BasicMicro Economics: An Outline

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Part II

ten Chapter

Production and Cost

Decisions about production require individual agents to make decisions about the allocation and use of physical inputs. Objectives of agents, technology, availability and quality of inputs determine the nature of these decisions. Since the objectives are often pecuniary, it is often necessary to relate the decisions about the physical units of inputs and outputs to the costs of production.

If the prices of the inputs and the production relationships are known (or understood), it is possible to calculate or estimate all the cost relationships for each level of output. In practice however, the decision maker will probably have partial information about some of the costs and will need to estimate production relationships in order to make decisions about the relative amounts of the different inputs to be used.

A. Production

Production is the process of altering resources or inputs so they satisfy more wants. Before goods can be distributed or sold, they must be produced. Production, more specifically, the technology used in the production of a good (or service) and the prices of the inputs determine the cost of production. Within the market model, production and costs of production are reflected in the supply function.

Production processes increase the ability of inputs (or resources) to satisfy wants by:

- a change in physical characteristics
- a change in location
- a change in time
- a change in ownership

At its most simplistic level, the economy is a social process that allocates relatively scarce resources to satisfy relatively unlimited wants. To achieve this objective, inputs or resources must be allocated to those uses that have the greatest value. In a market setting, this is achieved by buyers (consumers) and sellers (producers) interacting. Consumers or buyers wish to maximize their utility or satisfaction given (or constrained by) their incomes, preferences and the prices of the goods they may buy. The behavior of the buyers or consumers is expressed in the demand function. The producers and/or sellers have other objectives. Profits may be either an objective or constraint. As an objective, a producer may seek to maximize profits or minimize cost per unit. As a constraint the agent may desire to maximize "efficiency," market share, rate of growth or some other objective constrained by some "acceptable level of profits. In the long run, a private producer will probably find it necessary to produce an output that can be sold for more than it costs to produce. The costs of production (Total Cost, TC) must be less than the revenues (Total Revenue, TR).

Given a production relationship (Q = f (labour, land, capital, technology, …)) and the prices of the inputs, all the cost relationships can be calculated. Often, in the decision making process, information embedded in cost data must be interpreted to answer questions such as;

- "How many units of a good should be produced (to achieve the objective)?"
- "How big should may plant be?' or How many acres of land should I plant in potatoes?"

Once the question of plant size is answered, there are questions,

- "How many units of each variable input should be used (to best achieve the objective)?"
- "To what degree can one input be substituted for another in the production process?"

The question about plant size involves long run analysis. The questions about the use of variable inputs relate to short-run analysis. In both cases, the production relationships and prices of the inputs determine the cost functions and the answers to the questions.

Often decision-makers rely on cost data to choose among production alternatives. In order to use cost data as a "map" or guide to achieve production and/or financial objectives, the data must be interpreted. The ability to make decisions about the allocation and use of physical inputs to produce physical units of output (Q or TP) requires an understanding of the production and cost relationships.

The production relationships and prices of inputs determine costs. Here the production relationships will be used to construct the cost functions. In the decision making process, incomplete cost data is often used to make production decisions. The theory of production and costs provides the road map to the achievement of the objectives.

(1) Production Unit

In the circular flow diagram found in most principles of economics texts, production takes place in a "firm" or "business." When considering the production-cost relationships it is important to distinguish between firms and plants. A plant is a physical unit of production. The plant is characterized by physical units of inputs, such as land (R) or capital (K). This includes acres of land, deposits of minerals, buildings, machinery, roads, wells, and the like. The firm is an organization that may or may not have physical facilities and engage in production of economic goods. In some cases the firm may manage a single plant. In other instances, a firm may have many plants or no plant at all.

The cost functions that are associated with a single plant are significantly different from those that are associated with a firm. A single plant may experience economies in one range of output and diseconomies of scale in another. Alternatively, a firm may build a series of plants to achieve constant or even increasing returns. General Motors Corp. is often used as an example of an early firm that used decentralization to avoid rising costs per unit of output in a single plant.

Diversification is another strategy to influence production and associated costs. A firm or plant may produce several products. Alfred Marshall (one of the early Neoclassical economists in the last decade of the 19th century) considered the problem of "joint costs. " A firm that produces two outputs (beef and hides) will find it necessary to "allocate" costs to the outputs.

Unless specifically identified, the production and cost relationships will represent a single plant with a single product.

(2) Production Function

A production function is a model (usually mathematical) that relates possible levels of physical outputs to various sets of inputs, eg.
Q = f (Labour, Kapital, Land, technology, . . .).
To simplify the world, we will use two inputs Labour (L) and Kapital (K) so,

$$Q = f(L, K, technology, ...).$$

Here we will use a Cobb-Douglas production function that usually takes the form; $Q = AL^aK^b$. In this simplified version, each production function or process is limited to increasing, constant or decreasing returns to scale over the range of production. In more complex production processes, "economies of scale" (increasing returns) may initially occur. As the plant becomes larger (a larger fixed input in each successive short-run period), constant returns may be expected. Eventually, decreasing returns or "diseconomies of scale" may be expected when the plant size (fixed input) becomes "too large." This more complex production function is characterized by a long run average cost (cost per unit of output) that at first declines (increasing returns), then is horizontal (constant returns) and then rises (decreasing returns).

(3) Time and Production

As the period of time is changed, producers have more opportunities to alter inputs and technology. Generally, four time periods are used in the analysis of production:

"**market period**" -
A period of time in which the producer cannot change any inputs nor technology can be altered. Even output (Q) is fixed.

"**Short-run**"-
A period in which technology is constant, at least one input is fixed and at least one input is variable.

"**Long-run**" -
A period in which all inputs are variable but technology is constant.

"**The very Long-run**" -
During the very long-run, all inputs and technology change.

Most analysis in accounting, finance and economics is either long run or short-run.

(4) Production in the Short-Run

In the short-run, at least one input is fixed and technology is unchanged during the period. The fixed input(s) may be used to refer to the "size of a plant." Here K is used to represent capital as the fixed input. Depending on the production process, other inputs might be fixed. For heuristic purposes, we will vary one input. As the variable input is altered, the output (Q) changes. The relationship between the variable input (here L is used for "labour") and the output (Q) can be viewed from several perspectives.

The short-run production function will take the form

$Q = f(L)$, K and technology are fixed or held constant

A change in any of the fixed inputs or technology will alter the short-run production function.

In the short run, the relationship between the physical inputs and output can be describes from several perspectives. The relationship can be described as the total product, the output per unit of input (the average product, AP) or the change in output that is attributable to a change in the variable input (the marginal product, MP).

Total product (**TP** or **Q**) is the total output. Q or TP = f(L) given a fixed size of plant and technology.

Average product (**AP_L**) is the output per unit of input. AP = TP/L (in this case the output per worker). **AP_L** is the average product of labour.

$$AP_L = \frac{\text{output}}{\text{Input}} = \frac{TP}{L} = \frac{Q}{L}$$

Marginal Product (**MP_L**) is the change in output "caused" by a change in the variable input (L),

$$MP_L = \frac{\Delta TP}{\Delta L} = \frac{\Delta Q}{\Delta L}$$

(a) Total and Marginal Product

Over the range of inputs there are four possible relationships between Q and L

(1) TP or Q can increase at an increasing rate. MP will increase, (In Figure V.1 this range is from O to L_A.)

(2) TP may pass through an inflection point, in which case MP will be a maximum. (In Figure V.1, this is point A at L_A amount of input.) TP or Q may increase at a constant rate over some range of output. In this case, MP will remain constant in this range.

(3) TP might increase at a decreasing rate. This will cause MP to fall. This is referred to as "diminishing MP." In Figure V.1, this is shown in the range from L_A to L_B.

(4) If "too many" units of the variable input are added to the fixed input, TP can decrease, in which case MP will be negative. Any addition of L beyond L_B will reduce output; the MP of the input will be negative. It would be foolish to continue adding an input (even if it were "free") when the MP is negative.

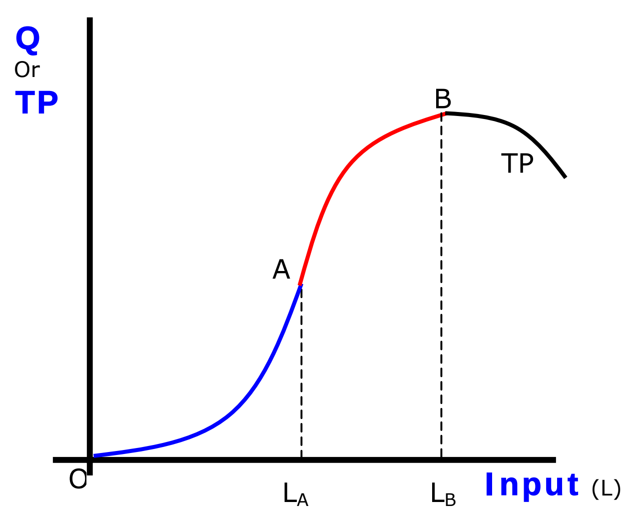


Figure V.1

The relationship between the total product (TP) and the marginal product (MP) can be shown. In Figure V.2, note that the inflection point in the TP function is at the same level of input (L_A) as the maximum of the MP. It is also important to understand that the maximum of the TP occurs when the MP of the input is zero at L_B.

Figure V.2

In **Figure V.2**, the total product (TP) function is shown in the upper panel. TP = f (L) This is a short run function which implies that there is a set of fixed inputs (a scale of plant) and a given state of technology.

The TP initially increases at an increasing rate. This may be caused by specialization and division of labour. At point A there is an inflection point in the TP function. Beyond L_A amount of labour, the TP increases at a decreasing rate until it reaches a maximum at point B. If additional units of labour are used beyond L_B, the output (Q or TP) will decline.

In the lower panel of Figure V.2 the average product (AP) and marginal product (MP) are shown. At L_A amount of labour (determined by the inflection point in TP at point A in the top panel), the MP will reach a maximum at point A′. When the TP increases at an increasing rate, MP rises. When TP increases at a decreasing rate, MP decreases. When TP is a maximum, MP is zero at point B′ in the lower panel. MP is the slope of the TP function.

At L_H amount of labour the AP will be a maximum at point H′. this is consistent with the tangency of the ray from the origin with the TP function in the top panel. When the AP is a maximum it will be equal to the MP.

When MP >AP, AP will increase. When MP<AP, the AP will decrease.

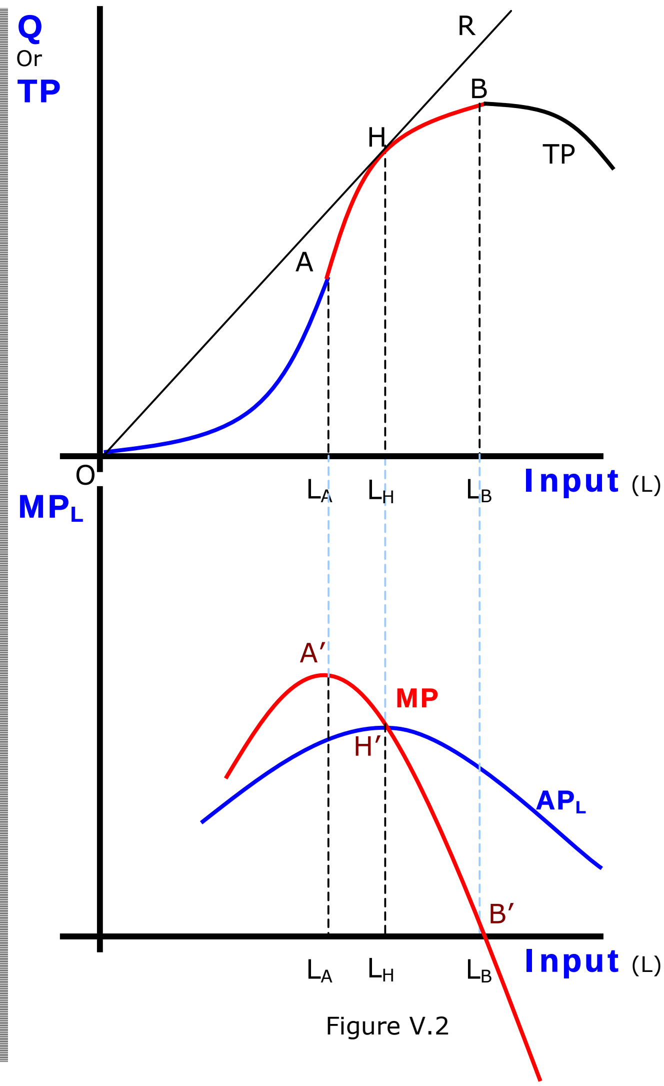


Figure V.2

(b) Average, Marginal and Total Product

The average product (AP) is related to both the TP and MP. Construct a ray (OR in Figure V.2) from the origin to a tangent point (H) on the TP. This will locate the level of input where the AP is a maximum, L_H. Note that at L_H level of input, AP_L is a maximum and is equal to the MP_L. When the MP is greater than the AP, MP "pulls" AP up. When MP is less than AP, it "pulls" AP down. MP will always intersect the AP at the maximum of the AP.

Technical efficiency was defined as a ratio of output to input,

$$\text{Technical Efficiency} = \frac{\text{Output}}{\text{Input}}$$

The AP is a ratio of TP or Q or output to a variable input and a set of fixed input(s).

$$AP = \frac{TP}{L} = \frac{output\ \ (Q)}{input\ \ (L,\ given\ \ K)}$$

The maximum of the AP is the "technically efficient" use of the variable input (L) given plant size. Remember that K is fixed in the short-run.

(c) Review of Production Relationships

In the short-run, as a variable input is added to a fixed input (plant size) the TP may increase at an increasing rate. As TP increases at an increasing rate MP for the variable input will rise. So long as the MP is greater than the AP of the variable input, AP will rise. This range is caused by a more "efficient mix" of inputs. Initially, there is "too much" of the fixed input and not enough of the variable input.

Eventually, as more variable inputs are added there may be an inflection point in the TP. It is also possible that the TP might increase at a constant rate in a range. An inflection point in the TP is where the "curvature" of the TP changes; it is changing from increasing at an increasing rate (concave from above or convex from below) to increasing at a decreasing rate (convex from above or concave from below). At this point, the MP of the variable input will be a maximum. AP will be rising.

At some point, the TP will begin to increase at a decreasing rate. There is "too much variable input" for the fixed input. Productivity of each additional input will be less; MP will fall in this range. AP of the variable input may be greater or less than the MP in this range. If MP is greater than AP, AP will be increasing. If MP is less than AP, AP will be decreasing.

A ray from the origin and tangent to the TP function (line OR in Figure V.2) will identify the level of variable input where the AP will be a maximum. At this point MP will equal AP. Since the fixed input is constant, AP is the equivalent of out measure of technical efficiency for a given scale of plant determined by the fixed input;

$$Tech.\ Efficiency = \frac{output}{input} = \frac{TP}{L\ \ (given\ \ fixed\ \ input)} = \text{AP of the varible input}$$

B. Cost

Producers who desire to earn profits must be concerned about both the revenue (the demand side of the economic problem) and the costs of production. Profits (Π) are defined as the difference between the total revenue (TR) and the total cost (TC). The concept of "efficiency" is also related to cost.

(1) Opportunity Cost

The relevant concept of cost is "opportunity cost." This is the value of the next best alternative use of a resource or good. It is the value sacrificed when a choice is made. A person who opens their own business and decides not to pay himself or herself any wages must realize that there is a "cost" associated with their labour, they sacrifice a wage that they could have earned in some other use.

A worker earns a wage based on their opportunity cost. An employer must pay a worker a wage that is equal to or greater than an alternative employer would pay (opportunity cost) or the worker would have an incentive to change jobs. Capital has a greater mobility than labour. If a capital owner can earn a higher return in some other use, they will move their resources. The opportunity cost for any use of land is its next highest valued use as well. It is also crucial to note that the entrepreneur also has an opportunity cost. If the entrepreneur is not earning a "normal profit" is some activity they will seek other opportunities. The normal profit is determined by the market and is considered a cost.

(2) Implicit and Explicit Cost

The opportunity costs associated with any activity may be **explicit**, out of pocket, expenditures made in monetary units or **implicit** costs that involve sacrifice that is not measured in monetary terms. It is often the job of economists and accountants to estimate implicit costs and express them in monetary terms. Depreciation is an example. Capital is used in the production process and it is "used" up, i.e. its value depreciates. Accountants assume the expected life of the asset and a path (straight line, sum of year's digits, double declining, etc) to calculate a monetary value.

In economics both implicit and explicit opportunity costs are considered in decision making. A "normal profit" is an example of an implicit cost of engaging in a business activity. An implied wage to an owner-operator is an implicit opportunity cost that should be included in any economic analysis.

C. Costs and Production in the Short-Run

If the short-run production function (Q =f(L) given fixed input and technology) and the prices of the inputs are known, all the short-run costs can be calculated. Often the producer will know the costs at a few levels of output and must estimate or calculate the production function in order to make decisions about how many units of the variable input to use or altering the size of the plant (fixed input).

Fixed Cost (**FC**) is the quantity of the fixed input times the price of the fixed input. FC is total fixed cost and may be referred to as TFC.

Average Fixed Cost (**AFC**) is the FC divided by the output or TP, Q, (remember Q=TP). AFC is fixed cost per Q.

Variable Cost (**VC**) is the quantity of the variable input times the price of the variable input. Sometimes VC is called total variable cost (TVC).

Average Variable Cost (**AVC**) is the VC divided by the output, AVC = VC/Q. It is the variable cost per Q.

Total Cost (**TC**) is the sum of the FC and VC.

Average Total Cost (**AC** or **ATC**) is the total cost per unit of output. AC = TC/Q.

Marginal cost (**MC**) is the change in TC or VC "caused" by a change in Q (or TP). Remember that fixed cost do not change and therefore do not influence MC. In Principles of Economics texts and courses MC is usually described as the change in TC associated with a one unit change in output,

$MC = \frac{\Delta TC}{\Delta Q}$ (for infinitely small ΔQ, $MC = \frac{dTC}{dQ}$)

MC will intersect AVC and AC at the minimum points on each of those cost functions.

D. Graphical Representations of Production and Cost Relationships

The short-run, total product function and the price of the variable input(s) determine the variable cost (VC or TVC) function.

In Figure V.3, the short-run TP function and VC function are shown.

Figure V.3

In Figure V.3 the production relationship is linked to the variable cost. In the upper panel, the TP function is shown.

$Q = f(L)$

(Given fixed input and technology)

TP initially increases at an increasing rate to point A where L_A amount of variable input is used. There is an inflection point at point A. TP then increases at a decreasing rate to a maximum at point B produced by L_B amount of input. Beyond L_B amount of input the TP declines.

Using L_A amount of labour, Q_A amount of output is produced. At input level L_B, Q_B output results.

In the lower panel, VC is expressed as a function of Q,

$VC = f(Q)$

(Given fixed input and technology)

The vertical axis, (TP or Q) in the upper panel becomes the horizontal axis (Q) in the lower panel.

The horizontal axis (L) in the upper panel is multiplied by the P_L and becomes the vertical axis in the lower panel.

The VC function is the TP function "rotated and looked at from the back side."

When the TP increases at an increasing rate, the VC increases at a decreasing rate.

When the TP increases at a decreasing rate, VC will increase at an increasing rate.

When the TP decreases at the quantity of input increases, the VC would

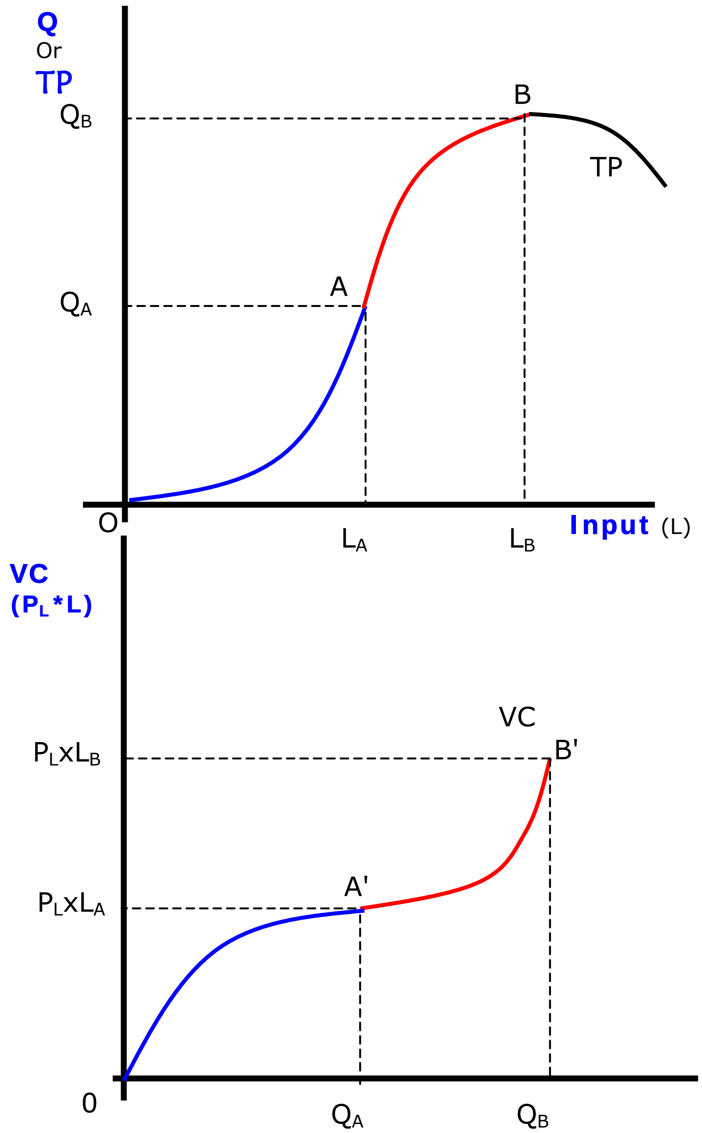


Figure V.3

In the range from 0 to L_A amount of labour, the output increases from 0 to Q_A. TP increases at an increasing rate in this range. The MP_L is increasing as additional units of labour are added. Since the VC (total variable cost) is the price of labour times the quantity of labour used (LP_L), VC will increase at a decreasing rate. The MC will be decreasing in this range.

In the range from L_A to L_B amount of labour the output rises from Q_A to Q_B, TP increases at a decreasing rate (MP will be decreasing in this range.). Variable cost (VC) will increase at an increasing rate (MC will be rising).

At the inflection point (A) in the production relationship, MP will be a maximum. This is consistent with the inflection point (A') in the VC function.

At the maximum of TP (L_B amount of labour, output Q_B) at point B, the VC function will "turn back" and as output decreases the VC will continue to rise. A "rational" producer would cease to add variable inputs when those additions reduce output.

(1) Variable Cost (VC or TVC) and Average Variable Cost (AVC)

The total variable cost is determined by the price of the variable input and the TP function. The average variable cost is simply the variable cost per unit of output (TP or Q), $AVC = \frac{VC}{Q}$.

Figure V.4

In Figure V.4 The relationships among the variable cost (VC), average variable cost (AVC) and the marginal cost (MC) are shown. In the top panel there are three points that are unique and can be used to identify what is happening to the MC and AVC. At point A' there is an inflection point in the VC. At point B' the slope of the VC is infinity. Point C is identified by a ray from the origin that is tangent to the VC at point C.

Point A' is associated with Q_A amount of output. At Q_A output the MC will be a minimum at A* in the lower panel. As the TP increases at an increasing rate, the MP rises, the VC increases at a decreasing rate and MC decreases. Beyond Q_A output, the TP increases at a decreasing rate, MP falls, VC increases at an increasing rate and MC will increase.

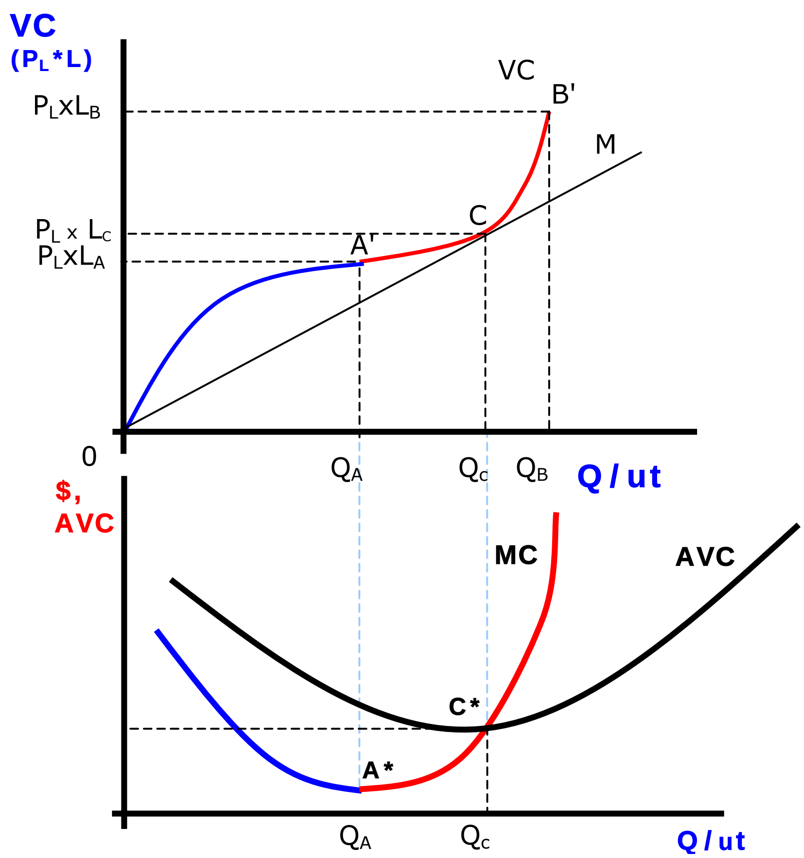


Figure V.4

In Figure V.4 the VC is shown with 3 points identified. A' is on the TVC at the level of output where there is an inflection point. This will be the same output level were the MC is a minimum. Point C is found by constructing a ray, OM from the origin to a point of tangency on the VC. The level of output will be the minimum of the AVC (also the maximum of the AP). At this point the MC will equal the AVC. When MC is less than AVC, AVC will decline. When MC is greater than AVC,C will rise. MC will always equal AVC at the minimum of the AVC.

(2) ATC, AVC, MC and AFC

The fixed cost is determined by the amount of the fixed input and its price. In the short-run the fixed cost does not change. As the output (Q) increases the average fixed cost (AFC) will decline. Since

$$AFC = \frac{Fixed\ Cost}{Q}$$

as long as Q increases, AFC will decrease, it approaches the Q axis "asymptotically."

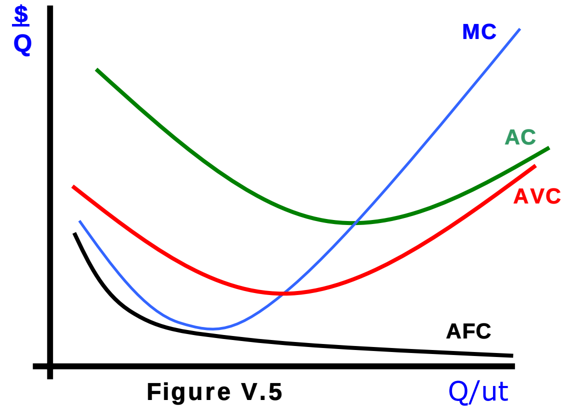


Figure V.5

The average total cost (ATC) is the total cost per unit of output.

$$ATC = \frac{TC}{Q} = AFC + AVC$$

In Figure V.5, the AFC is shown declining over the range of output. The vertical distance between the ATC and AVC is the same as AFC. The location or shape of the AVC is not related to the AFC.

The MC is not relate to the AFC but will intersect both the AVC and ATC at their minimum points.

E. Relationship of MC and AVC to MP_L and AP_L

In Figure V.6 there are three panels. The first shows the TP or short-run production function. The second is the marginal (MP) and average (AP) product functions associated with the short-run production function. In the third panel the related marginal cost (MC) and average variable cost (AVC) function are shown.

There are three points easily identifiable on the TP function; the inflection point (A), the point of tangency with a ray from the point of origin (H) and the maximum of the TP (B). Each of these points identifies a level (an amount) of the variable input (L) and a quantity of output. These points are associated with characteristics of the MP and AP functions

At point A, with L_A amount of labour and Q_A output the inflection point in TP is associated with the maximum of the MP. This maximum of the MP function is associated with the minimum of the MC;

$$MC = \frac{1}{MP_L}(\text{price of labour}).$$

Since MP>AP, the AP is increasing. MC<AVC, so AVC is decreasing.

At point H, the AP is a maximum at this level of input (L_H). At this level of input use the output (QH) has a minimum of the average variable cost (AVC). At this AVC, the MC will equal the MC. Point B represents the level of input (L_B) where the output (Q_B) is a maximum. At this level the MP_L will be zero.

Figure V.6

The average variable cost (AVC) and the average product (AP) are closely related. Marginal cost (MC) reflects the marginal product (MP). In the three panels of Figure V.6, the total product (TP in the upper panel) is related to the AP and MP in the middle panel. The lower panel shows the relationship of average variable cost (AVC) and marginal cost (MC) to AP, MP and TP.

As the variable input (L in this example) increases to L_A the TP in the upper panel increases at an increasing rate to point A. In this range the marginal product (in the middle panel) will rise. When more than L_A amount of the labour input is used, the MP will decrease for each additional unit. The inflection point at A in the upper panel is consistent with the maximum of the MP at point A* in the middle panel.

Point H on the TP function (in the upper panel) is constructed by passing a ray from the origin to a point of tangency on the TP function. This identifies L_H amount of labour. In the middle panel the AP of labour will rise up to L_H amount of input. Notice that in this range the MP is above or greater than the AP. When MP>AP, the AP will be increasing. At point H* in the middle panel, AP will be a maximum. At this point MP = AP.

As the input is increased above L_H, the AP will fall. When MP < AP, AP will be decreasing. At the maximum of the TP (at point B in the upper panel) L_B amount of the variable input is used. At this level of labour (L_B), the MP will be zero. It is important to note that when MP = 0, TP is a maximum.

In the lower panel, MC will be at a minimum at the output level (Q_A) where MP is a maximum (Q_A output at L_A input). AVC will be a minimum at Q_H output. This is where MC=AVC. This is at output level Q_H produced by L_H labour. When AVC is a minimum, AP will be a maximum.

When MC<AVC, AVC will be decreasing. When MC>AVC, AVC will increase. When AVC=MC, AVC is a minimum.

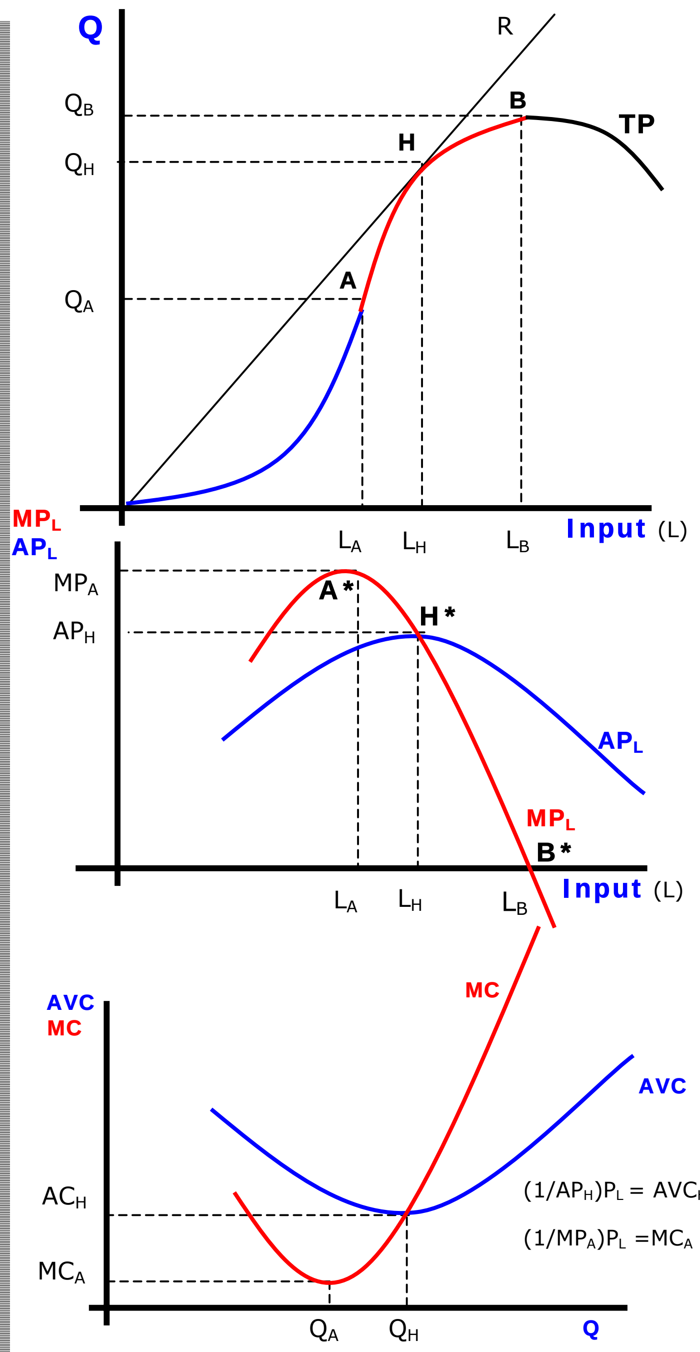


Figure V.6

F. Production and Cost Tables

The data from production functions and the prices of inputs determines all the cost functions. In the following example a short-run production function is given. In the table below the columns K, L and TP reflect short-run production. The plant size is determined by capital (K) that is 5 in the example. Since the P_K = \$3, the fixed cost (FC) is \$15 at all levels of output. The price of the variable input (L) is \$22. The variable cost (VC) can be calculated for each level of input use and associated with a level of output (Q). Total cost (TC) is the sum of FC and VC. The average cost functions can be calculated: AFC = FC/Q, AVC - VC/Q and ATC =AFC + AVC =TC/Q. Given the production function and the prices of the inputs, the cost functions are shown in Table V.1.

Marginal cost in the table is an estimate. Remember that $MC = \frac{\Delta TC}{\Delta Q}$. Since quantity is not changing at a constant rate of one, the MP will be used to represent ΔQ. This is not precisely MC but is only an estimate.

Table V.1
Production and Cost

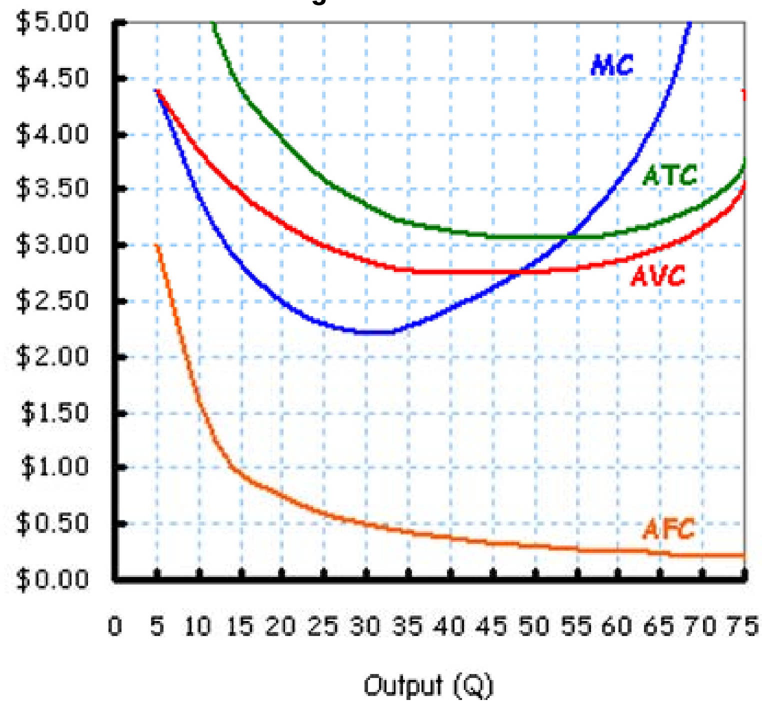
K	L	TP (Q)	MP	FC	VC	TC	≈ MC	AFC	AVC	ATC
5	0	0		$15	$0	$15		#DIV/0!	#DIV/0!	#DIV/0!
5	1	5	5	15	22	37	$4.40	$3.00	$4.40	$7.40
5	2	12	7	15	44	59	3.14	1.25	3.67	4.92
5	3	21	9	15	66	81	2.44	0.71	3.14	3.86
5	4	31	10	15	88	103	2.20	0.48	2.84	3.32
5	5	40	9	15	110	125	2.44	0.38	2.75	3.13
5	6	48	8	15	132	147	2.75	0.31	2.75	3.06
5	7	55	7	15	154	169	3.14	0.27	2.80	3.07
5	8	61	6	15	176	191	3.67	0.25	2.89	3.13
5	9	66	5	15	198	213	4.40	0.23	3.00	3.23
5	10	70	4	15	220	235	5.50	0.21	3.14	3.36
5	11	73	3	15	242	257	7.33	0.21	3.32	3.52
5	12	75	2	15	264	279	11.00	0.20	3.52	3.72
5	13	76	1	15	286	301	22.00	0.20	3.76	3.96
5	14	76	0	15	308	323	#DIV/0!	0.20	4.05	4.25
5	15	75	-1	15	330	345	-22.00	0.20	4.40	4.60

The cost functions constructed from the data inTable V.1 are shown in Figure V.7. Note that the MC intersects the AVC and ATC at the minimum points on those functions. The vertical distance between ATC and AVC is the same as the AFC. The AFC is unrelated to the MC and AVC. In this example the average fixed cost is less than the average variable cost and MC at every level of output. This is coincidence. In some other production process it might be greater at each level of output.

It is relevant that the AVC and MC are equal at the first unit of output. This will always be true. This also means that

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}.$$

Figure V.7



G. Production and Cost in the Long-run

Long-run Production describes a period in which all inputs (and Q) are variable while technology is constant. A Cobb-Douglas production function can be used to describe the relationships. There are a variety of other forms production functions can take, however the Cobb-Douglas is the simplest to describe. A short-run production function (Q = f(L) is a cross section from a long-run production function.

(1) Long Run Production

The long run production function is multidimensional, two or more inputs and output changes. If there are 2 inputs and one output, the long run production relationship is 3 dimensional. Using a topological map of "isoquants," three dimensions can be shown in two dimensions.

Figure V.8 is a representation of a long run production model using isoquants and isocosts. This model is an attempt to represent a three-dimensional model in two dimensions. It can be thought of as a "topological map" of production. In Figure V.8, two different levels of output of the good are shown. The term "isoquant" means equal quantity. In the graph two isoquants are shown. Q_1 and Q_2 represent two different levels of output. There are an infinite number of isoquants, one for each possible level of output but only two are shown. The isoquant (Q_1) represents all combinations of labour (L) and capital (K) that will produce Q_1 amount of output. Three input combinations that will produce Q_1 output are identified in the graph (points J, B and H). while there are an infinite number of input combinations that lie along the isoquant (Q_1), only these three are marked.

Isoquant Q_2 is a larger output than Q_1. Only input combination L_A, K_A at point A is identified.

Two isocost functions are also shown in Figure V.8. These are TC_1 and TC_2. "Isocost" means equal cost. All output combinations that lie on TC_1 require the same expenditure. All output combinations that cost less than TC_1 lie inside the isocost. Output combinations that cost more than TC_1 lie outside the isocost. TC_2 represents a greater cost than TC_1. The isocost function can be located by finding the intercepts on the K-axis (capital axis) and L-axis (labour axis). The L-intercept is found by dividing the total cost (TC_1 by the price of labour. If TC_1 were

\$200 and the price of labour were \$5 the L-intercept (L*) would be 40 units of labour, i.e. 40 units of labour at \$5 each will cost \$200. If the price of capital were \$4, the K-intercept for TC_1 is K* (200/4 = 50). A straight line between these two intercepts identifies all combinations of labour and capital that cost \$200.

In Figure V.8, the isoquants are represented as Q_1 and Q_2. There are an infinite number of isoquants, one for each possible quantity of output. In our example only two are shown. Along a given isoquant (Q_1) there is a constant level of output. Q_2 represents a greater output level. Along Q1 the output remains constant for different combinations of inputs (L and K) Input combination depicted by point J (K_A capital and L_J labour) results in the same output as the input combination K_B and L_B at point B. The slope of the isoquant between point J and B represent the marginal rate of substitution, the rate at which one input can be substituted for another holding output constant. The line TC_1 represents a given expenditure or isocost. Each point along the isocost represents different combinations of inputs that costs the same amount (TC_1). Point B (using K_B and L_B) is the lowest cost combination of inputs to produce Q_1 amount of output.

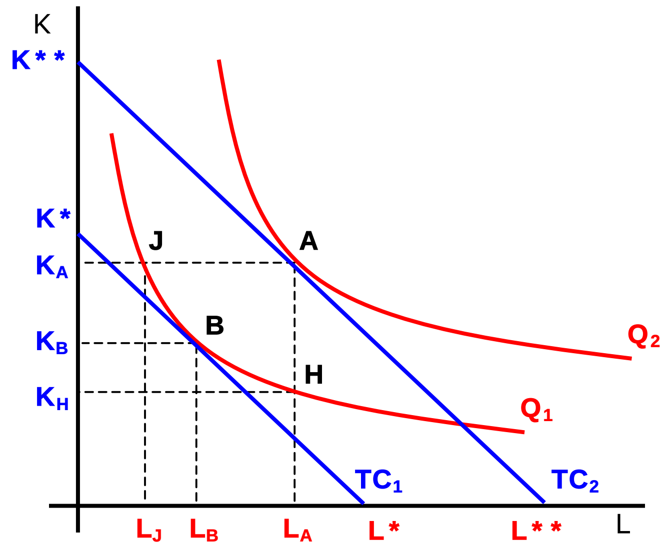


Figure V.8

Q_1 output could be produced by using K_A capital and L_J labour (point J on Q_1). Point H (L_A labour and K_H capital will also result in Q_1 output. Notice that both points J and H lie outside the isocost TC_1. Since point B lies on TC_1, that input combination cost less than those at point J and H. If Q_1 output is desired, TC_1 is the lowest cost of production that can be attained. This is accomplished by using L_B labour and K_B capital. The lowest cost of producing Q_2 given the price of labour and capital is at point A.

The slope of the isoquant represents the rate at which one input can be substituted for another and still produce the same output. The slope of the isocost represents relative price so of the inputs. The lowest cost combination of inputs is at the point of tangency between the isocost and the isoquant. When the isocost function is tangent to an isoquant, it identifies the combination of inputs that minimizes the cost per unit for that level of output.

The short-run production relationships are cross-sections taken out of the isoquant map. In intermediate microeconomics you will study the cost and production relationships in the isoquant map.

(2) Long-run Costs

The long-run costs are derived from the production function and the prices of the inputs. No inputs are fixed in the long run, so there are no fixed costs. All costs are variable in the long run. The long run can be thought of as a series of short-run periods that reflect the cross-sections taken out of Figure V.8. Consequently, the long run costs can be derived from a series of short-run cost functions. In principles of economics the "envelope curve" is used as an approximation of the long run average cost function In Figure V.9, there are series of short run average cost (AC) functions shown. Each represents a different size plant. Plant size A is represented by AC_A. As the plant gets larger, up to plant size $AC_{D,}$, the short-run AC function associated with each larger plant size is lower and further from the vertical axis. This range is sometimes referred to as "economies of scale." Generally it happens from specialization and/or division of labour. Plant D, represented by AC_D, represents the plant with the lowest cost per unit. As the plant size increases above D, the short-run average cost begins to rise. This region is often referred to as "diseconomies of scale." Lack of

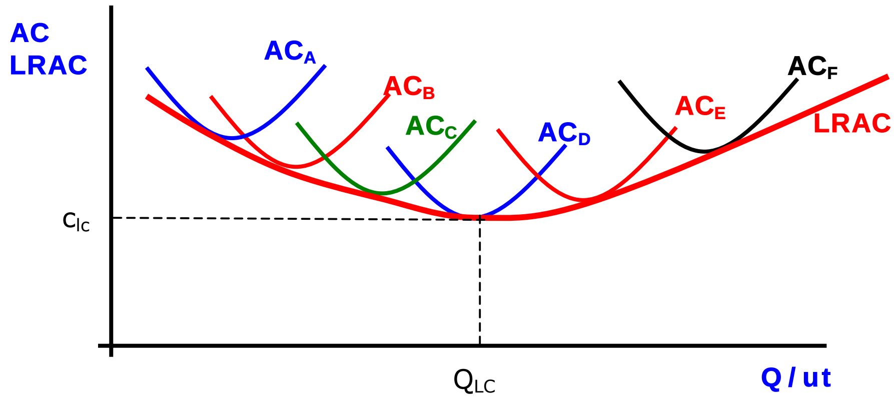


Figure V.9

information to make wise production choices is usually given as the reason for the increasing cost per unit as plant size increases above plant D.

The envelope curve or LRAC is constructed by passing a line so it is smooth and just touches each of the short-run AC functions. Within each short-run period there is a short run AC, MC, AVC and AFC. The firm or plant will move from one set of short-run curves by changing the fixed input. In Figure V.8 this would be the same as moving from one cross section to another.

Returns to Scale

The terms "economies of scale," "increasing returns to scale," "constant returns to scale," "decreasing returns to scale" and "diseconomies of scale" are frequently used. These terms involve subtle and complicated concepts that apply to the long run production process. In principles of economics they are simplified. Conceptually, returns to scale implies that all inputs are variable. Given a Cobb-Douglas production function of the form $Q = A\,L^{\alpha} K^{\beta}$. Q is output or quantity, L is quantity of labour and K is the quantity of capital. A, α and β are parameters that are determined by the technology of producing a specific product. When **$\alpha+\beta$ = 1**, the production process demonstrates "constant returns to scale." If L and K both increased by 10%, output (Q) would also increase by 10%. This is consistent with a long run average cost (envelope) function that has a slope of 0.

When **$\alpha+\beta$ > 1**, production has increasing returns. A 10% increase in both L and K results in a larger percentage (say 20%) increase in output (Q). This is consistent with the declining portion of the long run average cost function (LRAC). This tends to be the result of specialization and division of labour. It is sometimes referred to as economies of scale or economies of mass production. There may be a variety of forces that cause the LRAC to decline. Not all these forces are actually economies of scale. A larger firm (or monopsonist) may be able to negotiate lower prices for inputs. This is not economies of scale, it is a transfer of income or wealth from one group to another.

When α**+**β **< 1**, decreasing returns are said to exist. As both inputs increase 10%, output (Q) will increase by a smaller percentage (say 6%). This condition is consistent with the rising portion of the long run average cost function (LRAC). As a firm gets larger it may lack the information about various aspects of the production process and be unable to coordinate all the processes. This is the reason that a planned economy does not always produce optimal results.

In Figure V.9 economies of scale are said to exist up to output Q_{LC}. Diseconomies of scale occur as output expands about Q_{LC}.