Journal of Marketing Research Article Postprint

DYNAMIC MARKETING BUDGETING FOR PLATFORM FIRMS: THEORY, EVIDENCE AND APPLICATION *

Shrihari Sridhar**
Assistant Professor of Marketing
Eli Broad College of Business
Michigan State University
Email: sridhar@bus.msu.edu

Murali K. Mantrala***
Sam M. Walton Distinguished Professor of Marketing, Trulaske College of Business
438 Cornell Hall
University of Missouri
Columbia, MO 65211-1200
Phone: (573) 884-2734
Email: mantralam@missouri.edu

Prasad A. Naik
Professor of Marketing, Graduate School of Management
University of California Davis
Email: panaik@ucdavis.edu

Esther Thorson
Professor of Strategic Communication, Missouri School of Journalism
University of Missouri
Email: thorsone@missouri.edu

* The authors sincerely thank Ruth Bolton and Ross Rizley for their comments on earlier version for the ***Marketing Science Institute*** Working Paper series.

** This paper is part of the first authorǿs dissertation research.

*** Contact author

DYNAMIC MARKETING BUDGETING FOR PLATFORM FIRMS: THEORY, EVIDENCE AND APPLICATION

Few studies address the marketing budgeting problems of platform firms operating in two-sided markets with cross-market network effects, i.e., the demand from one customer group of the platform influences the demand from its other customer group. Yet such firms, e.g., media firms like newspapers whose customers are subscribers and advertisers, are prevalent in the marketplace and invest significantly in marketing. To enable such firms to make effective marketing decisions, the authors delineate the desired features of a platform firm's marketing response model, specify a new response model, and validate it using market data from a local newspaper. Results show that the firm faces reinforcing cross-market effects, its demand from both groups is impacted by marketing investments, and the model exhibits good forecasting capability. Then, the authors utilize the estimated response model to determine optimal marketing investments over a finite planning horizon and find that the firm should significantly increase its newsroom and sales force investments. Based on this model-based recommendation, the firm's management increased their newsroom budget by 18%. Further normative analysis sheds light on how cross-market and carryover effects alter classical one-sided marketing budgeting rules.

Keywords: Marketing Budgeting, Marketing Dynamics, Kalman Filter, Two-Sided Markets

A fundamental responsibility of marketing managers is to determine the optimal levels and allocation of scarce marketing resources. Consequently, a large volume of work in the marketing models literature has focused on developing normative rules for marketing resource allocation decisions (e.g. Ingene and Parry 1995; Raman 2006), investigating the effectiveness of firmsô marketing efforts empirically (e.g. Hanssens and Ouyang 2002; Manchanda, and Honka 2005), and building implementable model-based tools for optimizing marketing investment decisions in specific settings (e.g. Pauwels et al. 2010). However, surveys of this literature (Hanssens, Parsons, and Schultz 2001; Mantrala 2002; Shankar 2008) reveal that research to date has largely ignored marketing budgeting and allocation decisions by a substantial segment of firms in the economy, namely, platform firms that do business in *two-sided* (e.g., Evans 2003) markets.

Platform-firm markets are distinguished from *one-sided* firm markets in that they have two or more different groups of customers (that is, end-users of their products or service offerings) that businesses aim to acquire and retain (Rochet and Tirole 2005). Examples include print media like newspapers and magazines (readers and advertisers), TV broadcasting (viewers and advertisers), shopping malls (shoppers and retailers), payment cards (cardholders and merchants), and sports clubs (spectators and sponsors). More specifically, platform firmsô two-sided markets are characterized by: (1) the existence of two or more distinct groups of customers interested in different offerings of the platform; (2) at least one customer groupôs main interest is gaining access to the other group; and (3) where the platform can facilitate that access more efficiently than bi-lateral relationships between the members of the groups (Evans 2003). Thus, a platform firmôs demand from one customer group depends upon the demand for it from the other customer group, i.e., platform firms face cross-market network effects (CMEs) (e.g., Chen and

Xie 2007). Intuitively, when CMEs are present, a platform firmøs marketing efforts to stimulate demand from one customer group can have repercussions on its other customer group.

Hereafter, we shall refer to the end-user group that is primarily interested in and consumes an offering of the platform itself, regardless of the presence or absence of any other end-user group, as *attractors* (e.g., consumers of a newspaperøs editorial content, i.e., readers). Further, we shall refer to the end-user group interested in accessing attractors via the platform as *suitors* (e.g., advertisers buying space in a newspaper for their advertisements to readers). Figure 1 provides a diagrammatic representation of a newspaper firm allocating marketing efforts to its attractors and suitors. For example, a newspaper invests in enhancing its product quality (newsworthy content) to retain and grow its number of readers. At the same time, it invests in a sales force to promote its ad-space to the suitors. An increase in the number of suitors can, in turn, impact attractorsô future demand for the newspaper. Specifically, an increase in the ratio of advertising to editorial content in the newspaper can potentially increase or decrease demand from the attractors. (Advertising has a positive effect on readers when the ad provides information deemed to be valuable by the readers. On the other hand, some readers are óad-averseö, see, e.g., Sonnac 2000). Thus, these two sources of revenue — readersô purchases of the newspaperøs editorial content and the advertisersô purchases of the newspaperøs ad-space — are interrelated.

Although many such firms exist, including Fortune 100 companies like Time Inc. (magazine) and FOX (television network), the literature on optimal marketing resource allocation by platform firms is scarce. Two novel and challenging aspects of these firmsô marketing budgeting decision problems that any model-based solution must address are: (1) the differential dynamic (e.g., carryover) effects of marketing on the demands from the dual or multiple sides of the platform firmȇs business and (2) the CMEs between the multiple end-user groups. Evans and Schmalensee (2007) effectively summarize these challenges: ñ... its [platformȇs] customer groups form a dynamic system and live in a non-linear world. ... Changes in customers of one type affect customers of the other type é ò and the firm ñé must consider the interdependence of these two groups of customers at every turn.ò

Motivated by these observations, we propose a model and develop a theory for optimal marketing investment decisions by platform firms. Specifically, the objectives of our research are three-fold:

1. Propose a platform-firm market response model taking into account the quintessential features of the two-sided firm setting, and validate the proposed response model using market data from an archetypal platform firm, namely, a daily print newspaper company.
2. Develop a model-based algorithm and decision tool to assist managers in determining the optimal paths (trajectories), over some finite planning horizon, of the marketing investments towards the two sides of their market, and demonstrate the application and benefits of this tool for the participating newspaper company.
3. Derive new normative results for platform firm marketing budgeting to extend current one-sided marketing budgeting theory.

To meet the first objective, we propose a dynamic response model that captures the key features of a platform firmȇs two-sided market setting, e.g., CMEs, dynamic effects, and interaction between the demand of one end-user group and marketing effort directed at the other group. Then, using data from the local daily newspaper firm, we estimate the proposed response model using state-space methods (e.g., Xie et al 1997; Naik, Mantrala, and Sawyer 1998). The important findings from this empirical analysis are, first, the data support the proposed two-sided market response model, i.e., a model that includes CMEs performs better than the one without CMEs. Second, the estimated attractor and suitor effects are both significantly positive, revealing that this newspaper is a *reinforcing* platform, in contrast to Wilburȇs (2008) empirical finding of ad-averse TV viewers. Third, the significant CMEs imply marketing efforts have both direct and indirect effects, i.e., efforts towards one end-user group also influence the other end-user group. Furthermore, we replicate the proposed response model using data from a different newspaper, held by a different company and obtained similar results. This replication study enhances our confidence in the validity of the proposed market response model.

To meet the second objective, we develop an algorithm to determine the investment trajectories (time-paths) that maximize discounted long-term profits of the focal platform firm over a specified planning horizon. Because of the finite horizon nature of the problem and the complex interdependent demand dynamics, we encounter a non-linear, two point boundary value problem. We solve this problem by extending the approach of Naik, Raman and Winer (2005) to our platform firm setting. We incorporate this algorithm in a model-based decision aid that managers can use to evaluate and compare the outcomes of any selected investment-mix trajectories to those of the optimal policies. Thus, we contribute toward reducing the ñacute shortage of normative studies developing navigation systems that allow managers to optimize marketing effortsé ò as noted by Leeflang et al. (2009, p. 16).

Subsequently, we demonstrate the practical benefits of this decision aid for the focal newspaper firm. Specifically, we found that its marketing investments towards readers and advertisers were suboptimal. The managers were under-spending on newsroom resources, i.e., investments in news quality that generates revenues from attractors (readers). By reallocating resources to achieve the optimal levels, they can increases profits by about 28%. Based on our recommendations, the companyôs management was not only persuaded of the value of resource allocation based on econometric and optimization methods, but also decided to increase the firmôs newsroom budget by approximately $500,000 (which represents an 18% increase in the current newsroom budget). This decision stands in stark contrast to the recent cuts in newsroom investments made by other US daily newspapers to improve their financial performance (e.g., Rosentiel and Mitchell 2004). Thus, we believe that other newspapers also would benefit from the proposed model and decision aid for making informed marketing decisions.

Finally, to meet the third objective, we perform a normative analysis and deduce three new propositions that shed light on how dynamic CMEs and carryover effects modify the traditional rules for optimal marketing investments in the absence of CMEs. For example, we establish that, compared to one-sided settings, optimal investment levels should be higher (lower) for a reinforcing (counteractive) platform firm where both CMEs are positive (where the two CMEs have opposite signs).

The rest of the paper is organized as follows. The next section reviews the literature on platform firms that reveals the gap in research on optimal marketing budgeting by such firms. We then summarize how the platform firm's setting differs from the one-sided firm's setting, delineating the relevant features to capture the institutional reality of the two-sided setting. Next, we specify and validate the proposed response model empirically. Subsequently, we develop the marketing-mix algorithm and demonstrate its application to aid decision-making. Finally, the normative analyses yield new propositions on optimal marketing investments by platform firms. We close the paper by summarizing the key takeaways and identifying avenues for model extensions and future research.

PLATFORM FIRM BUSINESS STRATEGIES: SELECTED LITERATURE REVIEW

Platform-firms focus on the idea that they cater to two (or more) distinct groups of customers with members of at least one group wishing to access the other group (Evans 2003). Economists have primarily focused on platform firm pricing strategies. For example, Parker and

Van Alstyne (2005), Rochet and Tirole (2005), Armstrong and Wright (2007), and Bolt and Tieman (2006) examine how standard pricing policies should be restructured in the presence of two-sidedness, while Caillaud and Jullien (2003) study how pricing rules should change in a setting of competing platforms. Roson (2004) provides a detailed review of pricing-related work on two-sided markets. His review highlights that when cross-market network effects (CMEs) are present: (a) prices applied to the two market sides are both directly proportional to the price elasticity of the corresponding demand (Rochet and Tirole 2005); (b) socially optimal pricing in two-sided markets leads to an inherent cost recovery problem, inducing losses for the monopoly platform (Bolt and Tieman 2006); and (c) in a duopoly, the platform charging the lower fees could potentially capture both sides of the market and result in market monopoly (Caillaud and Jullien 2003). Eisenmann, Parker, and Alstyne (2006) summarize a number of the strategic pricing management takeaways from the preceding economics literature. They note that many emerging platform-firms struggle to establish and sustain their two-sided networks due to a common mistake: ñIn creating (pricing) strategies for two-sided markets, managers have typically relied on assumptions and paradigms that apply to products without network effects. As a result, they have made many decisions that are wholly inappropriate for the economics of their industries.ò

The marketing literature on platform firms is small but growing in recent years. For example, Chen and Xie (2007) examine the relationship between high levels of attractor loyalty and platform firm profits under competition. Wilbur (2008) estimates a structural model of suitor (advertiser) demand for attractors (viewers) and viewer demand for advertisers in the television industry and finds evidence for ad aversion among viewers. Gupta, Mela, and Vidal Sanz (2009) develop a model to calculate the customer lifetime value (CLV) of the buyers (attractors) and sellers (suitors) in an auction house, and find that buyer CLV is higher than that of the seller. Kind, Nilssen, and Sørgard (2009) investigate how competitive forces may influence the way media firms like TV channels and newspaper firms raise revenue ï from advertisers, or via direct payments from attractors, or from both sources. They show that that the less differentiated the media firms' content, the larger is the fraction of their revenue coming from advertising. On the other hand, direct payment from the media consumers becomes more important as the number of competing media products becomes large.

The marketing-mix problem of the platform-firm has received limited attention in the literature. A notable exception is the work by Mantrala, Naik, Sridhar, and Thorson (2007) who employ a static model to study the optimality of marketing expenditures towards subscribers and advertisers of daily newspaper firms. However, their model specification meets only few of the requirements for an on-going platform-firm that we delineate in the next section. Moreover, in this paper, we perform a longitudinal analysis, develop an optimal allocation algorithm and demonstrate its use to assist the firmŝ managers to plan dynamic marketing-mix investment policies over finite planning horizons.

Next, we present the key elements of platform firm market response models and how they differ from one-sided (ñclassicò) firmŝ response model and allocation problems studied in the extant literature (see, e.g., Mantrala 2002). We focus on monopoly models consistent with the empirical setting of local daily newspapers.

ONE-SIDED VS TWO-SIDED FIRMS' MARKETING RESPONSE MODELS

A monopoly firmŝ marketing budgeting and planning problems are addressed by formulating a market response model that relates demand from one revenue source (e.g., a product, region or end-customer group) to one or more marketing variables (e.g., advertising, sales force). Further, we distinguish between a firmôs marketing budgeting and allocation problems that involve only a single sales entity (e.g., a single product) versus multiple sales entities (e.g., the multi-product problem treated by Doyle and Saunders 1990). Lastly, some optimal marketing budgeting analyses are static (e.g., Dorfman and Steiner 1954), while others are dynamic with sales decay or carryover effects (e.g., Nerlove and Arrow 1962). Thus, classic monopoly firm budgeting problems are of the following types: static or dynamic, single sales entity problems (see Table 1 for market response model types I and II); and static or dynamic, multiple sales entity problems (see Table 1 for model types III and IV). Notably, in classic Model Types III and IV, a marketing input set for one sales entity (e.g., advertising or price for one product) could have a cross-price effect, e.g., Reibstein and Gatignon (1984), or ñspilloverò effect, on demand for the other product, e.g., Ingene and Parry (1995), Erdem and Sun (2002). Thus, the marketing effort, say advertising, aimed at one market segment (e.g., consumers in Germany) can directly impact (via spillover effect) another segment, say, consumers in Belgium (see, e.g., Gensch and Welam 1973, Brody and Finkelberg 1997).

As noted, all platform firms cater to at least two end-user groups or sales entities, making them inherently multiple sales entity problems. Then, how do they differ from Model Types III and IV? First, all platform firm problems involve *two or more distinct end-user (customer) groups* as sales entities, each having its own budget constraint and seeking primarily a different offering from the firm. In contrast, classic firmsô marketing budgeting decision problems that involve multiple sales entities need not involve distinct customer groups, e.g., a problem involving spending on promotion of complementary products, say cake mix and frosting, to the *same* end-users. Of course, classic firm problems sometimes do involve two or more distinct end-user groups (with distinct budgets) as in Brody and Finkelberg (1997). Hence, we distinguish such multi-group classic firm problems from those of platform firms. The main difference is that, for a classic firm, the level of demand from one group does not directly impact the demand from the other group. In contrast, in a platform firm problem, the level of demand from one group *directly impacts* the demand from the other group and vice versa, i.e., *cross network* effects arise in platform firm, but not in the classic firm. Model Types V and VI in Table 1 indicate this distinction in the market response models for platform firms. Moreover, because of these network effects, one groupøs demand increases as a function of the otherøs demand when the network effect is positive (i.e., a reinforcing platform). When one of the network effects is negative, the platform is counteractive. Also, the marketing effort toward one group and the level of the demand of the other side may have an interactive effect on the demand from the first group. For example, the effect of selling effort aimed at a newspaperøs advertisers is likely to vary with the number of its subscribers. Third, in multi-group classic firm problems, the demand from either group remains positive even if the demand from the other side goes to zero. For example, demand for a product in Germany exists even if the demand for it in Belgium is zero (and vice versa). However, in a platform firm setting, the demand from at least one side, the suitors, (e.g., advertisers) vanishes if attractorsø (e.g., readersø) demand for the platform disappears. Table 1 indicates this distinguishing feature of platform firms. To summarize, the essential features of a response model for a platform firmøs marketing are:

1. The demand from one side should directly impact the other sideøs demand for the relevant offering of the platform (and vice versa).
2. At least one sideøs (attractorsø) demand should have a positive direct effect on the other sideøs (suitorsø) demand level.
3. The suitorsø demand should be zero when the attractorsø demand is zero.

4. Attractorsô demand should remain non-zero and finite even when suitorsô demand is zero.
5. An interactive effect between marketing to one side and demand from the other should be allowed.
6. One sideôs demand should be a monotonic function of the other sideôs demand.

Additionally, for optimal budgeting by a platform firm, desirable model features are:

7. Previous period demand should affect current demand on the same side.
8. Demands should increase at a decreasing rate as contemporaneous marketing efforts are increased.

In the next section, we propose a platform response model with the above features. Note that Mantrala et al. (2007) do not incorporate the features (3) as well (5) through (7). Thus, by generalizing their static model, we deepen the understanding of planning marketing investments by platform-firms in the presence of dynamic cross-market network effects.

MODEL DEVELOPMENT

A two-sided market response function

We consider a monopolist platform firm such as a local daily newspaper (98% of daily newspapers are the only ones published in their market, Picard 1993). Additionally, we focus on how the sales on both sides of the platform grow in response to marketing communications investments (e.g., Simon and Arndt 1980, Naik, Prasad, and Sethi 2008) rather than price because prices stay fixed for long durations. For example, newspaper retail prices remained constant over four to seven years (Bils and Klenow 2002), and advertising rates for local newspapers remain unchanged for a year after they are set (Warner and Buchman 1991, p 205).

Let A_t and S_t denote the dollar sales revenues at time t from the attractor and suitor sides of the market, respectively. Also, let u_t and v_t represent the marketing investments of the platform towards its attractor and suitor sides, respectively. Then we specify the platform's dynamic sales-marketing effort response system as follows:

$$A_t = A_{t-1}^{\lambda_A} \tilde{S}_{t-1}^{\theta_{SA}} u_t^{\beta_u} e^{\varepsilon_{At}} \quad (1)$$

$$S_t = \tilde{S}_{t-1}^{\lambda_S} A_{t-1}^{\theta_{AS}} v_t^{\beta_v} e^{\varepsilon_{St}} \quad (2)$$

where $\tilde{S}_{t-1}$ = Max{ S_{t-1} , D_t } and D_t = {1 when S_{t-1} = 0, 0 when $S_{t-1} > 0$}.

Equation (1) states that sales from the attractor side is a product of attractor sales in the previous period (A_{t-1}), suitor sales in the previous period (S_{t-1}), and contemporaneous attractor market-focused marketing investment (u_t), e.g., investment in news quality. The exponent of A_{t-1}, i.e., λ_A , represents the attractor dynamics effect ; while the exponent of u_t, i.e., β_u, represents the attractor marketing-sales elasticity. More specifically, we capture diminishing returns to the current-period marketing investment when $0 < \beta_u < 1$. Next, θ_{SA} denotes the cross-market effect (CME) of suitor sales in the previous period on current attractor sales (or *suitor-repercussion effect*). This parameter value can be positive or negative depending on whether attractors value suitors' use of the platform or not, e.g., newspaper readers may be "ad-lovers" (Sonnac 2000) or TV viewers may be "ad-averse" (Wilbur 2008).

Similarly, in Equation (2), the exponents λ_S, β_v, and θ_{AS} represent respectively suitor dynamics effect, direct effect of current investment v_t, and the dynamic effect of attractor demand on suitor demand. Diminishing returns to marketing spending in Equations (1) and (2) rules out unbounded growth beyond a certain sales level because incremental marketing dollars do not draw additional readers (or advertisers) profitably, thus leading to finite optimal spending. We expect the CME θ_{AS} to be positive because suitors seek access to attractors, and their demand for the medium of the platform should increase when they observe a higher level of attractors' demand for the platform. Thus, Equations (1) and (2) together represent the market response model for the platform firm setting, which is *reinforcing* when $\theta_{AS} > 0$ and $\theta_{SA} > 0$ and *counteractive* when $\theta_{AS} > 0$ and $\theta_{SA} < 0$.

Importantly, the complete specifications of Equations (1) and (2) allow attractor sales to remain non-zero and finite when $S_{t-1} = 0$. That is, the model allows the platform to grow its attractor base even if it intermittently obtains zero revenues from suitors. For example, some media platforms may not secure non-zero advertiser revenues every period.

More specifically,

- when S_{t-1} is *non-zero*, attractor demand in period *t* depends on the cross-market effect of suitor demand in period *t-1* as well as level of attractor demand in *t-1* and investments in marketing towards attractors in period *t*; and suitor demand in period t depends on the cross-market effect of attractor demand in period *t-1* as well as level of suitor demand in *t-1* and investments in marketing towards suitors in period *t*. That is,

  [illegible] = [illegible] (1)

  [illegible] = [illegible] (2)

  (since [illegible] is 0, [illegible] is 1 and [illegible] = S_{t-1});

  - when S_{t-1} is *zero*, attractor demand in period *t* depends only on the level of attractor demand in the previous period and investments in marketing towards attractors in period *t*; while suitor demand in period *t* arises only because of cross-market network effect of attractor demand in *t -1* and investments in marketing towards suitors in period *t*, i.e. [illegible] = [illegible] and [illegible] = [illegible] (since [illegible] is 1, [illegible] = 1, and 1[illegible] is 1; ə$_1$ and ə$_2$ being freely estimable intercepts).

In summary, the general system of Equations (1) and (2) constituting the platform firm response model possesses the desirable features:

- The parameters θ_{AS} and θ_{SA} allow for direct demand interdependence;
- With θ_{AS} being positive, suitors demand increases as attractors demand increases, and is zero when attractor sales vanish, i.e., $S_t = 0$ when $A_{t-1} = 0$;
- Attractor demand remains non-zero and finite even when suitors demand is zero;
- The multiplicative form of Equations (1) and (2) implicitly incorporates the interactions between marketing investment towards one side of the market and demand from the other side of the market. That is, the total effect of a contemporaneous marketing effort toward one side of the market augments the accumulated sales on the other side.

The system of Equations (1) and (2) pose novel estimation and optimization challenges. On the estimation side, given their interdependent nature, the platform firmős demands are affected by correlated shocks over time. Therefore, ordinary least squares (OLS) based estimates are biased because OLS ignores the inter-temporal dependence (Naik, and Tsai 2000). On the optimization side, to obtain dynamically optimal marketing budgets with the response system Equations (1) and (2), we need to solve a multivariate non-linear boundary value problem, with direct interdependence between the states (revenues) and controls (marketing investments), a problem thatős new to the extant marketing literature. We explicate how we resolved these estimation and optimization challenges in the next two sections. In our empirical application, the case of $S_{t-1} = 0$ does not arise for any t (although we emphasize that the proposed estimation and optimization approaches hold for the general system).

EMPIRICAL ANALYSIS

We present the data from a major newspaper company, describe an estimation approach, conduct model selection and diagnostics, and furnish the empirical results. In addition, we replicate the findings by using a different newspaperős data to lend further validity to the proposed model.

Data

A privately-held media company, which has diversified holdings in newspaper and magazine publishing and wishes to remain anonymous, provided the data for its major newspaper. Medium-sized newspapers with subscriptions less than 85,000 form its core business. The particular print newspaper we examine is a monopolist in its city-region, producing differentiated content due to its local features. A third-party audit bureau verifies the subscription figures and provides demographic information (e.g., age, gender, income, home ownership) on the newspaperǒs readers (attractors) to its prospective advertisers (suitors) who may purchase ad space in the future. The newspaper appeals mainly to advertisers who seek to reach audiences older than 50, and these advertisers include financial companies and assisted living centers. Because the newspaper invests heavily in marketing to these advertisers, its share of local advertisersô print advertising budgets is quite high.

The dataset contains information on revenues from attractors (readers) and suitors (advertisers). In addition, the monthly marketing efforts towards these two revenue sources, namely, dollar spending on newsroom and ad-space sales force are provided. Prior work in the journalism literature suggests that investments in the newsroom are akin to investments in product quality (Litman and Bridges 1986), as the newsroom department is responsible for providing accurate and engaging news stories to its diverse local readers. Newsroom investments vary due to a) the continual hiring or termination of part-time employees in the newsroom; b) changes in the population and demographics trends in the county; c) changes in the amount of retailer-based economic activity in the county; and d) the occurrence of important events in the county (e.g. local government elections). The field sales forceǒs main task is to provide recent figures on the size and composition of the attractor base to the suitors as well as inform them about the potential benefits of purchasing ad-space in certain sections of the newspaper that their targeted attractors might read. Table 2 shows that the newspaper spends about equally in the newsroom and on the sales force. Panels A and B in Figure 2 plot the attractor and suitor sales over time. We observe from Panel B in Figure 2, that the special case of suitor sales being zero for some *t* does not apply in our empirical setting. Panel C in Figure 2 plots the marketing investments over time.

Estimation Approach

We apply filtering theory (e.g., Jazwinski 1970, Harvey 1994) to calibrate the proposed model using market data. Specifically, our response model represents a system of difference equations with non-linear decision variables, inter-temporal dependence of demand, and potentially correlated error structures. Also, in practice, the observed data on attractor and suitor sales may contain measurement errors. Naik and Tsai (2000) show the importance of separating the dynamics of market response from measurement errors when estimating market response functions. Therefore, using the Kalman Filter (KF), we develop an algorithm consisting of three steps: i) the transition equation step that specifies the sales dynamics; ii) the observation equation step that links the sales dynamics to the actual sales data; and iii) a likelihood function built recursively, and subsequently maximized to obtain the parameter estimates and to infer statistical significance. We describe the three steps in turn. (If suitor sales equal zero, we would augment the variable space via the definitions of ([illegible], [illegible]) and correspondingly extend the parameter vector to include $ə_1$ and $ə_2$.)

Step 1. The transition equation specifies the model dynamics and captures the influence of marketing efforts. We log-transform the response model to get,

$$\begin{bmatrix} Ln(A_t) \\ Ln(S_t) \end{bmatrix} = \begin{bmatrix} \square_{\square} & \square_{\square\square} \\ \square_{\square\square} & \square_{\square} \end{bmatrix} \begin{bmatrix} Ln(A_{t-1}) \\ Ln(S_{t-1}) \end{bmatrix} + \begin{bmatrix} \beta_u Ln(u_t) \\ \beta_v Ln(v_t) \end{bmatrix} \tag{3}$$

Denoting Z_{At}= Ln(A_t), Z_{St}=Ln(S_t), w_{ut} =Ln(u_t), w_{vt} =Ln(v_t), we specify the transition equation as

$$\begin{bmatrix} Z_{At} \\ Z_{St} \end{bmatrix} = \begin{bmatrix} \square_{\square} & \square_{\square\square} \\ \square_{\square\square} & \square_{\square} \end{bmatrix} \begin{bmatrix} Z_{A,t-1} \\ Z_{S,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{u,t} \\ \beta_v w_{v,t} \end{bmatrix} + \begin{bmatrix} \square_{A,t} \\ \square_{S,t} \end{bmatrix} \tag{4}$$

where A_t and S_t represent the attractor and suitor revenue respectively; $w_{u,t}$, and $w_{v,t}$ represent log-transformed investments towards the attractors and suitors respectively; and β_u and β_v represent marketing effectiveness. The transition error vector $\square_t = (\square_{A,t}, \square_{S,t})'$ follows $N(0, Q)$, where Q is the 2 x 2 covariance matrix, which can be non-diagonal to allow for correlated shocks in the system. The initial means of the transition vector $\square_0 = (\square_{A,0}, \square_{S,0})'$, which are analogous to regression-intercepts, are estimated from the market data.

Attractor sales exhibit a downward trend (see Figure 2), consistent with the general decline in print newspaper readership (e.g., Patterson 2007). Also, suitor sales exhibit seasonality during the year. To incorporate these aspects, we augment the transition equations with a time-trend variable for the attractor sales dynamics and seasonal dummies for the suitor sales dynamics. The augmented transition equation is,

$$\begin{bmatrix} Z_{At} \\ Z_{St} \end{bmatrix} = \begin{bmatrix} \square_{\square} & \square_{\square\square} \\ \square_{\square\square} & \square_{\square} \end{bmatrix} \begin{bmatrix} Z_{A,t-1} \\ Z_{S,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{u,t} \\ \beta_v w_{v,t} \end{bmatrix} + \begin{bmatrix} \gamma_1 t \\ \gamma_2 D_{1,t} + \gamma_3 D_{2,t} \end{bmatrix} + \begin{bmatrix} \square_{A,t} \\ \square_{S,t} \end{bmatrix} \tag{5}$$

where γ_1 captures the trend effects on Z_{At}, while γ_2 and γ_3 control for the seasonal year-end and beginning effects via the dummy variables D_{1t} and D_{2t} defined as follows:

$$D_{1t} = \begin{cases} 1, \text{if } t = \{11,12\}, \{23,24\}, \ldots\ldots \{119,120\} \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

$$D_{2t} = \begin{cases} 1, \text{if } t = \{1,2\}, \{13,14\}, \ldots\ldots \{109,110\} \\ 0 \text{ otherwise} \end{cases} \tag{7}$$

Step 2. We link the transition equation to observed data as follows:

$$\begin{bmatrix}\square_{\square\square}\\ \square_{\square\square}\end{bmatrix} = \begin{bmatrix}\square_{\square\square}\\ \square_{\square\square}\end{bmatrix} + \begin{bmatrix}\square_{\square,\square}\\ \square_{\square,\square}\end{bmatrix} \tag{8}$$

where Y_{At} and Y_{St} represent the actual log-transformed observed values of attractor and suitor revenues, and the observation error vector $\square_{\square} = (\square_{\square,\square}, \square_{\square,\square})'$ follows *N(0, H)*, where *H* represents a 2 x 2 matrix that can be non-diagonal to allow for correlated shocks to the system.

Step 3. Using the KF recursions (e.g., see Harvey 1994, p. 88) and denoting $Y_t = (Y_{At}, Y_{St})'$, we compute the log-likelihood function,

$$LL(\Psi) = \sum_{t=1}^{T} Ln(p(Y_t \mid \Im_{t-1}) \tag{9}$$

where $p(\cdot|\cdot)$ denotes the conditional density of Y_t given the history of information up to the previous period, $\Im_{t-1} = \{Y_1, \cdots, Y_{t-1}\}$. The vector Ψ contains the model parameters $(\square_{\square}, \square_{\square}, \square_{\square\square}, \square_{\square\square}, \square_{\square}, \square_{\square}, \square_{\square}, \square_{\square}, \square_{\square})'$ together with the observation and transition covariance matrices and the initial means. By maximizing Equation (9) with respect to Ψ, we obtain the parameter estimates $\hat{\Psi} = \text{argmax}(LL(\Psi))$ and infer their statistical significance via the information matrix.

Model selection and diagnostics

Applying the above approach to newspaper data, we estimated Equations (5) and (8) that include trend and seasonality, dynamic effects, marketing effectiveness, CMEs, and correlated errors. To determine the variables to be retained in the model, we apply model selection theory (see Burnham and Anderson 2002). The central idea of model selection is to balance parsimony (i.e., include few variables) and fidelity (i.e., improve goodness of fit). By including additional variables in the model, we can improve the model's fit to the observed data, but at the cost of over-parameterizing the model, thereby reducing estimation precision and forecasting accuracy.

Information criteria such as Akaike Information Criterion (AIC), bias-corrected Akaike information criterion (AIC_c), and Bayesian Information Criterion (BIC) are commonly used to compare nested or non-nested models. A smaller value of an information criterion indicates a better model. Besides information criteria, we conduct diagnostic checks by computing mean absolute deviation (MAD) of the modelôs predicted attractor and suitor outcomes. In addition, we test for exogeneity of newsroom and sales force investments by applying the approach due to Engle, Hendry, and Richard (1983) —see Appendix A for details.

We compare the four nested models that incrementally introduce the phenomena of interest. Model 1 includes trend, seasonality and carryover effects; Model 2 adds marketing variables to Model 1; Model 3 introduces CMEs to Model 2; and Model 4 admits correlated error terms to Model 3.

Table 3 reports the results of model selection. First, Model 2 with marketing effectiveness variables outperforms the one with lagged effects, trend and seasonality variables only (Model 1); e.g. the AIC value improves by 34.2% from -771 to -1035. Second, Model 3 with CMEs outperforms both Model 1 and Model 2; e.g. the AIC value (= -1120) improves by 45.2% over Model 1 and 8% over Model 2. Finally, Model 4 with CMEs and correlated errors outperforms the other three models; e.g. the AIC value (= -1122) improves by 45.5% over Model 1 and 8.4% over Model 2 and 0.2% over Model 3. Hence, we retain Model 4 for further analyses.

The retained model with CMEs and correlated errors also fits the data well; e.g., Table 3 shows low in-sample MAD (0.26%). Furthermore, to assess predictive ability, we estimate the models using 90 observations and forecast the remaining 30 observations in the hold-out sample using the calibrated model. We find that its predictive ability is also high, as evidenced by low out-of-sample MAD (0.35%). Figure 2 presents the visual evidence for the proximity of actual vs. estimated sales. Finally, the Engle at al. (1983) test for exogeneity shows that newsroom and sales force investments are weakly exogenous (see Appendix A for details). Thus, the above model selection and diagnostic checks furnish evidence that the proposed model is a parsimonious specification, fits the data well, and forecasts satisfactorily. We now present the empirical results.

Empirical Results

Control Variables. Table 4 presents the key parameter estimates and *t*-values. A significant value of γ_1 (-0.001, $p < 0.05$) indicates a declining trend in attractor revenues. The significant estimates γ_2 (0.04, $p < 0.05$) and γ_3 (-0.11, $p < 0.05$) suggest seasonality in suitor revenues. Specifically, we find a statistically significant increase in suitor revenue in the Thanksgiving and Christmas season followed by a drop-off in the beginning of the year. This finding comports with the experience of many small newspapers in the U.S.; for example, the *Monroe County Advocate* designs a ñChristmas Carolò supplement to accommodate more ad-space during holiday months because about 41% of news readers find ads most helpful during shopping sales (Newspaper Association of America Report 2006).

Cross-market effects. We find that the attraction effect (θ_{AS}) and the suitor effect (θ_{SA}) are *both positive* and significant (θ_{AS} = 0.16, $p < 0.05$; θ_{AS} = 0.12, $p < 0.05$), suggesting that this particular newspaper is a *reinforcing platform*. Positive suitor effects suggest that, unlike T.V. viewers who are ad-averse (Wilbur 2008), newspaperôs readers value advertising. Several factors support this finding: newspapers are a high-attention medium not suitable for multi-tasking; the newspaper ads are ñkeepableò since they can be cut out and used at a later period; and newspapers are viewed as a less-intrusive and more trustworthy source of information (Conaghan 2006).

*Sales dynamic effects***.** Considering the estimated values are less than 1 in magnitude, both parameters represent sales carryover effects, and the attractor carryover coefficient (λ_A) and the suitor carryover coefficient (λ_S) are positive and significant ($\lambda_A = 0.69$, $p < 0.05$; $\lambda_S = 0.63$, $p < 0.05$). A moderate value of $\lambda_A = 0.69$ suggests that newly acquired attractors may not stay with the newspaper for extended periods of time. This finding corroborates with the general trend of declining readership and the idea that local readers may not find enough community content in the newspaper (Project for Excellence in Journalism Report 2008).

Marketing effectiveness. The effectiveness of newsroom investments with respect to attractor revenues (β_u) and the sales force with respect to suitor revenues (β_v) are both positive and significant ($\beta_u = 0.25$, $p < 0.05$; $\beta_v = 0.18$, $p < 0.05$). These results support journalism scholarsô conceptual assertions that the cuts in newsroom investments adversely affect newspaper performance (Overholser 2004).

In sum, the empirical analyses show that market data support the proposed model, furnish strong evidence of the presence of CMEs, and shed light on the indirect marketing elasticities induced by CMEs. The empirical results not only are new to the marketing literature but also are of considerable value to the newspaper firm in particular, and the daily newspaper industry in general.

Replication. To further validate the model, we obtained additional data from a different privately-held media company. The dataset contains information on revenues from attractors (readers) and suitors (advertisers) as well as investments in the newsroom and sales force.

Applying the Kalman Filter approach, we repeated the analysis in the previous subsections. Once again, we found that the proposed platform-firm response model fits the data well, outperforms competing specifications (those without CMEs and/or with uncorrelated error terms), and forecasts satisfactorily. For brevity, we summarize the main findings below.

First, we find evidence for reinforcing CMEs (θ_{AS} = 0.27, p < 0.05; θ_{AS} = 0.18, p < 0.05). Second, we again find moderate values of carry-over in the attractor and suitor revenues (λ_A = 0.48, p < .05, and λ_S = 0.50, p <0 .05, respectively). Third, the newsroom investments significantly affect attractor revenues (β_u= 0.67, p < 0.05) and sales force investments significantly impact suitor revenues (β_v = 0.34, p <0.05). Because of the implicit interaction effects in the multiplicative model, newsroom investments have an indirect impact on suitor revenues, and sales force investments have an indirect impact on attractor revenues. Finally, we find evidence for a declining trend in the attractor revenues ($ɔ_1$ = -0.003, p < 0.01), an increase in the same in the November-December months ($ɔ_2$ = 0.09, p < 0.01), and a fall in the suitor revenues in July ($ɔ_3$ = -0.06, p < 0.01). Thus, this replication of previous findings —based on a different newspaper of a different company— enhances our confidence in the validity of the proposed model. Next, we show how this validated model can be used to determine the dynamically optimal marketing budgeting and allocations.

A MANAGERIAL DECISION TOOL AND REAL-WORLD APPLICATION

Problem motivation

Two-sided media firms such as daily newspapers, magazines, and radio stations must decide how much they should invest in news quality directed at readers or listeners, and in sales force effort directed at advertisers, over some finite planning horizon. However, many newspapers tend to view investments in the newsroom as costs that they can cut in order to improve profits. Such cutbacks are questionable because they are not based on a systematic assessment of the long-term consequences for circulation (attractor revenues) and, in turn, advertising revenue (suitors). In contrast, the model-based decision tool we develop below accounts for both long-term effects (λs), and CMEs (θs), in deriving optimal marketing investment trajectories over a pre-specified planning horizon.

Decision Tool

The platform firmôs goal is to maximize the total profit $J(u, v)$ over the planning horizon T. More formally, we capture this goal via the objective function:

$$Maximize \quad J(u,v) = \sum_{t=1}^{T} e^{-\rho t} \pi(A_t, S_t, u_t, v_t) \tag{10}$$

where $\pi(A,S,u,v) = m_A A + m_S S - u - v$. To maximize Equation (10), we apply Hamiltonôs maximum principle (e.g., see Kamien and Schwarz 1992, Sethi and Thompson 2006) and derive the necessary conditions (see Appendix B); the optimal controls

$$\begin{bmatrix} w_{ut}^* \\ w_{vt}^* \end{bmatrix} = \begin{bmatrix} Ln(\beta_u \mu_{1t}) \\ Ln(\beta_v \mu_{2t}) \end{bmatrix}, \tag{11}$$

and the co-state dynamics

$$\begin{bmatrix} \mu_{1t} - \mu_{1,t-1} \\ \mu_{2t} - \mu_{2,t-1} \end{bmatrix} = \begin{bmatrix} \rho + 1 - \lambda_A & -\theta_{AS} \\ -\theta_{SA} & \rho + 1 - \lambda_S \end{bmatrix} \begin{bmatrix} \mu_{1,t-1} \\ \mu_{2,t-1} \end{bmatrix} + \begin{bmatrix} -m_A e^{Y_{At}} \\ -m_S e^{Y_{St}} \end{bmatrix}, \tag{12}$$

where (w_{ut}^*, w_{vt}^*) represent the optimal investments towards the attractors and suitors, and (μ_{1t}, μ_{2t}) are the co-state variables corresponding to the attractor and suitor sales dynamics, respectively. To find the best investments strategies, we have to not only solve jointly the optimal controls in Equation (11), the co-state equations in Equation (12), and the state equations in Equation (5), but also account for the initial conditions given by

$$\begin{bmatrix} Y_{A0} \\ Y_{S0} \end{bmatrix} = \begin{bmatrix} \ln(\bar{A}) \\ \ln(\bar{S}) \end{bmatrix} \tag{13}$$

and the terminal conditions given by

$$\begin{bmatrix} \mu_1^* \\ \mu_2^* \end{bmatrix} = \frac{1}{(\rho + 1 - \lambda_A)(\rho + 1 - \lambda_S) - \theta_{AS}\theta_{SA}} \begin{bmatrix} \rho + 1 - \lambda_S & -\theta_{SA} \\ -\theta_{AS} & \rho + 1 - \lambda_A \end{bmatrix} \begin{bmatrix} m_A \bar{A} \\ m_S \bar{S} \end{bmatrix} \tag{14}$$

which represent the steady-state of Equation (12) evaluated at the market conditions. Due to the above initial and terminal conditions, the resulting dynamic maximization problem is called a "two-point boundary value" (TPBV) problem.

To solve this TPBV problem, we adapt the numerical algorithm proposed by Naik, Raman, and Winer (2005). First, we augment the state vector to contain both the state and co-state variables. That is, we define the 4 x 1 vector $z = (Y_A, Y_S, \mu_1, \mu_2)'$, whose transition is given by

$$\begin{bmatrix} Y_{At} - Y_{A,t-1} \\ Y_{St} - Y_{S,t-1} \\ \mu_{1t} - \mu_{1,t-1} \\ \mu_{2t} - \mu_{2,t-1} \end{bmatrix} = \begin{bmatrix} -(1-\lambda_A) & \theta_{SA} & 0 & 0 \\ \theta_{AS} & -(1-\lambda_S) & 0 & 0 \\ 0 & 0 & \rho + 1 - \lambda_A & -\theta_{AS} \\ 0 & 0 & -\theta_{SA} & \rho + 1 - \lambda_S \end{bmatrix} \begin{bmatrix} Y_{A,t-1} \\ Y_{S,t-1} \\ \mu_{1,t-1} \\ \mu_{2,t-1} \end{bmatrix} + \begin{bmatrix} \beta_u w_{ut}^* + \gamma_1 t \\ \beta_v w_{vt}^* + \gamma_2 D_{1,t} + \gamma_3 D_{2,t} \\ -m_A e^{Y_{A,t-1}} \\ -m_S e^{Y_{S,t-1}} \end{bmatrix} \tag{15}$$

Note that Equation (15) is nonlinear due to the exponential terms and the optimal controls (w_{ut}^* , w_{vt}^*). Then, using Equation (15), we define a term E_t as

$$E_t = z_t - z_{t-1} - g(z_{t-1}, \hat{\Psi}) \tag{16}$$

where $\hat{\Psi}$ contains the estimated parameters, and the nonlinear function $g(\cdot)$ is informed by Equation (15). Note that Equation (16) generates (T−1) equations, each of which is a 4 x 1 vector.

Next, we incorporate the initial and terminal conditions. Specifically, based on Equation (13), we obtain two equations via □□ = □□′ [□□ − □□ − □□□□, □ □], where k_1 = (1,1,0,0)′; similarly, based on Equation (14), we obtain two more equations □□□□ = □□′ [□□□□ − □□ − □□□□, □ □], where k_2 = (0,0,1,1)′. Note that we have equations E_1, E_2, é , E_T, E_{T+1}, where E_1 and E_{T+1} are 2 x 1 vectors, and the other E_t are 4 x 1 vectors. By stacking them one below another, we create a long vector G of dimension (4T) x 1. This resulting vector G is a function of (4T) x 1 variables x = vec(z_1, z_2,..., z_T).

To obtain the optimal state and co-state trajectories, we solve this large system of nonlinear difference equations $G(x) = 0$ by applying a quasi-Newton root-finding procedure (e.g., eqSolve in Gauss 7.0). We initiate this procedure by starting from the actual sales trends and the co-states trajectories implied by actual sales, margin, and spending data. Finally, using the converged solution, we compute via Equation (11) the optimal marketing investments over time. We now demonstrate the application of this decision tool.

Real-world Application. The newspaper company provided the data on not only sales and marketing investments, but also margins from sales to subscribers and to advertisers. We use the first 90 months of the data for model calibration and the last 30 months as the implementation period. Applying the above decision tool to these market data, we computed the optimal trajectories of □$^*_□$ and □$^*_□$for the 30-month period; the corresponding attractor and suitor sales trajectories □$^*_□$ and □$^*_□$ and the resulting optimal total profits trajectory □$^*_□$. We compared the optimal investments with the actual spends over time (□$_□$ and □$_□$),the associated revenues (□$_□$ and □$_□$) and the profits □$_□$.Our results indicated that the managers were, on average, under-spending in the last 30-month period on the sales force (□*/□ = 1.25) and much more so in the newsroom (□*/□= 1.43). Thus, by following the optimal spending paths, the firm could significantly increase revenues and profit — attractor revenue can increase by 50%, suitor revenue by 51%, and the overall profit by 28%.

In sum, the newspaper could increase investments in the newsroom (i.e., investments that increase readership, which may not necessarily increase the objective news “quality”) and achieve higher profitability, a finding that resonates with many journalism scholars over the years (e.g., Lacy and Martin 2004, Rosentiel and Mitchell 2004). The newspaper’s senior management reviewed these recommendations from the decision tool, were convinced of its value, and so decided to increase investments in the newsroom by $500,000, representing an average monthly increase of 18%. This decision not only represents a significant reversal in direction for the firm, but also is contrary to the current national trends of slashing newsroom investments. Since the newspaper management could only obtain additional funds of $500,000, they decided to invest all of it in the newsroom because they felt they were closer to the optimal sales force investment than they were to the optimal newsroom investment. Furthermore, management is currently sourcing additional funding to invest in sales force department as well.

Finer or coarser temporal aggregation. Managers’ decision calendars may follow a different frequency than the one used in estimating the model. For example, managers may make marketing budgeting decisions over a coarser (e.g. quarterly, yearly) or finer (e.g. weekly) frequency. We augment our marketing budgeting algorithm to resolve situations when the model calibration and decision frequencies are different. Specifically, Equation (16) suggests that the change of z_t from time point t–1 to t is given by g([illegible], [illegible]). This assumes that the difference between t–1 and t is one month. To allow for marketing investment decisions on a finer or coarser decision interval, we can choose R grid points in the planning calendar, denoting them by $r = 1, 2, \ldots, R$ and re-writing Equation (16) as

$$\square_{\square} = \square_{\square} - \square_{\square\square\square} - \square\square(\square_{\square\square\square}, \square) \qquad (17)$$

where $\square = \square_{\square} - \square_{\square\square\square}$ denotes the time interval between the two grid points *r-1* and *r*. The use of Equation (17) transposes the problem into one where the optimal solutions are found on *r* points in the planning calendar. Depending on the choice of *r* (e.g., weekly or quarterly), we can obtain solutions over a finer (*r* < one month) or coarser (*r* > 1 one month) planning calendar.

ANALYTICAL INSIGHTS

To gain normative insights to guide the marketing investments of an ongoing platform-firm with two end-user groups, we adopt the long-run perspective of a firm that expects to remain in business for the foreseeable future. Hence, we analyze optimal investments under steady-state conditions. Specifically, we maximize discounted profits over a long planning horizon (*T* Ÿ Ð). Appendix C shows that the optimal marketing investments are then expressed as

$$\begin{bmatrix}\square^*_{\square} \\ \square^*_{\square}\end{bmatrix} = \frac{1}{[\rho + 1 - \lambda_{\square}][\rho + 1 - \lambda_{\square}] - \theta_{\square\square}\theta_{\square\square}} \begin{bmatrix} \square_{\square}(\square_{\square}\square^*_{\square}\square\square + 1 - \square_{\square}\square + \square_{\square}\square^*_{\square}\square_{\square\square}) \\ \square_{\square}(\square_{\square}\square^*_{\square}\square_{\square\square} + \square_{\square}\square^*_{\square}\square\square + 1 - \square_{\square}\square) \end{bmatrix} \qquad (18)$$

Equation (18) reveals that the platform firmôs optimal marketing investments differ from those for a classic firm. For the platform firm, the marginal return of *each* marketing investment (*u* and *v*) depends on both cross-market effects and dynamic effects on both sides of the market. Consequently, the optimal marketing investment to sales ratio depends on the inter-dependencies in the system.

A comparative statics analysis of Equation (18) shows how the presence of CMEs alters the investment levels compared to the classic firm with the same sales carryover dynamics and discount rate, but CMEs close to zero ($\theta_{SA}, \theta_{AS} \rightarrow 0$).

Result 1. *All else equal, optimal marketing efforts by reinforcing platform firms (both θ_{AS} and $\theta_{SA} > 0$) directed at both attractors and suitors are greater than those by classic firms (θ_{AS} , $\theta_{SA} \rightarrow 0$).*

An example of a reinforcing platform is a local newspaper with ad-loving readers (e.g., Sonnac 2000). Result 1 offers the insight that when CMEs are mutually reinforcing, a profit-maximizing platform firm spends more on marketing —not less— than that by its counterpart classic firm since managers should account for the long-term benefits of own market carryovers and CMEs.

Result 2. *All else equal, optimal marketing efforts by counteractive platforms ($\theta_{AS} > 0$, $\theta_{SA} < 0$) directed at attractors are greater than those by classic firms (θ_{AS} , $\theta_{SA} \rightarrow 0$) provided the margin ratio m_S/m_A exceeds the critical value m^*.*

This result reveals that an important trade-off exists in marketing by counteractive platforms. Increasing marketing towards attractors (u) leads to an increase in attractor revenue (A) and, subsequently, an increase in suitor revenue (S) through the attraction effect (θ_{AS}). However, an increase in suitors, and, therefore, in suitor revenues, deters the long-term revenue from attractors, such as in a setting with ad-avoiding newspaper and magazine readers (Sonnac 2000). The amount of loss depends on the magnitude of the negative suitor-effect θ_{SA} and the long-term own-effect of attractors (λ_A).

The critical margin ratio is given by [illegible] where

([illegible], [illegible], [illegible]) are counteractive suitor, counteractive attractor sales, and classic attractor sales respectively. At this ratio, the long-term profit contribution of the suitor revenue outweighs the lost contribution due to lower attractor revenues that the counteractive effect induces. The critical margin ratio increases as the suitor effect or the carryover effect increases, and it decreases with the discount rate.

Result 2 suggests that rather than indiscriminately adding attractors, managers of counteractive platforms should tailor their marketing messages to gain attractors who may be more tolerant to suitors. For example, past research indicates that significant heterogeneity in ad-avoidance exists among the potential readers of magazines and newspapers (Sonnac 2000). In such situations, managers should target market segments that are less ad-averse.

Result 3. All else equal, optimal marketing effort by a counteractive platform ($\theta_{AS} > 0$, $\theta_{SA} < 0$) directed at suitors is lower than that of a classic firm (θ_{AS} , $\theta_{SA} \rightarrow 0$).

That is, although the effect of v in increasing the number of suitors may be large, the negative value of θ_{SA} reduces its overall long-term effectiveness, which reduces its optimal spending level. Result 3 has implications for investments in ad-selling effort of platforms like radio broadcasters. News radio stations commonly employ salespeople to sell ñpiggybackò slots to retailers, i.e., multiple slots that are scheduled back-to-back. While these significantly increase revenue for the station, they increase the number of ads heard during a program and increase the clutter of messages (Warner and Buchman 1991, pg. 229). Increased clutter contributes to wasted coverage (i.e., listeners not buying from the advertisers) and/or high turnover (i.e., listeners switching stations). In such situations, increasing investment in the sales force may not be optimal for the station even if salespeople are effective in selling piggyback slots to retailers.

CONCLUSION

Marketing managers bear the responsibility to plan their investment budget and its allocation optimally and demonstrate that these investments generate appropriate returns for the firm. Although considerable research on this topic exists, the literature so far has largely ignored resource allocation by platform firms operating in two-sided markets characterized by cross-market effects (CMEs). This gap in research motivates us to investigate two-sided platform firmsô marketing decisions both theoretically and empirically.

We developed a platform-firm response model that takes into account the quintessential features of the two-sided market, e.g. demand interdependence, zero demand from attractors resulting in zero demand from suitors (but not vice versa). Subsequently, we estimated and validated the proposed model using data from an archetypal platform firm, namely, a daily newspaper company with two end-user groups: readers and advertisers. We also replicated the proposed model using data from another newspaper platform firm. Both the original and replication analyses revealed the presence of dynamic CMEs between these newspapersô readers and advertisers. We then developed a novel algorithm to assist platform-firm managers determine dynamically optimal marketing investment paths over a finite planning horizon. Finally, we conducted a normative analysis and derived new propositions on how optimal marketing toward end-user groups is impacted by CMEs that arise in platform firm. The key takeaways for managers and academics are as follows:

Takeaway 1: In developing a platform firm response model, it is crucial to take into account dynamic demand interdependence between the firmôs two markets, and the idea that zero demand from attractors resulting in zero demand from suitors (but not vice versa). We develop and empirically validate such a platform-firm market response model in the context of daily newspapers.

Takeaway 2: We develop a marketing-mix algorithm that solves the implied non-linear boundary value problem to obtain dynamically optimal marketing budgeting faced by the managers. Our results show that the presence of CMEs substantially increases the net long-term worth of the newspaperôs spending on newsroom quality because this investment attracts readers and in turn achieves higher advertiser revenues. Thus, newspapers should increase investments in news quality, which is contrary to the practice of cutting newsroom investments to shore up profits followed by many troubled newspaper companies today.

Takeaway 3: Our recommendation based on the proposed marketing-mix algorithm was formally accepted by the newspaper management, and they increased their annual newsroom budget by $500,000 (a substantial increase of 18%).

Takeaway 4: In general, it is crucial for platform managers to account for both effects — CMEs and carryover — when making marketing investment decisions. The CME structure may imply higher marketing investments in the case of reinforcing platforms (Result 1); in counteractive platforms, managers should weigh the gain from adding suitors against the loss of some attractors when setting marketing investment levels (Result 2).

Takeaway 5: Due to the interplay between own-market and cross-market effects, it may be optimal for a platform firm to invest in marketing to a lower-margin group (Result 2).

We hope that platform managers, especially those from the troubled newspaper companies, use our proposed model-based approach to determine the dynamically optimal marketing investments towards the two sides of their firmsô markets.

We conclude by identifying four avenues for future research. First, our model applies to monopoly platform marketing and, therefore, the results need not generalize to competitive markets. Future research should aim to extend our analyses to competitive markets. (We thank an anonymous reviewer for this suggestion). A second extension would be to incorporate time-varying CMEs, allowing for both reinforcing and counteractive effects over different ranges of the data. A third extension would be to apply the model to settings with possibly increasing returns to scale. Finally, estimations and applications of our model in other platform settings would be worthwhile. For example, one research objective could be to investigate optimal marketing investment decisions over time in TV broadcast markets where existing evidence indicates viewers are ad-averse and take deliberate actions to avoid ads (e.g., Gustafson and Siddharth 2007).

REFERENCES

Armstrong, Mark, and Julian Wright (2007), ñTwo-Sided Markets, Competitive Bottlenecks and Exclusive Contracts,ò *Economic Theory,* 32 (2), 353-80.

Bils, Mark, and Peter J. Klenow (2002), ñSome Evidence on the Importance of Sticky Prices,ò Cambridge, Mass: *National Bureau of Economic Research*, Working Paper No. 906

Bolt, Wilko, and Alexander F. Tieman (2006), ñSocial Welfare and Cost Recovery in Two-Sided Markets,ò *Review of Network Economics,* 5 (1), 103-17.

Brody, Edward J. and H. Finkelberg (1997). ñAllocating Marketing Resource,ò *in Brand Valuation*, ed. R. Perrier, London, England: Premier Books.

Burnham, Kenneth. P., and David R. Anderson, (2002), *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach*, Springer.

Caillaud, Bernard, and Bruno Jullien (2003), ñChicken & Egg: Competition among Intermediation Service Providers,ò *RAND Journal of Economics,* 34 (2), 309-28.

Chen, Yuxin, and Jinhong Xie (2007), ñCross-Market Network Effect with Asymmetric Customer Loyalty: Implications for Competitive Advantage,ò *Marketing Science,* 26 (1), 52-66.

Conaghan, James (2006), ñNewspapersô Reach: The Competitive Edge,ò *Newspaper Association of America Report*.

Dorfman, Robert, and Steiner, Peter O. (1954), ñOptimal Advertising and Optimal Quality,ò *American Economic Review,* 44 (5), 826ï 36.

Doyle, Peter, and John Saunders (1990), ñMultiproduct Advertising Budgeting,ò *Marketing Science,* 9 (2), 97-113.

Eisenmann, Thomas, Geoffrey Parker, and Marshall W. Van Alstyne (2006), ñStrategies for Two-sided Markets,ò *Harvard Business Review,* 84(10), 92-101

Engle, Robert. F., David. F. Hendry, and Jean-Francois Richard (1983), ñExogeneity,ò *Econometrica,* 51 (2), 277-304.

Erdem, Tülin and Baohung Sun (2002), ñAn Empirical Investigation of the Spillover Effects of Advertising and Sales Promotions in Umbrella Branding,ò *Journal of Marketing Research*, 39(4), 408-420.

Evans, David (2003), ñThe Antitrust Economics of Multi-Sided Platform Industries,ò *Yale Journal on Regulation*, Handbook- Volume 20, 325ï 82.

Evans, David, and Robert Schmalensee (2007), *Catalyst Code: The Strategies Behind the World's Most Dynamic Companies*. Boston, Mass.: Harvard Business School Press.

Gensch, Dennis H., and Ulf Peter Welam (1973), ñAn Optimum Budget Allocation Model for Dynamic, Interacting Market Segments,ò *Management Science,* 20 (2), 179-87.

Gupta, Sunil, Carl F Mela, and Jose Vidal-Sanz (2009), ñThe Value of a óFreeô Customer,ò Cambridge, Mass: Working Paper, Harvard Business School.

Gustafson, Paul and S. Siddarth (2007), ñDescribing the Dynamics of Attention to TV Commercials: A Hierarchial Bayes Model of the Time to Zap an Ad,ò *Journal of Applied Statistics*, 34(5), 585-609.

Hanssens, Dominique M. and Ming. Ouyang (2002), ñHysteresis in Market Response: When is Marketing Spending an Investment?,ò *Review of Marketing Science*, (419), archived in Social Science Research Network Electronic Library.

Hanssens, Dominique M., Leonard Parsons, and Randall Schultz (2001), *Market Response Models: Econometric and Time Series Analysis*, 2nd ed. Boston, Mass: Kluwer Academic Publishers.
Harvey, Andrew C. (1994), *Forecasting, Structural Time Series Models and the Kalman Filter.* New York, N.Y.: Cambridge University Press.
Ingene, Charles A, and Mark E Parry (1995), ñA Note on Multi-Regional Marketing,ò *Management Science,* 41(7), 1194-207.
Jazwinski, Andrew.H. (1970), *Stochastic Processes and Filtering Theory.* New York, 1970.
Kamien, Morton, and Nancy Schwartz (1991), *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, 2nd ed. Amsterdam, Elsevier.
Kind, Hans Jarle, Tore Nilssen, and Lars Sorgard (2009), ñBusiness Models for Media Firms: Does Competition Matter for how they Raise Revenue?ò *Marketing Science*, 28(6), 1112-1128.
Lacy, Stephen and Hugh J. Martin (2004), ñCompetition, Circulation, and Advertising,ò *Newspaper Research Journal*, 25(1), 18-39.
Leeflang, Peter S.H. , Tammo H.A. Bijmolt, Jenny van Doorna, Dominique M. Hanssens, Harald J. van Heerde, Peter C. Verhoef and Jaap E. Wieringa (2009), ñCreating Lift Versus Building the Base: Current Trends in Marketing Dynamics,ò *International Journal of Research in Marketing*, 26 (1), 13-20.
Litman, Barry. R. and Janet A. Bridges (1986), ñAn Economic Analysis of Daily Newspaper Performance,ò *Newspaper Research Journal,* 7(3), 9-22.
Manchanda, Puneet and Elisabeth Honka (2005), ñThe Effects and Role of Direct-to-Physician Marketing in the Pharmaceutical Industry: An Integrative Review,ò *Yale Journal of Health Policy Law and Economics,* 5(2), 785-822.
Mantrala, Murali (2002), ñAllocating Marketing Resourcesò In *Handbook of Marketing*, eds. Barton A. Weitz and Robin Wensley, London Sage Publications.
Mantrala, Murali K., Prasad A. Naik, Shrihari Sridhar, and Esther Thorson (2007), ñUphill or Downhill? Locating the Firm on a Profit Function,ò *Journal of Marketing,* 71(2), 26-44.
Naik, Prasad A., Murali K. Mantrala, and Alan. Sawyer (1998), ñPlanning Media Schedules in the Presence of Dynamic Advertising Quality,ò *Marketing Science,* 17 (3), 214-235.
Naik, Prasad A., Ashutosh Prasad, and Suresh P. Sethi (2008), ñBuilding Brand Awareness in Dynamic Oligopoly Markets,ò *Management Science,* 54 (1), 129-138.
Naik, Prasad A., Kalyan Raman, and Russell S. Winer (2005), ñPlanning Marketing-Mix Strategies in the Presence of Interaction Effects,ò *Marketing Science*, 24 (1), 25-34.
Naik, Prasad A. and Chih-Ling Tsai (2000), ñControlling Measurement Errors in Models of Advertising Competition,ò *Journal of Marketing Research*, 37(1), 113-124.
Nerlove, Marc, and Kenneth Arrow (1962), ñOptimal Advertising Policy under Dynamic Conditions,ò *Economica,* 29 (3), 129-42.
Newspaper Association of America Marketing Department Report (2006), *How America Shops and Spends 2006*. Minneapolis, Minn.: MORI Research.
Overholser, Geneva (2004), ñGood Journalism and Business: An Industry Perspective,ò *Newspaper Research Journal*, 15(1), 8-17.
Parker, Geoffrey G., and Marshall W. Van Alstyne (2005), ñTwo-Sided Network Effects,ò *Management Science,* 51(10), 1494ï 504.

Patterson, Thomas E. (2007), ñYoung People and News,ò report from the Joan Shorenstein Center on the Press, Politics and Public Policy, John F. Kennedy School of Government, Harvard University.

Pauwels, Koen, Joep Arts and Thorsten Wiesel (2010), ñMarketingõs Profit Impact: Quantifying Online and Offline Funnel Progressionò, *Marketing Science Institute Report,* [10-01].

Picard, Robert G. (1993), ñEconomics of the Daily Newspaper Industry.ò In *Media Economics: A Reader,* 2nd ed., eds. Alison Alexander, James Owers, and Rodney Carveth, 111-29. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Project for Excellence in Journalism Report (2008), ñ*The Changing Newsroom: What is Being Gained and What is Being Lost in America's Daily Newspapers?"* Accessed at http://www.journalism.org/node/11961.

Raman, Kalyan (2006), ñBoundary Value Problems in Stochastic Optimal Control of Advertising,ò *Automatica*, 42(8), 1357-1362.

Reibstein, David J., and Hubert Gatignon (1984), ñOptimal Product Line Pricing: The Influence of Elasticities and Cross-Elasticities,ò *Journal of Marketing Research*, 21(3) 259-67.

Rochet, Jean Charles, and Jean Tirole (2005),ñTwo-Sided Markets: A Progress Report,ò *RAND Journal of Economics*, 37(3), 645-667.

Rosentiel, Tom, and Amy Mitchell (2004), ñThe Impact of Investing in Newsroom Resources,ò *Newspaper Research Journal,* 25 (1), 84-97.

Roson, Roberto (2004), ñTwo-Sided Markets: A Tentative Survey,ò *Review of Network Economics,* 4 (2), 142-60.

Sethi, Suresh P. and G. L. Thompson (2006), *Optimal Control Theory: Applications to Management Science and Economics*. Norwell, Mass.: Kluwer Academic Publishers.

Simon, Julian L., and Johan Arndt (1980), ñThe Shape of the Advertising Function,ò *Journal of Advertising Research,* 20 (3), 11-28.

Sonnac, Nathalie (2000), ñReadersô Attitudes towards Press Advertising: Are they Ad-lovers or Ad-averse?ò *Journal of Media Economics,* 13(4), 249ï 59.

Shankar, Venkatesh (2008), *"*Strategic Allocation of Marketing Resources: Methods and Managerial Insights,ò *Marketing Science Institute Report [08-207],* Cambridge, Massachusetts.

Warner, Charles, and Joseph Buchman (1991), *Broadcast and Cable Selling,* 2nd ed., Belmont, CA: Wadsworth.

Wilbur, Kenneth C. (2008), ñA Two-Sided Empirical Model of Television Advertising and Viewing Markets,ò *Marketing Science,* 27 (3), 356-79.

Xie, Jinhong, Michael Song, Marvin Sirbu, and Qiong Wang (1997), ñKalman Filter Estimation of New Product Diffusion Models,ò *Journal of Marketing Research,* 34 (3), 378-93.

FIGURE 1

NEWSPAPER ALLOCATING MARKETING EFFORTS

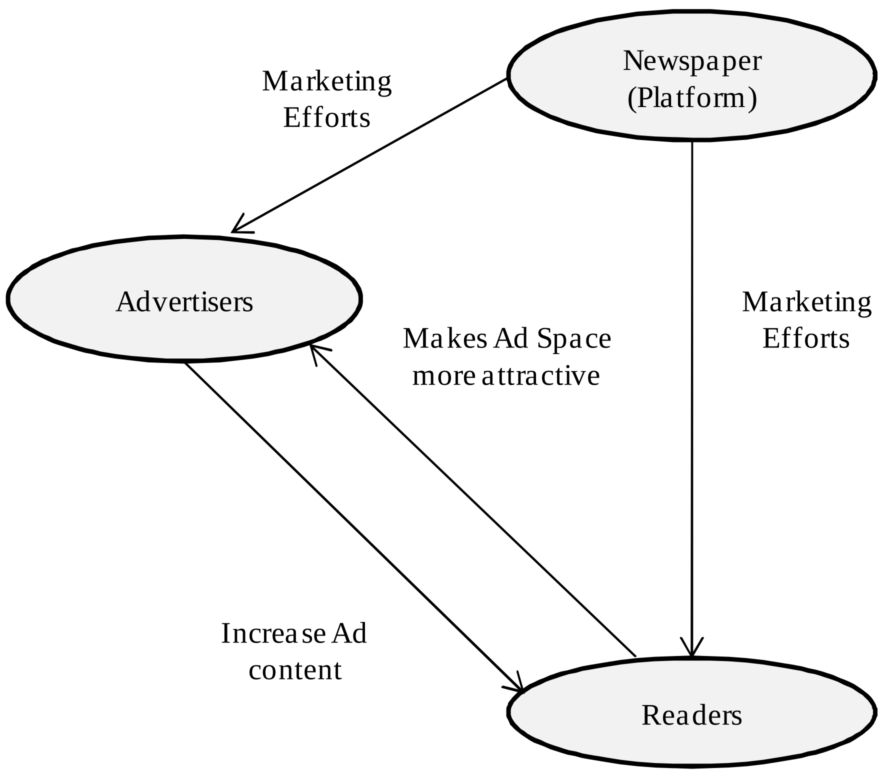
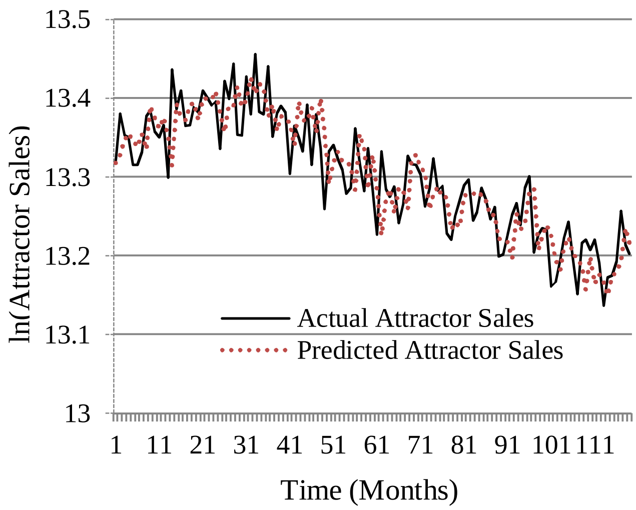
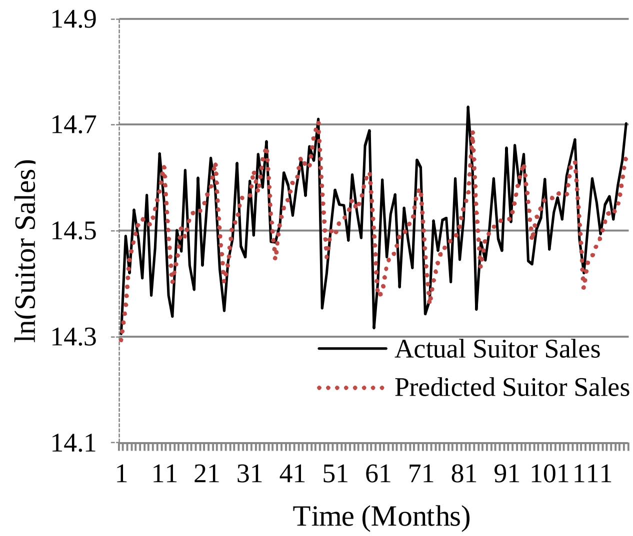


FIGURE 2

PANEL A. OBSERVED ATTRACTOR PATTERNS



PANEL B. OBSERVED SUITOR PATTERNS



PANEL C. NEWSROOM/SALESFORCE INVESTMENTS

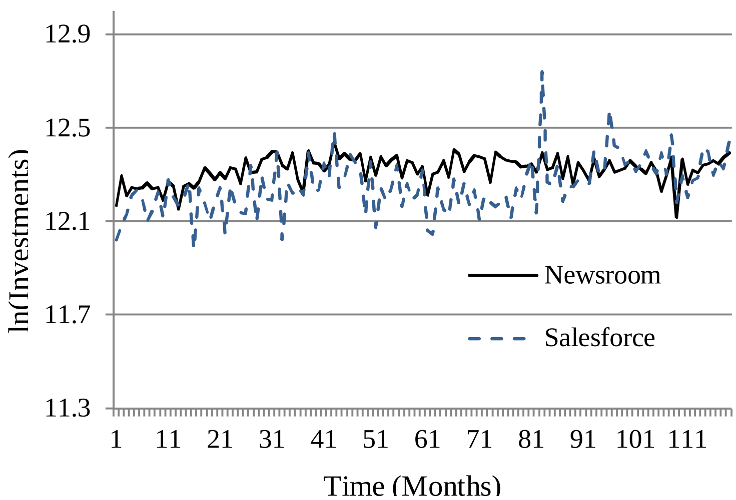


Table 1. Classic ('one-sided') vs. Platform Firm Marketing Budgeting and Response Models

Model Type	*Description*	*Specification*	*Example*
I	Static Single Entity	[illegible] = [illegible].	Dorfman and Steiner (1954)
II	Dynamic Single Entity	[illegible] = [illegible], [illegible].	Nerlove and Arrow (1962)
III	Static Multiple Entity	[illegible] = [illegible], [illegible]. [illegible] = [illegible], [illegible]. [illegible]$(Sales_{it})/$[illegible]$(Sales_{jt}$[illegible] $0,$ [illegible] $\neq$ [illegible]	Ingene and Parry (1995)
IV	Dynamic Multiple Entity	[illegible] = [illegible], [illegible], [illegible]. [illegible] = [illegible], [illegible], [illegible]. [illegible]$(Sales_{it})/$[illegible]$(Sales_{jt-1}$[illegible] $0,$ [illegible] $\neq$ [illegible].	Gensch and Welam (1973)
V	Static, Platform-Firm	[illegible] = [illegible], [illegible]. [illegible] = [illegible], [illegible]. [illegible] = 0, [illegible] = 0, [illegible] $\neq$ [illegible], for at least one side. [illegible]$(Sales_{it})/$[illegible]$(Sales_{i,t-1}) = 0,$ [illegible], [illegible].	Mantrala et al. (2007): (they do not allow [illegible] = 0 $\Leftrightarrow$ [illegible] = 0, [illegible] $\neq$ [illegible]) for at least one side.
VI	Dynamic, Platform-Firm	[illegible] = [illegible], [illegible], [illegible]. [illegible] = [illegible], [illegible], [illegible]. [illegible] = 0, [illegible] = 0, [illegible] $\neq$ [illegible], for at least one side. [illegible]$(Sales_{it})/$[illegible]$(Sales_{j,t-1}$[illegible] $0,$ [illegible] $\neq$ [illegible]	This paper

Table 2. Descriptive Statistics

Variable *	*Mean*	*Std. Deviation*
Attractor Revenues (Subscription)	60.04	4.43
Suitor Revenues (Advertising)	202.4	19.45
Newsroom Investments	22.14	1.30
Salesforce Investments	21.02	2.56

* All variables in 10, 000 U.S. dollars per month.

Table 3.Model Selection

Nos.	*Models*	*AIC*	*AICc*	*BIC*	*MAD (Fit)*	*MAD (Forecast)*
1	Trend, Seasonality and Lagged Effects Only	-771	-767	-739	0.61%	0.63%
2	Trend, Seasonality and Lagged Effects, Marketing Variables, No CMEs	-1035	-1030	-998	0.40%	0.42%
3	Trend, Seasonality, Lagged Effects, Marketing Variables, CMEs, Uncorrelated Errors	-1120	-1115	-1080	0.39%	0.43%
4	Trend, Seasonality, Lagged Effects, Marketing Variables, CMEs, Correlated Errors	-1122	-1116	-1079	0.26%	0.35%

Table 4. Estimation Results

Parameter	*Estimate*	*t-value*
Trend in Attractor Revenue (γ_1)	-0.001	-14.35
Year End Ad Revenue Rise (γ_2)	0.04	2.43
Year Beginning Ad Revenue Drop(γ_3)	-0.11	-12.06
Attractor Revenue Intercept (α_{A0})	13.27	7.88
Suitor Revenue Intercept (α_{S0})	14.51	11.38
Attractor Revenue Carry-over (λ_A)	0.69	26.05
Suitor Revenue Carry-over (λ_S)	0.63	26.56
Attractor Cross-Market Effect (θ_{AS})	0.16	6.74
Suitor Repercussion Cross-Market Effect (θ_{SA})	0.12	2.26
Effectiveness of Attractor -Directed Marketing (β_u)	0.25	7.60
Effectiveness of Suitor-Directed Marketing (β_v)	0.18	8.63

APPENDIX A: TEST FOR EXOGNEITY

Applying Engle, Hendry, and Richardôs (1983) approach, we test for exogeneity of newsroom and sales force investments. Let $p_1(Y_A, w_u)$ be the joint density of attractor revenues and newsroom investments, $p_2(Y_A|w_u)$ denote the conditional density of attractor revenues given newsroom investments, and $p_3(w_u)$ represent the marginal density. Then we factorize $p_1(Y_A, w_u) = p_2\ (Y_A|w_u)$ x $p_3(w_u)$, and weak-exogeneity means that a precise specification of $p_3(\cdot)$ is not needed and no loss of information occurs when the estimation is based on the conditional density $p_2(\cdot)$. To verify this, we first estimated marginal models of newsroom and sales force investments (see Engle et al. (1983, p. 289), Naik, Raman and Winer 2005).

$$w_{ut} = \breve{U}_{0n} + \breve{U}_{in}\ w_{ut-ni} + \breve{U}_{jn} Y_{At-nj} + \breve{U}_{kn} Y_{St-nk} + \mathring{U}_{u,t} \qquad \text{(A1)}$$

$$w_{vt} = \breve{U}_{0s} + \breve{U}_{is}\ w_{vt-si} + \breve{U}_{js} Y_{St-sj} + \breve{U}_{ks} Y_{At-sk} + \mathring{U}_{v,t} \qquad \text{(A2)}$$

We determined *ni,nj,nk,si,sj* and *sk* based on AIC. We computed the residuals pertaining to newsroom and sales force investments (i.e. □ and □). Next, we obtained the residuals from the estimated system of sales revenues from the KF estimation. We then examined the correlations between the marginal model and the conditional model residuals. For newsroom investments, we examined the correlations from the marginal model of newsroom investments and the subscriptions revenue equation. The resulting correlation (p-value) is -0.059 (0.532). Given it lacks significance at the 95% confidence level, newsroom investments are weakly exogenous. Similarly, for sales force investments, we examined the correlations from the marginal model of sales force investments and the ad revenue equation. The resulting correlation (p-value) is 0.067 (0.476). Given that it lacks significance at the 95% confidence level, sales force investments also are weakly exogenous. Thus, we find that both newsroom and sales force investments are weakly exogenous. Consequently, the empirical results are valid in the sense of efficient estimation (Engle et al. 1983, p. 290).

APPENDIX B: DYNAMIC OPTIMIZATION

Given the presence of CMEs (θ_{AS} and θ_{SA}), how should platform-firm managers determine their overall marketing budget trajectory and the budget allocation across attractors and suitors? Let u_t and v_t denote the marketing investments toward attractors and suitors, respectively. For a given discount rate [illegible], the platform firm seeks to find the investment levels that maximize the total discounted profits, which is expressed as

Maximize $J(u,v) = \sum$ [illegible] $\pi($[illegible]$)$, (B1)

where π[illegible] $=$ [illegible] $+$ [illegible] S[illegible] $- u$[illegible] $- v$[illegible] (B2)

subject to the dynamic market-response functions of the platform-firm:

[illegible] and (B3)

[illegible] (B4)

where [illegible] = Max{ S_{t-1} , [illegible] }and [illegible] = {1 when $S_{t-1} = 0$, 0 when $S_{t-1} > 0$}.

Because (B3) and (B4) are non-linear, we transform them via logarithms as follows:

[illegible] , (B5)

where [illegible] and [illegible] represent log-transformed attractor and suitor sales respectively, and [illegible] and [illegible] represent log-transformed investments towards the attractors and suitors respectively. By including trend and seasonality terms, we obtain

[illegible] . (B6)

Next, subtracting [illegible] from the first row and [illegible] from the second row of (B6), we get

[illegible] , (B7)

which can be expressed as the following when $S_{t-1} > 0$:

[illegible] , (B8)

If $S_{t-1} = 0$ for some t then replace [illegible] by 0 and [illegible] by unity and add [illegible] and [illegible] in (B8).

To solve this dynamic optimization problem, we apply the discrete-time Maximum principle to derive the optimal effort levels. When $S_{t-1} > 0$, the Hamiltonian at each instant t is

$$[\text{illegible}] \tag{B9}$$

where [illegible] and [illegible] are the co-state variables corresponding to the two state equations. If $S_{t-1} = 0$ for some t then replace [illegible] by 0 and [illegible] by unity and add [illegible] and [illegible] in (B9). The conditions for optimality are

$$[\text{illegible}] = 0 \text{ and } [\text{illegible}] = 0 \tag{B10}$$

$$\Delta[\text{illegible}] = [\text{illegible}] \tag{B11}$$

$$\Delta[\text{illegible}] = [\text{illegible}] \tag{B12}$$

From (B10) we obtain the optimal controls:

$$[\text{illegible}]^* = \mathrm{Ln}([\text{illegible}]), \text{ and } [\text{illegible}]^* = \mathrm{Ln}([\text{illegible}]) \tag{B13}$$

which can be exponentiated to transform back to the actual marketing investments. To derive analytical insights, we obtain the stationary co-state variables by setting (B11) and (B12) to zero.

$$[\text{illegible}] = \frac{1}{[\text{illegible}]\rho + 1 - \lambda[\text{illegible}]\rho + 1 - \lambda[\text{illegible}] - \theta\theta} [\text{illegible}] \tag{B14}$$

where [illegible] and [illegible] represent the optimal attractor and suitor values, respectively. Substituting (B14) into (B13), we obtain the optimal marketing investments:

$$[\text{illegible}] \tag{B15}$$

APPENDIX C: PROOFS OF ANALYTICAL RESULTS

Proof of Result1. Based on (18) in the manuscript,

$$\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}^{*}\square\square + 1 - \square_{\square}\square + \square_{\square}\square_{\square}^{*}\square_{\square\square})}{\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square - \theta_{\square\square}\theta_{\square\square}},\ \square_{\square\square}, \square_{\square\square} > 0 \tag{C1}$$

$$\text{And } \square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{0}^{*})}{\square\rho + 1 - \lambda_{A}\square},\quad \square_{\square\square}, \square_{\square\square} \rightarrow 0, \tag{C2}$$

where the subscript *r* refers to reinforcing and the subscript *0* refers to classic firm. Rewriting (C2), we get

$$\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}^{*})\square\square + 1 - \square_{\square}\square}{\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square} \tag{C3}$$

A comparison of (C3) and (C1) reveals that the denominator of (C3) is higher than (C1) ($\square_{\square\square} > 0, \square_{\square\square} > 0$, in (C1)) and the numerator of (C3) is lower than (C1). Furthermore, $\square_{\square}^{*} > \square_{\square}^{*}$, $\square_{\square}^{*} > \square_{\square}^{*}$, thus proving the claim.

Proof of Result 2. Based on (18) in the manuscript,

$$\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}^{*}\square\square + 1 - \square_{\square}\square + \square_{\square}\square_{\square}^{*}\square_{\square\square})}{\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square - \theta_{\square\square}\theta_{\square\square}},\ \square_{\square\square} > 0, \square_{\square\square} < 0 \tag{C4}$$

$$\text{and } \square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}^{*})}{\square\square\square\ \square\square\ \square_{\square}\square},\quad \square_{\square\square}, \square_{\square\square} \rightarrow 0, \tag{C5}$$

where the subscript *c* refers to counteractive and the subscript *0* refers to classic firm.
A comparison of (C4) and (C5) reveals that $\square_{\square}^{*} > \square_{\square}^{*}$ only when

$$\square_{\square}\square_{\square}^{*}\square\rho + 1 - \lambda_{\square}\square\theta_{\square\square} > \square_{\square}((\square_{\square}^{*} - \square_{\square}^{*})\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square - \square_{\square}^{*}\theta_{\square\square}\theta_{\square\square}), \tag{C6}$$

which is equivalent to

$$\frac{\square_{\square}}{\square_{\square}} > \frac{(\square_{\square}^{*} - \square_{\square}^{*})\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square - \square_{\square}^{*}\theta_{\square\square}\theta_{\square\square}}{\square_{\square}^{*}\square\rho + 1 - \lambda_{\square}\square\theta_{\square\square}} = \square^{*} \tag{C7}$$

This inequality shows that $\square_{\square}^{*} > \square_{\square}^{*}$ when $\square_{\square}/\square_{\square} > \square^{*}$ as posited.

Proof of Result 3. Based on (18) in the manuscript,

$$\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}{}^{*}\square_{\square\square} + \square_{\square}\square_{\square}^{*}\square\square + 1 - \square_{\square})\square}{\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square - \theta_{\square\square}\theta_{\square\square}},\ \square_{\square\square} > 0, \square_{\square\square} < 0 \tag{C8}$$

and $\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{0}^{*})}{\square\rho + 1 - \lambda_{S}\square},\ \square_{\square\square}, \square_{\square\square} \rightarrow 0$ (C9)

Rewriting (C9), we get

$$\square_{\square}^{*} = \frac{\square_{\square}(\square_{\square}\square_{\square}^{*})\square\square + 1 - \square_{\square}\square}{\square\rho + 1 - \lambda_{\square}\square\square\rho + 1 - \lambda_{\square}\square} \tag{C10}$$

A comparison of (C10) and (C8) reveals that the denominator of (C10) is lower than (C8) ($\square_{\square\square} > 0, \square_{\square\square} < 0$ in C(8)) and the numerator of (C10) is higher than (C8). Hence $\square_{\square}^{*} < \square_{\square}^{*}$, which proves the claim.