal contract $t_i \neq t^{SB}$ must be at least as great as his payoff from implementing t^{SB} . Hence if the lender deletes all contracts $t_i \neq t^{SB}$ from the menu the borrower's expected payoff cannot fall. In fact, we show in the Appendix that it must strictly rise. Holding the cutoff signal s^{SB} fixed, this means the lender can adjust the (only) remaining contract t^{SB} in her favor such that the borrower's expected payoff drops back to \bar{V} . Denote the adjusted contract by \hat{t}^{SB} . Clearly, as the lender just breaks even at (t^{SB}, s^{SB}) , replacing t^{SB} by \hat{t}^{SB} implies that she must make a positive profit at (\hat{t}^{SB}, s^{SB}) . Since the lender's expected payoff is increasing in s , this in turn means that—holding \hat{t}^{SB} fixed—she can lower the cutoff signal from s^{SB} to $\hat{s}^{SB} < s^{SB}$, contradicting the optimality of the original menu. (Note that lowering the cutoff from s^{SB} to \hat{s}^{SB} while holding \hat{t}^{SB} fixed increases the borrower's payoff and is therefore feasible.)

Proposition 2. *The unique optimal menu of contracts consists of a single contract.*

4.2 Ordering of Cash-Flow Distributions

Assumption 1 can be relaxed at the cost of strengthening Assumption 4. Consider the following weaker assumption concerning the ordering of $G_\theta(x)$:

¹⁴This setting is similar to Maskin and Tirole (1990, 1992).

¹⁵If $\bar{V} = 0$ the unique optimal contract is $t(x) = x$. Any optimal menu must then contain $t(x) = x$, in which case this contract is always chosen.

Assumption 1a. For any pair $(\theta, \theta') \in \Theta$ with $\theta' > \theta$ the ratio $[1 - G_{\theta'}(x)]/[1 - G_{\theta}(x)]$ is strictly increasing in x for all $x \in [\underline{x}, \bar{x}]$.

It is easily established that Assumption 1a implies Assumption 1.¹⁶ We next strengthen Assumption 4 by requiring that—in addition to $t(x)$ being nondecreasing—the borrower’s *ex post* profit $x - t(x)$ be nondecreasing in x . (If $x - t(x)$ were decreasing the borrower could make a gain by destroying cash flow *ex post*.)

Assumption 4a. $t(x)$ and $x - t(x)$ are both nondecreasing.

It is now straightforward to show that, with Assumptions 1 and 4 being replaced by Assumptions 1a and 4a, respectively, all our previous results continue to hold.¹⁷

Proposition 3. Proposition 1 continues to hold if Assumptions 1 and 4 are replaced by Assumptions 1a and 4a, respectively.

4.3 Fully Informative Signal

Our results also hold if the signal s is fully informative. Consider the following relabeling of our model. There is a continuum of types $s \in S$. Each type represents a “portfolio” consisting of all projects $\theta \in \Theta$ with type-dependent portfolio weights $h(\theta | s)$. Upon observing the signal s the lender now knows exactly the borrower’s type. And yet, the underlying fundamental uncertainty is the same as before since a type now represents a weighted average of cash-flow distributions. It is straightforward to show that, once we introduce the necessary changes in notation, all our previous results obtain. In particular, the cutoff signal now becomes a “cutoff type” representing the lower bound of the set of all acceptable types.

This has interesting implications. There is a large literature concerned with the development and evaluation of statistical methods to improve the predictive power of credit risk models.¹⁸ Our result suggests that—no matter how good the predictive power is—the associated credit

¹⁶As noted in Nachman and Noe (1994), Assumption 1a can be alternatively expressed in the following way: For all $x' > x$ in X and all $\theta' > \theta$ in Θ it must hold that $\frac{G_{\theta}(x') - G_{\theta}(x)}{1 - G_{\theta}(x)} > \frac{G_{\theta'}(x') - G_{\theta'}(x)}{1 - G_{\theta'}(x)}$. Nachman and Noe refer to this requirement as Conditional Stochastic Dominance.

¹⁷As for Lemmas 1-3 this is immediate since there we only need first order stochastic dominance, which is implied both by Assumptions 1 and 1a.

¹⁸For an overview of this literature see Saunders and Allen (2002).

decision will remain inefficient for the simple reason that the lender does typically not capture all the surplus.

4.4 Private Information about Project Quality

Finally, our results extend to the case where the borrower has private information about the project quality. If the borrower and lender have common priors the borrower's participation constraint needs to be satisfied only in expectation (*ex ante* individual rationality). If the borrower has private information about his type, however, the participation constraint must be satisfied for each type the lender wants to attract (*interim* individual rationality). Moreover, if borrowers know their types they may well have different, type-dependent reservation values $\bar{V}(\theta) \geq 0$. Whether a given type can—and will—be attracted by the lender's offer depends on the precise shape of the function $\bar{V}(\theta)$. In particular, it need not necessarily be true that there exists a threshold type $\hat{\theta}$ such that borrowers will be attracted if and only if $\theta > \hat{\theta}$. In what follows we sketch the argument for the simple two-type case.

As in the earlier example, let $\theta = 1, 2$ denote the low- and high-type project, respectively. To make lending profitable the high-type borrower's reservation value $\bar{V}(2)$ must be sufficiently less than $\mu(2)$. In contrast—and consistent with our earlier assumptions—lending to the low-type borrower is assumed to be strictly unprofitable. Any payoff realized by the low-type borrower is therefore wasted from the lender's point of view as it does not relax the participation constraint of the high-type borrower. Accordingly, the lender has two objectives: (i) minimizing the efficiency loss from excessive project rejection, and (ii) minimizing the rent extracted by the low-type borrower.¹⁹ As for (i) we have shown that a debt contract maximizes the lender's payoff from low-type borrowers, thus making her less conservative in her credit decisions. But this is the same as saying that debt minimizes the payoff captured by low-type borrowers. Hence debt accomplishes both objectives, implying that our results continue to hold if the borrower has private information.

¹⁹The second objective is equivalent to minimizing the “mispricing” of high types. See Myers and Majluf (1984) and Nachman and Noe (1994) for details.

5 Private Wealth

We now relax the assumption that the borrower has no private wealth. Private wealth can be used in two ways: as collateral and to co-finance the investment. As we show below, the two uses have virtually identical effects. Drawing on the borrower's private wealth is likely to be costly. If the wealth is held in the form of illiquid assets there are liquidation costs. On the other hand, if the wealth is liquid there are foregone returns from alternative investments. As it turns out, a debt contract minimizes both the efficiency loss from excessive project rejection *and* the (costly) use of the borrower's private wealth. Explicitly introducing costs of private wealth therefore has no effect on the security design, which is why we abstract from it. Only in case of indifference, i.e., if two different levels of collateral or co-investment implement the same cutoff signal, we select the solution that draws less on the borrower's wealth.

To rule out trivial cases where the first-best credit policy can be implemented for all possible values of \bar{V} we assume that

Assumption 5. $w < k - \underline{x}$.

Under Assumption 5 the first-best credit policy can be implemented for low, but not for high values of \bar{V} .

5.1 Collateral

Denote the transfer to the lender made out of the project cash flow by $t^P(x)$ and that made out of the collateral by $t^C(x)$, where $t^P(x) \leq x$ and $0 \leq t^C(x) \leq w$. By Assumption 4 the total transfer $t(x) = t^P(x) + t^C(x)$ must be nondecreasing.

To see the role played by collateral consider again equations (2)-(3) characterizing the first- and second-best cutoff signal, respectively:

$$\sum_{\theta \in \Theta} h(\theta | s^{FB}) \mu(\theta) = k$$

and

$$\sum_{\theta \in \Theta} h(\theta | s^{SB}) U_{\theta}(t) = k. \tag{5}$$

In the absence of collateral it is necessarily true that $U_{\theta}(t) \leq \mu(\theta)$ for all θ , implying that the second-best credit policy is inefficient for all $\bar{V} > 0$ (Lemma 3). This is different if the

borrower can post collateral. Specifically, by liquidating collateral in low cash-flow states it is possible to have $U_\theta(t) > \mu(\theta)$ for low θ , i.e., the lender's payoff from low type-projects can exceed the projects' expected cash flow. (At the same time it must hold that $U_\theta(t) < \mu(\theta)$ for high θ , however, or else the borrower will not break even.) Accordingly, with collateral (5) can potentially be satisfied with $s^{SB} = s^{FB}$ even if the lender does not capture the full surplus.

There are two ways of transferring funds to the lender in low cash-flow states, thereby raising his payoff from low cash-flow projects: taking funds out of the project cash flow and liquidating collateral. A debt contract, by maximizing the funds transferred out of the project cash flow in low cash-flow states, economizes on the use of collateral. Debt thus accomplishes two objectives: it makes the lender less conservative and it ensures that collateral is liquidated only when absolutely necessary. More precisely, if the total transfer to the lender in low cash-flow states is $x + \Delta(x)$, then x is taken out of the project cash flow and the residual $\Delta(x)$ is taken out of the collateral.

Making the lender less conservative improves efficiency. For low \bar{V} it is now possible to implement the first-best credit policy. For this only a fraction of the borrower's wealth must be pledged as collateral, i.e., $\max \{t^C(x)\} < w$. For intermediate \bar{V} the first-best is still attainable, but now the entire wealth must be pledged, i.e., $\max \{t^C(x)\} = w$. Finally, if \bar{V} is sufficiently high the first-best credit policy is no longer implementable. In this case we have the familiar result that $s^{SB} > s^{FB}$, where s^{SB} is strictly increasing in \bar{V} . As \bar{V} increases, more and more collateral must be pledged to implement the first-best credit policy.

Figure 1 depicts the optimal transfer made out of the collateral, $t^C(x)$, as a function of the project cash flow. The left picture depicts the case where \bar{V} is low; the right picture depicts the case where \bar{V} takes on intermediate and high values.

Proposition 4. *If the borrower can pledge private wealth as collateral the unique optimal transfer made out of the project cash flow, $t^P(x)$, takes the form of a standard debt contract. The unique optimal transfer made out of the collateral, $t^C(x)$, is depicted in Figure 1.*

Given Proposition 4 and the preceding discussion, the following corollary is immediate.

Corollary 2. *If the borrower can pledge private wealth as collateral the first-best credit policy is implementable if and only if \bar{V} is sufficiently low. The fraction of wealth pledged as collateral is weakly increasing in \bar{V} . Finally, an increase in the borrower's wealth weakly improves the optimal credit policy.*

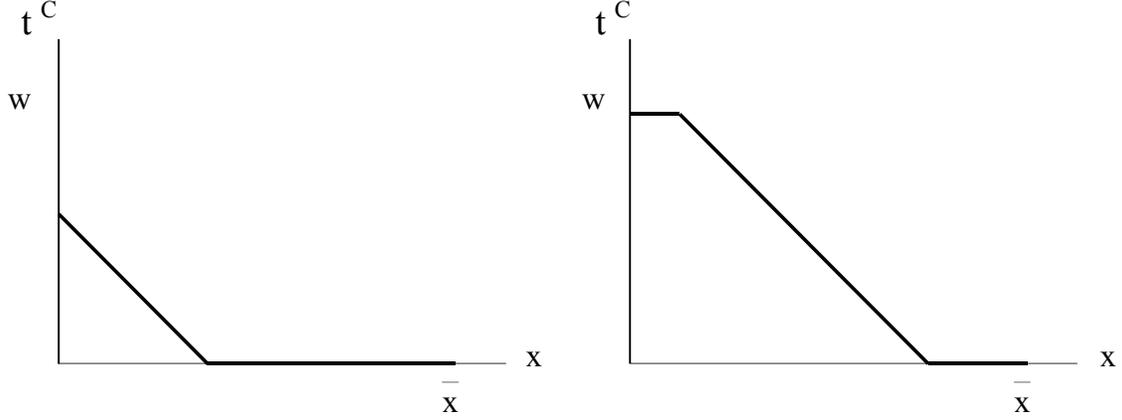


Figure 1: Optimal collateral policy for low \bar{V} (left) and intermediate and high \bar{V} (right).

5.2 Co-Financing

We now consider the other polar case where the borrower's wealth is used to co-finance the investment. Denote the fraction of the investment financed by the lender by L . The remainder $k - L$ is financed by the borrower, where $k - L \leq w$.

To see why co-financing helps, consider again the two equations characterizing the first- and second-best cutoff signal, respectively, where (6) now takes into account that lender finances only a fraction $L \leq k$ of the investment:

$$\sum_{\theta \in \Theta} h(\theta | s^{FB}) \mu(\theta) = k$$

and

$$\sum_{\theta \in \Theta} h(\theta | s^{SB}) U_{\theta}(t) = L. \quad (6)$$

As was shown earlier, if $L = k$ we necessarily have that $s^{SB} > s^{FB}$ whenever $\bar{V} > 0$ (Lemma 3). By contrast, if $L < k$ equation (6) can potentially be satisfied with $s^{SB} = s^{FB}$ even if $U_{\theta}(t) < \mu(\theta)$, i.e., even if the lender does not capture all the surplus. Co-financing, like collateral, reduces the lender's risk exposure and thus makes her less conservative.

The analysis is similar to that with collateral. If \bar{V} is low the first-best credit policy can be implemented by reducing the lender's investment share to $L < k$. The lender's investment share is a decreasing function of \bar{V} , i.e., the greater is \bar{V} the more private wealth is needed to attain

the first-best. At some point the borrower’s wealth constraint $k - L \leq w$ binds. We then have the familiar result that $s^{SB} > s^{FB}$, where s^{SB} is strictly increasing in \bar{V} .

Proposition 5. *If the borrower can co-finance the investment the unique optimal transfer made out of the project cash flow, $t^P(x)$, takes the form of a standard debt contract.*

Given Proposition 5 and the preceding discussion, the following corollary is evident.

Corollary 3. *If the borrower can co-finance the investment the first-best credit policy is implementable if and only if \bar{V} is sufficiently low. The fraction of the borrower’s wealth used to co-finance the investment is strictly increasing in \bar{V} . Finally, an increase in the borrower’s wealth weakly improves the optimal credit policy.*

Corollaries 2 and 3 suggest that an increase in the borrower’s wealth improves the optimal credit policy, either by increasing the amount of collateral that can be posted or by reducing the lender’s investment. This has implications for the relation between wealth and financing constraints. For instance, if wealth is held in the form of real estate—which can be potentially pledged as collateral—Corollary 2 implies that for high \bar{V} a decrease in real estate prices (inefficiently) reduces credit volume. Existing arguments drawing a link between wealth and financing constraints typically rest on borrower moral hazard.²⁰ By contrast, in our model the borrower’s wealth is used to improve the incentives of the lender.

6 Concluding Remarks

Lending decisions are inevitably discretionary and subjective, even if they are based on “hard” information such as financial statements. We consider a stylized model of lending where the lender can assess the borrower’s credit risk prior to the loan decision. Based on the outcome of the credit risk analysis the lender is free to deny the applicant, i.e., the credit decision is fully discretionary. We show that—regardless of the type of security in place—the lender’s credit decision will be inefficient in the sense that she will reject borrowers that would have been accepted in a first-best world where credit decisions are contractible. We then explore the effect of the security in place on the lender’s credit decision. Specifically, we ask what security minimizes the inefficiency arising from excessive project rejection. (This is also the security that maximizes the lender’s expected payoff subject to the borrower’s participation constraint.)

²⁰See, e.g., Bernanke and Gertler (1989), Holmström and Tirole (1996), and Kiyotaki and Moore (1997).

In a model with continuous cash-flow distribution we show that the unique optimal security is standard debt. Debt maximizes the lender’s payoff in low cash-flow states, thereby maximizing her return from low-type borrowers, i.e., borrowers with a high probability mass on low cash-flow outcomes. By making the lender more lenient, debt thus implements constrained efficient credit decisions.

Unlike other models of security design the incentive problem in our model is with the lender, not the borrower. Specifically, the security in place affects the lender’s incentives to approve or deny the loan. Using standard terminology, the problem is one of *ex ante* lender moral hazard. In the paper we argue that the optimality of debt is robust with respect to introducing private information on the part of the borrower. Furthermore, our assumptions imply that debt also remains optimal if we introduce *ex post* moral hazard on the part of the borrower. (See Innes (1990) and the assumptions therein.) The outcome is less clear if—in addition to the *ex ante* moral hazard problem considered here—there is *ex post* moral hazard both on the part of the borrower and the lender. Such double-sided moral hazard is frequently assumed in models of venture capital security design. There, the conclusion is typically that some equity-linked security such as, e.g., convertible debt is optimal (Schmidt (2000), Hellmann (2001), Repullo and Suarez (2000)). Exploring the tension between double-sided *ex post* moral hazard (which appears to favor equity) and *ex ante* lender moral hazard (which favors debt) in a single model seems to be an interesting avenue for future research.

7 Appendix A: Implications of Assumption 3

Denote the posterior distribution function for some signal s by $H(\theta | s) := \int_0^s h(\theta | s') ds'$. Assumption 3 implies now that for all $s' > s$ in S the posterior distribution $H(\theta | s')$ dominates that of $H(\theta | s)$ in the sense of First-Order Stochastic Dominance. This follows as the posterior distribution also satisfies the MLRP property, which holds by Assumption 3 and as the ratio

$$\frac{h(s' | \theta)}{h(s | \theta)} = \frac{f_\theta(s') \sum_{\theta' \in \Theta} h(\theta') f_{\theta'}(s)}{f_\theta(s) \sum_{\theta' \in \Theta} h(\theta') f_{\theta'}(s')}$$

strictly increases in θ in case $s' > s$. (That $f_\theta(s')/f_\theta(s)$ strictly increases in θ follows as Assumption 3 states that $f_{\theta'}(s')/f_{\theta'}(s) > f_\theta(s')/f_\theta(s)$ holds if $\theta' > \theta$ and $s' > s$.) As is well-known, by First-Order Stochastic Dominance and as the values $\mu(\theta)$ are strictly increasing in θ , the conditional expected cash flow is strictly increasing in the observed signal. In this sense, observing a

higher signal is good news about the project. In what follows, we show that—as asserted in the main text—also the reverse is true. Hence, unless Assumption 3 is satisfied, we can not say that observing a higher signal is goods news. Together with the previous arguments we then obtain the following assertion.

Claim. *The posterior distribution of $s' \in S$ strictly dominates that for $s \in S$, where $s < s'$, for all prior distribution of types if and only if Assumption 3 holds.*

Proof. We have already shown sufficiency. To prove necessity, observe first that by First-Order Stochastic Dominance it must hold for all $s' > s$ in S and for all $\theta < \bar{\theta}$ in Θ that

$$H(\theta | s') = \frac{\sum_{\tilde{\theta} \leq \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s')}{\sum_{\tilde{\theta} \in \Theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s')} < H(\theta | s) = \frac{\sum_{\tilde{\theta} \leq \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s)}{\sum_{\tilde{\theta} \in \Theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s)}. \quad (7)$$

Transforming (7), we obtain the requirement

$$\left[\sum_{\tilde{\theta} \leq \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s') \right] \left[\sum_{\tilde{\theta} > \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s) \right] - \left[\sum_{\tilde{\theta} > \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s') \right] \left[\sum_{\tilde{\theta} \leq \theta} h(\tilde{\theta}) f_{\tilde{\theta}}(s) \right] < 0. \quad (8)$$

Now suppose that Assumption 3 does not hold. Hence there exist signals $s' > s$ in S and types $\theta'' > \theta'$ in Θ such that $f_{\theta''}(s')/f_{\theta'}(s') \leq f_{\theta''}(s)/f_{\theta'}(s)$. If the prior distribution now puts all probability mass on the two types θ'' and θ' and if we choose $\theta' \leq \theta < \theta''$, it then follows that the requirement (8) is not satisfied. Q.E.D.

8 Appendix B: Proofs

Proof of Lemma 1. The first-best credit policy prescribes to approve the loan whenever, given the observed signal s , the expected cash flow $\sum_{\theta \in \Theta} \mu(\theta) h(\theta | s)$ exceeds k and to reject the loan whenever it falls below k . By Assumptions 1 and 3 the conditional expected cash-flow is strictly higher after observing a higher signal. (As shown in Appendix A, Assumption 3 implies that the posterior distribution satisfies the MLRP property.) Moreover, the posterior beliefs $h(\theta | s)$ change continuously in s , which holds by continuity of the densities $f_{\theta}(s)$ of the signal generating technology. Finally, recall that the expected cash flow is strictly higher than k at $s = 1$ and strictly lower than k at $s = 0$. We thus obtain a unique threshold $0 < s^{FB} < 1$, which is given by (2), such that the borrower should be rejected for $s < s^{FB}$ and accepted for $s > s^{FB}$. Q.E.D.

Proof of Lemma 2. The lender's optimal credit policy is to reject the project if, conditional on the observed signal s , the expected repayment is lower than the loan size k and to accept

if it is higher than k . By Assumption 4 $t(x)$ must be nondecreasing. If $t(x)$ is flat, $U_\theta(t) < k$ holds for all $\theta \in \Theta$, which makes it optimal to always reject. If $t(x)$ is not flat everywhere, it must be strictly increasing over a set of positive measure. By Assumption 1 $U_\theta(t)$ is then strictly increasing in θ . Given Assumption 3, the lender's expected payoff when approving the loan is then strictly increasing in the observed signal s . Moreover, it changes continuously in s , which follows from continuity of $f_\theta(s)$, while we have also $\sum_{\theta \in \Theta} U_\theta(t)h(\theta | 0) < k$. Existence and characterization of the cutoff signal $s^{SB}(t) > 0$ follow then immediately. Q.E.D.

Proof of Proposition 1 and Corollary 1. For $\bar{V} = 0$ the lender simply obtains all proceeds, i.e., $t(x) = x$. Suppose thus that $\bar{V} > 0$ holds. We show first that any repayment scheme t that is not debt cannot be optimal. We argue to a contradiction. Suppose thus that a contract t that is not debt was optimal. We proceed as follows. We first construct a debt contract \tilde{t} that would give the lender and the borrower the same expected payoff if the lender applied the same threshold as under t , i.e., $s^{SB}(t)$. Subsequently, we show that the true threshold implemented under \tilde{t} satisfies $s^{SB}(\tilde{t}) < s^{SB}(t)$. This will imply that t cannot be optimal.

Suppose thus for a moment that the threshold applied by the lender is kept fixed at $s^{SB}(t)$. Using this threshold, the lender's expected payoff from a debt contract with repayment R equals

$$\sum_{\theta \in \Theta} h(\theta) [1 - F_\theta(s^{SB}(t))] \left[\int_{x \in X} \min\{x, R\} g_\theta(x) dx - k \right], \quad (9)$$

which is continuous and strictly increasing in R . Requiring that the expected payoff is equal to that under t yields thus a unique debt contract. We denote this contract by \tilde{t} .

It is now helpful to introduce some additional notation. Define

$$z(\theta) := \int_{x \in X} [\tilde{t}(x) - t(x)] g_\theta(x) dx$$

and observe that by construction of \tilde{t} it holds that

$$\sum_{\theta \in \Theta} h(\theta) [1 - F_\theta(s^{SB})] z(\theta) = 0. \quad (10)$$

If it could only be observed whether the borrower passes some threshold s or not, after passing this threshold the conditional distribution would put on type θ the probability

$$\bar{h}(\theta | s) := \frac{h(\theta) [1 - F_\theta(s)]}{\sum_{\theta' \in \Theta} h(\theta') [1 - F_{\theta'}(s)]}.$$

With this definition the requirement (10) transforms to

$$\sum_{\theta \in \Theta} z(\theta) \bar{h}(\theta | s^{SB}) = 0. \quad (11)$$

In what follows we show that (11) implies $\sum_{\theta \in \Theta} z(\theta)h(\theta | s^{SB}) > 0$, from which it follows immediately that the true cutoff signal under \tilde{t} must be lower than that under t . We need the following auxiliary result.

Claim 1. *There exists a type $\tilde{\theta} \in \Theta$ with $1 \leq \tilde{\theta} < \bar{\theta}$, such that the following assertions hold:*

i) If $\bar{\theta} > 2$ it holds that $\tilde{\theta} > 1$, $z(\tilde{\theta}) \geq 0$, and $z(\theta) > 0$ for all $\theta < \tilde{\theta}$. For $\bar{\theta} = 2$ it holds that $z(\tilde{\theta}) > 0$ with $\tilde{\theta} = 1$.

ii) For all $\theta > \tilde{\theta}$ it holds that $z(\theta) < 0$.

Proof. Note first that by construction of \tilde{t} and $z(\theta)$ there must be types $\theta \in \Theta$ such that $z(\theta) < 0$ and types $\theta \in \Theta$ such that $z(\theta) > 0$. Observe next that by Assumption 4 and construction of \tilde{t} there exists a value \tilde{x} in the interior of X such that $\tilde{t}(x) \geq t(x)$ holds for $x < \tilde{x}$ and $\tilde{t}(x) \leq t(x)$ holds for $x > \tilde{x}$, where the inequalities hold strictly over sets of positive measure. Given some pair $\theta' > \theta$ in Θ we can now transform $z(\theta')$ into

$$z(\theta') = \int_{\underline{x}}^{\tilde{x}} [\tilde{t}(x) - t(x)] g_{\theta'}(x) \frac{g_{\theta'}(x)}{g_{\theta}(x)} dx + \int_{\tilde{x}}^{\bar{x}} [\tilde{t}(x) - t(x)] g_{\theta}(x) \frac{g_{\theta'}(x)}{g_{\theta}(x)} dx.$$

As $g_{\theta'}(x)/g_{\theta}(x)$ is strictly increasing in x by Assumption 1, the construction of \tilde{x} implies

$$z(\theta') < \frac{g_{\theta'}(\tilde{x})}{g_{\theta}(\tilde{x})} z(\theta). \quad (12)$$

By (12) $z(\theta) \leq 0$ must imply $z(\theta') < 0$ for all higher types $\theta' > \theta$. This proves existence of the asserted type $\tilde{\theta}$. Q.E.D.

Claim 2. $\sum_{\theta \in \Theta} z(\theta)h(\theta | s^{SB}) > 0$.

Proof. Using (11) and the definitions of $h(\theta | s^{SB})$ and $\bar{h}(\theta | s^{SB})$, we obtain

$$\begin{aligned} \sum_{\theta \in \Theta} z(\theta)h(\theta | s^{SB}) &= \sum_{\theta \in \Theta} z(\theta) [h(\theta | s^{SB}) - \bar{h}(\theta | s^{SB})] \\ &= \sum_{\theta \in \Theta} z(\theta)\bar{h}(\theta | s^{SB}) \left[\frac{f_{\theta}(s^{SB})}{1 - F_{\theta}(s^{SB})} \frac{\sum_{\theta' \in \Theta} h(\theta')F_{\theta'}(s^{SB})}{\sum_{\theta' \in \Theta} h(\theta')f_{\theta'}(s^{SB})} - 1 \right]. \end{aligned}$$

We next denote the term in rectangular brackets by $\alpha(\theta)$. By Assumption 3 $\alpha(\theta)$ is nonincreasing for all $\theta \in \Theta$ and strictly increasing for at least one $\theta \in \Theta$. (Note that the Monotone Likelihood Ratio Property implies (strict) monotonicity of the hazard rate.) Rewriting

$$\sum_{\theta \in \Theta} z(\theta)h(\theta | s^{SB}) = \sum_{\theta \leq \tilde{\theta}} \alpha(\theta)z(\theta)\bar{h}(\theta | s^{SB}) + \sum_{\theta > \tilde{\theta}} \alpha(\theta)z(\theta)\bar{h}(\theta | s^{SB})$$

and using the monotonicity of $\alpha(\theta)$ together with Claim 1, we finally obtain

$$\sum_{\theta \in \Theta} z(\theta) h(\theta | s^{SB}) > \alpha(\tilde{\theta}) \sum_{\theta \in \Theta} z(\theta) \bar{h}(\theta | s^{SB}) = 0.$$

Q.E.D.

By Claim 2, the construction of $z(\theta)$, and the definition of the optimal credit policy, the following result is then immediate.

Claim 3. $s^{SB}(t) > s^{SB}(\tilde{t})$.

We are now in the position to argue that the lender is strictly better off by offering \tilde{t} instead of t . Observe first that by construction the lender is indeed strictly better off if the borrower accepts the offer. It thus remains to show that the borrower realizes at least \bar{V} , which is the case if his payoff is not lower than under the original contract t . By construction the borrower's payoff would remain unchanged if the lender applied the threshold $s^{SB}(t)$. Given $s^{SB}(\tilde{t}) < s^{SB}(t)$ from Claim 3, i.e., that the ex-ante probability of acceptance is increased, and given that the borrower's expected payoff from acceptance is strictly positive for all values s , the borrower becomes indeed strictly better off under \tilde{t} .

Summing up, we have thus shown that an optimal contract must be debt. We show next that, given some value \bar{V} , there exists a unique optimal debt contract. Note first that by optimality the borrower's participation constraint must bind. Given a debt contract with repayment requirement R , his expected payoff equals

$$\sum_{\theta \in \Theta} h(\theta) [1 - F_{\theta}(s^{SB}(t))] \int_{x \in X} \max\{0, x - R\} g_{\theta}(x) dx,$$

which is continuous in R and equal to zero at the two boundaries $R = \underline{x}$ and $R = \bar{x}$. We thus obtain for all feasible values \bar{V} a compact set of values R for which the participation constraint binds. Denote this set by $\mathbf{R}(\bar{V})$. As the lender's payoff is strictly increasing in R , $\max \mathbf{R}(\bar{V})$ must be the unique solution. We denote this solution by $R^{SB}(\bar{V})$. This completes the proof of Proposition 1.

By construction it follows immediately that $R^{SB}(\bar{V})$ is strictly decreasing. To prove Corollary 1 it thus remains to establish that this implies an increase in the applied cutoff signal s^{SB} . But this follows immediately from the definition of s^{SB} in (3) and as an increase in the repayment requirement R increases $U_{\theta}(t)$ for all $\theta \in \Theta$. Q.E.D.

Proof of Proposition 2. By Assumptions 1-4 it follows again that, even when offering a menu, the lender's optimal policy is described by a unique cutoff signal. (Compare Lemma 2.) In a slight abuse of notation denote by s^{SB} the lowest signal for which the lender does not reject the borrower. Recall that, unless the borrower is always rejected, the lender just breaks even at $s = s^{SB}$. By the argument in the main text it holds that $s^{SB} > s^{FB}$ in case $\bar{V} > 0$. The optimal menu will minimize the resulting inefficiency from a too conservative policy.

In a slight deviation from what we specified previously for the case with a single contract, we now specify that the lender approves the loan if he observes $s = s^{SB}$. As this is a zero-probability event this change is without consequences. But it allows to identify some contract t^{SB} that is implemented at the lowest acceptable signal, i.e., at $s = s^{SB}$.²¹

Suppose now a menu is offered from which the lender will pick with positive probability contracts different to t^{SB} . Denote this menu by $T = \{t_i\}_{i \in I}$. We will now argue that offering T can not be optimal. Construct $T' = \{t^{SB}\}$ by deleting all contracts but t^{SB} from the original menu T . Note that this does not affect s^{SB} , while the borrower's expected payoff can not decrease. (The latter observation follows directly from the revealed preferences of the lender under the original menu.) We distinguish now between the two cases where t^{SB} is a debt contract and where this is not the case.

Suppose first that t^{SB} is not a debt contract. Then we can construct a new offer $T'' = \{\tilde{t}^{SB}\}$, where the debt contract \tilde{t}^{SB} is constructed from t^{SB} in analogy to the proof of Proposition 1. Note that this ensures that the new cutoff signal is strictly lower than s^{SB} . If the borrower's participation constraint is slack, we can adjust the repayment level to further decrease the cutoff threshold until the participation constraint binds. As the borrower's participation constraint now binds under the new menu T'' and as the cutoff signal is strictly lower than that under T , it follows that T can not be optimal.

It remains to discuss the case where t^{SB} is debt. We argue now that in this case the borrower's participation constraint will not bind under T' , implying that we can again create a new offer $T'' = \{\tilde{t}^{SB}\}$ with a higher repayment level in the single contract \tilde{t}^{SB} . This again contradicts optimality of T . The remaining step to be proved is that the borrower's participation constraint is relaxed if we exchange T by T' . This follows from the following observation.

Claim. *Suppose the lender is indifferent between a debt contract t^D and another contract $t \neq t^D$*

²¹In case of mixing t^{SB} is in the support of the lender's distribution if s^{SB} is realized.

if observing a signal $s < 1$. Then the lender must strictly prefer t over t^D for all signals $s' > s$.

Proof. The argument is analogous to that used in the proof of Proposition 1. We are therefore brief. By Assumption 4 and the definition of a debt contract, there exists a value x^D in the interior of X such that $t^D(x) \geq t(x)$ holds for $x < x^D$ and $t^D(x) \leq t(x)$ holds for $x > x^D$, where the inequality holds strictly over sets of positive measure. Define the difference $Z(\theta) := \int_{x \in X} [t^D(x) - t(x)] g_\theta(x) dx$. Then Claim 1 in the proof of Proposition 1 holds also for the function $Z(\theta)$. Observe next that by assumption $\sum_{\theta \in \Theta} Z(\theta) h(\theta | s) = 0$. We then obtain

$$\sum_{\theta \in \Theta} Z(\theta) h(\theta | s') = \sum_{\theta \in \Theta} z(\theta) h(\theta | s) \left[\frac{f_\theta(s')}{f_\theta(s)} \frac{\sum_{\theta' \in \Theta} h(\theta') f_{\theta'}(s)}{\sum_{\theta' \in \Theta} h(\theta') f_{\theta'}(s')} - 1 \right].$$

Given Assumption 3 and the argument in Claim 2 of the proof of Proposition 1, this proves that $\sum_{\theta \in \Theta} Z(\theta) h(\theta | s') > 0$, from which the assertion follows directly. Q.E.D.

This completes the proof of Proposition 2. Q.E.D.

Proof of Proposition 3. The proof is analogous to that of Proposition 1. The only difference, where we made previously use of the stronger Assumption 1, is in the proof of Claim 1. We therefore restrict ourselves to showing that Claim 1 is still valid under Assumptions 1a and 4a.

By Assumption 4a $t(x)$ is now continuous and nondecreasing, while satisfying $t(x') - t(x) \leq x' - x$ for all $x, x' \in X$. By monotonicity it is also a.e. differentiable. Where the derivative exists, we denote it by $d(x)$. Partial integration obtains then

$$\int_{\underline{x}}^{\bar{x}} t(x) dG_\theta(x) = t(\underline{x}) + \int_{\underline{x}}^{\bar{x}} d(x) [1 - G_\theta(x)] dx, \quad (13)$$

where we have also used that $G_\theta(\underline{x}) = 0$. Denote the repayment level specified under the debt contract \tilde{t} by $\underline{x} < \tilde{x} < \bar{x}$. Then by definition of \tilde{t} the derivative equals one for $x < \tilde{x}$ and zero for $x > \tilde{x}$. Using the definition of $z(\theta)$ and (13), we obtain

$$z(\theta) = [\underline{x} - t(\underline{x})] + \int_{\underline{x}}^{\tilde{x}} [1 - d(x)] [1 - G_\theta(x)] dx - \int_{\tilde{x}}^{\bar{x}} d(x) [1 - G_\theta(x)] dx$$

such that, given two types $\theta' > \theta$ in Θ , we can transform $z(\theta')$ to

$$\begin{aligned} z(\theta') &= \underline{x} - t(\underline{x}) + \int_{\underline{x}}^{\tilde{x}} [1 - d(x)] [1 - G_\theta(x)] \left[\frac{1 - G_{\theta'}(x)}{1 - G_\theta(x)} \right] dx \\ &\quad - \int_{\tilde{x}}^{\bar{x}} d(x) [1 - G_\theta(x)] \left[\frac{1 - G_{\theta'}(x)}{1 - G_\theta(x)} \right] dx. \end{aligned}$$

Recall now that $0 \leq d(x) \leq 1$ holds by Assumption 4a. Moreover, by construction of \tilde{t} it follows that $d(x) > 0$ holds strictly for a set of positive measure $x > \tilde{x}$, while either $d(x) < 1$ holds

strictly for a set of positive measure $x < \tilde{x}$ or it holds that $t(\underline{x}) < \underline{x}$. Together with Assumption 1a we then obtain

$$z(\theta') < \frac{1 - G_{\theta'}(\tilde{x})}{1 - G_{\theta}(\tilde{x})} z(\theta), \quad (14)$$

which is equivalent to (12) in Claim 1 of the proof of Proposition 1. Q.E.D.

Proof of Proposition 4 and Corollary 2. We need the following auxiliary result.

Claim 1. *Take some debt contract t with repayment level $R \in [\underline{x}, \bar{x}]$. Then $U_{\theta}(t) - \mu(\theta)$ is strictly decreasing in θ for all $\theta \in \Theta$.*

Proof. Define $\Delta(R) := U_{\theta'}(t) - U_{\theta}(t)$ for two arbitrary types $\theta' > \theta$ in Θ . Note that $d\Delta/dR = G_{\theta}(R) - G_{\theta'}(R)$, which by Assumption 1 is strictly positive for all $R \in (\underline{x}, \bar{x})$. Moreover, we obtain $\Delta(\bar{x}) = \mu(\theta') - \mu(\theta)$. It thus follows that $\Delta(R) < \mu(\theta') - \mu(\theta)$ holds for all $R < \bar{x}$, which implies $U_{\theta'}(t) - U_{\theta}(t) < \mu(\theta') - \mu(\theta)$ for all $R < \bar{x}$. This proves the assertion. Q.E.D.

We next set up the lender's problem more formally. The lender's payoff must be maximized subject to the borrower's participation constraint and Assumption 4. (This problem was solved in Proposition 1, albeit with the restriction to $w = 0$ and therefore to $t^C(x) = 0$ for all $x \in X$.) If the set of solutions to this problem is not a singleton, we have to choose the contract that minimizes the expected liquidated wealth, which we denote by

$$C := \sum_{\theta \in \Theta} h(\theta) [1 - F_{\theta}(s^{SB})] \int_{\underline{x}}^{\bar{x}} t^C(x) g_{\theta}(x) dx. \quad (15)$$

We next prove two more auxiliary results.

Claim 2. *For all $x \in X$, $t^C(x) > 0$ implies $t^P(x) = x$.²²*

Proof. Suppose $t^C(x) > 0$ and $t^P(x) < x$ over some set $X' \subseteq X$. This implies existence of a set $X'' \subseteq X'$ and a value $\varepsilon > 0$ such that $t^C(x) > \varepsilon$ and $t^P(x) < x - \varepsilon$ for all $x \in X''$. We can then construct a new contract satisfying $\tilde{t}^P(x) = t^P(x) + \varepsilon$ and $\tilde{t}^C(x) = t^C(x) - \varepsilon$ for all $x \in X''$ and $\tilde{t}^P(x) = t^P(x)$ and $\tilde{t}^C(x) = t^C(x)$ for all $x \in X \setminus X''$. This operation leaves total transfers unchanged for all outcomes, but decreases the expected transfers from private wealth. Q.E.D.

Claim 3. *$t^C(x) - t^C(x') \leq x' - x$ holds for all $x' > x$.*

²²In what follows, we use the assertion that a result holds for all $x \in X$ interchangeably for the assertion that the result holds for all $x \in X$ but a set of measure zero.

Proof. The assertion follows by combining Assumption 4 with Claim 2. Note first that by Assumption 4 and the definition of transfers it must hold that

$$t^C(x) - t^C(x') \leq t^P(x') - t^P(x). \quad (16)$$

The assertion follows immediately if $t^C(x) \leq t^C(x')$. For $t^C(x) > t^C(x')$ we distinguish between two cases. Suppose first that $t^C(x) > 0$ and $t^C(x') > 0$, implying by Claim 2 that $t^P(x) = x$ and $t^P(x') = x'$. Substituting these values into (16) proves the assertion. Finally, suppose that $t^C(x) > 0$ and $t^C(x') = 0$, implying $t^P(x) = x$. In this case the right side of (16) transforms to $t^P(x') - x$, which is surely bounded from above by $x' - x$. Q.E.D.

We prove next that transfers (t^P, t^C) are uniquely determined and that they are characterized as follows. First, t^P is a debt contract with some repayment level R^P . Second, t^C follows the description in Figure 1 and can thus be fully described by some value x^C from which on it holds that $t^C(x) = 0$. Precisely, for $x^C \leq w$ it holds that $t^C(x) = \max\{x^C - x, 0\}$, while for $x^C > w$ it holds that $t^C(x) = w$ if $x \leq x^C - w$ and $t^C(x) = \max\{x^C - x, 0\}$ if $x > x^C - w$. We denote the respective set of transfers satisfying these characteristics by $(t^P, t^C) \in T^P \times T^C$.

Claim 4. *The solution must be an element of $T^P \times T^C$.*

Proof. Assume that a pair $(t^P, t^C) \notin T^P \times T^C$ solves the problem. We transform this contract in the following way. First, we transform t^P into \tilde{t}^P . Assuming that s^{SB} is kept constant, $\tilde{t}^P \in T^P$ is uniquely determined by the requirement that payoffs for both sides remain constant. (Note that both payoffs are continuous and strictly monotonic in the repayment level \tilde{R}^P , which characterizes \tilde{t}^P) Second, we transform t^C into \tilde{t}^C . Assuming that s^{SB} is kept constant, $\tilde{t}^C \in T^C$ is uniquely determined by the requirement that payoffs for both sides remain constant. (Note that both payoffs are continuous and strictly monotonic in the value \tilde{x}^C , which characterizes \tilde{t}^C .)

From the argument in Proposition 1 we know that the first operation to obtain \tilde{t}^P leads to a decrease in the lender's optimal threshold. (Recall that this followed from Assumption 1 and the fact that t^P and \tilde{t}^P cross exactly once unless $t^P \in T^P$.) We argue next that the same holds for the second operation to obtain \tilde{t}^C . To see this, note that by Claim 3 t^C and \tilde{t}^C cross exactly once, i.e., there exists a single value $\tilde{x} \in X$ such that $\tilde{t}^C(x) \geq t^C(x)$ holds for $x \leq \tilde{x}$ and $\tilde{t}^C(x) \leq t^C(x)$ holds for $x \geq \tilde{x}$, unless $t^C \in T^C$.

If $s^{SB}(\tilde{t}) < s^{FB}$ holds, we have to perform yet another operation. In this case we have to adjust \tilde{R}^P , which characterizes \tilde{t}^P , and \tilde{x}^C , which characterizes \tilde{t}^C , in the following way. Keeping again s^{SB} constant, i.e., at $s^{SB}(t)$, we increase \tilde{R}^P and simultaneously adjust \tilde{x}^C such that payoffs remain constant until the true threshold $s^{SB}(\tilde{t})$ satisfies $s^{SB}(\tilde{t}) = s^{FB}$. By Lemma 3 a solution satisfying $\tilde{x}^C > 0$ exists. Note that this operation reduces the expected transferred wealth. Denote the respective realization by \tilde{C} .²³

We are now in the position to complete the proof, comparing t with the newly constructed contract \tilde{t} . We discuss three cases in turn.

Case 1. Assume first that $s^{SB}(t) = s^{FB}$, implying by our operations that also $s^{SB}(\tilde{t}) = s^{FB}(t)$ and that payoffs are therefore unchanged. As, however, $\tilde{C} < C$ holds by construction of \tilde{t}^C , we obtain a contradiction to the optimality of (t^P, t^C) .

Case 2. Assume next that $s^{SB}(t) > s^{FB}$. Note first that our construction implies $s^{FB} \leq s^{SB}(\tilde{t}) < s^{SB}(t)$. As the lender optimally readjusts his threshold following the change of contracts, it follows by construction that the lender is strictly better off under \tilde{t} . It thus remains to show that the borrower is not worse off such that his participation constraint is still satisfied. Given our construction, the borrower's payoff would not be affected if the lender still implemented his policy of only accepting types $s \geq s^{SB}(t)$. Hence, it is sufficient to prove that the borrower's expected payoff is non-negative after being accepted for realizations $s \in [s^{SB}(\tilde{t}), s^{SB}(t)]$. Note that this is no longer immediate as the borrower pledges collateral.

Assertion: For Case 2 the borrower's payoff is not lower if we replace t by \tilde{t} .

Proof. At $s = s^{SB}(\tilde{t})$ it holds by definition of the cutoff signal that $\sum_{\theta \in \Theta} h(\theta | s) U_\theta(\tilde{t}) = k$, while by $s^{FB} \leq s^{SB}(\tilde{t})$ we have $\sum_{\theta \in \Theta} h(\theta | s) \mu(\theta) \geq k$. This implies that the borrower's expected payoff at $s = s^{SB}(\tilde{t})$ is nonnegative. We now contradict the assumption that it becomes strictly negative at some value $s' > s^{SB}(\tilde{t})$. Define $y(\theta) := U_\theta(\tilde{t}) - \mu(\theta)$. Using continuity of payoffs in the signal, the assertion

$$\sum_{\theta \in \Theta} h(\theta | s') y(\theta) > 0 \tag{17}$$

²³Note that we do not claim for this operation that $s^{SB}(\tilde{t})$ is monotonic in \tilde{x}^C . If there are more than one solutions yielding $s^{SB}(\tilde{t}) = s^{FB}$, we may choose the one with the lowest realization of \tilde{C} .

then implies existence of some signal $s \in [s^{SB}(\tilde{t}), s']$ where

$$\sum_{\theta \in \Theta} h(\theta | s)y(\theta) = 0. \quad (18)$$

We argue first that $y(\theta)$ is strictly decreasing in θ . To see this, note that $U_\theta(\tilde{t}) = U_\theta(\tilde{t}^P) + U_\theta(\tilde{t}^C)$. As \tilde{t}^C is by construction nonincreasing, it follows from Assumption 1 that $U_{\theta'}(\tilde{t}^C) \leq U_\theta(\tilde{t}^C)$ holds for all $\theta' > \theta$ in Θ . As a consequence, $y(\theta)$ is surely strictly decreasing if it holds for all $\theta' > \theta$ in Θ that $U_{\theta'}(\tilde{t}^P) - U_\theta(\tilde{t}^P) < \mu(\theta') - \mu(\theta)$. As \tilde{t}^P is a debt contract, this follows by Claim 1. We have thus shown that $y(\theta) = U_\theta(\tilde{t}) - \mu(\theta)$ is strictly decreasing in θ . Hence, to ensure that (17) holds, it must be the case that $y(\theta) > 0$ for $\theta = 1$. Moreover, by construction of \tilde{t} it follows that $y(\theta) < 0$ holds for sufficiently high values of θ . We can thus find a value $1 \leq \tilde{\theta} < \bar{\theta}$ such that $y(\theta) > 0$ holds for all $\theta \leq \tilde{\theta}$ and $y(\theta) < 0$ holds for all $\theta > \tilde{\theta}$. We can now use this result to contradict the claim that (17) holds. Using (18), we obtain

$$\begin{aligned} \sum_{\theta \in \Theta} h(\theta | s')y(\theta) &= \sum_{\theta \in \Theta} h(\theta | s)y(\theta) \left[\frac{h(\theta | s')}{h(\theta | s)} - 1 \right] \\ &< \left[\frac{h(\tilde{\theta} | s')}{h(\tilde{\theta} | s)} - 1 \right] \sum_{\theta \in \Theta} h(\theta | s)y(\theta), \end{aligned}$$

where the last step follows by monotonicity of $y(\theta)$, the definition of $\tilde{\theta}$, and by Assumption 3. Using again (18), this contradicts (17). Q.E.D.

This completes the discussion of Case 2.

Case 3. It remains to consider the case where $s^{SB}(t) < s^{FB}$. (In contrast to the analysis for $w = 0$ such a contract is now feasible.) By our operations this implies $s^{SB}(\tilde{t}) = s^{FB}$. Hence in contrast to the original contract there is now no acceptance for signals $s \in [s^{SB}(t), s^{FB})$. Recall that by construction payoffs would unchanged if the original cutoff signal $s^{SB}(t)$ was still implemented. As the lender optimally adjusts his policy, his payoff is again strictly higher under the new threshold s^{FB} . It thus again remains to show that the borrower is not made worse off, which is surely the case if his expected payoff is non-positive for realizations $s \in [s^{SB}(t), s^{FB})$. This follows from an argument analogous to that used for the case $s^{SB}(t) > s^{FB}$ in the previous Assertion.

Combining the Cases 1-3, we obtain a contradiction. This completes the proof of Claim 4 that an optimal contract must satisfy $(t^P, t^C) \in T^P \times T^C$. Q.E.D.

By Claim 4 the lender's program has only the two variables x^C and R^P . Moreover, Assumption 4 transforms to the requirement $R^P \geq x^c$. By optimality the borrower's participation constraint must be binding, which implies that the lender maximizes the joint surplus subject to the program's constraints. It is then immediate that the optimal contract implements a credit policy satisfying $s \geq s^{FB}$.

If the first-best policy cannot be achieved, i.e., if $s^{SB} > s^{FB}$ holds under an optimal contract, it must hold that $x^C = R^P$, i.e., the constraint due to Assumption 4 must be binding. If this did not hold, i.e., if $x^C < R^P$ and $s^{SB} > s^{FB}$, we could by the arguments in Claim 4 find a feasible contract under which the cut-off signal becomes lower as we increase x^C and decrease R^P .

If it is possible to implement the efficient credit policy, we argue next that the optimal contract is still unique. This follows from the second objective to minimize C . While in this case there may be many pairs (x^C, R^P) implementing the efficient policy, there is clearly a single pair with the lowest value x^C . (Note that by Lemma 3 this must satisfy $x^C > 0$.)²⁴

We have thus obtained the following results.

Claim 5. *There is a unique optimal contract under which $s^{SB} \geq s^{FB}$ is implemented. If $s^{SB} > s^{FB}$, then $x^C = R^P$.*

We now introduce some additional notation. We claim that there is a threshold $\widehat{V} > 0$ such that the efficient credit policy is only chosen if $\overline{V} \leq \widehat{V}$.

Claim 6. *There exists $\widehat{V} > 0$ such that for $\overline{V} > \widehat{V}$ it holds that $s^{SB} > s^{FB}$, while for $\overline{V} \leq \widehat{V}$ it holds that $s^{SB} = s^{FB}$.*

Proof. By Claim 5 we know that an efficient policy can be implemented if and only if this is possible by a contract t that can be written as

$$t(x) = w + \max \{x, R\} \tag{19}$$

for some value R . The assertion follows now directly from the following construction of the threshold \widehat{V} . The equation

$$\sum_{\theta \in \Theta} h(\theta | s^{FB}) \left[\int_{\underline{x}}^{\overline{x}} \min \{x, R\} g_{\theta}(x) dx \right] = k - w \tag{20}$$

²⁴Existence follows by continuity of payoffs in x^C and R^P .

has by Assumption 5 a unique solution for R , which we denote by \widehat{R} , such that for all $R < \widehat{R}$ the first-best credit policy can not be obtained. Substituting \widehat{R} into the borrower's expected payoff, we obtain

$$\widehat{V} := \sum_{\theta \in \Theta} h(\theta) [1 - F_{\theta}(s^{FB})] \left[\int_{\underline{x}}^{\overline{x}} \max \{x - \widehat{R}, 0\} g_{\theta}(x) dx \right]. \quad (21)$$

Q.E.D.

Combining Claims 4-6 proves Proposition 4. We turn now to the proof of Corollary 2.

Claim 7.

i) For $\overline{V} \geq \widehat{V}$ it holds that s^{SB} increases in \overline{V} .

ii) \widehat{V} increases in w , while for $\overline{V} > \widehat{V}$ an increase in w strictly increases s^{SB} .

Proof. We prove first assertion i). From Claim 5 we know that for $\overline{V} \geq \widehat{V}$ it holds that $x^C = R^P = R$. (In fact, the optimal contract can be written as in (19).) An increase in \overline{V} must now lead to a decrease in R . (Otherwise, as the lender's payoff is strictly decreasing in R , the previous contract would not have been optimal.) But s^{SB} is by previous arguments (see the proof of Proposition 1) strictly decreasing in R .

Turn next to assertion ii). Recall the definition of \widehat{V} by (20)-(21) in Claim 6. The strict monotonicity in w is then immediate. Suppose next that $\overline{V} > \widehat{V}$ and consider a marginal increase in w . Recall also that for $\overline{V} > \widehat{V}$ the optimal contract can be written as in (19). As we adjust w and R to keep the borrower's payoff equal to \overline{V} , it follows from by now standard arguments that this must strictly reduce the cutoff signal s^{SB} . Q.E.D.

We turn next to the size of the pledged collateral. The maximum value of pledged wealth, i.e., $\max_{x \in X} t^C(x)$, is given by $t^C(\underline{x})$. We know that $t^C(\underline{x}) = w$ holds for all $\overline{V} \geq \widehat{V}$. To complete the proof of Corollary 2 it thus remains to show that $t^C(\underline{x})$ weakly increases over $\overline{V} \leq \widehat{V}$. This follows from the following assertion.

Claim 8. For $\overline{V} \leq \widehat{V}$ it holds that x^C weakly increases in \overline{V} .

Proof. Recall that for $\overline{V} \leq \widehat{V}$ it holds that $s^{SB} = s^{FB}$. While for $\overline{V} < \widehat{V}$ the efficient cutoff signal could be obtained by various combinations (R^P, x^C) , there is a unique such combination that minimizes x^C and therefore C . Note next that as \overline{V} increases the program of the lender

becomes more constrained. It follows thus directly from optimality that the respective value x^C – and thus C^- – can not be strictly decreasing in \bar{V} . Q.E.D.

Proof of Proposition 5 and Corollary 3. It is helpful to note that a contract specifying t and the loan size L is equivalent to a contract under which the lender provides the full loan k and where we adjust the payout to $\tilde{t}(x) = F + t(x)$, where $F = k - L$. In what follows we restrict consideration to such contracts where the repayment is characterized by a pair (F, t) .

The arguments to prove Proposition 5 and Corollary 3 are analogous to those used to prove Propositions 1 and 4 and Corollary 2. We are therefore short. The optimal contract again maximizes the lender's expected payoff. If this generates multiple solutions, which will again hold only if $s^{SB} = s^{FB}$, then an optimal contract will reduce the borrower's share of co-financing F .

Claim 1. *There exists a unique optimal contract, which takes the form of standard debt.*

Proof of Claim 1. The proof that t must be standard debt proceeds in analogy to that of Claim 4 in the proof of Proposition 3. Hence if we transform any other payout t into a debt contract we can either implement a more efficient credit policy or, otherwise, at least reduce F . Uniqueness follows then immediately. Q.E.D.

Claim 2.

i) There exists $\hat{V} > 0$ such that for $\bar{V} > \hat{V}$ it holds that $s^{SB} > s^{FB}$, while for $\bar{V} \leq \hat{V}$ it holds that $s^{SB} = s^{FB}$.

ii) For $\bar{V} \geq \hat{V}$ it holds that s^{SB} increases in \bar{V} .

iii) \hat{V} increases in w , while for $\bar{V} > \hat{V}$ an increase in w strictly increases s^{SB} .

Proof. Assertion i) follows from Claim 6 in the proof of Proposition 4. (Recall that for $s^{SB} > s^{FB}$ the optimal contract with collateral can be written as $t(x) = w + \max\{x, R\}$.) Likewise, we can appeal to Claim 7 in the proof of Proposition 3 to obtain assertions ii) and iii). Q.E.D.

Claims 1-2 together with the following assertion complete the proof.

Claim 3. *For $\bar{V} \leq \hat{V}$ it holds that F strictly increases in \bar{V} .*

The proof of Claim 3 is analogous to that of Claim 8 in the proof of Proposition 3. This completes the proof of Proposition 5 and Corollary 3. Q.E.D.

9 References

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