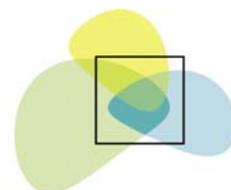


RISK AND UNCERTAINTY IN COST BENEFIT ANALYSIS

TOOLBOX PAPER

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Written by: Karsten Stæhr

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For further information please contact:

Institut for Miljøvurdering /

Environmental Assessment Institute

Gl. Kongevej 5, 1st floor

DK - 1610 Copenhagen V

Phone: +45 7226 5800

Fax: +45 7226 5839

E-mail: imv@imv.dk

Web: www.imv.dk

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ABSTRACT IN ENGLISH

This toolbox paper discusses ways to incorporate risk and uncertainty into cost benefit analyses. Risk is randomness which is measurable and can be described by a probability distribution, while uncertainty is randomness without a well-defined distribution.

A cost benefit analysis informs the decision-making process by estimating the net present value of a project or policy. By incorporating risk and uncertainty into the analysis, the reliability of the estimated expected net present value can be assessed. Sometimes the expected net present value can also be estimated more precisely.

It is in most cases useful to carry out a risk and uncertainty assessment as part of a cost benefit analysis. Many government projects – like environmental policy, regulation and programmes – are subject to much randomness. Against this stands that the government can pool the risks and uncertainties from many projects and furthermore spread the remaining risks and uncertainties on many individuals.

No method of risk and uncertainty analysis fits all cases. Various *ad hoc* methods can be useful as an aid to a risk averse decision-maker. Costs can be adjusted upwards or benefits downwards, the period for which benefits are counted can be shortened and benefits can be discounted heavily.

Gross sensitivity analyses seek to assess the sensitivity of the expected net present value to changes in the variables entering the calculation. Stress testing produce worst and best case scenarios, as the variables are changed from their lowest to their highest values.

Risk analyses based on Monte Carlo simulation can be useful if it is meaningful to attach distributions to the variables entering the cost benefit analysis. The simulations depict the distribution of the net present value. Confidence intervals of the expected net present value can be found and the probability that the project has a positive net present value can be estimated.

When risks and uncertainties are important, projects that can be postponed or reversed will generally be more valuable to society than less flexible projects.

The treatment of risk and uncertainty should be thought into the cost benefit analysis at an early stage, in part to ensure that sufficient data on risk and uncertainty is collected.

DANSK RESUMÉ

Dette papir i IMVs værktøjskasseserie drøfter metoder til at vurdere betydningen af risiko og usikkerhed i en cost benefit analyse. Risiko er her defineret som variabilitet, der er målbar og kan beskrives med en statistisk sandsynlighedsfordeling. Usikkerhed er variabilitet, som ikke kan tillægges en veldefineret statistisk fordeling.

I en cost benefit analyse udregnes den forventede nutidsværdi af et projekt eller et politiktiltag. Ved at inddrage risiko og usikkerhed i cost benefit analysen kan man få et billede af, hvor præcist den forventede nutidsværdi er bestemt, og samtidig kan den forventede nutidsværdi bestemmes mere nøjagtigt.

Det er i næsten alle tilfælde nyttigt at gennemføre en vurdering af risiko og usikkerhed ved en cost benefit analyse. Mange offentlige projekter, herunder miljøtiltag, afhænger af mange meget usikre antagelser. Omvendt kan myndighederne sprede risiko eller usikkerhed ud over mange projekter og over mange individer.

Der er ikke nogen metode til at inddrage risiko og usikkerhed, som er hensigtsmæssig i alle tilfælde. Forskellige ad hoc metoder kan understøtte beslutningsprocessen for en risikoavers beslutningstager. Omkostninger kan justeres ned eller fordele justeres op, perioden med fordele kan forkortes, eller fordelene kan diskonteres kraftigt.

Følsomhedsanalyse bruges til at bedømme, hvor sensitiv den forventede nutidsværdi er over for ændringer i de variabler, som indgår i beregningen. Ved en stress test ændres variablerne fra deres laveste til den højeste værdi for at fastlægge effekten på den forventede nutidsværdi.

Risikoanalyser baseret på Monte Carlo simulering er nyttigt, når fordelingerne kendes til de variabler som indgår i beregningen. Simuleringen bruges til at fastlægge hele fordelingen af nutidsværdien. Dermed kan konfidensintervaller for den forventede nutidsværdi bestemmes, og man kan finde sandsynligheden for, at et projekt har en positiv nutidsværdi.

Når fremtidig risiko og usikkerhed i stor udstrækning påvirker cost benefit analysens resultat, vil relativt fleksible projektmodeller i mange tilfælde være mest hensigtsmæssige fra en samfundsøkonomisk synsvinkel.

Vurderingen af risiko og usikkerhed bør tænkes ind i cost benefit analysen på et tidligt stadie, bl.a. for at sikre at tilstrækkeligt datamateriale bliver indsamlet.

1 INTRODUCTION

“It is trivial to note that the future is uncertain. It is, however, far from trivial to analyze that uncertainty”, Maler & Fisher (2005, p. 574)

Cost benefit analysis (CBA) – or social cost benefit analysis as the methodology is sometimes called – is an important tool for socio-economic assessment of government projects and policies (Petersen & Busk 2004). The aim is to help assess whether or not a given project or policy will benefit society. By incorporating risk and uncertainty into the analysis, the reliability of the estimate of the expected net present value can be assessed using different methods. This information can help inform the decision-making process and reduce the number of policy errors, i.e. ensure that socially beneficial projects are implemented and that socially unfavourable projects never leave the drawing board.

This toolbox paper discusses ways to incorporate risk and uncertainty into cost benefit analyses. The aim is to discuss how risk and uncertainty affect the reliability of a CBA, but also how this information may be used in the decision-making process. The main target group includes analysts, students and other practitioners who seek guidance on how to take into account risk and uncertainty when undertaking a CBA. The toolbox paper aspires to place the discussion of practical procedures into a theoretical framework, partly to facilitate the discussion on how risk and uncertainty may affect the decision-making.

The first step in a CBA is to identify and quantify all relevant costs and benefits as seen from society’s viewpoint. The net present value (NPV) is then found as the sum of the discounted flows of costs and benefits over the presumed lifespan of the project. Absent risks and uncertainties, a NPV above 0 suggests that the project entails a potential efficiency improvement as benefits exceed costs, implying that possible losers can be compensated from the gain of the winners.

In practice all CBAs make use of estimates of variables which can only be assessed or forecast imprecisely. The risk or uncertainty of the variables entering a CBA will affect the precision of the estimated expected NPV and often also the expected NPV itself. It is therefore important to consider the effects of risk and uncertainty when undertaking cost benefit analyses.

The returns on both private and public projects are affected by risk and uncertainty. The main difference is that some of the costs and benefits entering a social CBA are inherently very uncertain as they pertain to non-marketed goods and services, to outcomes very far into the future and with very complex cause-effect relations (Hanley 1992). Where a private company might need to consider the sales prospects a few years ahead, governments seeking to select socially beneficial environmental policies must incorporate a broad range of costs and benefits over a long time horizon. Other differences in project assessments between a private and a public decision-maker include different attitudes to risk and uncertainty and different diversification possibilities.

CBA is an important tool for assessing the welfare economic effects of decisions regarding regulation, taxation or individual projects. Society attains an efficient resource utilisation when only projects and policies, which *ex ante* have a positive expected NPV, are implemented. This however does not preclude that projects *ex post* result in a NPV which is lower than the expected NPV, possibly much lower. It is therefore important to assess the precision with which the expected NPV is estimated. While the use of CBA may be on the rise, it is less common that CBAs include a thorough evaluation of the impact of risk and uncertainty on possible outcomes of NPV.¹ It is also illustrative that the recent authoritative and thorough overview of CBA methods, Pearce *et al.* (2006), allocates only 3 out of 275 pages to the treatment of risk and uncertainty in CBA.

This toolbox paper discusses the need for assessing the reliability of cost benefit analyses. For this purpose cost benefit analyses must be considered in light of the government's functions in a market economy and its ability to manage the risks and uncertainties of its activities. The core of the toolbox paper comprises brief introductions to the methods used to ascertain how risk and uncertainty affect results of CBA. The focus is on the methods considered most useful in analyses undertaken when undertaking environmental CBA in practice, while other methods are treated cursorily.² The toolbox paper focuses on the treatment of risk and uncertainty in cost benefit analysis, but many of the insights and methods are also

¹ An example is the metro project in Copenhagen. In 2000 the Danish National Audit Office criticised that the profitability of project had not been subjected to a thorough risk assessment as both capital costs and revenue streams of similar projects in other countries had exhibited significant discrepancies between planned and realised outcomes (Statsrevisoratet 2000).

² Additional methods applicable in practice can be found in Treasury Board of Canada (1998, chs. 7-9).

applicable for other project or policy assessment methods, e.g. cost effectiveness analysis.

Brief treatments of risk and uncertainty in CBA are available in many standard textbooks on cost benefit analysis, e.g. Campbell & Brown (2003, ch. 9), Johansson (1993, ch. 8), Boardman *et al.* (2006, ch. 7), Brent (1998, ch. 11). Several government guides to CBA also discuss the treatment of risk and uncertainty, e.g. Finansministeriet (1999, app. G), Møller *et al.* (2000), Treasury Board of Canada (1998, chs. 7-9), HM Treasury (2003, annex 4), U.S. Environmental Protection Agency (1983, ch. IV) and NCEDR (2005).

This toolbox paper argues that it is important to undertake analyses of risk and uncertainty in most CBAs and that the results should be clearly stated in the CBA reporting. The treatment of risk and uncertainty should be taken into account in the cost benefit analysis at an early stage, in part to ensure that sufficient data on risk and uncertainty is collected.

No method of risk and uncertainty analysis fits all cases of cost benefit analyses. Depending on the case it can be useful to apply various *ad hoc* methods. A risk averse decision-maker may seek to avoid overestimating the NPV by adjusting costs upwards or benefits downwards, by shortening the period for which benefits are counted or by discounting benefits heavily.

Gross sensitivity analyses seek to ascertain the sensitivity of the expected NPV estimate to specified changes in the variables entering the calculation. Worst/best case scenarios, where the variables are changed from their lowest to their highest values, also provide important information about the importance of the different variables that affect the expected NPV. Graphical representations of the sensitivity and worst/best case results can be illustrative.

Risk analyses based on Monte Carlo simulations can be useful in cases where it is meaningful to attach distributions to the variables entering the CBA. First, in some cases where random variables enter the NPV calculation in a non-linear manner, a simulation is needed to obtain a reliable estimate of the expected NPV. Second, the resulting simulated NPV distribution gives a depiction of the risk associated with the calculation. From the distribution one can find confidence intervals of the NPV estimate and one can estimate the probability that the project has a positive net present value, i.e. that $NPV > 0$. The simulation approach has the advantage of taking into account the entire distribution of the variables in the CBA calculation, while the sensitivity analyses and, especially, stress testing often pay undue attention to

extreme observations. In many cases an estimation of the NPV distribution using Monte Carlo simulation also makes it easy to communicate risks to a broad audience.

It is usually beneficial if the decision-maker can react to risks or uncertainties affecting the net present value of a project. In some cases it may be beneficial to postpone the implementation of a project in order to learn more about factors affecting the project. In other cases it is beneficial to reverse a project if it turns out to bring about very harmful results. In case of risks and uncertainties, flexible projects that can be postponed or reversed will generally be more valuable than less flexible projects.

An important added value of risk and uncertainty calculations is that they impel the analyst to identify and quantify sources of risk and uncertainty at an early stage of the CBA. It can lead to extra data sampling or new calculation methods in order to attain a more precise expected NPV estimate. Explicit quantification of the imprecision of the expected NPV estimate can thus lead to a deeper understanding of the problems of the CBA in question and contribute to a better and more reliable selection of environmental projects and policies.

2 R I S K A N D U N C E R T A I N T Y

In everyday use, the two terms “risk” and “uncertainty” are used interchangeably to indicate that future outcomes are not deterministically known. Clearly, this applies to very many situations, not least to the outcome of environmental projects and policies.

Knight (1921) introduced a distinction between risk and uncertainty, which in some cases can be useful when considering the reliability of cost benefit analyses. Risk is defined as randomness that is measurable or quantifiable, i.e. can be described by a probability distribution. Uncertainty, on the other hand, is a more fundamental form of randomness which cannot be measured or captured by a probability distribution. This fundamental form of randomness is sometimes labelled *Knightian* uncertainty to emphasise the distinction between risk and uncertainty.

Using these definitions, risk is variability or randomness that can be quantified. Risk can lead to an unfavourable outcome, but also to a favourable outcome (i.e. there can be a “risk of a positive outcome”). Risk often stems from a process that is repeated many times. Uncertainty is randomness which cannot be described by a distribution. Uncertainty often stems from an infrequently occurring, discrete event.

Examples of variables subject to risk include future oil prices, the soybean production in Brazil next year and the number of future typhoid cancer cases in a population after exposure to a given dose of radiation. Clearly, no estimate of the future realisation of the variables can be precise, but – at least in theory – it is possible to make educated guesses of the distributions of the variables in question.

Examples of Knightian uncertainty include the possibility of an earthquake hitting Sweden next year, the invention within the next 10 years of a technology curing cancer or sudden changes in consumer tastes. In these cases, it is essentially impossible to attach probabilities to the possible outcomes. Knightian uncertainty also includes hazards which are not even perceivable; the terrorist attacks 11 September and the emergence of hitherto unknown diseases are possible examples. Clearly, one cannot attach a probability distribution to an event which hardly anybody anticipates.

In everyday use – and in countless cost benefit analyses – the two terms risk and uncertainty are used indistinguishably. This reflects that it is not always important to distinguish between the two terms; e.g. if no risk or sensitivity analyses are re-

quired. It also reflects that it is often difficult to distinguish between risk and uncertainty in practice. This point can be illustrated by considering a CBA that is based on, *inter alia*, the demand for oil in the next 10 years. The demand will be subject to risks as it depends on the oil price which has historically fluctuated within a certain interval. The oil demand is also subject to uncertainty as a new invention might make oil completely superfluous. In practice, it is thus not easy to distinguish between risk and (Knightian) uncertainty.

Any environmental project or policy – or human activity for that matter – is affected by many variables subject to (Knightian) uncertainty. It does not imply, however, that it is practically impossible to assess the reliabilities of findings from cost benefit analyses. In many cases, it might be useful to focus on risks which are well-defined and quantifiable and disregard uncertainties which are very unlikely to materialise. When considering whether or not to build a treatment plant for waste water, the CBA might take into account risks with respect to the quantity of waste water, while essentially ignoring the consequences of possible meteor fallouts, war or entirely unforeseeable technical problems.

Another issue is how to estimate the distribution of a variable subject to risk. Many cost benefit analyses of environmental projects or policies build on scientific results where only little explicit information is available on the distribution of the risk. By way of example, many dose-response studies are subject to a wide margin of error and a certain degree of arbitrariness, but does this imply that the dose-response factor cannot be described by a probability distribution? It is argued in chapter 4 that it is often possible to make rough estimates of the distribution of risks, especially if the need for such information is incorporated in the early exploratory stages of the cost benefit analysis.

In sum, the distinction between measurable risk and unmeasurable (Knightian) uncertainty is useful when assessing the reliability of cost benefit analyses. Risk is measurable randomness which can be quantified by a distribution function. Uncertainty is unmeasurable randomness and therefore difficult to incorporate into a CBA. Randomness should to the largest extent be quantified when CBAs are undertaken, but it should also be kept in mind that the calculations can be affected by shocks and structural changes which are essentially impossible to predict and/or assess.

3 RISK AND UNCERTAINTY IN CBA IN THEORY

The idea of CBA was introduced in the 19th century, but its use for socio-economic assessments only gained importance in the second part of the 20th century essentially mirroring developments within welfare economics (Pearce *et al.* 2006, ch. 1). The fundamental welfare theorems were proved within a mathematical deductive framework. The decentralised market equilibrium was shown – under ideal circumstances – to be Pareto efficient, i.e. no resources are wasted in the sense that nobody can be made better off without somebody else being made worse off. It was also shown that the market solution will be inefficient or wasteful in case of market failures like externalities and public goods. In these cases, government intervention can lead to a Pareto improvement and increase social welfare.³

The government possesses a range of instruments to affect the market solution and hence social welfare, e.g. taxes, subsidies, standards, mandatory injunctions and specific projects. In principle, the government faces a colossal maximisation problem: choose for all future periods all different instruments contingent on all possible sources of randomness in order to maximise social welfare subject to the functioning of the economy – including the impacts of all market failures – and the reactions of firms and individuals. Even in theory, this maximisation problem is clearly too complex to be solved. It has therefore become customary to split up the government's problem into smaller parts in order to reduce its complexity (Musgrave 1969). The convention is to consider the effects on efficiency or social welfare of policies within a certain field, e.g. competition or environmental policies.

The use of cost benefit analysis as a tool for practical assessment of government projects and policies mirrors the above partial approach: instead of solving a large unmanageable welfare maximisation problem, CBA seeks to determine the *potential* of a project or policy to *contribute* to social welfare. The CBA generally does not require that an explicit social welfare function is defined (but only the societal welfare increases in the welfare of individuals; see section 3.1). The purpose is to determine whether a project or policy creates additional surplus or value which has the potential to increase social welfare (Petersen & Busk 2004).

³ Government intervention might also be warranted if the government has distributional or paternalistic objectives.

3.1 Cost benefit analysis under certainty

The principles of applying CBA are relatively straightforward when all variables are deterministically known; the variables can vary over time but in a perfectly predictable manner. Assume that the project lifespan is $T + 1$ years: the first period in which the project affects society is period $t = 0$ and the last period is $t = T$. The total benefits and the total costs – monetary and non-monetary – pertaining to society are calculated for each period $t = 0, \dots, T$. The net present value of the project is then found by discounting the net benefits for each period to the initial period using the possibly time-dependent discount rate δ_t .⁴ In algebraic terms, NPV is found as:

$$\text{NPV} = \sum_{t=0}^T \frac{1}{(1 + \delta_t)^t} (\text{Benefits}_t - \text{Costs}_t) \quad (1)$$

The calculation of a project's NPV is a “mechanical” exercise and does not address the normative question whether the project should be undertaken or not. In particular, even if $\text{NPV} > 0$, some individuals will be better off but others could be worse off. A new sewage treatment plant might e.g. improve the environment for most people in the area, but worsen it for those living close to the new plant. Should such a plant be built?

The “Kaldor-Hicks compensation principle” states that the criterion $\text{NPV} > 0$ is relevant even if a project produces both winners and losers. The argument is that the losers can be compensated by the winners as the surplus pertaining to the winners exceeds the loss experienced by the losers. Thus, a project with $\text{NPV} > 0$ has the *potential* to make some individuals better off while no individuals are worse off, i.e. to constitute a Pareto improvement.⁵

Taking as a decision rule, the Kaldor-Hicks compensation principle states that if several projects are mutually exclusive, the project with the highest NPV should be chosen. If the projects are *not* mutually exclusive, all projects with $\text{NPV} > 0$ would be potentially Pareto improving and should be implemented.

A project or policy with a positive NPV will generally be able to improve social welfare, provided social welfare can be expressed by a Paretian social welfare func-

⁴ Kjellingbro (2004) discusses discounting in detail.

⁵ Such redistributive compensation can be expensive, difficult or virtually impossible in practice. The Kaldor-Hick compensation principle, however, does not require that the compensation can or actually will be carried out in practice.

tion, i.e. social welfare is an increasing function in the welfare of every individual in society (Ng 2004: 3). The potential Pareto improvement implies that welfare need not decrease for anybody but will increase for somebody. Thus, given that all variables are known with certainty, a sufficient but not necessary condition for a project to have the potential to improve (Paretian) social welfare is that $NPV > 0$.⁶

3.2 Cost benefit analysis under risk and uncertainty

The discussion above assumed that all variables are deterministic, i.e. there are no risks involved in the lifespan of the project or policy. This is clearly not a realistic assumption in any case. The question is how the incorporation of risks changes the CBA and the normative use of the methodology.

The NPV calculated by (1) will now depend on the realisation of the variables subject to risks. A natural starting point is therefore to consider the expected net present value of the project:

$$E[NPV] = E_0 \left[\sum_{t=0}^T \frac{1}{(1 + \delta_t)^t} (\text{Benefits}_t - \text{Costs}_t) \right] \quad (2)$$

The expectations operator $E_0[\cdot]$ signifies the mathematical expectation conditional on information prior to the initial period, i.e. prior to the decision of whether or not to implement the project. Note that formula (2) assumes that the project remains in place during the entire project horizon and cannot be discontinued or reversed e.g. in case the performance of the project deteriorates. This assumption is reasonable for many projects, but not for others. Chapter 5 discusses briefly the implications of project reversibility.

The net present values presented in most cost benefit analyses are actually *expected* net present values. The expected NPV is usually found by using the expectation to the value of all the variables entering the calculation. When two or more random variables enter the CBA calculation non-linearly, i.e. in the form of a product or a fraction, this method for estimating the expected net present value will generally only approximate the mathematically correct expectation.⁷ If the random

⁶ Note that $NPV > 0$ is a *sufficient* but not a *necessary* condition for a project to improve social welfare. Depending on the social welfare function and its implied distributional objectives, a project with $NPV < 0$ might improve social welfare if it makes socially important individuals better off.

⁷ The “true” or mathematically correct expectation is often called the unbiased expectation, i.e. the expectation where no systematic error is made.

variables enter multiplicatively, the method of finding $E[\text{NPV}]$ using expected values for all random variables will only be correct if the correlations between the variables entering non-linearly are zero.⁸ If a random variable enters as the denominator in a fraction, then the method generally produces a result which is not the mathematically correct expectation.⁹ Thus, the standard way of finding the expected NPV is usually only an approximation. To obtain a mathematically correct estimate of the expected net present value, one would need to include knowledge of the *distributions* of the variables entering non-linearly in the NPV calculation. The method used in practice is described in section 4.4.

Let us now turn to the normative aspects of the cost benefit analysis when random variables enter the calculation of the expected net present value. In particular, is it in this case reasonable for a government to use the equivalent of the “Kaldor-Hicks compensation principle” and use $E[\text{NPV}] > 0$ as a condition for project selection? The question is still unsettled in the theoretical literature but it is possible to draw together some guidance.

Social welfare is usually presumed to be an aggregate of the well-being or utilities of individuals in society. The starting point must therefore be the attitude of individuals toward risk. It is generally assumed that individuals are risk averse and concerned about their expected utility.¹⁰ Individuals are willing to pay for insurance which limits their loss in case an unfavourable event takes place, e.g. their home burns down. In other words, individuals usually do not only consider the expected return, but also the distribution of the return. Being exposed to a risk constitutes a cost to risk averse individuals and they are willing to pay in order to reduce or eliminate the risk.

Assuming that individuals are risk averse, the question is how to account for risk in a project or policy which affects all risk averse individuals similarly. This problem

⁸ The expectation of the product of two random variables X and Y is $E[X \cdot Y] = E[X] \cdot E[Y] + \text{Cov}(X, Y)$. The expectation of the product of two random variables is only equal to the product of the expectations if the covariance between the variables is zero.

⁹ If X is a random variable, the expectation $E[1/X]$ is generally not equal to $1/E[X]$. This is easily illustrated by an example where $X = 1$ with probability 0.5 and $X = 3$ with probability 0.5. In this case, $E[1/X] = 0.67$ while $1/E[X] = 0.5$.

¹⁰ The two assumptions are likely violated in a range of practical situations: First, many individuals buy lottery tickets where they are likely to lose money, but which give them a small chance of winning a large sum of money. This reflects risk-loving behaviour. People engaging in base jumping or motor cycle racing also appear to seek risks. Second, the expected utility framework is clearly restrictive. Many individuals are for example more concerned about an event with a large cost but extreme low probability than expected utility would suggest. Third, individuals' *perception* of risks appears to be highly subjective and not necessarily corresponding to “objective” measures of risk. See also Viscusi (1989).

was analysed in Arrow & Lind (1970) where they derived the so-called Arrow-Lind theorem based on the idea of risk spreading. Assume that all n individuals in society are identical and that all benefits and costs are shared equally. When the number of individuals increases, the share of the risk carried by each individual decreases and, correspondingly, the individual's welfare cost from the risky project decreases. More surprisingly, Arrow & Lind (1970) prove that also society's *total* welfare loss (i.e. the aggregate loss of the n individuals) from the risky project decreases.¹¹ On the margin, when n approaches infinity, the randomness of the project does not affect social welfare at all. In other words, the spreading of risks to many individuals implies – under a number of conditions – that a project can be evaluated only on the basis of its expected net present value (Foldes & Rees 1977; Pearce 1986, ch. 6). The result questions the need to take into account risks when undertaking cost benefit analyses. However, the Arrow-Lind theorem builds on a number of assumptions which are unlikely to be met in practice (Pearce 1986, ch. 6):

First, in no cases can risks be spread among an infinite number of individuals. A country has always a finite number of individuals and, hence, there will still remain risks borne by society.

Second, the theorem assumes that all risks are shared equally by all individuals. In practice, this assumption is unrealistic. Most projects will likely expose some individuals to more risks than others. For example, the economic profitability of an electricity producing dam might be shared by everybody in society, but environmental problems from the dam might affect disproportionately individuals living close to the dam. Arrow & Lind (1970) consider this situation and show that if there are actuarially fair insurance markets, then risk averse individuals will insure away the idiosyncratic risks, i.e. the risks specific to the individual, so that eventually all individuals will only be exposed to the economy-wide risks associated with the project. It is clear, however, that insurance markets do not exist for very many contingencies, so in general risky projects will affect some individuals disproportionately so that the spreading of risks will be imperfect.

Third, the risk spreading argument breaks down if the risks take the form of an externality affecting everybody equally – irrespective of the number of individuals in society. A project leading to ozone depletion will likely affect everybody inde-

¹¹ As the number of individuals, n , increases, the welfare loss of each individual decreases faster than n increases.

pendently of the number of individuals in society. When a risk in this way takes the form of a public good (or rather a “public bad”), then the societal risk will not be reduced when the number of individuals increases.

3.3 Project selection as portfolio choice

The previous section considered the treatment of risks in a CBA of a single project or policy. It was argued that there might be theoretical arguments for focusing on the expected NPV, but in practice society should also consider risks when choosing whether or not to implement a project. This section goes one step further and considers the overall project and policy determination of a government being averse to risk.

In practice, authorities undertake a large number of environmental projects and policies. Each project will each period have a net benefit (benefits minus costs) and the net benefit can vary from period to period. In other words, each project or policy is a non-financial – and often also non-monetary – asset with expected returns, variances and co-variances varying over time. This implies that a government choosing projects and policies in order to maximise society’s welfare function is essentially solving a portfolio selection problem resembling similar problems in financial economics. Standard portfolio theory can thus be used to shed light on the treatment of risk in CBA (Harberger 1996, Brent 1998: 217).

The main insight from portfolio theory is that risks can be *pooled* so that the overall risk of the portfolio is lowered. The intuition is straightforward: at instances when some assets have low returns, other assets might have high returns. On the margin where the number of projects approaches infinity, the variance of the individual assets becomes unimportant (“The law of large numbers”). In practice, no portfolio will contain an infinite number of assets. In this case portfolio theory tells us that the co-variances between the individual asset returns will affect the expected return and variance of the portfolio. The pooling of risks suggests that covariances (correlations) across projects within and between periods will be of importance for social welfare (Wall 2004).

In practice, covariances between the socio-economic returns to different projects are seldom calculated. Still, the idea is applied in some cases where a decision-maker “bets on more than one horse” in order to reduce the likelihood of an unsatisfactory result. Congestion problems in a city might be sought by both building wider roads and expanding public transportation. In case of uncertainty with re-

spect to peoples' commuting preferences, such a policy would increase the probability of a successful outcome.

Whether or not the government takes into account covariances, the point is that a *portfolio* of projects or policies pools the risks of the individual projects or policies. The welfare cost of the risks of the portfolio is less than the cost if the risky projects had been considered independently. There are thus strong arguments for considering several projects or policies in combination when undertaking CBA, especially if the net benefits from the projects are correlated (Arrow & Lind 1970). This insight is carried out in practice in some cases: when considering a new bridge with adjoining roads, the CBA would comprise both the bridge and the roads. In other cases, however, it is too demanding to undertake a combined CBA of numerous projects as the calculations could be excessively complex. It is still, however, important to keep in mind that the risks of a single project to some extent will be pooled and thus will affect welfare less than if the project is considered independently.

3.4 Should risk be considered in CBA?

In practice, all CBA calculations are subject to many sources of risk (and uncertainty). Individuals are usually assumed to be risk averse. The question is then to which extent the decision-maker should be concerned about variability and whether risk and uncertainty should be considered in a CBA.

There are two important arguments for paying relatively little attention to risks in CBA, namely risk pooling and risk spreading. Society undertakes many projects and policies and society's assessment of the pooled portfolio of projects and policies will generally be different from the total assessments of the individual projects and policies. The risks of an individual project or a portfolio of projects will be spread to many individuals and reduce society's concern about risks. In the limit, when the number of projects or the number of individuals approaches infinity, it is sufficient for society to consider the expected net present value.

There is never an infinite number of projects or individuals, and the limit results are thus not immediately applicable in practice. There are many situations where it is useful to know the distribution of the NPV of a project and policy: (1) when the distribution of NPV must be known to find the mathematically correct value of the expected net present value; (2) when the size or the risk of a project implies that its risks cannot be effectively pooled; (3) when the risks are borne disproportionately by some individuals and the affected individuals cannot spread the risks to all

individuals by purchasing actuarially fair insurance; (4) when risks to a large extent have the form of “public bads”.

There are also a number of other factors which in practice can be important for the treatment of risk in CBA. First, for a project of such a size that the risk cannot be pooled, the government might seek to buy insurance internationally. To assess the insurance premium, the risk profile of the project must be known. The same applies to cases of “implicit insurance”, where a government assumes that it will receive help from abroad in case of harmful contingencies, e.g. an oil spill. The value of foreign aid in case of a harmful contingency will depend on the likelihood of such an event occurring. Second, a decision-maker might for political reasons have preferences with respect to the risk profile of different projects. A project with a high probability of failure might be *politically* unacceptable even if it has a high positive expected NPV and all risks are effectively pooled, e.g. if public debate focuses on negative project outcomes.

4 PRACTICAL METHODS

There are many cases where it is useful to assess the reliability of the expected net present value from a cost benefit analysis. This section presents a number of methods used in practice to incorporate risk and uncertainty into CBAs.

4.1 Adjusting the expected NPV to take account of risk aversion

This section discusses a number of methods used to adjust the calculated expected NPV in order to take into account risk aversion when assessing a project or policy.¹² In other words, the methods are used to add caution to the decision-making process when risks and uncertainties are present.

In practice, the specific form of the social welfare function and/or the preferences of the decision-maker are not known. This means that the methods discussed below essentially represent ad hoc approaches to adjustments for risk and uncertainty. This makes it important to consider what is meant by 'caution' or prudence in case of e.g. an environmental project or policy. The decision-maker can make two types of errors:

- i) They can choose not to undertake a project which would have benefited society.
- ii) They can undertake a project which is not socially favourable.

Measures which seek to reduce type I errors will frequently lead to more type II errors. Similarly, measures to reduce type II errors will lead to more type I errors. Different adjustment methods lead to different probabilities for making each of the two types of errors.

4.1.1 Cut off period

Many projects and policies involve large negative net benefits in the early stages and positive net benefits in the later stages. A crude way to reduce the risk of adopting socially unfavourable projects or policies (error type I above) is to cut off the period of positive net benefit flows e.g. after three or four years (Mishan 1976, ch. 27). The method implies that net benefits beyond the cut-off period are perceived to have a social value equal to zero. Note that the cut off method will increase the risk of implementing socially unfavourable projects if the discharged

¹² Where no other sources are indicated, this section builds on Treasury Board of Canada (1998, ch. 8).

periods are mainly periods with negative net benefits (error type II). The cut off method is often used in private project assessments.

4.1.2 Risk-adjusted discount rate

It has been suggested that the discount rate used should be adjusted upwards to reduce the weight of later periods in the calculation of the expected NPV (Mishan 1976, ch. 27). The rationale is that in some cases it is easier to forecast developments in the near future than in more distant periods. The discount rate should be higher in the case with risk and/or uncertainty than in the deterministic case. The method is clearly not appropriate if the main risks and uncertainties stem from the early periods. This could be the case if e.g. the construction costs of a sewage treatment plant are highly uncertain, while the future benefits in the form of environmental improvement are less uncertain. More generally, the use of a higher discount rate than under certainty is based on the implicit assumption that the risk and/or uncertainty compound itself geometrically over time. The method is also conceptually a bit unappealing as it blends accounting for the true time preference and for risk aversion (Treasury Board of Canada 1998, sec. 8.3).¹³

It is sometimes argued that it is reasonable to use a decreasing discount rate if the discount rate itself is subject to risk and the time horizon is long (Kjellingbro 2004). This is the result of the non-linear way the discount rate affects the net present value. When the discount rate is subject to risk, the expected net present value is larger than the net present value calculated using the expected discount rate.¹⁴ The difference increases as the time horizon is extended. Using a decreasing discount rate would in principle ensure that net present value calculated using the expected discount rate yields a mathematically correct $E[\text{NPV}]$ estimate.

4.1.3 Certainty equivalents

The calculation of certainty equivalents is a method to adjust the expected NPV to take into account risky and/or uncertain net benefits based on an explicit welfare theoretic foundation. For each period of the project lifespan, the certainty equiva-

¹³ See also Kjellingbro (2004) for a fuller discussion of a possible risk adjustment of the discount rate. The key point is whether the discount rate is viewed as a true time preference or as an opportunity cost of holding an asset (the prescriptive vs. the descriptive view).

¹⁴ A net benefit of 1 in period t discounted to period 0 takes the value $(1 + \delta)^{-t}$, cf. also formula (2). Assume that the discount rate δ is random (and not degenerated to a fixed value). The discount function $(1 + \delta)^{-t}$ is strictly convex for $\delta > 0$ and it therefore follows from Jensen's inequality that it will always hold that $E[(1 + \delta)^{-t}] > (1 + E[\delta])^{-t}$.

lent is the fixed (non-random) net benefit, which would make the decision-maker indifferent between the fixed value and the random net benefits. The certainty equivalent is smaller than the expected (or average) net benefits if the decision-maker is risk averse. The difference between the certainty equivalence and the expected net benefit is often called the risk premium; i.e. the amount the decision-maker would be willing to pay to avoid having to carry the risk of the project.

The main challenge of using the method is to find a reasonable estimate of the certainty equivalent. This would ideally require knowledge of both the distribution of the net benefits in each period of the project lifespan and also the decision-maker's preferences with respect to risk. For practical use, the certainty equivalent must be found without the use of highly complex calculations. It is therefore often assumed that the decision-maker has an "exponential utility function" (Walls 2004). This function implies a constant risk aversion, i.e. the risk premium will not change as a function of the expected net benefit. The certainty equivalent CE_t of the net benefit NB_t when NB_t is distributed with variance σ^2 can then be found as:

$$CE_t = E_0[NB_t] - \frac{\sigma^2}{2R} \quad (3)$$

The operator $E_0[.]$ denotes the expectation based on information prior to the initial period. The variable $R \geq 0$ captures the risk tolerance value of the decision-maker. The value $R = \infty$ signifies risk neutrality, where the certainty equivalent is equal to expected net benefit; a low risk tolerance means that CE_t is much smaller than $E[NB_t]$. In practice, σ^2 will depend on the specific project considered. The value R must be estimated based on the government's previous choices, international practice or other sources of information. Tabulations of CE_t as a function of different values of $E_0[NB_t]$ and R may help pin down the decision-maker's risk tolerance value (Walls 2004).

4.1.4 Downward revision of benefits, upward revision of costs

The use of certainty equivalents implies that a project with risky net benefits have a lower expected NPV than otherwise. The result is fewer type II errors where a socially unfavourable project is undertaken. The method, however, requires some computations and a prior knowledge of the distributions of the variables entering the CBA calculation. This has resulted in the suggestion that expected values of risky or uncertain *benefits* are adjusted downwards on an ad hoc basis, e.g. by

10%. Alternatively, the expected value of risky or uncertain costs can be adjusted upwards. Experiences from other countries or previous projects might provide some guidance for the appropriate adjustments. In particular, the adjustments might be relatively small if the cost and benefit flows are known with a relatively large degree of certainty, e.g. if there is considerable experience with similar projects or the project uses mainly known technology. The adjustments might be larger if the project is based on an untested model, is complex or relies on new technologies.¹⁵

4.1.5 Safety margin

A rough but widely used way to let risk and uncertainty affect the evaluation of the expected NPV is to demand a sizeable safety margin. Instead of requiring that $E[\text{NPV}]$ should simply be positive, it is often assumed that $E[\text{NPV}]$ should be larger than a preset positive value. Alternatively, a large positive $E[\text{NPV}]$ is interpreted as providing a desired “safety margin” for acceptance of a project with the implicit premise that the main result would not change had risk or uncertainty been incorporated.

As before, the imposing of a safety margin is only a safety margin with respect to one type of error, while it increases the possibility of errors of the other type. The stipulation of a very high $E[\text{NPV}]$ reduces the change of making a type II error, i.e. to undertake a project which is not socially favourable, but increases the chance of making a type I error, i.e. to pass over a socially favourable project.

4.1.6 Precautionary principle

The costs of environmental damage can potentially be very high, i.e. if damage to an ecosystem is irreversible. This has led to the conception that a precautionary principle should be applied whenever projects or policies affecting the environment are assessed (Hansen *et al.* 2003). The idea behind the precautionary principle is to ensure that projects or policies do not bring about excessive harm to the environment, i.e. only projects or policies that are on the “safe side” are implemented.

The precautionary principle can be implemented by ad hoc adjustments to costs and benefits in directions so that the expected net present value is changed in a

¹⁵ In addition, experience shows that when the project is inherently risky or uncertain, the assessments of the expected costs are often biased downwards and the expected benefits are biased upwards. The phenomenon is sometimes called “optimism bias” (HM Treasury 2003, annex 4).

direction making it less likely that an environmentally unfriendly project is accepted and making it more likely that an environmentally friendly project is accepted. The precautionary adjustments may be estimated based on assessments of gain to society from not damaging the environment irreversibly. (See section 5.1 for further details.)

4.2 Sensitivity analysis

This section presents methods which can be used to assess the sensitivity of the expected NPV to changes in the variables entering CBA.¹⁶ In this sense, sensitivity analyses can expose some characteristics of the distribution of NPV, but – as opposed to the methods discussed in section 4.1 – sensitivity analyses do not explicitly address the normative question of project selection.

Sensitivity analysis has the advantage that it can take into account not only risks, but also – to some extent – unmeasurable uncertainty. The sensitivity analysis shows what happens to $E[\text{NPV}]$ when a variable is changed, but the chosen values for the variable need not be drawn from any well-defined distribution. In this sense, the effects of uncertainty in the form of sudden changes in existing variables can be analysed. Evidently, uncertainty in the form of entirely unforeseeable events cannot be addressed adequately in any framework, including the use of sensitivity analysis.

4.2.1 Gross sensitivity analysis

The starting point for any sensitivity analysis is a baseline scenario where the expected net present value is computed, given that all variables take their expected value.¹⁷ The next step is to vary one of the variables and then find the new $E[\text{NPV}]$ *conditional* on the changed value of the variable. The exercise can be undertaken for all variables entering the CBA and repeated with different changes in the variables. The method is also called variable-by-variable sensitivity analysis. The analysis reveals how *sensitive* the estimated $E[\text{NPV}]$ is to given changes in the considered variables. It is computationally easy and also has the advantage that it

¹⁶ Where no other sources are indicated, this section builds on Treasury Board of Canada (1998, ch. 7).

¹⁷ As argued in section 3.2, this way of calculating the expected net present value amounts to an approximation. See also section 4.3.

does not require any knowledge of the distributions of the variables entering the CBA. It can be useful to present the results in a table.

A gross sensitivity analysis is a descriptive exercise without immediate normative implications. Sensitivity analyses are, however, often used to show whether a given change in a variable changes the sign of the expected net present value. If this is the case, the value of the variable for which $E[NPV]$ changes sign is called the *switching value*. Note that the switching value for a given variable is conditional on all other variables retaining their expected values. It is often useful to establish *how large* a change in a variable that is required to change the sign of $E[NPV]$. The relative change needed to bring about a sign change is sometimes called the *switching ratio*. A decision-maker may exercise caution if the $E[NPV]$ is positive in the baseline, but the construction costs has a low switching ratio.

In some cases, it might be useful to undertake a gross sensitivity analysis changing two or more variables at the same time. This so-called scenario analysis is particularly useful if it is known that the chosen variables are closely correlated. Consider an example of an environmental project seeking to reduce traffic congestion by building an additional road. The expected net present value of the project depends on, inter alia, the number of cars that start using the new road and the number that still uses the old road. A gross sensitivity analysis can then show the effect on $E[NPV]$ of different uses of the new road. However, if the overall number of cars is broadly constant, then a gross sensitivity analysis considering the combined effect on $E[NPV]$ of changed uses of both the new and the old road may be useful.

When undertaking a sensitivity analysis, it is often important to distinguish between different types of variables. Some variables are within the control of the decision-maker and the sensitivity analysis can then be used as a basis to make changes to these. Other variables are essentially outside the control of the decision-maker and the sensitivity analysis then provides information on how risks and uncertainties affect the project.

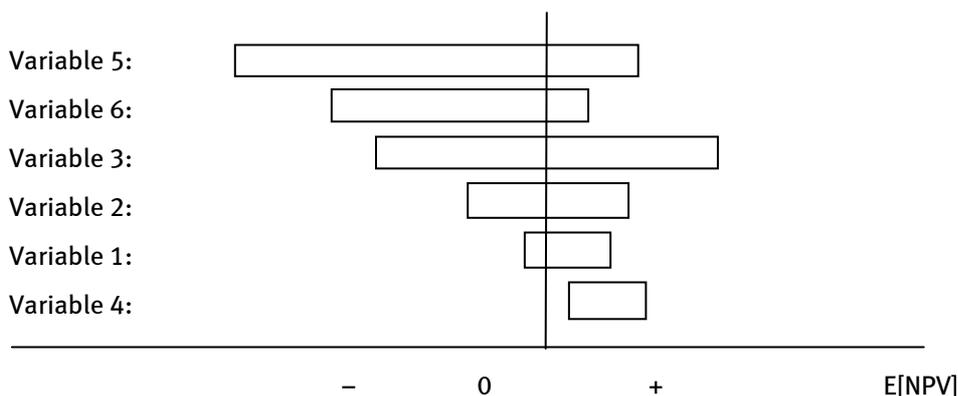
4.2.2 Stress testing

The discussion above focused on the gross sensitivity of $E[NPV]$ to changes in the variables. It is possible, however, that a risk averse decision-maker will be especially concerned about the prospect of large or very likely losses. One way to address this issue is to *stress test* the CBA calculations by calculating worst/best case scenarios. Such stress testing is also called an *analysis of extremes*.

The starting point is again the baseline $E[NPV]$ calculated using expected values for all variables. The $E[NPV]$ is then recalculated using, respectively, the smallest and the largest value for each of the variables. These worst/best case scenarios help pinpoint where a very unfavourable (or favourable) development of a variable affects the expected net present value most strongly.

The many items of information imply that it is useful to present data in a table or figure. Figure 1 shows a so-called tornado chart where the span in NPV from the worst to the best case scenario is shown for all variables. The variables are ordered so that the variable associated with the worst case scenario (i.e. giving the lowest $E[NPV]$) is shown first, then the second worst case variable and so on.

Figure 1: Tornado chart



Note: The tornado chart shows the values of the expected NPV when individual variables are changed from their minimum to maximum value, while the remaining variables retain their mean value

The tornado chart is a convenient way to present the results of the stress test as the chart gives a quick overview of the variable(s) which might cause concern for a risk averse decision-maker. In Figure 1, variables 5, 6 and 3 are those that can lead to the lowest expected net present values. The stress testing does not tell anything about how likely it is that a variable takes its worst case value.

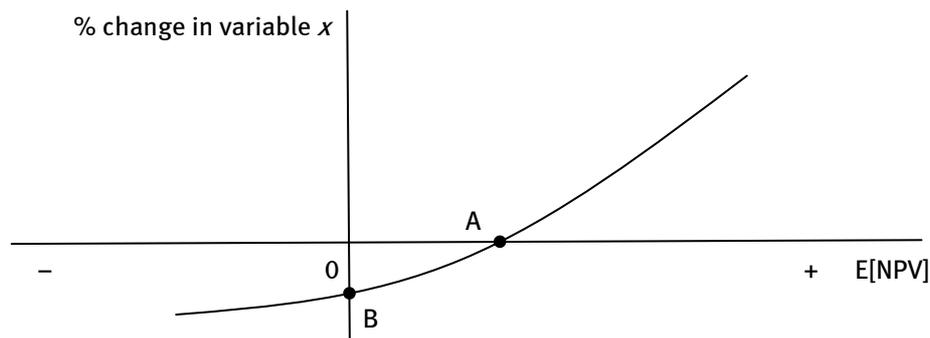
It is of course also possible to calculate the NPV resulting if all variable take their worst case values. This would inform the decision-maker of the absolute worst outcome of the project, but clearly such an event has a very low probability of occurring (Mishan 1976, ch. 27).

4.2.3 Combining gross sensitivity analysis and stress testing

A sensitivity analysis often considers what happens to $E[\text{NPV}]$ when the variables change and when they take their extreme values, i.e. the analysis comprises both a gross sensitivity analysis and a stress test.

It is sometimes convenient to present the results of the gross sensitivity analysis and the stress testing in conjunction, cf. Figure 2. The x locus shows how *relative* changes in the variable x from its baseline expected value $E[x]$ affect the expected net present value. The locus should be drawn from what is perceived to be its minimum to what is perceived to be its maximum. Figure 2 is drawn so that when x takes its expected value, the $E[\text{NPV}]$ is positive (point A), but for sufficiently low values of x , the $E[\text{NPV}]$ turns negative. The switching ratio can be found as the intersection of the x locus and the second axis (point B).

Figure 2: Expected net present value as a function of change in variable



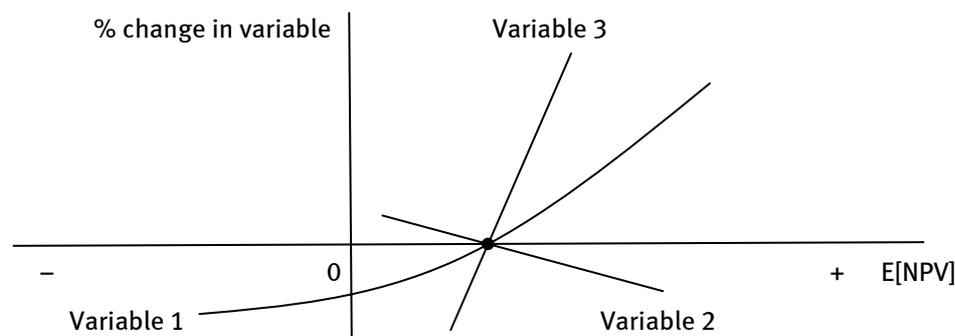
Note: The figure shows the expected NPV conditional on relative changes in the variable x from its expected value $E[x]$

Sensitivity analyses will usually be carried out for all or most of the variables entering the CBA calculation and it will often be convenient to present the results in a table or figure. An example of the latter is the spider plot, where more variables are included in a figure like Figure 2. Figure 3 shows a spider plot with three variables.

The spider plot makes it easy to compare the sensitivities of different variables on the expected net present value. In Figure 2 changes in variables 1 and 2 have relatively large effects on $E[\text{NPV}]$, while variable 3 has only little effect. If each locus is drawn from its minimum to its maximum, the spider plot also reveals which variables that can cause a sign reversal of $E[\text{NPV}]$. In Figure 3 changes in variable 1 can

alter the sign of $E[NPV]$, while neither variable 2 nor 3 can make $E[NPV]$ negative as long as all other variables are held at their expected values.

Figure 3: Spider plot



Note: The figure shows the expected NPV conditional on relative changes in each of the variables

4.3 Risk analysis using Monte Carlo simulation

Sensitivity analyses as discussed in 4.2 exhibit several limitations. First, a gross sensitivity analysis can be somewhat arbitrary as long as no probabilities are associated with the experiments. Second, stress testing has the drawback that while it is unrealistic to assume that a variable takes its expected value constantly, it is generally uncommon for the variable to take any of its extreme values. Stress testing may put too much emphasis on outcomes that are very unlikely. Third, both methods consider only one variable at a time (with a few exceptions; see subsection 4.2.1) and thus may put too little emphasis on unfavourable developments affecting several variables simultaneously.

4.3.1 Assigning probabilities

To address these weaknesses, it has been suggested that the different values of the variables be weighted based on their likelihood of occurring (Mishan 1976, ch. 27; Brent 1998, ch. 11). Consider an example with three variables entering the calculation of $E[NPV]$. Each of the three variables takes its worst-case value with probability 10%, its best-case value with probability 10%, and its central value (the expected value) with probability 80%. With this information it is possible to calculate different values of NPV and its corresponding probability of occurrence. In the

example, the probability of all three variables coming out with their worst case values is 0.1% (assuming independence) to which corresponds a given NPV. This method gives some measure of the probability distribution of NPV, but it is difficult to implement in practice (computational demanding) and at the same time relatively arbitrary. A better alternative is to carry out Monte Carlo simulations.

4.3.2 Undertaking Monte Carlo simulations

Risk analysis or probability-weighted sensitivity analysis can be carried out using Monte Carlo simulation of the NPV calculation.¹⁸ The idea is to attach distributions to the variables entering the NPV calculation and then simulate a large number of draws from these distributions in order to find the resulting distribution of NPV. Knowledge of the NPV distribution makes it possible to assess the reliability of the CBA. Monte Carlo simulations can be carried out in the spreadsheet that is used to calculate the NPV using an add-in spreadsheet tool for Monte Carlo simulations. Monte Carlo simulations can also be undertaken in many econometric packages and accounting programmes.

The main challenge is to determine the distribution of the variables entering the calculation of the NPV. Some variables might be known with large precision and might be treated as deterministic variables. However, most variables are random and each should be assigned a distribution.

There are essentially two methods of determining the distributions of the individual variables entering the NPV calculation.¹⁹ One method is based on fitting a distribution to the historical data of the variable. This is made easier by tailor-made modules integrated within the simulation add-ins used for Monte Carlo simulations. The applicability of this method rests on an assumption that the historically observed picture will continue in the future.

If historical data is not applicable – or is not available – then one might rely on expert assessments. These assessments could be derived from engineering calculations or environmental impact studies. Engineers designing a bridge would e.g. be able to estimate the risk of weather conditions leading to serious structural

¹⁸ Where no other sources are indicated, this and the following subsections build on Treasury Board of Canada (1998, ch. 9). Campbell & Brown (2003) give a detailed depiction on how to carry out the analyses in practice.

¹⁹ Theoretically, but unlikely in practice, the variable could also follow a known objective distribution, e.g. based on a flip of coin.

damage to the bridge.²⁰ An environmental impact study may find that the particle concentration in a city follows a log normal distribution. In some cases, the distributional characteristics follow directly from the way the variable is found. When the variable is estimated using statistical or econometric methods, its expected value and standard deviation and – in some cases – the entire distribution result from the analysis (Brent 1998: 222-223).

More typically, the assessments will be informal and based on experts' experience and intuition. Experts might suggest an interval of likely outcomes, perhaps supplemented with a most likely outcome. This still leaves the choice of a distribution. If the variable is the sum of many underlying contributions, then there are strong theoretical arguments for choosing the normal distribution ("the central limit theorem"). The disadvantage of this distribution is that it is unbounded, i.e. the variable can take any value from minus infinity to plus infinity. This inconvenience can be addressed by truncating the distribution. The parameterisation of the normal distribution can be aided by the fact that approximately 95% of the mass will lie within interval ranging from the expected value minus two times the standard deviation to the expected value plus two times the standard deviation.

If minimum and maximum values as well as a "central" or average value are available, a triangular distribution might be an expedient choice (Campbell & Brown 2003: 210). A uniform distribution over the range of perceivable values might be a good choice if no information about a central value is available (Boardman *et al.* 1996: 203). Some variables might only take discrete values in which case discrete distributions might be warranted. One can either choose a common discrete distribution or alternatively attach probabilities to each possible outcome of the variable considered.

Finally it must be considered whether any of the variables entering the NPV calculation are correlated. Correlation can be present in several forms:

- i) Across two or more variables within one period. An example would be particle pollution and noise from traffic which will often be present at the same time.
- ii) Across periods for one variable, i.e. serial correlation. An example could be consumption patterns which often show a high degree of persistence.

²⁰ Brent (1998: 210-213) presents an illustrative example where engineers estimate the distribution of labour productivity for harbour workers at a port in Mogadiscio, Somalia. An initial very rough assessment was refined using an iterative procedure where the engineers were repeatedly asked to reassess their distributional conjectures.

iii) Across periods for two or more variables. An example is gas prices where many contracts stipulate that the gas price is adjusted according to the oil price of the previous year.

Most programmes for Monte Carlo simulation (including spreadsheet add-ins) allow for the specification of correlations between two or more variables. The correlation coefficients can be derived from statistical analyses of past data or be suggested by experts.

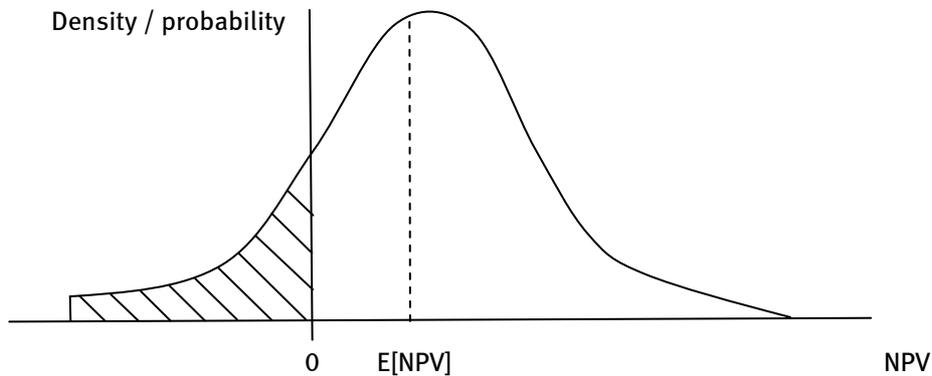
When the distributions are chosen, the Monte Carlo simulations can be undertaken. It is customary to use 1000-10000 simulations. Each simulation draws a value of all the variables based on the assigned distributions and returns the resulting NPV. With 1000-10000 simulations, the entire distribution of NPV will be traced. The fit of the simulated distribution to the theoretical distribution improves as the number of simulations increases. The results from the simulations of the NPV distribution can be represented in different ways.

4.3.3 Presenting the results

The average or expected value of the net present value is important. As argued in section 3.2, the $E[\text{NPV}]$ found by insertion of the expectations to the variables will generally be incorrect because of non-linearities. The expected net present value found via Monte Carlo simulations will be mathematically correct (given that the chosen distributions for the variables are correct).

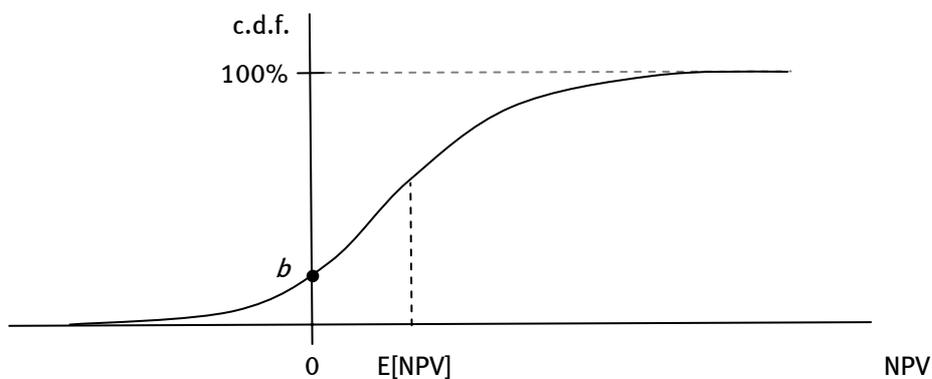
The variance or standard deviation should also be presented. The standard deviation can be used to assess the dispersion of the NPV distribution. A large standard deviation relative to the expected value indicates that the expected value is imprecisely estimated. The standard deviation can be used to map out a confidence interval. If the NPV is a normal distribution, then there is 95% probability that the true value of NPV lies within the interval spanned by expected NPV plus/minus two times the standard deviation. Using a confidence interval makes it relatively easy to communicate the risk of the project as it can be stated that there is a 95% chance that the NPV will be within the estimated confidence interval.

It can be illustrative to present the simulated probability density function of NPV. The simulation programme will usually make available this figure. Figure 4 shows an example where the distribution of NPV is bell-shaped; $E[\text{NPV}]$ is positive, but a part of the distribution falls below $\text{NPV} = 0$.

Figure 4: Simulated probability density function

The perhaps most intuitive communication of the results from Monte Carlo simulations is to indicate the expected NPV supplemented by the probability of NPV taking a negative value: “The project has an expected NPV of a kroner, but there is a b percent probability that NPV turns out to be negative”. The probability of NPV being negative is sometimes called the *expected loss ratio*. The expected loss ratio is the hatched area in Figure 4.

One can also present the cumulative distribution function which makes it easy to read the expected loss ratio from the graph. The expected loss ratio is found where the function intersects the second axis. Figure 5 shows a simulated cumulative distribution function with expected loss ratio b .

Figure 5: Simulated cumulative distribution function

Section 3.3 brought up that a decision-maker should focus on the NPV from the *total* portfolio of projects and policies. To some extent, it is possible to incorporate this insight into risk analyses based on Monte-Carlo simulations. If two or more projects are known to have net benefits, which are correlated within a period (and possibly also across periods) then the CBA could seek to calculate *one* NPV for the projects in combination. The Monte Carlo simulation would then reveal the expected NPV for the combined project portfolio along with its distribution of the NPV.

4.4 Which method?

When a CBA is subject to risk and uncertainty, a range of practical tools can be employed to assess the ensuing reliability of the CBA and possibly also help guide the decision making. Various ad hoc methods could be used to adjust the expected NPV in light of possible welfare consequences of the risk and uncertainty (section 4.1). Sensitivity analyses can reveal how the expected NPV reacts to changes in the variables (section 4.2). Risk analyses using Monte Carlo simulation can give an mathematically correct estimate of the expected NPV and uncover the distribution of NPV (section 4.3).

The question is then which method to employ. It is tempting to answer as Winnie the Pooh: "Yes please, both" – or in this case perhaps all available methods – but this answer is not always appropriate. A number of points must be considered.

- Ad hoc adjustments to the expected net present value tend to mix up the "technical" calculation of the CBA and the decision-making. An adjustment affects the expected NPV and thus changes the basis for the decision-making. It follows that such adjustments should be only in cases where there are very good arguments for their use. In addition, the ad hoc adjustments should be clearly explained in the CBA documentation.
- If the aim is to examine the impact of genuine Knightian uncertainty, then a gross sensitivity analysis is the main applicable method, possibly supplemented by a worst/best case scenario if information about extremes of the variables is available. If the goal is to examine the impact of risk, then Monte Carlo simulations should be used in addition to the beforehand mentioned methods.
- If the primary aim is to describe how risk and uncertainty affect the calculated expected NPV, then gross sensitivity analyses may be sufficient. However, if the aim is to provide more specific policy guidance on the selection of projects or

policies, then Monte Carlo simulations will likely be most useful. Some of the ad hoc methods may also be suitable ways to make sure that risk and uncertainty are incorporated into the decision making process.

- The different methods require different resources in terms of time, data and computing power and this can also affect the choice of method. More resources are required to undertake detailed Monte Carlo simulations than e.g. an ad hoc shortening of the CBA horizon. Ideally, one would strive for some proportionality between the resources laid down in analysing risk and uncertainty and the expected return to the results achieved.

5 THE VALUE OF INFORMATION: POSTPONEMENT AND REVERSIBILITY

The complexities of undertaking a cost benefit analysis imply that a number of simplifications must be employed. Almost all cost benefit analyses (implicitly) assume the following two:

- The decision of whether or not to undertake a proposed project is taken once and for all, i.e. the decision and project implementation cannot be postponed.
- If the project is undertaken, the project remains in place for the entire pre-planned project horizon, i.e. project reversibility is not considered.

The risk and uncertainty analyses discussed in chapter 4 were undertaken on CBAs where these assumptions were also implicit. The assumptions are convenient from a computational viewpoint. They imply in some sense a very “passive” attitude to risks and uncertainties; the decision-maker chooses whether or not to undertake a project and then sticks to it. They can be rationalised if everything of importance to the CBA calculation essentially replicates itself every period.²¹ In this case, no new information will appear at any time, which would alter the expected net present value resulting from the CBA.

In many cases, however, the assumptions that the project decision cannot be postponed and that the project cannot be reversed are unrealistic. This chapter discusses briefly how risks and uncertainties affect the calculation of the net present value when it is assumed that the project decision can be delayed or that the project can be reversed within the horizon of the CBA. Unfortunately the complexities also increase and the discussion will therefore focus on the main conceptual issues.²²

5.1 The value of waiting when effects are irreversible

In this section, it is assumed that the decision of whether or not to undertake a proposed project can be taken at any time, but if it is undertaken the project or its effects cannot be reversed.²³ The irreversible effects are typical for many environ-

²¹ This also means that risks should be drawn from the same distribution every period.

²² Dixit & Pindyck (1994) is a standard reference on the theory of investment under risk and uncertainty. Maler & Fisher (2005) consider the topic in the context of environmental assessment.

²³ The basic ideas were developed independently by Arrow & Fischer (1974). This section builds on Johansen (1991, ch. 10), Johansen (1993, ch. 8), NCEDR (2005, module 5) and Boardman *et al.* (2006, ch. 7).

mental projects, which cause irreversible harm to the environment. Examples include projects which lead to the extinction of a species or the destruction of irreplaceable nature. Deforestation could thus lead to depletion of tropical forest ecosystems that cannot be recreated. Other projects or their effects may in principle be reversible, but the costs are for all practical purposes so high that reversibility is ruled out. Pollution with DDT or heavy metals may be examples in this respect.

For convenience, only two periods are considered, namely $t = 0$ and $t = 1$. (An extension to a multiple period framework raises no new issues.) Consider a project where the NPV depends on information, i.e. the realisation of risk or uncertainty, which is revealed only at the end of period $t = 0$. The decision of whether or not to undertake the project can be taken either at the beginning of period $t = 0$ (i.e. prior to the shock occurring) or the beginning of period $t = 1$ (i.e. after the shock has occurred).

The problem of the decision-maker in period 0 is whether to undertake the project already in period 0 or to postpone the decision until period 1 when the shock is known.²⁴ Postponing the project decision brings about a gain in period 1 because the shock is then known and the decision-maker can make the best possible project decision. In other words, waiting makes available additional information and provides more options for the decision-maker.²⁵ The postponement of the project has an option value – or *quasi*-option value to evoke that such waiting options are seldom traded. Put differently, making the project in period 0 has an opportunity in terms of possible welfare forsaken in period 1. Clearly, the quasi-option value cannot be negative as the decision-maker could always choose in period 1 the project which would otherwise have been chosen in period 0.

Returning to the decision-maker's problem, postponement of the project decision has an option value, but also a possible disadvantage as there will be no contribution to NPV from period 0. By undertaking the project in period 0, there can be a contribution to NPV already in that period, but the decision-maker foregoes the chance to make the best possible decision in period 1. This line of reasoning suggests that the quasi-option value is important for the decision of whether to implement the proposed project or to postpone the decision.

²⁴ The decision-maker could also decide not to undertake the project under any circumstances, i.e. either in period 0 or 1, but that amounts to postponing it in period 0 and then not undertaking it in period 1.

²⁵ See Maler & Fisher (2004, sec. 585) for a discussion of option values vs. option prices.

Waiting options are usually not traded and this implies that the option value must be assessed based on the decision-maker's risk preferences and discounting as well as an assessment of the risks and uncertainties affecting the costs and benefits of the project. One clear-cut result is that more risks and uncertainties increase the option value of waiting and makes it more favourable to postpone the project decision.

5.2 The value of reversibility of the project or its results

This section discusses how full or partial reversibility of a project – or its effects – can affect the distribution of the NPV.²⁶ As discussed in section 5.1, some projects or the effects of the projects will be impossible or virtually impossible to reverse; the reversibility costs are infinite or at least so large that a reversal is unthinkable. Other projects have smaller reversibility costs, i.e. the project or its effects can be fully or partial reversed at a cost less than infinity. Examples may be the building of a road or regulation of the use of chemicals. The road can be removed and regulation can be revoked; while the costs may be substantial, they are not infinite.

There are cases where the project or its effects is not immediately reversible, but where the effects could be fully or partly remediated through mitigation measures. A new airport might bring about much more noise than expected. To close the airport would likely be impermissibly expensive, but the noise effects could still be mitigated at a cost, i.e. by changing flight patterns, installing noise-reducing windows or moving people away from the affected areas. The costs of such remediating measures or compensatory projects may be high. It may be appropriate to examine whether the compensatory projects are socially beneficial.

Reversibility of a project or its effects may truncate the lower tail of the NPV distribution as conventionally calculated. The project can be reserved in case the resolution of a risk or uncertainty leads to net benefits much lower than anticipated. The standard use of CBA leads to estimates of the NPV distribution with too low an expected NPV and too large variability.

A simple example can illustrate the importance of reversibility of a project or its effects. Consider a bridge construction project with a two period horizon. In period $t = 0$ there is a construction cost and in period $t = 1$ there are benefits in the form

²⁶ This section builds on Maler & Fisher 2005, sec. 3; Abel *et al.* 1996; Dixit & Pindyck 1998 and Drazen 2001, ch. 13.

of reduced transportation time and costs in the form of environmental degradation. If the net benefit in period $t = 1$ is subject to substantial risk and the project is irreversible, the resulting NPV distribution will be highly dispersed. If, however, the project is reversible, then a very bad outcome in period 1 may be averted if the project is reversed in that period. In other words, if the project turns out to be so environmentally damaging that the net benefits in period 1 would be highly negative, then it might be preferable to incur the reversibility and tear down the bridge before the excessive damage is done.

The possibility of reversing a project after the revelation of information on the net benefits of the project has a quasi-option value to the decision-maker. Put differently, when undertaking a project which is irreversible the decision-maker incurs an opportunity cost by foregoing the potential benefit of reversing project. The quasi-option value of being able to reverse the project (or its effects) will always be positive. On the other hand, projects with reversibility may be more expensive to implement. The value of the quasi-option value of reversibility will be larger, the more risky or uncertain future net benefits are and the more risk averse the decision-maker is. Project reversibility will generally make it more favourable for the decision-maker to implement a project.

5.3 In practice

This chapter has discussed the possibility of postponing the project decision and the possibility of reversing the project. In practice, many projects will comprise both possibilities. Both waiting and reversibility bring about advantages for the decision-maker and therefore carry (quasi-)option values.²⁷

The possibility of postponement of irreversible projects implies that some – apparently socially beneficial – projects should be passed by. A project with an expected net present value clearly above 0 and with little variance could still be unfavourable if the (option) value of waiting is sufficiently large. Conversely, a project, which a standard CBA suggests is unfavourable, may be favourable if it is reversible and the gain from reversibility is sufficiently large. Thus, flexible project proposals that can be postponed or reversed will generally be more advantageous to a decision-maker than less flexible projects.

²⁷ In technical terms the decision maker has both a call and a put option (Abel *et al.* 1996).

In practice, it is demanding to estimate the quasi-option value of postponing a project or retaining the possibility of reversing it. Estimates are likely to exhibit a large degree of arbitrariness. For some large projects, the estimation of the value of options to postpone or reverse projects will be useful, while in other cases it may suffice to discuss possible implications of these additional sources of flexibility. Boardman et al. (2006, pp. 192-193) recommends: "... when insufficient knowledge is available to formulate a decision problem for explicitly calculating the magnitude of quasi-option value [...], it should be discussed as a possible source of bias rather than added as an arbitrary adjustment to expected net benefits". The discussion can be important if a standard CBA yields a rather inconclusive result, e.g. if the expected net present value is close to 0 or if the result is very sensitive to changes in the variables.

6 FINAL COMMENTS

The cornerstone of socio-economic assessment is the cost benefit analysis. The expected net present value provides important guidance on whether or not to undertake a proposed project or policy. Shocks – in the form of measurable risks and immeasurable uncertainties – will affect the distribution of the net present value and sometimes also the decision of whether or not to undertake the project. To ensure that decisions are made on a solid basis, risk and uncertainty must be incorporated into the cost benefit analysis.

This toolbox paper has discussed a number of ways to take into account risk and uncertainty when undertaking a CBA. The methods included simple *ad hoc* methods, sensitivity analyses, stress testing, risk analyses based on Monte Carlo simulation and complex valuation of quasi-options. See Table 1 for a list of possible steps when incorporating risks and uncertainties in a CBA.

A thorough assessment of risk and uncertainty in CBA invokes difficult balancing acts: While it is important to identify and quantify risks and uncertainties, too much weight should not be put on events which are highly unlikely. It is also important to retain some form of proportionality between the resources spent on the risk and uncertainty analyses relative to the expected returns. This suggests that risk and uncertainty analyses should be more thorough when the proposed project has a large socio-economic impact, when the net present value depends strongly on risks and uncertainties and/or when the project is expected to be hit by large shocks.

An important additional payoff derived from undertaking risk and uncertainty analyses is that they encourage the analyst to identify and possibly quantify sources of risk and uncertainty during all stages of the CBA. It can lead to extra data sampling or new calculation methods to obtain a more precise NPV estimate. The analyses can lead to a deeper understanding of the problems of a proposed project or policy and contribute to a better and more reliable selection of projects.

The incorporation of risk and uncertain in a CBA can also bring about important feedback to the design and implementation phases of the project. The decision-maker may be presented with different alternative project proposals with different profiles of expected net present values and variances. In some cases it might be possible to device alternative technologies or new approaches that reduce the variability of the net present value while retaining the expected value. The decision-maker might also consider the option value of postponing a project or of seeking

ways to reduce the degree of reversibility of a project or its results. In some cases compensation measures may be able to reduce the costs of projects exposed to unforeseen adverse effects.

Table 1. Risk and uncertainty in cost benefit analysis – step by step

<p>1) Set up the equation generating NPV</p> <ul style="list-style-type: none"> • Are all relevant factors accounted for? • Are there sources of risk or uncertainty that is not captured by the variables entering the NPV? • Is there uncertainty with respect to the specification of the equation generating NPV?
<p>2) List all variables (and parameters) entering the calculation of NPV. Determine for each variable whether it is:</p> <ul style="list-style-type: none"> • Deterministic • Subject to risk • Subject to (Knightian) uncertainty <p>NB: Only categorise as uncertainty if clearly indicated.</p>
<p>3a) For variables subject to risk, determine the distribution, including possible end points. If appropriate, assign correlation coefficients between variables. The distributions and correlations can be based on:</p> <ul style="list-style-type: none"> • Historical experiences • Expert assessments
<p>3b) For variables subject to uncertainty, determine a “central point” (or expected value) and, if possible, the maximum and minimum values.</p> <p>NB: If the expected value and the end points are known, then the variable will often be subject to risk.</p>
<p>4) Sensitivity analyses</p> <ul style="list-style-type: none"> • Gross sensitivity analysis: change one-by-one all variables entering the NPV calculation, e.g. by 10%. Identify variables of particular importance for the NPV • Stress testing: set one-by-one all variables at their minimum and maximum values • Present results in tables and figures. A spider plot combines gross sensitivity testing and stress testing
<p>5) Monte Carlo risk analysis</p> <ul style="list-style-type: none"> • Use the distributions and correlations of the variables subject to risk • Set all variables subject to uncertainty at their central value • Simulate to find the expected value, the standard error of the NPV and the probability that the expected NPV is below 0. • Possibly redo Monte Carlo simulation with other value for the uncertain variables
<p>6) Ad hoc adjustments in expected net present value</p> <ul style="list-style-type: none"> • Adjust costs, benefits, the discount rate or the horizon of the CBA in order to adjust the expected NPV • Explain the purpose of and the reason for an ad hoc adjustment • Ad hoc adjustments should be explained, e.g. by precautionary principle or quasi-option values
<p>7) Feedback on results</p> <ul style="list-style-type: none"> • Is the assessment of risk and uncertainty adequate and reasonable? • Are there ways to reduce the variability of NPV?

Finally, it should be underscored that also *ex ante* risk and uncertainty analyses as discussed in this toolbox paper are subject to risk and uncertainty. Such analyses should never “stand alone”, but be used as part of the basis for decisions. It is important to undertake *ex post* evaluations of projects and their preceding CBA. Ex post analyses make it possible to assess whether the CBA and its sensitivity and risk analyses were reasonably accurate, but also to learn more about different sources of risk and uncertainty.

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