

Break-even Analysis

An enterprise, whether or not a profit maximizer, often finds it useful to know what price (or output level) must be for total revenue just equal total cost. This can be done with a break-even analysis. Strictly speaking, this analysis is to determine the minimum level of output that allows the firm to break even, but it could be used for some other tasks.

In this Appendix, we introduce:

- The algebra of break-even analysis
- Break-even diagram
- Operating leverage

I. THE ALGEBRA OF BREAK-EVEN ANALYSIS

Let Q_{BE} denote the break-even output level. By definition

$$TR \text{ (at } Q_{BE}) = TC \text{ (at } Q_{BE})$$

or
$$TR \text{ (at } Q_{BE}) = TFC + TVC \text{ (at } Q_{BE}) \quad (1)$$

The **break-even condition** (1) holds true for *any* cost and demand functions. Hence, in general, when costs and demand are complex, the analysis of this condition might not be any simpler than the analysis of profit maximization. Yet, what is widely known in business as *break-even analysis* is indeed much easier than profit analysis, although it also starts with the above identity, because it makes a very important assumption: that *price and average variable cost do not change with output level*.

Thus, if we **assume that price and AVC are constant**, (1) can be rewritten as follows

$$P \cdot Q_{BE} = TFC + AVC \cdot Q_{BE}$$

which yields:

$$Q_{BE} = \frac{TFC}{P - AVC} \quad (2)$$

• The difference “ $P - AVC$ ” is often called the **average contribution margin**¹ (ACM) because it represents the portion of selling price that “contributes” to paying the fixed costs.

• Formula (2) can be generalized to deal with the situation where the firm has determined in advance a **target profit**. The output quantity Q^* that will yield this profit is implicitly given

¹The *total contribution margin* is simply $(P - AVC)Q = TR - TVC$.

by²

$$P \cdot Q^* = \text{Target profit} + \text{TFC} + \text{AVC} \cdot Q^*$$

hence

$$Q_{BE} = \frac{\text{TFC} + \text{Target Profit}}{P - \text{AVC}}$$

TABLE 1 Effects of TFC, AVC, and P on break-even output

Variable	Direction of Change	Break-even Output
Total Fixed Cost (e.g., cost of equipment)	Up	Up
	Down	Down
Average Variable Cost (e.g., cost of material)	Up	Up
	Down	Down
Product Price	Up	Down
	Down	Up

§ Example 1. Calculate the break-even output for TFC = \$20,000, P = \$7, and AVC = \$5

Answer:

$$Q_{BE} = \frac{20,000}{7 - 5} = \frac{20,000}{2} = 10,000$$

§ Example 2. Suppose TFC = \$10,000, P = \$5, AVC = \$2. What is the output necessary to earn \$5000 total profit? What is the “average contribution margin”?

Answer:

$$Q = \frac{10,000 + 5,000}{5 - 2} = \frac{15,000}{3} = 5,000$$

$$\text{ACM} = P - \text{AVC} = \$3$$

²We still assume that both P and AVC do not vary with output level.

II. BREAK-EVEN DIAGRAM

Figure 1 is a typical *break-even diagram* (also known as *break-even chart*). The total revenue and total cost curves are straight lines because price and AVC are assumed to be constant.

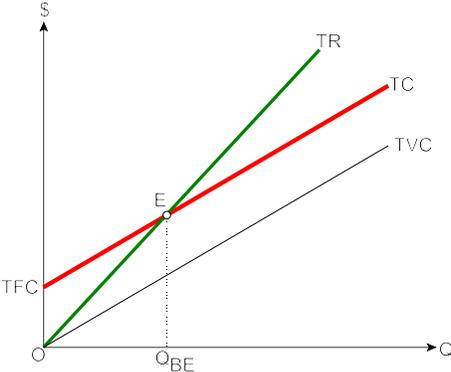


Figure 1. A break-even diagram with constant price and constant unit cost.

The break-even diagram can be employed to see the effects of various exogenous changes on the break-even point. Here are a few scenarios:

TABLE 2.

Initial change	Which curve is affected	What happens to Q_{BE}
Increase in output price	TR curve, counterclockwise	Decrease
Increase in the price of a variable input	TVC and TC curves, both counterclockwise	Increase
Higher TFC	TFC curve, parallel-shift up	Increase

■ The break-even diagram can also be modified to determine the output required to meet a non-zero target profit level (Figure 2). Effects on this output due to various changes (as in Table 2) can also be similarly studied with this diagram.

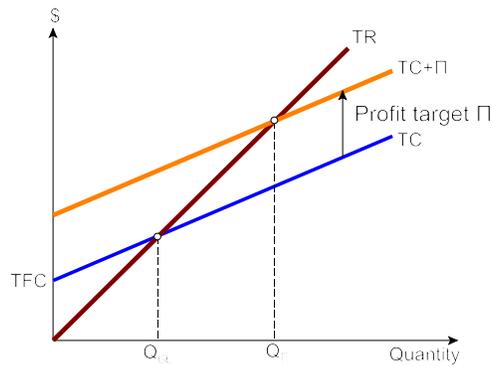


Figure 2. Break-even diagram with a non-zero target profit.

§ **Example 3** . Suppose that product price decreases from P_1 to P_2 . Show on the diagram how much output would change to *maintain the same level of profit target Π* .

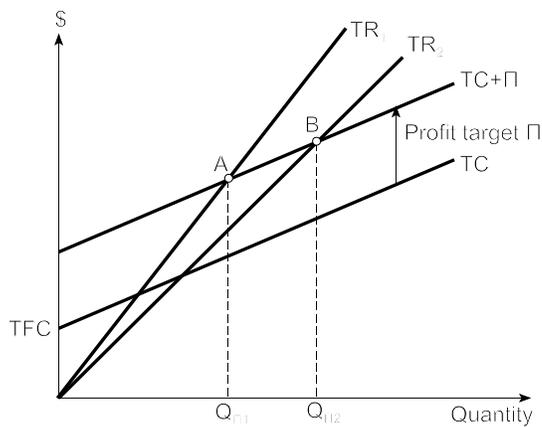
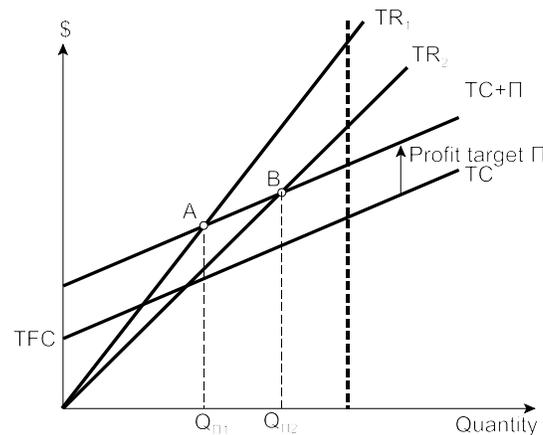


Figure 3

Answer: Draw a line parallel to TC through A. This line cuts the TR_2 line at B, drop a vertical line from B to determine the new output level. [Note: The solution may not be feasible if we are told that the market size \bar{Q} is such that $\bar{Q} < Q_{\Pi 2}$.]

§ Example 4 Given TR and TC, a firm is currently operating at 50% of its capacity at some profit target Π_0 . How much of a price drop would cause the firm to operate at 75% capacity at the same level of profit?



Answer: the vertical dotted line on the right marks the firm's capacity. Current output is half of that. Draw the line through A and parallel to TC, this is the "isoprofit" line. From 75% capacity output point, draw the vertical line. B is the intersection. Draw the line TR₂ through B. The slope of TR₂ gives the required output price.

III. OPERATING LEVERAGE

We have seen several types of elasticities: of demand, supply, etc. There is another kind that is quite popular among financial economists: the elasticity of total profit with respect to output level, also known as the **degree of operating leverage**.

● **Definition:** The *Degree of Operating Leverage* (DOL) at a given output level Q is the percentage change in total profit that results from a one-percent change in units sold:

$$\text{Degree of Operating Leverage} = \frac{\text{Change in Profit (in percentage)}}{\text{Change in Output (in percentage)}}$$

For example if, as a result of a 1% increase in output, profit increases by 3% then the DOL is $3\%/1\% = 3$.

● Algebraically, this may be expressed as:

$$DOL = \frac{Q}{\Pi} \frac{\partial \Pi}{\partial Q}$$

■ Under the special assumptions adopted by break-even analysis (that is, if *price and AVC*

are constant), it can be readily derived that:³

$$\text{DOL} = \frac{\text{TFC}}{\text{Profit}} = \frac{\text{TR} - \text{TVC}}{\text{TR} - \text{TC}} = \frac{\text{TR} - \text{TVC}}{\text{TR} - \text{TVC} - \text{TFC}} \quad (3)$$

Several conclusions can be drawn:

- (1) The DOL generally depends on the output level, and is equal to zero at the profit-maximizing output (because $\partial\Pi/\partial Q = 0$ at that point)
- (2) The DOL is negative (positive) below (above) the break-even level. Equation (3) also yields an interesting result:
 - **Rule:** For the same total cost, the Degree of Operating Leverage increases with fixed costs and decreases with variable costs.⁴
- (3) Consider 2 (almost identical) plants, except that

Plant A (capital intensive): low fixed costs, high variable costs

Plant B (labor intensive) high fixed costs, low variable costs

Since fixed costs are outlays already made, if the firm chooses to build Plant A instead of B, it can be said that the firm “**gets more leverage**” out of the resources it already has.

- (4) A plant with high fixed costs and low variable costs will also have a higher break-even point than a plant with low fixed costs and high variable costs.

The significance of this relationship is that a firm with **large fixed costs** usually breaks even at a higher output level. However, this firm’s DOL is also higher, its profits rises at a relatively high rate when production rises above break-even. Likewise, its profits declines more quickly during economic downturns, and the firm would become unprofitable at a relatively large output quantity (since the break-even

³*Proof:* Using the definition of Total Profit (Π) we can take the partial derivative of it with respect to output: $\partial\Pi/\partial Q = \partial(\text{TR} - \text{TVC} - \text{TFC})/\partial Q$. Since TFC is a constant (its derivative is thus zero), we obtain $\partial\Pi/\partial Q = \partial(\text{TR} - \text{TVC})/\partial Q$. Now, if P and Q do not change with Q, this would become $\partial\Pi/\partial Q = (P - \text{AVC})(\partial Q/\partial Q) = P - \text{AVC}$. This simply says that for an unit increase in output, profit will increase by an amount equal to the difference between price and average variable cost. Substituting $\partial\Pi/\partial Q = P - \text{AVC}$ back into the definition of DOL ($= (Q/\Pi)(\partial\Pi/\partial Q)$) and simplifying, we get the boxed property given above.

⁴*Proof:* Using, say, the definition $\text{DOL} = (\text{TR} - \text{TVC})/(\text{TR} - \text{TC})$ above, we take the partial derivative of DOL with respect to AVC, obtaining: $\partial(\text{DOL})/\partial(\text{TVC}) = -1/(\text{TR} - \text{TC})$ which is clearly negative if the firm is making profit. Indeed, since Q is a constant, we can rewrite this as $\partial(\text{DOL})/\partial(\text{AVC}) = -Q/(P - \text{ATC}) < 0$.

quantity will be high). On the other hand, a plant with **low fixed costs** and high variable costs will break even at lower quantities, and its profits will tend to be less sensitive to output level.

Thus, the break-even quantity and the DOL could have significant influences on a firm deciding whether or not to convert from an old--labor-intensive--manufacturing facility to a more modern, automated (i.e., capital-intensive) plant.

§ *Example 5.* What is the degree of operating leverage at an output level where profit is \$500,000, given that TFC is \$400,000?

Answer:
$$DOL = \frac{400,000}{500,000} = 0.8$$

§ *Example 6.* The firm is contemplating a new investment of $\Delta(TFC)$, how much the new AVC would have to go down for it to achieve the same profit target at *twice* the output level as before? How does the DOL at the new point compare with the old DOL?

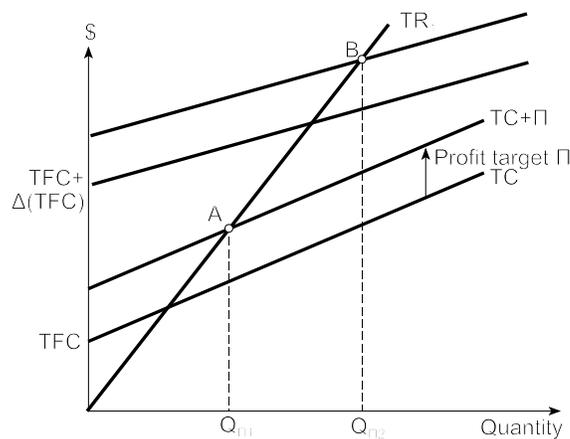


Figure 5

Answer: Mark the point $TFC + \Delta(TFC)$ on the vertical axis. Then mark the point $TFC + \Delta(TFC) + \Pi$ on the same axis. From the output level $Q_{\Pi 2}$ (which is twice $Q_{\Pi 1}$) draw the vertical line which cuts TR_1 at B. The slope at the line through B gives the new AVC. (Additional question: If both $\Delta(TFC)$ and the new AVC are given, what is the break even output level with the new investment?)

IV. PROFIT VS. BREAK-EVEN ANALYSIS

Break-even analysis or profit analysis are not mutually exclusive, both deserve a place in the manager's toolbox.

TABLE III

Quick way to determine the bare minimum for survival of firm?	Break-even analysis
Easy to do "what ifs" (What if TFC is different? What if AVC is different? What if P is different?)	Break-even chart
Not much information available?	Break-even analysis (Profit analysis aims at determining the profit-maximizing level, and thus requires information about cost and demand at <i>all output levels</i>)
Emphasis on long run?	Profit analysis (Reason: in the long run there are no fixed costs, and without fixed costs break-even points are indeterminate)
Most profitable output?	Profit analysis

Although a break-even analysis often simpler than a profit analysis, it does not answer many questions, and the questions that it answers are subject to numerous qualifications.

- Break-even analysis often fails to take into consideration many economic costs and benefits (especially those, e.g., from R&D, that will not materialize until far into the future). It is also difficult to distinguish between fixed and variable costs.
- Break-even analysis achieves its simplicity by assuming that both price and average variable cost are constant. Thus, its results would be erroneous if the firm is not a price taker and has output-sensitive average cost.
- It is difficult to do break-even analysis for a multiproduct firm, primarily because the total fixed costs are shared by many products and the product mix changes over time.

SUMMARY

1. The main purpose of break-even analysis is to determine the output level at which TR is equal to TC. Break-even analysis is easy to do (and thus its popularity) if both the product price and the average cost are constant. Graphically, the break-even output is where the TR curve and the TC curve intersect.
2. Break-even analysis is a popular tool to examine the effects of TFC and TVC on break-even output level and on the **degree of operating leverage**. The degree of leverage expresses the sensitivity of total profit with respect to output.

EXERCISES

1. The average variable cost is constant at \$5.00 per unit. The firm is selling 1000 units a week. Average fixed cost is also \$5 per unit. The market price for the product is \$12.00 per unit.
 - a. Calculate total profit.
 - b. Derive an equation for total cost.
 - c. Calculate the break-even level of output.
 - d. If the firm sets a target of \$3400 as their weekly profit, how many units of output should it sell?

ANS:

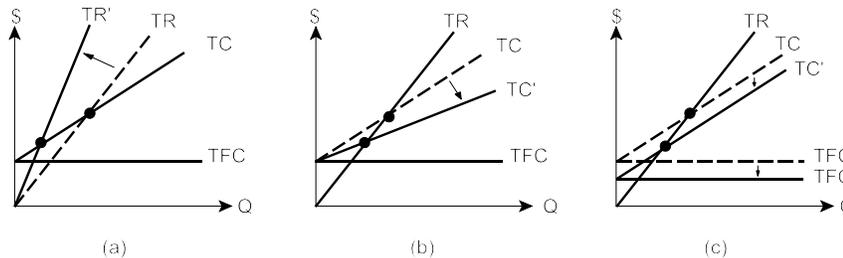
- a. Profit = $12(1000) - 1000(5 + 5) = \2000
- b. $TC = TFC + TVC = 5000 + 5Q$
- c. Break-even: $TFC / (P - AVC) = 5000 / 7 = 714.28$ units
- d. Target Profit $\$3400 = (5000 + 3400) / 7 = 1200$ units

2. Fill in the following table

TFC	AVC	P	Break-even output
100	5	8	Answer: 133.3

3. Draw a break-even diagram to illustrate each of the following scenarios:
 - a. An increase in output price
 - b. A decrease in the price of a variable input
 - c. A lower total fixed costs.

ANSWER:



4. The QuietBlow Company has a small plant that manufactures noise suppressors for leafblowers. Its annual fixed costs are \$30,000, and its variable costs are \$10 per unit. It can sell a suppressor for \$25.

- a. How many suppressors must the company sell to break even?
 - b. What is the break-even revenue?
 - c. The company sold 3,000 units last year. What was its profit?
 - d. Due to a new lump-sum tax, next year's fixed costs are expected to rise to \$37,500. What will be the break-even quantity?
 - e. If the company will sell the number of units obtained in part (d) and wants to maintain the same profit as last year, what will its new price have to be?
5. Writers' Pleasure, Inc. produces gold-plated pen and pencil sets. It has a fixed annual cost of \$50,000, and the average variable cost is \$20. It expects to sell 5,000 sets next year.
- a. In order to just break even, how much will the company have to charge for each set?
 - b. Based on its plant investment, the company requires an annual profit of \$30,000. How much will it have to charge per set to obtain this profit? (Quantity sold will still be 5,000 sets.)
 - c. If the company wants to earn a markup of 50 percent on its variable costs, how many sets will it have to sell at the price obtained in part b?

TFC	AVC	P	Q at target profit
50,000	20	_____	5,000, $\Pi = 0$
50,000	20	_____ (say, P*)	5,000, $\Pi = 0$
50,000	20	1.5×AVC	P*

6. Easy-Barrows, Inc., produces wheelbarrows. Its costs have been analyzed as follows:

VARIABLE COST

Materials	\$30/unit
Manufacturing labor 3 hours/unit (\$8/hour)	
Assembly labor	1 hour/unit (\$8/hour)
Packing materials	\$3/unit
Packing labor	20 minutes/unit (\$6/hour)
Shipping cost	\$10/unit

FIXED COSTS

Overhead labor	\$50,000/year
Utilities	\$5,000/year
Plant operation	\$65,000/year

SELLING PRICE \$100/unit

- a. Calculate the break-even quantity.
 - b. Calculate the break-even revenue.
 - c. Develop a chart to show profits at quantities of 2,000, 4,000, 6,000, 8,000, and 10,000.
7. SoundNow Co. is considering discontinuance of its line of music tapes due to stiff competition from CDs and other new, technologically advanced recordings. The variable cost of its tapes last year was about 40 percent of its tape revenue, and the

allocated fixed cost equaled \$100,000 per year. Last year's sales were \$250,000, but it is expected that in the future, annual revenue will drop by 20 percent and variable costs will rise to 50 percent of revenue (because of price reductions). Will tapes still be profitable for the company?

8. The Wico company sells widgets at \$9 each; variable unit cost is \$6, and fixed cost is \$60,000 per year.
 - a. What is the break-even output quantity?
 - b. How many units must the company sell per year to achieve a profit of \$15,000?
 - c. What will be the degree of operating leverage at the quantity sold in part a? In part b?
 - d. What will be the degree of operating leverage if 30,000 units are sold per year?

ANSWER: (a) 20,000, (b) 25,000, (c) zero, 4, (d) 2.

9. Two companies, Lawnman Inc. and Tauro Co., are competing in the manufacture and sale of a new type of kryptonite-powered lawnmowers. Lawnman has a somewhat older plant and requires a variable cost of \$150 per lawnmower; its fixed costs are \$200,000 per year. Tauro's plant is more automated and thus has lower unit variable costs of \$100; its fixed cost is \$400,000. Since the two companies are close competitors, they both sell their product at \$250 per unit.

	Lawnman Inc.	Tauro Co
TFC (\$/year)	\$200,000	\$400,000
AVC (\$/unit)	\$150	\$100
Price	\$250	\$250

- a. What is the break-even quantity for each?
 - b. At which quantity would the two companies have equal profits?
 - c. At the quantity obtained in part b, what is each company's degree of operating leverage?
 - d. If sales of each company were to reach 4,500 units per year, which company would be more profitable? Why?
10. Saddam Toaster Co. is contemplating a modernization of its antiquated plant. It now sells its toasters for \$20 each; the variable cost per unit is \$8, and fixed costs are \$840,000 per year.
 - a. Calculate the break-even quantity.
 - b. If the proposed modernization is carried out, the new plant would have fixed costs of \$1,200,000 per year, but its variable costs would decrease to \$5 per unit.
 - (1) What will be the break-even point now?
 - (2) if the company wanted to break even at the same quantity as with the old plant, what price would it have to charge for a toaster?
 - c. If the new plant is built, the company would want to decrease its price to \$19 to improve its competitive position.
 - (1) At which quantity would profits of the old and the new plants be equal (assuming the price of a toaster is \$20 for the old plant but \$19 for the new)? How much

- would the profit be at this quantity?
- (2) Calculate the degree of operating leverage for each plant at the quantity obtained in part (1).
 - (3) If sales are projected to reach 150,000 units per year in the near future, would you recommend construction of the new plant? Why or why not? (Assume that both plants have the capacity to produce this quantity.)
11. The Green Thumb Company produces and sells a special type of fertilizer for yellow roses. Its annual fixed cost was \$10,000. During the past year, the company sold 8,000 bags of its product. It estimates that at this level of sales its degree of operating leverage is 1.5.
 - a. How much was Green Thumb's profit last year?
 - b. At which level of production would the company just break even?
 12. A manager of a firm facing an ACM (average contribution margin) equal to \$5 and total fixed costs of \$10,000 claims that the firm can make a total profit of \$20,000 if it produces 6,000 units. Is he correct?

ANSWER: Yes.

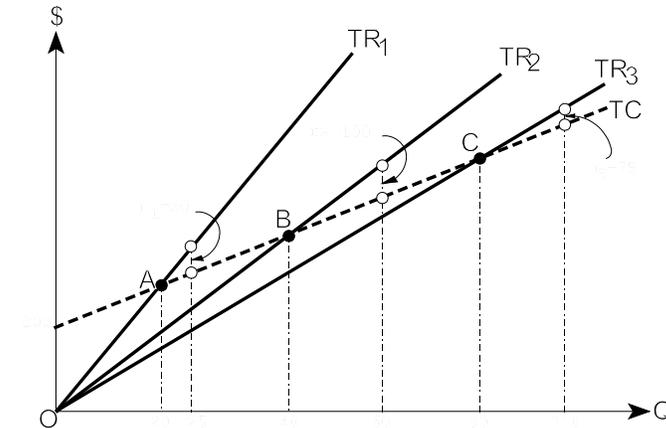
13. The Smith Company made and sold 10,000 picnic tables last year. When output was between 5,000 and 10,000 tables, its average variable cost was \$24. In this output range, each table contributed 60 percent of its revenue to fixed costs and profits.
 - (a) What was the price per table?
 - (b) If the Smith Company increases its price by 10 percent, how many tables will it have to sell next year to obtain the same profit as last year?
 - (c) If the Smith Company increases its price by 10 percent, and if its average variable cost increases by 8 percent as a result of wage increases, how many tables will it have to sell next year to obtain the same profit as last year?
14. Consider a firm which has a TFC of \$200 and an AVC of \$10 per unit. The expected sales at three alternative prices are:

P	Q
20	25
15	60
12.5	110

Which of these three prices would bring it the highest profit? What price should it charge to increase market share without making less than \$50 per period?

● *Solution*

Each of these prices would yield a different total revenue curve, as shown in Figure 5.19.



As price is reduced, the break-even level of sales increases from 20 (at A, for $P = 20$) to 40 (at B, for $P = 15$) and finally to 80 (at C, for $P = 12.5$); more important, the corresponding profit increases from $\pi_1 = 50$ to $\pi_2 = 100$ and then decreases to $\pi_3 = 75$. Consequently, the price of \$15 should be selected if the objective is to maximize profits. On the other hand, if the objective is to increase market share subject to making at least \$50 profit per time period, then a price of \$12.50 should be chosen.

15. ABC Company is contemplating manufacturing a product which can be sold for \$10 per unit on the market. It knows of two production processes, between which it has to choose one, and only one. The following data have been collected for $Q = 150,000$ units.

	Production Process 1	Production Process 2
TVC	800,000	950,000
TFC	400,000	250,000

- Calculate the break-even point for each process.
- Which process should be used if there was a high probability of exceeding sales of 150,000 units? Why?
- Which process should be used if there was a high probability of selling considerably less than 150,000 units? Why?

● *Solution*

a. Break-even point for process 1:

$$Q_1 = \text{TFC}/[\text{P}-\text{AVC}] = 400,000/[10-(800,000/150,000)]$$

$$= 85,713.67$$

Break-even point for process 2:

$$Q_2 = 250,000/[10-(950,000/150,000)]$$

$$= 68,181.82$$

b. If sales are going to exceed 150,000 units, the firm should use process 1 because the average variable cost (assumed to be constant) for each unit above 150,000 will be only \$5.33 for process 1 but will be \$6.33 for process 2.

c. Process 2 should be used if the firm expects to operate below 150,000 units because the variable cost will fall more quickly with this process and profits will decline less slowly than with process 1. █

16. Floyd's Coffee Shop collects on the average \$8 per customer. Its variable cost per customer averages \$5, and its annual fixed cost is \$60,000.
- In order to break even, it must serve _____ customers per year. (Answers: 20,000)
 - If Floyd wants to make a profit of \$20,000 per year, it will have to serve _____ customers per year. (Answer: 26,667)
 - If Floyd serves 30,000 customers per year, its profit will be _____ (Answer: \$30,000)
 - If Floyd serves 25,000 customers per year, the degree of operating leverage is _____ (Answer: 5)
17. Company A sells its product for \$4 per unit, has variable costs per unit of \$2.50, and its fixed cost is \$50,000 per period. Company B sells a product similar to A's for \$3.80 per unit, has variable costs per unit of \$1.80, and its fixed cost is \$80,000 per period.
- The break-even quantity for A is _____ and for B is _____ (Answers: 33,333, 40,000)
 - The break-even revenue for A is _____ and for B is _____ (Answers: \$133,333, \$152,000)
 - The two companies will have the same profit when the production for each reaches units, and the amount of profit for each will be _____ (Answers: 60,000, \$40,000)
 - At the quantity produced in question a, the degree of operating leverage for A will be _____, and for B it will be _____ (Answers: 2.25, 3)
 - If production reaches 70,000 units per period, then
 - A's profit will be higher than B's.
 - B's profit will be higher than A's.
 - both will earn the same profit.
 - cannot tell which will make the higher profit.
18. There are three alternative types of plants to manufacture a certain product

Plant	TFC	AVC
Type A	\$3 million/year	\$500/ton
Type B	\$2 million/year	\$750/ton
Type C	\$1 million/year	\$1000/ton

Suppose the product can be sold regardless of quantity at a constant price of \$1,000/unit.

Fill in the following table:

	Break-even output	Profit at tons/month				
		5000	4000	3000	2000	1000
Type A						
Type B						
Type C						

Assuming that the firm is predicting that it can sell 5,000 tons/month, which plant should be used?

● **Answer:**

- a. *Type A plant: $Q_{BE} = 6000$ tons/year*
Type B plant: $Q_{BE} = 8000$ tons/year
Type C plant: $Q_{BE} = \infty$ (impossible to break even)
- b. *For $Q = 5000$ tons/month = 60,000 tons/year*
 $\Pi_A = \$27$ million/year, etc...