Construction Project Scheduling with Time, Cost, and Material Restrictions Using Fuzzy Mathematical Models and Critical Path Method

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Abstract: This article evaluates the viability of using fuzzy mathematical models for determining construction schedules and for evaluating the contingencies created by schedule compression and delays due to unforeseen material shortages. Networks were analyzed using three methods: manual critical path method scheduling calculations, Primavera Project Management software (P5), and mathematical models using the Optimization Programming Language software. Fuzzy mathematical models that allow the multiobjective optimization of project schedules considering constraints such as time, cost, and unexpected materials shortages were used to verify commonly used methodologies for finding the minimum completion time for projects. The research also used a heuristic procedure for material allocation and sensitivity analysis to test five cases of material shortage, which increase the cost of construction and delay the completion time of projects. From the results obtained during the research investigation, it was determined that it is not just whether there is a shortage of a material but rather the way materials are allocated to different activities that affect project durations. It is important to give higher priority to activities that have minimum float values, instead of merely allocating materials to activities that are immediately ready to start.

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Introduction

Construction management decisions are made based on schedules that are developed during the early planning stage of projects, yet many possible scenarios should be considered during construction. Decisions could be made that rely solely on the expertise of DMs that use commercial software such as Primavera Project Planner (P3), Primavera Project Management (P5), or Microsoft Project but sometimes the assumptions that are made during the planning stage of a project change during construction. These decisions, however, need to be supported by a risk management plan. In many cases, even though allowances are considered during the planning stage to minimize the risks, they may not be sufficient to cover all possibilities and the planner will still have to react when changes occur. For instance, suppose that at a pavement facility, the raw materials that are coming from a particular quarry are unexpectedly insufficient, or that abnormal weather makes it too difficult to perform tasks outdoors. These, and many other unpredictable events, constantly affect project schedules. In many cases, DMs are required to make decisions quickly during construction.

Even though there are important resources whose restricted availability could affect project schedules such as equipment, labor, or subcontractors, this article mainly discusses how fuzzy mathematical models may be used to generate construction project schedules and how to incorporate restrictions that are defined by DMs on items such as materials, time, and cost. Time-cost trade-offs may also be incorporated into schedules using fuzzy mathematical models, which facilitate time-cost trade-off analysis. In this study, linear relations are assumed in fuzzy mathematical models.

Benefits of this research to practitioners are related to showing the viability of using fuzzy mathematical models for determining construction schedules and for evaluating the contingencies created by schedule compression and delays due to unforeseen material shortages. The proposed approach can help planners and schedulers allocate available resources to competing tasks in such a fashion that two conflicting objectives can be satisfied using fuzzy math modeling. Networks were analyzed using three methods: manual critical path method (CPM) scheduling calculations, Primavera Project Management software (P5), and mathematical models using the Optimization Programming Language (OPL) software. The practitioners usually have little time to react to changes and the proposed approach can provide fast solutions for small to moderate size problems (up to a couple of thousand activities, depending on the computer used). Another benefit is that the models can easily be modified based on schedulers' preferences and can be used to solve the problem under different circumstances without much difficulty. Researchers may benefit from this methodology by verifying commonly used methodologies (e.g., manual CPM scheduling calculations, Primavera

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Project Management software, mathematical models using the OPL software) for finding the minimum completion time for projects. The proposed approach can also be extended to include evolutionary computational techniques to deal with much larger construction project networks.

Literature Review

CPM scheduling is a technique that has been used since the 1950s, and the construction industry benefits from its use in some areas such as the planning and controlling of projects, communicating plans, and training new managers. Project managers use commercial project management software based on critical path analysis, such as Primavera Project Planner (P3) or Microsoft Project, which are based on heuristic methods to plan and control schedules (Liberatore et al. 2001; Kelleher 2004; Karaca and Onargan 2007). In one research investigation, labor productivity improved by 6% when resources were considered in CPM schedules and an additional 4–6% improvement was obtained when using computerized systems (Perdomo-Rivera 2004). Other efforts have attempted to integrate Computer-Aided Drafting, Primavera Project Management software (P3), and Geographical Information Systems to generate three-dimensional (3D) drawings and show synchronized P3 schedules that permit a faster and better conceptualization of projects and these may be useful in scheduling, planning, controlling, and decision-making processes; furthermore, innovative technology such as four-dimensional (4D) models (i.e., 3D technology involving scheduling, planning, procurement, and other areas) link components in 3D models with activities from design, procurement, and construction schedules (Riley 2000; Poku and Arditi 2006).

Schedules may not simulate reality if they do not incorporate material constraints. Besides material shortages and hence delays in project completion times, some other variables also affect construction projects such as activity durations, early start time, late start time, early completion time, late completion time, normal costs, and crash costs. The weather, traffic, and the limited availability of other resources such as skilled workers, machines, equipment, etc., also cause some of these problems. Therefore, float calculated using CPM techniques will lose its significance and new critical sequences will develop (Wiest 1964; Kim and de la Garza 2005). Schedules that neglect material constraints might mislead planners and affect the control of projects. Commercial project management software packages based on CPM schedules integrated with compatible software provide DMs with valuable alternatives to prevent, or minimize, the effects of probable delays, such as the software used for the Delay Analysis System (Yates 1993). A different approach has been followed by other writers, who stress that considering project constraints is not adequate because constraints need to be analyzed and prioritized depending on their repercussions on the entire project (Chua and Shen 2005).

Optimization models have been used in construction projects, but they have not been successful when used on large networks. CPM techniques with discrete information instead of continuous membership functions have proven to be more efficient and they provide not optimal, but usable solutions (Moder et al. 1983). However, some optimization techniques present the opportunity for analyzing more than one objective at a time and this permits a more realistic approach. Uncertainties have been analyzed by using fuzzy goal programming and optimal solutions have been achieved while simultaneously considering two objectives and using membership functions (Deporter and Ellis 1990; Arikan and Gungor 2001; Suer et al. 2008a,b). Furthermore, uncertainties have been considered in diverse project settings using the fuzzy set theory, which provides possible completion times for each activity in a network (Ayyub and Haldar 1984; Lorterapong and Moselhi 1996). To avoid dealing with potential uncertainties, a new tool was formulated by Ordoñez-Oliveros and Fayek that provides DMs with the opportunity to create an updated schedule and to evaluate the consequences of delays in order to make decisions when they are required during a project (Ordoñez-Oliveros and Fayek 2005). DMs (e.g., project managers, construction managers) are responsible for providing realistic schedules because they are the ones who should understand what activities are critical and know how much pressure to apply to increase worker efficiency (Nepal et al. 2006).

Applicability of Fuzzy Mathematical Models in Construction Projects

When the plans for a construction project are going to be used to build a different project in another location the planning, controlling, and execution are performed in a different manner due to the uniqueness of each project. Therefore, having similar characteristics such as the plans and project duration, and using the same equipment, standard conditions, and resources for two projects does not imply the application of the same procedures nor guarantee similar results. Conditions could change and there could be delays and unexpected situations that arise during construction. The volatility in the construction materials market makes scheduled material deliveries uncertain. For example, as the demand for steel or cement increases, deliveries could be delayed and projects may have to be cancelled. DMs may use their experience to help achieve goals or objectives as effectively as possible by replanning and rescheduling projects when it becomes necessary to do so. However, even though project managers may have experience in planning and executing construction projects, imprecision and uncertainty in their decision-making practices still exists in the scheduling of processes. The efficiency of a construction project depends on many variables (e.g., early start time, early completion time, late start time, late completion time, normal cost, crash cost), conditions (e.g., priorities, milestones, budget, expected duration, and material requirement) and uncertainties (e.g., delays, schedule growth, cost growth, and material constraints) that have to be accounted for by providing forecasts to realistic construction networks that generate a favorable schedule, minimize the project completion time and costs and that also consider material constraints. Even when construction companies use commercial resource allocation software they might have material shortages, and it is in these situations when having a mathematical model with material constraints is useful. Personnel in the construction industry frequently have to address material management issues, such as materials not being available where and when they are needed, and a lack of information about where materials are located at job sites. These problems may increase expenses and the required time to complete a project. Using fuzzy mathematical models allows project managers to try to achieve the two main project objectives of minimizing costs and time under material restrictions, and it helps them consider ambiguities in decisions by using membership functions. Membership functions are used for ranking objectives using similar terms. For instance, there could be a grade of association to the membership function or a

gradual transition from 0 (the nonsatisfaction level) to 1 (the maximum satisfaction level), which allows new constraints to be added to fuzzy mathematical models.

In a typical situation, DMs initially want to minimize project completion times. By using project networks that include time and cost data, the objective of achieving a minimum completion time could be reached. A DM's aspiration level cannot be expressed deterministically; therefore, the completion time may not be satisfactory. To compensate for the reduction in time, a DM would tolerate a fuzzy amount of cost, turning the situation into a multiobjective network problem. Fuzzy aspiration levels are assigned to both objectives and the conflicting objectives are optimized simultaneously within this framework. The level of aspiration of a DM is the possible goal which the DM sets for time and cost. Finally, for given scenarios of material shortages along the network, the fuzzy mathematical model will recalculate the project duration and costs and provide DMs with new values of satisfaction, in accordance with the fuzzy aspiration levels previously defined in the model.

Research Methodology

In order to achieve the objectives of the research the following tasks were performed:

1. A construction project was selected to demonstrate the applicability of using fuzzy mathematical programming models to verify network schedule calculations. Sample data from a 60 activity construction networks for a two-story building were obtained that included normal and crash costs and durations (Mubarak 2005).
2. The computer software programs to be used for the research were selected and they included Primavera Project Management (P5), and the commercial mathematical modeling software known as the OPL.
3. The CPM schedule was drawn and manual calculations were performed in order to determine the critical path of the project, the project duration, and activity float times.
4. A mathematical model (Model 1) for generating optimal completion times was written based on the Activity-on-arrow logic diagramming method networks. Running the optimization software OPL model generated the same solution for the project duration as was obtained by using CPM scheduling techniques.
5. Membership functions for time as an objective were created based on the information from the completion time determined by the model. The normal completion time for the case study project was 142 days, but the objective was to try and reduce extra costs involved in the project related to indirect costs; therefore, the optimal amount of time that the project could be crashed was determined using the model.
6. A cost analysis based on network-crashing calculations was also performed using the mathematical model (Model 2). Since Primavera Project Manager does not perform network-crashing calculations, a mathematical model was used to perform this function. An analysis was performed in order to determine the minimum cost for a construction network by using the OPL mathematical model.
7. The membership function for cost as an objective was generated by using the information from the cost analysis and the DM criteria.
8. A fuzzy mathematical model was created that combined all of the results from the previous phases, including minimum completion time, minimum cost, and the membership functions. In addition, the previous objectives of time and cost were considered as constraints. The model optimized each objective individually by using membership functions and maximized the satisfaction level between them.
9. Time-cost trade-off calculations were performed using different variations of material allocations and constraints to determine the optimal duration of a sample project.
10. Material restrictions were added to the fuzzy mathematical model and several cases were tested to validate how the project network was affected each time.

Results

This section discusses the results that were obtained for the research investigation. Three mathematical models were used for analyzing construction projects in the research:

1. Model 1: linear programming model to minimize project completion times.
2. Model 2: linear programming model to minimize crashing costs.
3. Model 3: linear programming model to solve fuzzy biobjective formulation based on minimizing project completion times and crashing costs.

Model 1: Minimizing Project Completion Time

$$\text{Objective function: } \ \text{Min}z = X_L - X_1 \tag{1}$$

$$\text{Constraints: } \ X_j \geqslant X_i + d_{ij} \ \text{ for all } (i,j) \tag{2}$$

$$X_j \geqslant \text{ESM}_j \ \text{ for all } j \tag{3}$$

$$X_j \geqslant 0 \ \text{ for all } j \tag{4}$$

where X_j=time that the event corresponding to node j occurs; X_i=time that the event corresponding to node i occurs; X_1=first node in the network; X_L=last node in the network; d_{ij} duration of activity (i,j); and ESM_j=earliest time that the event corresponding to node j can occur based on material availability.

The project completion time (the time between the last node and the first node) is minimized in Eq. (1). An activity is denoted by a pair of nodes (i,j) and node i must occur before node j occurs. Eq. (2) guarantees that the completion time at node j is equal to or greater than the completion time at node i plus the duration of activity (i,j). Eq. (3) establishes the earliest time node j can occur considering material availability. Eq. (4) enforces nonnegativity restrictions for the decision variables.

Model 2: Minimizing Crashing Costs

$$\text{Objective function: } \ \text{Min}z = \sum_{\text{For all } (ij)} C_{ij}X_{ij} \tag{5}$$

$$\text{Constraints: } \ X_j \geqslant X_i + d_{ij} - X_{ij} \ \text{ for all } (i,j) \tag{6}$$

$$X_j \geqslant \text{ESM}_j \ \text{ for all } j \tag{7}$$

$$X_{ij} \leqslant \text{UL}_{ij} \ \text{ for all } (i,j) \tag{8}$$

$$X_L - X_1 \leqslant \text{DD} \tag{9}$$

$$X_j \geqslant 0 \quad \text{for all } j \tag{10}$$

where X_{ij}=crashing time for activity (i,j); UL_{ij}=maximum crashing time for activity (i,j); DD=maximum allowable completion time for the project; and C_{ij}=unit crashing cost for activity (i,j).

The crashing costs for all activities are minimized in Eq. (5). Eq. (6) guarantees that the completion time at node j is equal to or greater than the completion time at node i plus the duration of activity (i,j) minus the amount of time by which activity (i,j) is crashed. Eq. (7) establishes the earliest time node j can occur considering material availability. The maximum crash time for each activity is limited by its upper bound as given in Eq. (8). The maximum allowable completion time of the project is dictated by Eq. (9) and nonnegativity restrictions are established by Eq. (10).

Model 3: Fuzzy Biobjective Model

$$\text{Objective function: } \ \mathrm{Max z} = \mathrm{fz} \tag{11}$$

$$\text{Constraints: } \ \mathrm{fzt} = \left[\frac{(\mathrm{UB}_t - z_t)}{\mathrm{UB}_t - \mathrm{LB}_t}\right] \tag{12}$$

$$\mathrm{fzc} = \left[\frac{(\mathrm{UB}_c - z_c)}{\mathrm{UB}_c - \mathrm{LB}_c}\right] \tag{13}$$

$$\mathrm{fz} \leqslant \mathrm{fzt} \tag{14}$$

$$\mathrm{fz} \leqslant \mathrm{fzc} \tag{15}$$

$$z_t = X_L - X_1 \tag{16}$$

$$z_c = \sum_{\text{For all } (ij)} C_{ij} X_{ij} \tag{17}$$

$$X_j \geqslant X_i + d_{ij} - X_{ij} \quad \text{for all } (i,j) \tag{18}$$

$$X_j \geqslant \mathrm{ESM}_j \quad \text{for all } j \tag{19}$$

$$X_{ij} \leqslant \mathrm{UL}_{ij} \quad \text{for all } (i,j) \tag{20}$$

$$X_j \geqslant 0 \quad \text{for all } j \tag{21}$$

where UB_t=upper bound on the project completion time; LB_t=lower bound on the project completion time; z_T=project completion time; X_1=first node in the network; UB_c=upper bound on the crashing costs; LB_c=lower bound on the crashing costs; z_C=total crashing costs; fzt=satisfaction with respect to completion time; fzc=satisfaction with respect to cost; and fz=overall satisfaction level [determined to be the minimum of fzt and fzc, i.e., fz=min(fzt,fzc)].

The objective is to maximize the minimum satisfaction level of project completion time and crashing costs as given in Eq. (11) (Bellman and Zadeh 1970). Eq. (12) is used to determine the satisfaction level due to project completion time and Eq. (13) does the same for crashing costs. The overall satisfaction level of the solution fz cannot exceed the satisfaction levels of the completion time and the crashing costs as determined as given in Eqs. (14) and (15). The project completion time is computed using Eq. (16) and the crashing cost is determined using Eq. (17). The relations in Eqs. (18)–(20) are the same as the relations in Eqs. (6)–(8) that were discussed in the previous model. Eq. (21) is used for nonnegativity restrictions.

Table 1. Twenty-Activity Network

Activities	Nodes	Precedence	Material demand	Duration (days) Normal	Duration (days) Maximum crash
Mobiliz.	10–20		0	1	0
A	30–20	Mob.	3	6	2
B	40–20	Mob.	4	3	1
C	50–30	A	3	5	3
D	60–30	A	4	4	2
E	70–40	B	2	8	3
F	80–50	C	5	7	0
G	90–60	D	4	2	0
H	100–60	D	4	9	3
I	110–70	E	3	4	2
J	120–80	F	4	4	2
K	140–90	G	5	3	1
L	130–100	H	3	5	0
M	110–100	H	2	3	1
N	140–120	J	6	2	0
O	150–130	L	5	8	3
P	150–110	M-I	3	7	2
Q	160–140	K-N	6	5	3
R	160–150	O-P	2	9	3
Punch list	170–160	Q-R	0	1	0

Material allocations and restrictions were analyzed by creating a 20-activity sample model. Precedence relationships, activity durations, maximum crashing time, and the required amounts of materials for each activity are shown in Table 1. The sample model was analyzed using five different material allocation scenarios.

The objective of the analysis is to study the influence of material allocation decisions for critical and noncritical activities. To achieve this, mathematical models for the minimum completion time and minimum crashing cost were developed and solved using the OPL software package. Considering the early start allocation results generated in the base case, the amount of material required by Day 20 was 29 units of material x_1. In the five cases analyzed, only 20 units of material x_1 were available and the rest would be delivered after Day 20. The criteria for making the changes in each case lies on establishing a base case with simple conditions and then gradually focusing on variations to such conditions implementing cost-time trade-offs, analysis of float, critical path considerations, and ultimately material constraints. The explicit determination of these conditions is originated from research curiosity as to being able to investigate the impact of such variations to the base case.

Base Case

The CPM calculations for the original network were performed manually and the results are shown in Fig. 1. A mathematical model, with a minimum completion time as objective, was generated and its completion time was 43 days. In this scenario, materials were unlimited; therefore, the only critical constraint was time.

Case One

In Case One the allocation was performed using CPM calculations, the optimal solution generated by the mathematical model, and an analysis of the float. This model primarily considered the

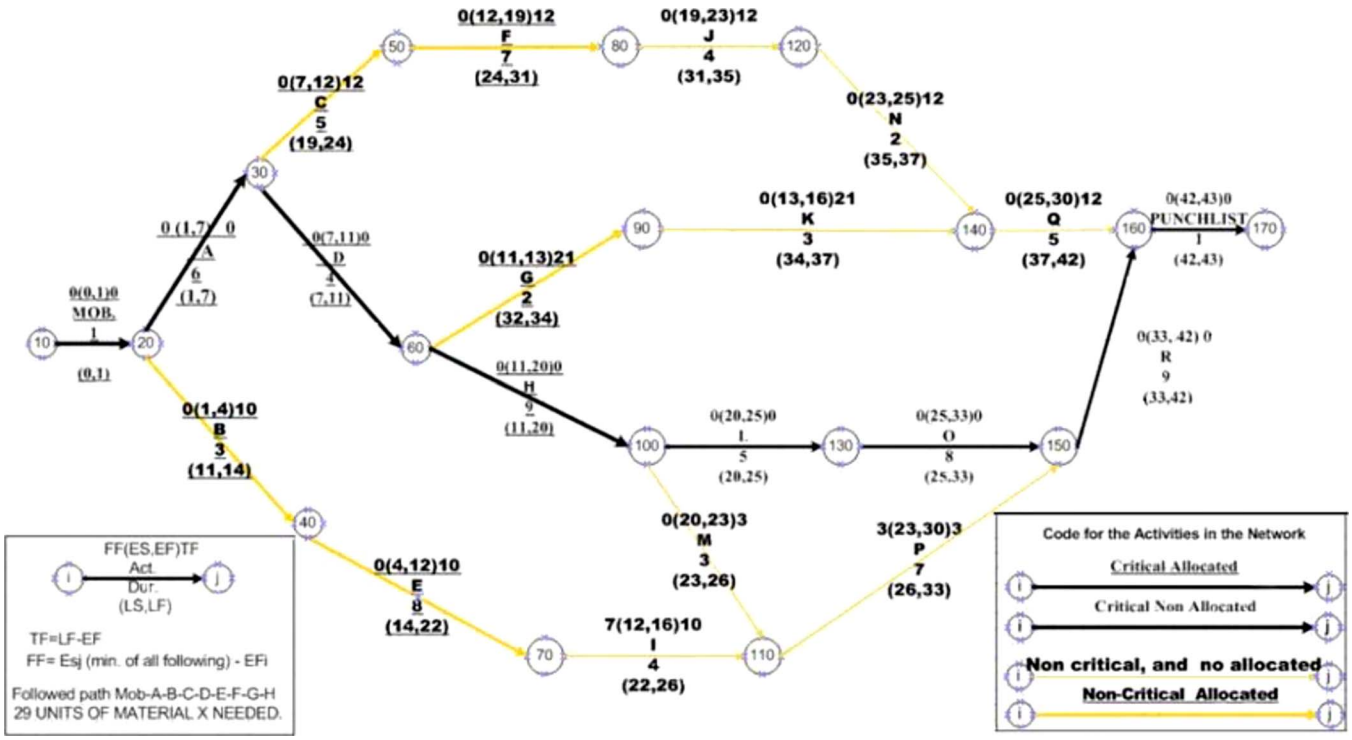

Fig. 1. Manual CPM calculations for the base case

critical path or activities on other paths that had the least amount of float. The material allocation for this case required 20 units by Day 20, which was the amount available. When the mathematical model was run for this case, the completion time of 43 days was not affected. When considering the critical path, and the activities with the least amount of float, the completion time did not increase. Therefore, the cost for completing the project on time was the original cost. In addition, the total number of days the project could be crashed was 6 with a cost of $15,600. Therefore, Case One is a viable alternative.

Case Two

For Case Two the allocation focused on the critical path and the paths that had the least amount of float. The completion time increased by 1 day, becoming 44 days, and the cost to finish on time was $1,400. The amount of units of material allocated was 20. The critical path was considered in this case, and Activity C was delayed. Activity C, starting after Day 20, created a new critical path and a new completion time of 44 days. The cost increased by 4.6% up to $1,400.

Case Three

For Case three the allocation was focused on the critical path and the next path chosen was the one that had higher activity float time. Activities with a float time equal or close to zero become critical since a delay on those activities would represent a delay in the project completion time. In most circumstances, critical activities form the critical path. When the project is crashed, noncritical activities become critical, therefore altering the critical path. When the critical path was considered, and the activities with higher activity float were the next activities to be considered, the completion time increased by 23.3% from 43 to 53 days, and the cost to finish on time without crashing increased by 99% up to $29,800, assuming that $30,000 was the maximum amount approved to be spent. The total number of days the project could be crashed after the normal completion time was 1 day. The cost for completing the project on Day 42 was $34,800. This happened because attention was not focused on the most critical activities.

Case Four

For Case Four the allocation strategy was to finish activities without considering critical floats. The amount of material x_1 allocated in this case was 19 units. The completion time in this case was 52 days and the cost to finish on time was $21,800. The total number of days the project could be crashed was 9 days and the cost to finish the project on Day 43 was $19,200.

Case Five

In Case Five, the allocation strategy was to consider the critical path and noncritical activities crossing it before Day 20. The total number of days the project could be crashed after the normal completion time of 43 days was 1 with a cost of $34,800. The amount of material x_1 allocated in this case was 18 units.

The information for the original network and the five cases is shown in Table 2. For each case a cost-time trade-off mathematical model was developed and the results are presented in Table 3. The results generated by the models were tabulated up to the maximum number of days that the project could be crashed. Therefore, the blank spaces on the table mean that it was not possible to crash the project for that time.

For the base case where there were no material constraints, the project could be crashed 13 days with a maximum cost of $33,800. The importance of considering the critical path is seen in Cases One and Two, where the maximum crashing time was reduced by approximately 50%. In Case Two, Activity I was finished 1 day early, but Activity P could not be started until Activity M was completed; therefore, it is best to start some other activities instead of Activity I in order to minimize the completion time. If the total float of an activity is not taken into account it will result in having more critical activities and in extending the

Table 2. Completion Times, Constraints, and the Allocation Path

Case	Generated restrictions/allocation path	Completion time (days)	Crashing cost ($)	x_1 allocated (units)
Original	Original/**Mob-A-B-C-D-E-F-H**	43	0	29
One	I>=20, G>=20, L>=20, M>=20, F>20/**Mob-A-B-C-D-E-H**	43	0	20
Two	C>=20, G>=20, L>=20, M>=20, P>=20/**Mob-A-B-D-E-H-I**	44	1,400	20
Three	B>=20, G>=20, J>=20, L>=20, M>=20/**Mob-A-C-D-F-H**	53	29,800	19
Four	B>=20, G>=20, H>=20, E>=20, J>=20/**Mob-A-B-C-D-F**	52	21,800	19
Five	B>=20, K>=20, F>=20, L>=20, M>=20/**Mob-A-C-D-G-H**	53	29,800	18

completion time. For Cases Three and Five, the free float of Activity B was 12 days and this activity was not allocated, which meant starting Activity B after Day 20 and completing the project 10 days later than the normal duration.

The previous analysis demonstrates that it is important for the success of projects that DMs be knowledgeable about allocation strategies and the consequences of having material constraints. The models developed in this research could support similar efforts on managing material allocation. Effectively manipulating the aspects of the project, such as critical activities, material constraints, time-cost trade-offs, and so forth is more beneficial than having the shortest schedule. Unless the scheduling tools are used according to the aspects listed earlier, the results could lead to additional problems.

Sensitivity Analysis

In order to check how sensitive the model was in terms of cost to variations in the amount of days that the critical path could be crashed, a sensitivity analysis was performed on the model. The sensitivity analysis consisted of analyzing how different crash time durations would affect the completion time and cost in six time scenarios (TSs), which are shown in Table 4.

In Time scenario I (TS-I) normal crash times were used; in TS-II the crash time of activities on the critical path was assumed to be zero; in TS-III the crash time was assumed to be high, which meant greater than or equal to 3 days and less than or equal to 5; in TS-IV the crash time was assumed to be low, which meant less than 3 days and greater than zero days; in TS-V the crash time was assumed to be high in the first stage of the network and low in the final stage; finally, in TS-VI the crash time was assumed to be low in the first part of the network and high at the end.

In the first sensitivity analysis, the critical path was evaluated for the base case by using the six crash TSs described in Table 4. The results of the evaluation are presented in Fig. 2.

Fig. 2 shows that in the base case TS-II cannot be crashed at all because the maximum crashing time in the original critical path is zero. Every TS had the same constant increasing cost for the first five crashed days. TS-IV is the most critical, allowing the smallest network-crashing time with the highest cost.

In the second sensitivity analysis, the critical path was evaluated for Case One by using the six crash TSs. The first time-cost curve for the base case was kept as a reference point and results are shown in Fig. 3. The results are shown up to their maximum possible crashing time for each TS. The results for TS-I from the

Table 3. Time-Cost Trade-Off Calculations

	Crashing cost ($)					
Time (days)	Original	Case One	Case Two	Case Three	Case Four	Case Five
43	0	0	1,400	29,800	21,800	29,800
42	1,000	1,000	3,800	34,800		34,800
41	2,000	2,800	7,820			
40	3,000	4,600	11,840			
39	4,000	7,600	17,860			
38	5,000	12,000	24,660			
37	6,000	16,400	31,460			
36	7,000					
35	8,000					
34	11,000					
33	14,000					
32	17,000					
31	25,000					
30	33,800					

Table 4. Time-Scenarios for the Critical Path Analysis

Crash TSs (in days)	
I	Normal crash time
II	Cannot be crashed
III	High crashing time (3⩽crash time⩽5)
IV	Low crashing time (0⩽crash time⩽2)
V	High first part and low second part
VI	Low first part and high second part

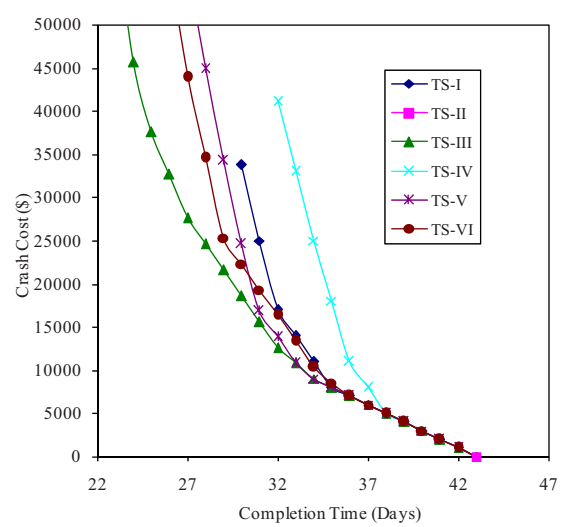

Fig. 2. Sensitivity curves for the base case

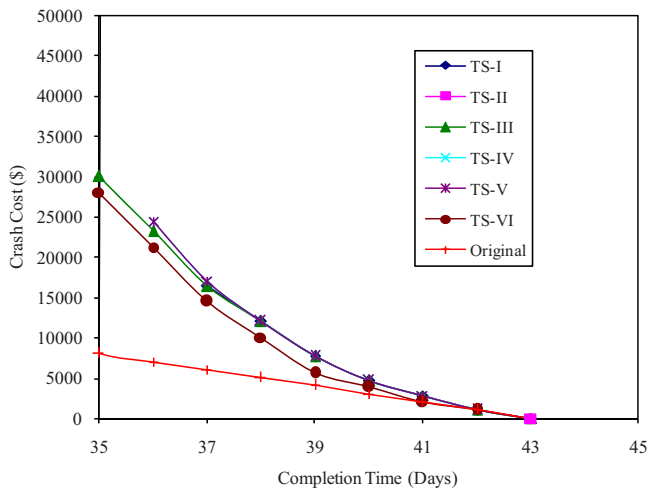

Fig. 3. Sensitivity curves for Case One

sensitivity analysis for Case One were the same as the values found in the cost-time trade-off shown in Table 4.

TS-III, TS-IV, and TS-V had the same constant increasing cost for the first six crashed days. When they are put side by side, the curve from the base case and the curves from this case show how the cost increases at a higher rate for all TSs.

In the third sensitivity analysis, the critical path was evaluated for Case Two by using the six crash TSs. In this case even though for TS-II the crash time for the original critical path was zero there is a cost for finishing on Day 43, as is shown in Fig. 4. This cost exists because the original critical path changed and the longest path has a duration of 44 days. The results generated for Case Two while evaluating TS-I, were the same as the results obtained for the cost-time trade-off for Case Two.

As can be seen in Fig. 4, the cost of crashing Case Two was almost identical in all of the different TSs and even TS-II had a 1-day crashing cost because it developed a new critical path. Therefore, constraining the original path would not affect the completion time.

In the fourth sensitivity analysis, the critical path was evaluated for Case Three by using the same scenarios as for the previous cases. The cost for the base case was presented up to the maximum crashing time period in Case Three, which was 4 days for TS-VI. It was not possible to crash the project up to that date or number of days, such as in TS-II where the project could not be completed before 43 days. The results of the evaluation are shown in Fig. 5.

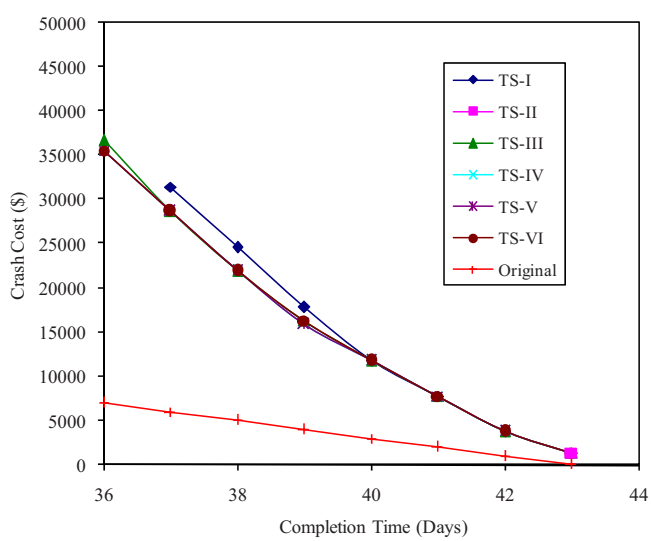

Fig. 4. Sensitivity curves for Case Two

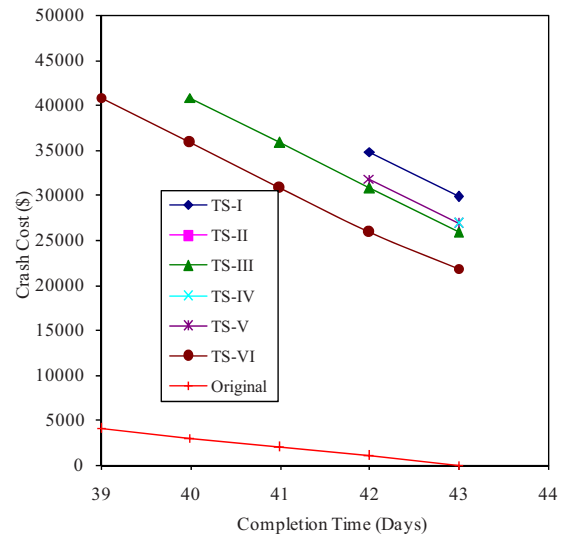

Fig. 5. Sensitivity curves for Case Three

The results indicate that TS-I and TS-III are the most critical for two reasons: the maximum crashing time period is reduced up to 2 days, and the crashing cost is the highest for these TSs. In general this sensitivity analysis presented a high crashing cost for every scenario, compared with the planned crashing cost information. TS-II was not possible to crash and TS-VI was the most favorable among the six scenarios.

Results from the evaluation of the critical path for Case Four by using the six crash TSs are shown in Fig. 6. The original time-cost curve for the base case was kept as a benchmark.

In this analysis TS-V was the most critical, because the maximum crashing time was reduced 1–42 days, and the cost was the highest among the TSs. TS-II and TS-IV could not be crashed at all. TS-III and TS-VI had similar behavior when crashed up to 3 days, but after that TS-VI became more expensive.

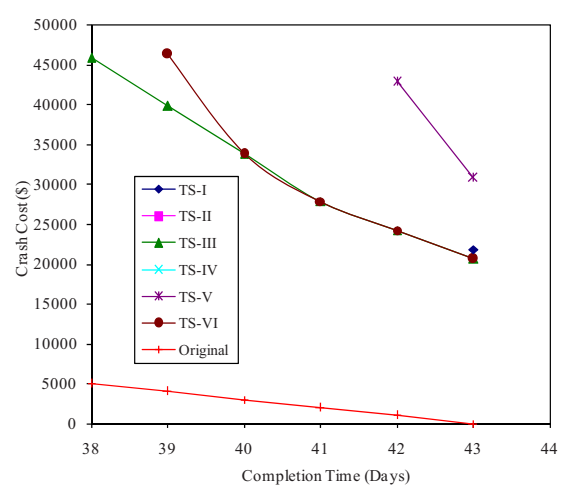

Fig. 6. Sensitivity curves for Case Four

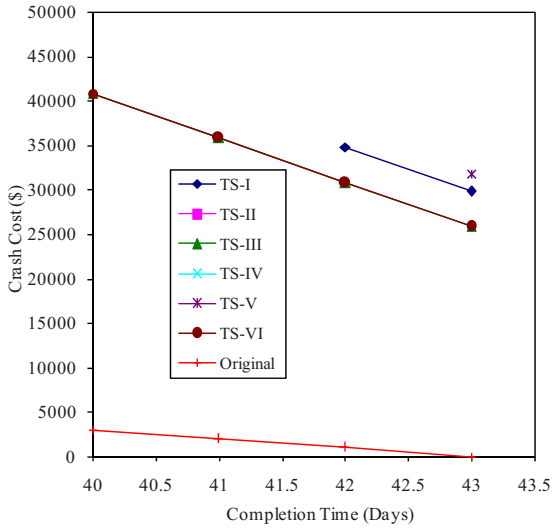

Fig. 7. Sensitivity curves for Case Five

Finally, the critical path was evaluated for Case Five by using the six crash TSs described previously. The maximum number of days that the project could be crashed was 3 days up to Day 40 in this case, as illustrated in Fig. 7. Therefore, the base case cost information was presented up to Day 40 to be compared with the results from the TSs in this case.

These results have a more constrained crashing time period, as shown in Fig. 7. The maximum number of days that the project could be crashed was 3 days for TS-III and TS-VI. For the cases of TS-IV and TS-V the project could not be crashed; in order to finish in the normal completion time, which is 43 days, the cost was high ($31,800). This is because TS-IV and TS-V have a low crashing time at the end of the critical path, in contrast with TS-III and TS-VI that have high crashing times.

In general, even though the TSs for each case presented had small crashing costs, it has to be considered that in some cases the project was already constrained by the initial allocations. For instance, Sensitivity analyses Three and Five reduced the amount of maximum crashing days to 1 day. However, with TS-III and TS-VI that amount increased from 1 to 4 days. The allocation of materials is critical as presented in the six cases analyzed, but when the maximum amount of crashing days was high, the effect was reduced. TS-III has had consistently favorable results, because it allowed high crashing times. TS-VI exhibited a similar behavior because at the end of the scenario, the maximum amount of crashing days was high and most of the first one-half of the project was already allocated.

Fuzzy Mathematical Model Analysis

Using the information from the completion time and cost models membership functions were generated. The normal completion time was 43 days and the objective was to finish 6 days early by day 37. Therefore, the time membership function was represented as

$$f(x)_T \begin{cases} 1 & \text{if} \rightarrow z_T \leqslant 37 \\ (43 - z_T)/6 & \text{if} \rightarrow 37 < z_T \leqslant 43 \\ 0 & \text{if} \rightarrow z_T > 43 \end{cases} \tag{22}$$

The independent variable z_T represents the time to finish the project. The original cost for crashing the project 6 days was

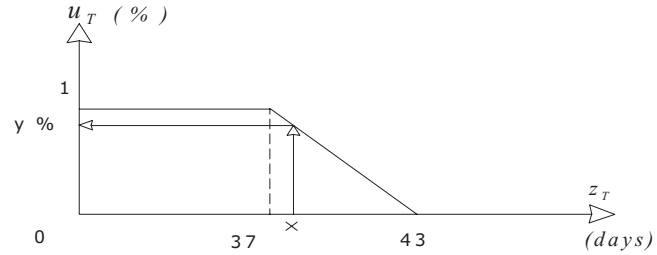

Fig. 8. Membership function for completion time

$6,000. Therefore, the cost membership function is shown below, where the dependent variable z_C represents the cost for the respective completion time. The graphical representation, where the y-axis represents the satisfaction level and the x-axis represents the completion time is given in Fig. 8.

$$f(x)_C \begin{cases} 1 & \text{if} \rightarrow z_C = 0 \\ (6{,}000 - z_C)/6{,}000 & \text{if} \rightarrow 0 < z_C \leqslant 6{,}000 \\ 0 & \text{if} \rightarrow z_C > 6{,}000 \end{cases} \tag{23}$$

Table 5 presents the results generated by single objective models for completion time and cost. Although a 100% satisfaction level is achieved in this model, just one of the objectives is considered. For instance, the completion time is 37 days, which means 100% satisfaction, but the cost of crashing the project is $6,000, which was defined in the membership function with zero satisfaction. In these mathematical models fz represents the final satisfaction level, fz_1 the satisfaction level for the minimum completion time, and fz_2 the satisfaction level for the minimum crashing cost. The result from the fuzzy mathematical model generated the same completion date (z_1=40 days) for all of the TSs with 0.5 satisfaction level (fz=50%), and minimum cost (z_2 =$3,000) with 0.5 satisfaction level (fz_1=50%).

For Case One, the project crashing cost was $16,400 and the membership function was

$$f(x)_C \begin{cases} 1 & \text{if} \rightarrow z_C = 0 \\ (16{,}400 - z_C)/16{,}400 & \text{if} \rightarrow 0 < z_C \leqslant 16{,}400 \\ 0 & \text{if} \rightarrow z_C > 16{,}400 \end{cases} \tag{24}$$

Table 6 shows the results generated by the single objective model that was used to define the membership function. The optimal solution for Case One reached a satisfaction level of 49.7 and 50%. All of solutions considered material constraints.

By using fuzzy mathematical models it was shown that although the models may have the same satisfaction level for the cases analyzed, the costs are different. In order to find the same satisfaction level, they have to be crashed 3 days (base case) and 4 days (Case One).

Table 5. Fuzzy Mathematical Model Results for the Base Case

	TSs					
Parameter	I	II	III	IV	V	VI
fz	50	0	50	50	50	50
z_1 (days)	40	43	40	40	40	40
fz_1 (%)	50	0	50	50	50	50
z_2 ($)	3,000	0	3,000	3,000	3,000	3,000
fz_2 (%)	50	100	50	50	50	50

Table 6. Fuzzy Mathematical Model Results for Case One

Parameter	TSs					
	I	II	III	IV	V	VI
fz (%)	53.7	0	53.7	50	50	53.7
z_1 (days)	39	43	39	40	40	39
fz_1 (%)	66.7	0	66.7	50	50	66.7
z_2 ($)	7,600	0	7,600	7,000	7,000	7,600
fz_2 (%)	53.7	100	53.7	57.3	57.3	53.7

Conclusions

Cost-time trade-off models provide DMs with an alternative decision tool that could be used to develop scheduling models, where additional constraints may be considered, as is possible in fuzzy mathematical models. Fuzzy mathematical models provide DMs with a range of time that is between the originally scheduled time with no constraints and the maximum crash time. This information is used as the basis for creating membership functions that include cost as an objective in the fuzzy mathematical models. When crashing a project, other paths in the project could become critical. Therefore, the critical path and the activities with low amounts of float have to be considered to avoid lengthy delays and to help reduce costs.

Fuzzy mathematical models allow the inclusion of time and cost in the schedule analysis process. In addition, material restrictions may be included as constraints that are incorporated into models to generate more realistic solutions. The fuzzy math model used in this paper allowed us to consider two objectives simultaneously to satisfy the DM.

For the fuzzy mathematical model research, sensitivity analysis techniques were used to analyze different paths in sample networks to determine the effects of material constraints on the cost of projects. Using allocation analysis for the case studies highlighted how important it is to monitor the critical path during an analysis. Assuming that the maximum possible cost for a sample project is $30,000, and it takes 43 days to complete the project with no constraints, the following results were obtained from the analysis. When the critical path was not considered, the project completion time was increased by 20% from 43 to 52 days, and the cost increased approximately 73% up to $21,800. This increased time and cost was due to the activities that were not critical were becoming critical.

On all construction projects there is always uncertainty about the dates for material deliveries; therefore, including material helps project managers to evaluate situations and make more efficient decisions. Traditional mathematical models generate usable solutions, using single objectives or multiobjectives, but they do not consider fuzziness. Computers could be used to process complex fuzzy mathematical models and to generate alternatives that include trade-offs between cost and time for network projects in construction.

As future work, this fuzzy math model can be expanded to include more fuzzy parameters and nonlinear relations. There are also restrictions in terms of how big models can be solved by using commercial optimization packages. Another possible extension is to use evolutionary computational approaches to solve fuzzy models for large construction projects.

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