4. PROJECT BENEFIT COST ANALYSIS

4.1. Least cost

This chapter deals with the comparison of alternative strategies or alternative solutions. To illustrate this the following example is discussed.

An rather old excavator, which will be replaced after two years, needs repairing (or overhaul) at an estimated costs of € 10,000 (investment). If the overhaul is not carried out it is anticipated that the operating expenses (M + R) for the excavator will involve an increase of € 5,600 for each of the two remaining years of its service life. It might appear that this is a good investment because an expenditure of € 10,000 leads to a total saving of € 11,200 over the next two years. This is, however, incorrect as it overlooks the time value of money.

The question is whether the **present value** of the anticipated savings on running costs is greater or less than the investment costs for overhaul of € 10,000. An alternative, but equivalent, way of framing the question is: "would the owner of the excavator earn more or less than the € 5,600 per year offered by the savings on running costs by placing the initial € 10,000 into an alternative investment (for example in stocks or bonds)". The first approach focuses upon the discounting of future euro's into present value, whereas the second approach concentrates on the translation of present euro's into a time stream of euro's (compounding). In principle these approaches are equivalent. The problem can only be solved if a certain interest rate is assumed.

Present value concept (discounting).
For an interest rate of **10 %** the present value of the savings in M + R costs is:

$$\frac{5{,}600}{1.10} + \frac{5{,}600}{1.10^2} = € \ 5{,}091 + € \ 4{,}628 = \underline{\underline{€ \ 9{,}719.}}$$

When the present values of the savings on M + R expenses is compared with the overhaul costs of € 10,000 it is obvious that the overhaul investment is not a wise decision. If, however, the interest rate is only **5 %** then the present value of the savings on M + R expenditure is:

$$\frac{5{,}600}{1.05} + \frac{5{,}600}{1.05^2} = € \ 5{,}333 + € \ 5{,}080 = \underline{\underline{€ \ 10{,}413}}.$$

In present euro's, which is more than the initial investment of € 10,000, making the repair at the beginning the better option.

Compounding approach.
Suppose that there is an opportunity to invest the € 10,000 in a stock or a bond offering an annual rate of return equal to **10 %** (after costs etc.). The future value is the initial investment plus the compounded interest earned. At the end of the first year the future value will be € 10,000 plus € 1,000 in interest earning (10%) = € 11,000. Assume (for simplicity) that the M + R extra costs are only paid once a year (at the end of the year); the balance after paying the bill for the extra M + R costs will be € 11,000 - € 5,600 = € 5,400.

If the balance is reinvested at 10% the future value at the end of the second year is € 5,400 x 1.10 = € 5,940 or a surplus of € 340 remains after payment of the second extra costs for M + R. Clearly the alternative investment is a stock or a bond is more attractive. Alternatively if the available return is **5 %** rather than 10 % the future value at the end of the first year will be € 10,000 x 1.05 = € 10,500 or € 4,900 after payment of the extra € 5,600 expenses for M + R. By the end of the second year there would only be € 4,900 x 1.05 = € 5,145 available from the original € 10,000 to pay the bill of € 5,600 for the extra M + R costs of the second year or a deficit of € 455. In this case the investment in the overhaul is justified.

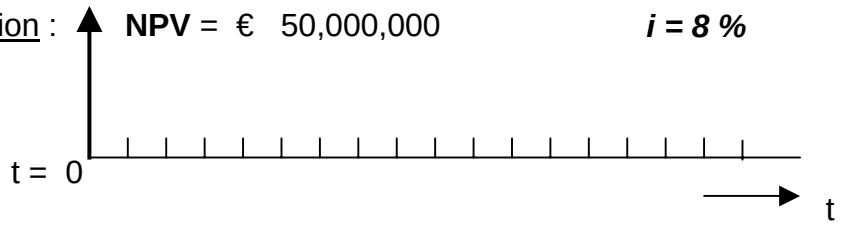
The following comparison of options is exactly the same as the discussed example above. An office building is to be replaced by a new building at the end of two years. Should the insulation in this old building be improved at a cost of € 10,000 if the anticipated savings in heating and cooling cost is € 5,600 for each of the remaining years in the building's life ?

Example

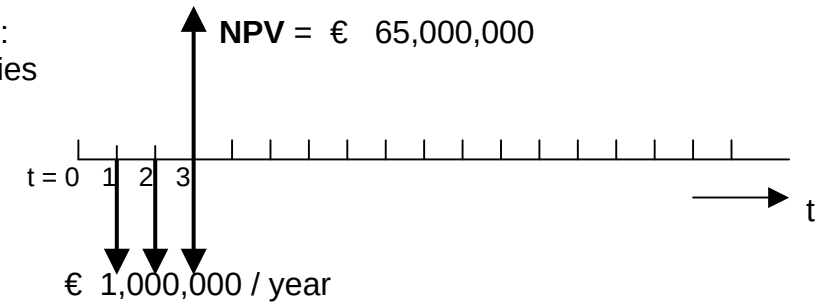
The expected **Nett Present Value NPV** (see next paragraph) of some project is € 50,000,000. Collecting more data and doing some additional studies during a period of 3 more years are expected to increase the NPV to a value of € 65,000,000 (discounted value at the end of the 3-years study period). The costs of these additional activities is € 1,000,000 at the end of each of these 3 years. Should the extra data collection and studies be done? The discount rate is **8 %.**

Answer

Original situation : **NPV** = € 50,000,000 ***i = 8 %***



New situation :
additional studies



Express new situation in **Present Value (P.V.)** (t = 0):

discount factor $1 / (1.08)^3$ = 0.794 (for NPV of € 65,000,000)

discount factor $[(1.08)^3 - 1] / [0.08 \times (1.08)^3]$ = 2.577

NPV € 65,000,000 x 0.794 - € 1,000,000 x 2.577 = **€ 49,023,000**

As the NPV for the new situation is lower than the original schedule the conclusion is that additional studies should not be carried out !

Capitalized costs

In construction works the precise life of an asset may be difficult to access with accuracy. An initial capital investment is made in order to shape the natural ground; the life of these earth works (for example a canal.a road cutting) may be very long or even forever. In such cases the computation of capital recovery takes a similar form to the computation of simple interest.

If the value of **n** increases , so the term $\frac{(1+i)^n}{(1+i)^n - 1}$ approaches to 1.

and the capital recovery formula becomes approximately :

A = i . P (for n = 100 years).

If the lifetime of an asset is considered to be 100 years and not in perpetuity, there will be only a very small difference in the resulting calculation between using the appropriate capital recovery factor itself and using the relevant interest rate.

The term ***capitalized costs*** is commonly used by engineers in cases where comparisons of costs are made over periods of time in perpetuity and annual costs are assumed to be incurred on a perpetual basis.

The Present Value of the annual costs becomes: **P =** $\frac{A}{i}$

Optimum of initial cost and maintenance cost

Example
A dike is proposed for river protection. The higher the dike the greater the costs, and the lower the risk of flooding. Estimated data are indicated in the following table:

Height of dike (m)	2	3	4	5	6	7	8
Cost of dike (€ 1,000)	10	25	43	67	100	150	225
Risk of flooding (times per year)	4	2	1	0.5	0.1	0.05	0.01

If the damage by flooding is estimated at € 10,000 each time it occurs, what design height should be selected if money can be borrowed at (a) **10 %**; and (b) **20 %** ?

Answer:
Capital costs: the dike would be everlasting, so only annual costs need to be considered

a. For **10 % interest** rate.

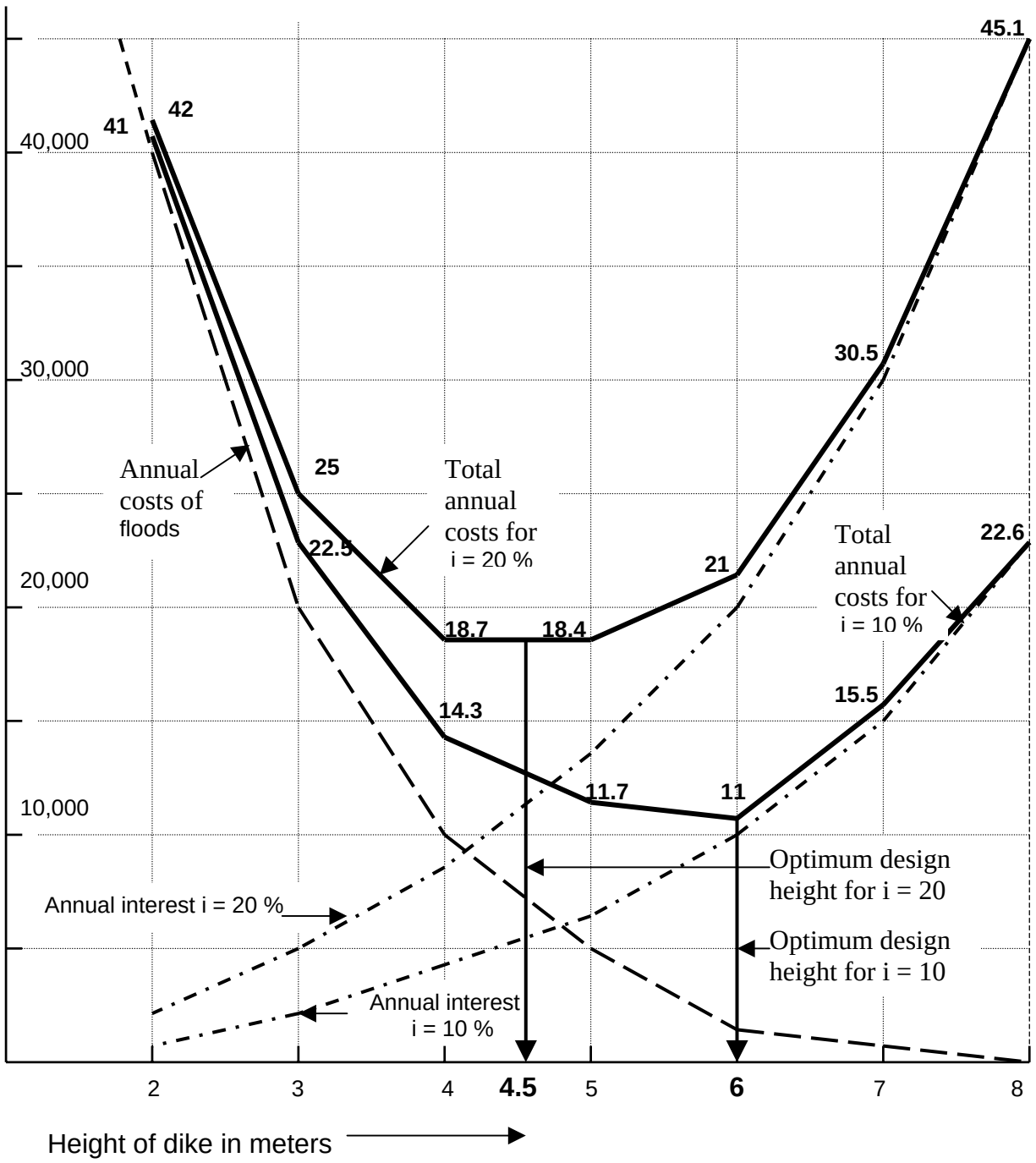
Height of dike (m)	2	3	4	5	6	7	8
Annual interest (€ 1,000)	1.0	2.5	4.3	6.7	10.0	15.0	22.5
Annual costs of Floods (€ 1,000)	40	20	10	5.0	1	0.5	0.1
Total annual Costs (€ 1,000)	41	22.5	14.3	11.7	11	15.5	22.6

The total annual costs are minimal, at 10 % interest rate, for a design height of **6 metres**.

b. For **20 % interest** rate.

Height of dike (m)	2	3	4	5	6	7	8
Annual interest (€ 1,000)	2.0	5.0	8.7	13.4	20.0	30.0	45.0
Annual costs of Floods (€ 1,000)	40	20	10	5.0	1	0.5	0.1
Total annual Costs (€ 1,000)	42	25	18.7	18.4	21	30.5	45.1

The total annual costs are minimal, at 20 % interest rate, for a design height of **4.5 metres**.



Optimum design height of a river dike for flood protection, optimisation of capital costs and the cost of flooding for different rates of interest (i = 10 % and i = 20 %).

4.2. Net Present Value (NPV)

The method of appraising alternative capital investment projects by the **net present value** (NPV) is long established and well tried. The net present value method is alternative known as the present value, the present worth or the net present worth method. The basis of this method is that all future costs and benefits concerned with an investment project are converted (discounted) to present value, using a selected interest rate.

$$\textbf{NPV} = \sum \frac{B_t}{(1+i)^t} - \sum \frac{C_t}{(1+i)^t}$$

B = Benefits
C = Costs

In all cases the NPV is uniquely defined. It is widely used in the selection of projects. If the NPV is positive, the project is considered to be profitable: it yields benefits and exceeds investments, operating costs and taxes. It is frequently more convenient and certainly more conventional to express all euro estimates in terms of present value.

For example consider two alternative projects, A and B, either of which would cost € 10,000 today and yield benefits over a four-year period as follows:

Year	1	2	3	4
Project A	€ 6,000	€ 2,000	€ 16,000	€ 4,000
Project B	€ 8,000	€ 1,000	€ 12,000	€ 4,800

Which of these projects is preferable? From a mere comparison of the annual benefits it is impossible to determine the answer, as project A is to be preferred for the 2nd and 3rd year, while project B is better for the first and last year.

Once a discount rate is selected these benefits can be converted into present values and a comparison made. Present value of the benefits for an interest rate of **5 %:**

<u>Project A</u> : $\frac{6{,}000}{1.05} + \frac{2{,}000}{1.05^2} + \frac{16{,}000}{1.05^3} + \frac{4{,}000}{1.05^4}$

= € 5,714 + € 1,814 + € 13,821 + € 3,291 = <u>€ 24,641</u>

<u>Project B</u> : $\frac{8{,}000}{1.05} + \frac{1{,}000}{1.05^2} + \frac{12{,}000}{1.05^3} + \frac{4{,}800}{1.05^4}$

= € 7,619 + € 907 + € 10,366 + € 3,949 = <u>€ 22,841</u>

Project A is superior to project B.
Furthermore both projects have a positive NPV and are therefore economically feasible.

Another meaning is that if € 24,641 is put in a bank today at **5 %** interest it would be possible to withdraw € 6,000, € 2,000, € 16,000 and € 4,000 in the first, second, third and fourth year respectively before the account would be depleted.

Example

Two different tenders have been received for works. Both quote a total price of € 50 million, but they demand **different payment schedules**:

Tenderer A demands the following schedule:

- initial payment (t=0): € 5 million,
- thereafter 9 equal 6-month instalments, each € 5 million.

The works will be completed at the end of year 5.

Tenderer B demands the following schedule:

- initial payment (t=0) € 2.5 million
- after 6 month € 10 million
- after 12 month € 15 million
- after 18 month € 5 million
- after 24 month € 5 million
- after 36 month € 5 million
- after 48 month € 7.5 million

The works will be completed at the end of year 4.

Which tender is to be preferred if:

a. The criterion of **least cost** is applied;

b. The criterion of **maximum NPV** is applied; the net benefits of the project, discounted at the moment of completion, are estimated at € 80×10^6.

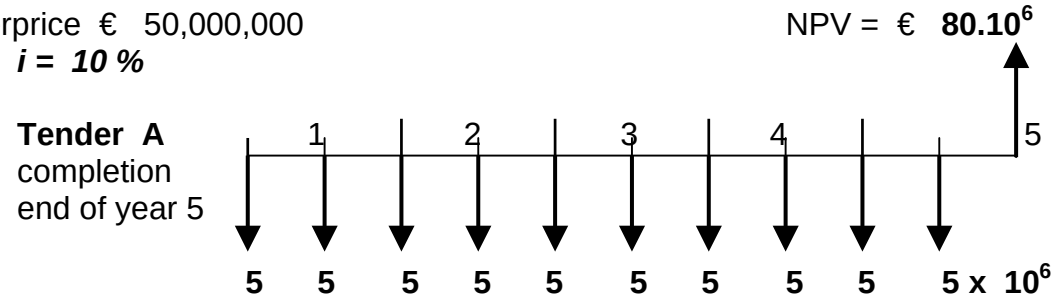
The discount rate is **10 %.**

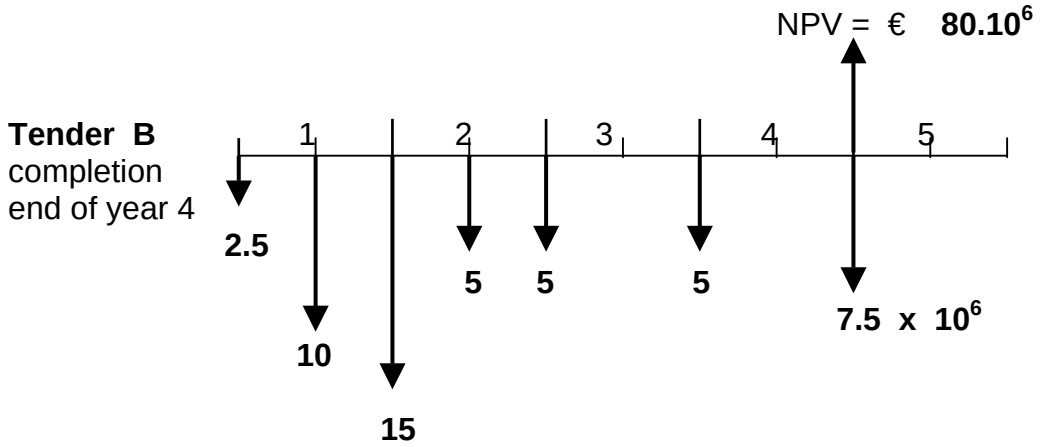
Answer

a. Criterion of least costs

tenderprice € 50,000,000

i = 10 %





Simplify (not fully correct) : half-yearly interest : 5 % and bring half-yearly payments to the beginning of the year:

Tender A

Total payment every year: € 5,000,000 + € $\frac{5{,}000{,}000}{1.05}$ = € 9,762,000

Present Value of all payments:

$$\left\{ 1 + \frac{1}{1.10} + \frac{1}{1.10^2} + \frac{1}{1.10^3} + \frac{1}{1.10^4} \right\} \text{ x € } 9{,}762{,}000 =$$

$$= 4.169 \text{ x € } 9{,}762{,}000 = \text{€ } \underline{\mathbf{40{,}700{,}000}}$$

Tender B

Present Value of all payments:

$$\left\{ 2.5 + \frac{10}{1.05} + \frac{15}{1.10} + \frac{5}{1.05x1.10} + \frac{5}{1.10^2} + \frac{5}{1.10^3} + \frac{7.5}{1.10^4} \right\} . 10^6$$

$$= \{ 2.5 + 9.524 + 13.636 + 4.329 + 4.132 + 3.757 + 5.123 \} . 10^6$$

$$= \text{€ } \underline{\mathbf{43{,}001{,}000}}$$

Conclusion: **Tender A is cheaper if criterion of least costs (cheapest) is applied**.

<u>b. Criterion of maximum NPV</u>

Tender A

Present Value of net benefits of the project at t = 0: 80. 10^6 x $\frac{1}{1.10^5}$ = € 49.67.10^6

Net Present Value = € 49.67 x 10^6 - € 40.7 x 10^6 = € **<u>8,970,000</u>**

Tender B

Present Value of net benefits of the project at t = 0: 80. 10^6 x $\frac{1}{1.10^4}$ = € 54.64.10^6

Net Present Value = € 54.64 x 10^6 - € 43.0 x 10^6 = € **<u>11,640,000</u>**

Conclusion: **Tender B is cheaper if criterion of maximum NPV is applied.**

4. 3. Equivalent annual cost method

In using the equivalent annual cost method for the purpose of comparison, all payments (costs) and receipts (benefits), are converted to their equivalent uniform annual costs. Again it is necessary to make an assumption about the required rate of return (freely interchangeable with the interest i) before it is possible to convert varaible cash flows to an uniform series of payments over the life of an investment proposal. The following example illustrates the application of the equivalent annual cost technique:

Example
To cross a river, a timber bridge has been designed, at an estimated cost of € 8 million. The lifetime of the bridge is estimated at 25 years and the annual costs for maintenance at 2.5 % of the construction costs. It is believed that a concrete bridge, with a lifetime of 50 years and annual costs for maintenance of 0.5 % of the construction costs, could be a better alternative. What are the maximum cost of a concrete bridge, in order to make this a viable alternative?
The discount rate is **7.5 %;** the residual value is in both cases zero.

Answer
The two designs represents mutually exclusive projects with identical benefits and constant annual costs the comparison can be made on annual costs basis.
Timber bridge

Annuity *[A/P, 7.5 %, 25]* =	0.0897 (say 9 %)	
Depreciation & interest	9 %	
Maintenance:	2.5 %	
Total annual costs:	**11.5 %**	of € 8 x 10^6

Concrete bridge (maximum construction cost X)

Annuity *[A/P, 7.5 %, 50]* =	0.0771 (say 7.7 %)	
Depreciation & interest:	7.7 %	
Maintenance:	0.5 %	
Total annual costs:	**8.2 %**	of **X** . 10^6

Therefore: $0.082 . \mathbf{X} . 10^6 < 0.115 \times € 8 \times 10^6$ **X < € 11.22 . 10^6**

Remark
If the NPV Method would have been used it has to be realised that the **service life** of the timber bridhe is shorter than the concrete bridge or with other words the two bridges do not offer the same 'service'. The timber bridge only provides a connection for 25 years, while the concrete bridge provides the same 'service' but for 50 years. So in order to make a correct comparison the timber bridge should be renewed after 25 years in order to provide the same duration of 'service'. In this example this is rather simple but usually the lifetime of one alternative is not equal or a multiple value of the other alternative. This problem is avoided by using the equivalent annual cost method.

Example. Equivalent annual cost comparison

A flood control pumping station is being designed. Three possible pumping stations are proposed and the relevant costs are shown in the table.
The cost of capital may be taken as **19 %.**

Scheme	**A**	**B**	**C**
Cost of pumps (€)	12,000	18,000	28,000
Life (years)	15	15	20
Maintenance Per annum (€)	1,000	1,500	1,500
Cost of pipes (€)	22,000	18,000	12,000
Life (years)	30	30	30
Cost of pumping (€ per hour)	1.20	0.90	0.80

Table: Costs of alternative schemes.

Questions
1. What is the most economic range of pumping times in hours/ year for each scheme (demonstrate your answer by a graph).

2. What is the most economical scheme if the expected frequency of pumping is according the following figure:

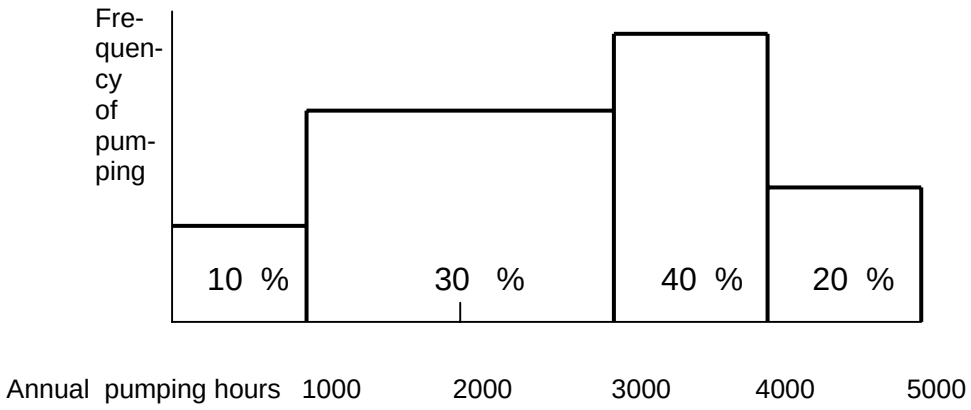


Figure: Frequency of pumping demand

Answer
The solution is to plot the equivalent annual costs of each scheme for different pumping demands and determine the range of pumping demands which are cheapest for each scheme.

Convert cost of installation of the pumps and the pipes to an annual cost by the annuity factor (or capital recovery factor), where **i = 19 %** and n = 15 or 20 years (pumps) or n = 30 years (pipes).

annuity for i = 19 % and n = 15 years: 0.20509
annuity for i = 19 % and n = 20 years: 0.19604
annuity for i = 19 % and n = 30 years: 0.19103

The maintenance cost of the pumps is already expressed in annual costs.
Calculate the annual 'fixed' costs, which are independent of the number of hours pumping.

Scheme A
The equivalent annual costs of installation and maintenance costs of the pumps and pipes = € 12,000 x 0.20509 + € 1,000 + € 22,000 x 0.19103 = € 7,663.74

Scheme B
The equivalent annual costs of installation and maintenance costs of the pumps and pipes = € 18,000 x 0.20509 + € 1,500 + € 18,000 x 0.19103 = € 8,630.16

Scheme C
The equivalent annual costs of installation and maintenance costs of the pumps and pipes = € 28,000 x 0.19604 + € 1,500 + € 12,000 x 0.19103 = € 9,281.48

The annual 'variable' cost depending on the number of hours pumping for each scheme are:

Pumping hours	*Scheme A*	*Scheme B*	*Scheme C*
0	0	0	0
1000	1,200	900	800
5000	6,000	4,500	4,000

These pumping costs vary linearly between 0 and 5000 hours.

Taking the 'fixed' equivalent annual cost and the 'variable' pumping cost the following figure can be plotted (see next page).

Economic break-even point between Scheme A and Scheme B at X pumping hours.

€ 7,663.74 + X hours x € 1.20 = € 8,630.16 + X hours x € 0.90

X hours x € 0.30 = € 966.42 **X** ⟶ **3,221** pumping hours.

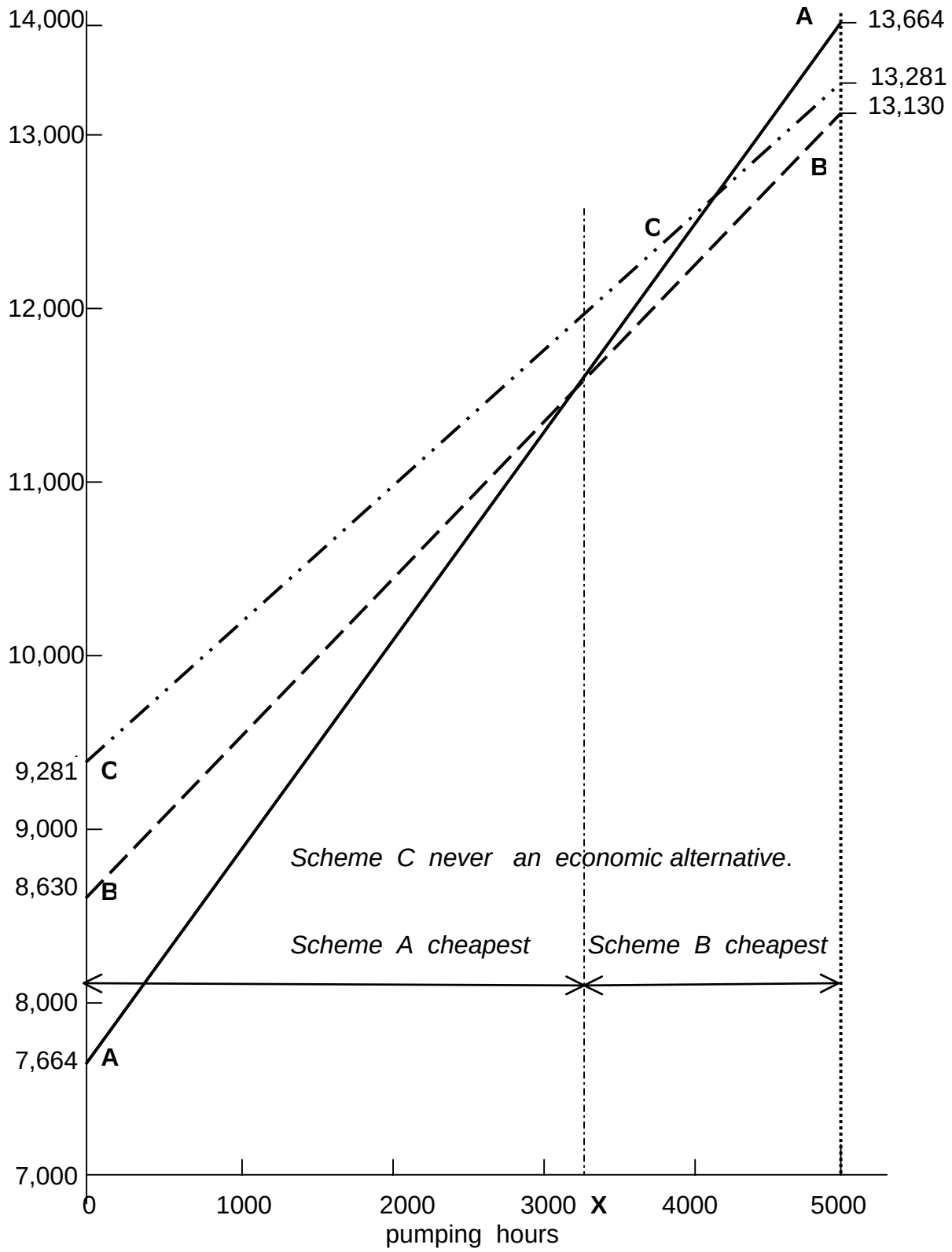


Figure: Annual costs versus pumping demand.

<u>Question 2</u>

For the given frequency of pumping demand the 'average' pumping hours is:
0.10 x 500 + 0.30 x 2,000 + 0.40 x 3,500 + 0.20 x 4,500 = 2,950
pumping hours; therefore Scheme A is the most economical solution.

4.4. Internal Rate of Return (IRR)

The IRR is defined as the **discount rate** at which the present value of benefits equals the present value of costs, or at which the **NPV = 0**.

Whereas the determination of the NPV is straightforward, the IRR as a rule cannot be calculated easily. Usually the IRR has to be determined by trial and error: by assuming some values for **i,** the NPV canbe calculated and by way of interpolation the value of **i** can be determined, for which the NPV = 0, thus yielding the IRR. Nowadays, various pocket calculators are programmed to determine quickly the IRR.

The IRR is a measure for the return on the investments that the project yields. Any project with an IRR exceeding the market rate of interest, i.e. the interest rate at which investible funds can be obtained, is acceptable. As such it can be used with other investment opportunities and in particular with the prevailing market rate of interest. The underlying assumption in the calculation of the IRR is that revenues generated by the project, can be re-invested against the same (high) rate as the IRR itself. This may be too an optimistic assumption, particularly if the IRR is high. There may not be other opportunities for investments which yield the same high returns.

<u>Example</u>
The construction of a water supply project is under construction and will be completed on January 1, 2006. The expenditure during construction are as follows:

January 1, 2002	€	150,000
January 1, 2003	€	200,000
January 1, 2004	€	250,000
January 1, 2005	€	300,000
January 1, 2006	€	200,000

A final payment to the contractor will be made on January 1, 2007 of € 100,000.

The useful life of the project is assumed at 20 years. The residual value of the project at the end of this period is nil. The interest that has to be paid on the borrowed capital is **7 %.** The annual cost of operation and maintenance at the end of every year is expected to be:

€	50,000	per year during the first five years,
€	100,000	per year during the second five years
€	150,000	per year during the last ten years.

It is expected that the sale of the water will be as follows:

1,000,000 m^3	per year during the first ten years,
2,000,000 m^3	per year during the second ten years.

Question a:
At what constant price should the water be sold in order to be able to liquidate the project at the end of the 20 years without debt, or profit ?
Question b:
The end of years receipts are assumed to be as follows:

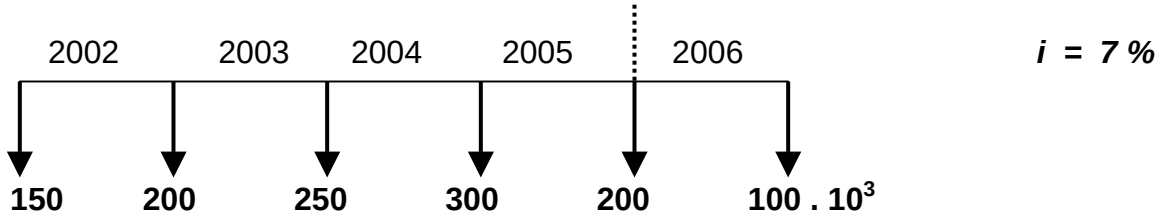
€	120,000	per year during the first five years
€	180,000	per years during the years 6 – 10
€	250,000	per year during the years 11 – 15
€	390,000	per year during the years 16 – 20

Determine the B/C ratio and the Net Present Value (NPV) (for **7 %** interest).
Determine the maximum interest rate for which the money could be borrowed whereby the project still is economically viable (**IRR** = Internal Rate of Return).

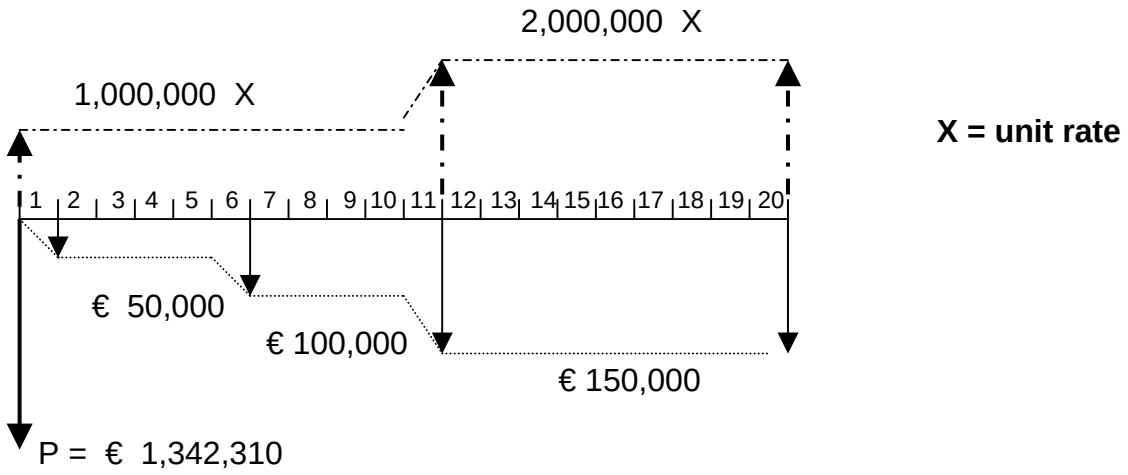
Answer

Question a

Present value at start of project (Jan. 1, 2006) of construction costs (in thousand euro's)



150 .*[F/P, i, 4]* + 200 .*[F/P, i, 3]* + 250 . *[F/P, i, 2]* + 300 .*[F/P, i, 1]* + 200 + 100 .[*P/F, i, 1]*
= 150 . 1.3108 + 200 . 1.2250 + 250 . 1.1449 + 300 . 1.07 + 200 + 100 . 0.9346
= 196.62 + 245 + 286.23 + 321 + 200 + 93.46 = **€ 1,342.31 . 10^3**



P.V. of Benefits : € 1,000,000 X *[P/A, i, 10]* + € 2,000,000 X *[P/A, i, 10] . [P/F, i, 10]*
= € 1,000,000 X . 7.0236 + € 2,000,000 X . 7.0236 . 0.5083
= **14,157,224 X**

P.V. of Costs : 1,342,310 + 50,000 *[P/A, i, 5]* + 100,000 *[P/A, i,5] [P/F, i, 5]* + 150,000 *[P/A, i, 10] [P/F, i, 10]* =
1,342,310 + 50,000 . 4.1002 + 100,000 . 4.1002 . 0.7130 + 150,000 . 7.0236 . 0.5083 =
1,342,310 + 205,010 + 292,338 + 535,565 = **€ 2,375,223**

Unit rate X : 2,375,223 / 14,157,224 = **€ 0.168 per m^3**

Question b

P.V. of Benefits : 120,000 *[P/A, i, 5]* + 180,000 *[P/A, i, 5] . [P/F, i, 5]* +
250,000 *[P/A, i, 5] . [P/F, i, 10]* + 390,000 *[P/A, i, 5] . [P/F, i, 15]*
= 120,000. 4.1002 + 180,000 . 4.1002 . 0.7130 + 250,000 .
4.1002. 0.5083 + 390,000 . 4,1002 . 0.3624 =
492,024 + 526,220 + 521,033 + 579,506 = **€ 2,118,783**

P.V. of Costs : **€ 2,375,223**

B/C ratio : **0.89**

NPV: 2.118,783 - 2,375,223 = (say**)** **€ 256,000** (for i = 7 %) (negative !)

IRR: **try 5 %:**

PV benefits: {120,000 + 180,000 . 0.7835 + 250,000 . 0.6139 + 390,000 .
0.4810}. 4.3295 = € 2.606.770

PV costs: Construction costs: € 1,300,000
Operation and maintenance:
50,000 . 4.3295 + 100,000 . 4.3295 . 0.7835 + 150,000 .
7.7217 . 0.6139 = € 1,366,744

NPV (5 %) € 2,606,770 - € 1,300,000 - € 1,266,770 = (say) + € 39,000

IRR therefore slightly above 5 % (by interpolation approximately 5.3 %).

4.4. Benefit-cost ratio

The B/C - ratio has been widely used in the early stages of benefit- cost analysis. It is defined as:

$$\text{B/C-ratio:} \quad \frac{\sum \frac{B_t}{(1+i)^t}}{\sum \frac{C_t}{(1+i)^t}}$$

Present value of B = Benefits
Present Value of C = Costs

the ratio of the present value of benefits to the present value of costs.

If the B/C ratio has a value of more than 1, then the project was considered to be attractive; if the value was less than 1, then the project could not earn back the inputs applied, and thus was not recommended for execution (for a certain value of i). For nearly 60 years the B/C ratio method has been the accepted procedure for making go / no-go decisions on independent projects and for comparing alternative projects in the public sector, even though the other methods as discussed will lead to identical recommendations, *assuming all these procedures are properly applied.*

$$\text{Conventional B/C-ratio:} \quad \frac{\sum \frac{B_t}{(1+i)^t}}{I + \sum \frac{O\&M_t}{(1+i)^t}}$$

Present Value of B = Benefits
Present Value of O & M = Operation and Maintenance Costs
Present Value of I = Initial Costs

$$\text{Modified B/C-ratio:} \quad \frac{\sum \frac{B_t}{(1+i)^t} - \sum \frac{O\&M_t}{(1+i)^t}}{I}$$

The resulting B/C-ratios will give consistent results in determining the acceptability of a project (B/C > 1 or B/C < 1 or B/C = 0). The magnitude however of the B/C ratio will differ between conventional and modified B/C. Therefore is the B/C – factor not used internationally anymore , for a number of reasons:

1. Without further information, the B/C ratio is not well-defined: are the benefits net of running costs, or are gross benefits considered?
2. The project with the highest B/C ratio does not always yield the highest value for other indicators (NPV, IRR), used in project appraisal.

Example

For the extension of the runway of an airport land needs to be purchased for **€** 350,000. Construction cost for the runway are projected to be **€** 600,000 and the additional annual maintenance cost for the extension are estimated to be **€** 22,500. If the runway is extended, a small terminal will be constructed at a cost of **€** 350,000. The annual extra operating and maintenance cost for this terminal is estimated at **€** 75,000. The operational cost of the airport itself will increase by **€** 100,000 for additional air traffic controllers to cope with the increased number of flights. The annual benefits of this extension, consisting of extra income from airlines leasing, airport tax, convenience benefit, additional tourism, is estimated at **€** 490,000. Apply with a study period of 20 years and 10 % interest rate

Conventional B/C-ratio:

$$\frac{€\ 490{,}000\ .\ (P/A,\ 10\ \%,\ 20\ years)}{€\ 1{,}200{,}000\ +\ €\ 197{,}500\ .\ (P/A,\ 10\ \%,\ 20\ years)} =$$

$$\frac{€\ 490{,}000\ .\ 8.5136}{€\ 1{,}200{,}000\ +\ €\ 197{,}500\ .\ 8.5136} = 1.448$$

Modified B/C-ratio:

$$\frac{€\ 490{,}000\ .\ (P/A,\ 10\ \%,\ 20\ years)\ -\ €\ 197{,}500\ .\ (P/A,\ 10\ \%,\ 20\ years)}{€\ 1{,}200{,}000} = 2.075$$

The difference between conventional and modified B/C –ratios is essentially due to subtracting the equivalent present value of operating and maintenance from both the numerator and the denominator of the B/C-ratio. Subtracting a constant (the present value of O & M costs) from both numerator and denominator does not alter the relative magnitudes of the numerator and denominator but the ratio is not the same.

An additional issue of concern is the treatment of *disbenefits* in benefit/cost ratio analysis. In the example of the runway extension project the increased noise level from commercial planes will be a serious nuisance to people living nearby the airport. The annual disbenefits of this '*noise pollution'* is estimated at **€** 100,000. Taken this into account the conventional B/C-ratio's will change as follows:

Disbenefits considered as reduced benefits:

$$\frac{[\ €\ 490{,}000\ -\ €\ 100{,}000\ ]\ .\ (P/A,\ 10\ \%,\ 20\ years)}{€\ 1{,}200{,}000\ +\ €\ 197{,}500\ .\ (P/A,\ 10\ \%,\ 20\ years)} = 1.152$$

Disbenefits treated as additional costs:

$$\frac{[\ €\ 490{,}000]\ .\ (P/A,\ 10\ \%,\ 20\ years)}{€\ 1{,}200{,}000\ +\ [\ €\ 197{,}500\ +\ €\ 100{,}000\ ]\ .\ (P/A,10\ \%,\ 20\ years)} = 1.118$$

4.6. Exercises Cost Benefit Analysis

1. At a long-term strip-mining coal site it is proposed to maintain temporary haulage roads serving the excavation by using hand labour. The annual wage bill is estimated to be **€ 105,000**. With other associated expenses, the total cost of labour to the contractor will be **€ 145,000** per year. The production of coal on the site is expected to last for 6 years, and alternative methods of constructing and maintaining haulage roads need to be investigated.

   The first alternative is to buy a motor-grader for **€ 95,000** and, as a consequence, reduce the labour force. Maintenance of the grader is estimated to average **€ 4,000** per year for the 6 years, after which it will have a salvage value or resale value of **€ 20,000**. The labour costs associated with the use of the grader amount to **€ 80,000** per year.

   The second alternative is to lay more substantial roads in the first instance, extending these after 2 years and again after 4 years. Initial costs are then **€ 80,000** , with further invest-ments of **€ 40,000** and **€ 37,000** after 2 and 4 years respectively. Total labour costs in this scheme amount to **€ 64,000** per year.

   If the return of at least **10 %** is desirable on the capital invested , which is the most economic scheme ?
   Make the comparisons based on :
   a. the equivalent annual cost method and
   b. the present value method.

2. The erection of a building for storage is under consideration. There are two technical acceptable alternatives: a reinforced concrete shell roof structure having an initial cost of **€ 2,700,000** and a steel-framed structure with brick cladding for an initial cost of € **1,800,000**. The life of the concrete building is estimated to be **60 years** and, while there will be no maintenance costs for the building during the first **10 years**, there will thereafter be an annual maintenance cost of **€ 35,000**. The life of the other building is estimated to be **20 years** with, an equivalent annual maintenance cost from completion of construction of **€ 40,000.** The salvage value of the concrete building is estimated at **€ 80,000** and that of the steel-framed building at **€ 27,000**.
   An acceptable rate of return is assessed at **10 %**.
   Which is the better economic proposition ?
   Make the comparisons based on :
   a. the equivalent annual cost method and
   b. the present value method.

3. A specialized piling rig is purchased by a contractor for one project only. The duration of the project is two years. The economic life of the rig is 10 years, but it is sold at the end of the project, that is after 2 years, then the contractor will be able to get half the purchese value. If the rig costs **€ 75,000** and the required rate of return is **10 %,** what is the annual cost of the rig to the contractor if operating expenses are ignored ?

4. A pumping scheme being developed has three different possible systems of pumps and pipeworks. If the life of the scheme is **20 years**, which scheme should be recommended as the most economic ?

Scheme	*Pipe diameter* (mm)	*Installation cost* (€)	*Annual running cost* (€)
A	500	24,000	9,500
B	600	26,000	6,000
C	700	31,000	5,200

Use **10 %** to represent the cost of capital. If the cost of capital was **6 %**, would the recommendation alter ?

5. A hydroelectric project, if completely developed now, will cost **€ 100,000,000.** Annual operation and maintenance charges will amount to **€ 5,000,000** per year. Alternatively, **€ 55,000,000** may be invested in the project now and the remainder of the work carried out in 12 years' time at a cost of **€ 53,000,000**. In this alternative case annual operation and maintenance charges will be **€ 3,400,000** per year for the first 12 years and **€ 5,600,000** per year thereafter. Both schemes are assumed to have perpetual life. Compare their equivalent annual costs with an interest at **12 %.**

6. Water for an irrigation scheme can be supplied either by gravity (Alternative A) or by pumping (Alternative B).
Alternative A requires a relatively long canal with intake from a reservoir. The total investment is estimated at **€ 300,000**. The annual costs for maintenance and operation are estimated at **€ 10,000.** Useful service life is estimated at 30 years.
Alternative B requires a pumping station with an intake from a nearby river. The invest-ments are estimated at **€ 90,000** for the civil engineering structures with a service life of 30 years and at **€ 25,000** for the mechanical and electrical equipment with a service life of 15 years. The annual costs for maintenance and operation are estimated at **€ 20,000**.
The net salvage of all investments at the end of their service life is assumed to be zero.
   a. Determine the most economic alternative for an interest rate is 6 %
   b. Determine the most economic alternative for an interest rate is 4 %
   c. Determine the unit cost per m^3 for an interest rate of 6 % if the estimated consump-tion is 1.5 million m^3 / year during the first 6 years and 2 million m^3 / year during the remaining 24 years.

7. In an economic assessment concerned with the alignment of a new road, one of the alternatives to be evaluated on the basis of annual cost consists of a bridge at an estimated cost of **€ 1,350,000**, an embankment costing **€ 215,000**, and other earthworks at an estimated cost of **€ 38,000.** Maintenance on the earthwork and the embankment is estimated to reach an annual cost of **€ 30,000** over the first 4 years of its service and then drop to **€ 14,000** for every year thereafter. Mainte-nance on the bridge is expected to remain constant throughout its life at a figure of **€ 70,000** a year.

What is the total equivalent uniform annual cost of this alterna-tive if the life of the bridge is estimated at 60 years, the life of the earthworks and the embankments is in perpetuity and the interest rate to be used is **15 %**?

8. A proposed highway project requires an initial investment of **€ 10 million** and a supplementary investment of **€ 5 million** at the end of the tenth year. The project will have an useful life of 50 years, counting from the date of the initial investment. The interest rate is 6 %. The cost of operation and maintenance is **€ 200,000** per year. The benefits of the project has been estimated to begin with **€ 1.0 million** per year for the first 15 years (at the end of each year), thereafter increasing at once to **€ 2.75 million** per year and remaining constant for the remaining 35 years. Determine the value of Benefit-cost (B/C) ratio, Net Present Value (B-C), and Internal Rate if Return (IRR).

9. In diverting river water for an irrigation project, two alternative schemes are prepared, as follows:
   Scheme 1. Open ditch and tunnel with a capital cost of **€ 2,500,000** and an annual maintenance cost of **€ 40,000** per year.
   Scheme 2. Pipework and open flume with a capital cost of **€ 1,750,000** and a maintenance cost of **€ 80,000** per year, with a major replacement cost of € **120,000** every 10 years.
   Either of the above schemes will provide the service required. If the current interest rate is **12 %**, compare the two schemes on the basis of capitalized cost (n is 100 = perpetuity).

10. In a remote wilderness in Africa a rich ore deposit has been discovered. It has been estimated that all ore can be mined during a period of 20 years. The most economical way to bring out the ore is by river. To make the river navigable there are two alternative projects:
    Plan A to regulate the river by training works, excavation and blasting of rock, with a total initial cost of **€ 10,000,000** and a cost of dredging of **€ 2,000,000** per year.
    Plan B to canalize the river by means of weirs and navigation locks: initial costs **€ 20,000,000** and cost of operation and maintenance of **€ 400,000** per year.
    Capital for both projects is available at **10 %** interest.
    The terminal value of the navigation works after 20 years is assumed to be nil.
    Question a: Make a cost comparison of annual costs.

    The cost of dredging of Plan A is now expected to be as follows:
    € 100,000 during the first year and then gradually increases by an amount of **€ 200,000** per year till it would reach a cost of **€ 3,900,000** during the twentieth year.
    Question b: Determine which of the two projects is more economic.

11. A new highway of 25 m wide is in the stage of being designed. A considerable portion of the highway has to be cut deeply (10 m) in the surrounding terrain of sandy soils. The problem is to determine the most economic side slope of the cut. If they are steep, they will require a lot of maintenance due to erosion during heavy rainfall. If they are flat, they require extra excavation during the construction of the highway. The capital cost of excavation and disposal of the soil is **€ 3.00** per m^3.

Slope	Total excavation (m^3) per km	Annual slope maintenance (**€**)
1 : 1 (n = 1)	250,000 + 100,000 = 350,000	**€ 80,000** per km
1 : 2 (n = 2)	250,000 + 200,000 = 450,000	**€ 50,000** per km
1 : 3 (n = 3)	250,000 + 300,000 = 550,000	**€ 34,000** per km
1 : 4 (n = 4)	250,000 + 400,000 = 650,000	**€ 24,000** per km

The capital cost of the road deck is **€ 250,000** per km. The useful life of the project is 50 years. Annual maintenance of the road deck costs **€ 3,000** per km. The interest rate is **5 %.**

12. An appraisal of three alternatives, mutually exclusive projects, A, B, and C, is being made for a company that requires a return of at least **10 %** on its invested capital. The estimated details of the investment are shown in the table below. Which investment should be recommended and why?
Support your recommendation and reasoning by calculation.

Euro	*Project A*	*Project B*	*Project C*
Initial cost	100,000	160,000	280,000
Scrap value	nil	nil	40,000
Net annual receipts	18,400	30,600	42,300
Life, years	8	8	10

13. A decision has to be made with regard to the installation of automatic control equipment on a concrete batching plant installed at the construction site. Quotations for the equipment show its cost to be **€ 300,000** , but its installation will have the effect of reducing annual labour cost from an estimated **€ 150,000** to **€ 45,000**.
Maintenance of the automatic plant is expected to amount to **€ 6,000** per year more than the manually controlled plant and only this excess cost need be considered in the analysis.

The automatic equipment, if installed, will have a salvage value of **€ 30,000** irrespectively of the length of time it is in use. The contractor carrying out the work state their rate of return on capital to be **10 %**. Will the selection of the automatic equipment for the contract with a duration of 3.5 years be justified, and what is the minimum contract period that will do this?

14. A public agency has sufficient funds available for a number of projects. One of these projects can be executed in four ways (A, B, C or D). The investments and the net annual benefits of the 4 alternatives are listed in the following table:

Alternative	Investment	Net annual benefits
A	100	20
B	200	30
C	300	50
D	500	75

All amounts are given in thousands of euros.
Assume that all alternatives have an infinitely long service life and that the net annual benefits remain constant in the future. Questions:

a. Which alternative has the highest rate of return ?
b. Which alternative is to be preferred if unused funds can be invested in other projects with a rate of return of **10 %** ?
c. Would you come to another conclusion than that given onder b, if unused funds could be invested in projects with a rate of return of **14 %** ?

15. For the installation of a pipeline connection two different payment schedules are offered:

a. an immediate payment of **€ 1,150** at the moment the connection is made, or
b. 8 annual payments of **€ 231.50** with the additional condition that these payments have to be made at the beginning of each year.

Questions:

a. Which payment proposal do you prefer if you can borrow **€ 1,150** now for 8 years at **12%** per year under normal conditions (payment at the end of the period).
b. What is the effective annual interest rate in the case of 8 annual payments?

16. The first cost of a project is **€ 100,000**. The annual equivalent operation and maintenance costs are **€ 15,000**. The annual equivalent benefits are **€ 26,500**. The life of the investment is 25 years. Its net salvage value is zero.
Questions:

a. Estimate the internal rate of return of the project.
b. Could the investment be made economically if funds are available at an interest rate of **4%** per year? Explain your answer briefly.
c. In how many years can a loan for the financing of this project be repaid, if the loan carries an annual interest rate of 4 % and the annual surplus is initially used for this repayment ?

17. Water has to be transported by gravity by means of a canal. The canal has a useful life of 20 years and requires an investment of **€ 1,000,000.** The interest rate is **10 %** per year. The net salvage value of the canal after 20 years of operation is assumed to be zero. The annual equivalent maintenance and operation costs are estimated at **€ 100,000**.
Calculate the constant transportion cost (unit cost) in **€ per m^3** for the following cases:
   a. the annual transport is 15 million m^3 throughout the 20 years' period;
   b. the annual transport is 13 million m^3 during the first period of 10 years and 17 million m^3 during the second period of 10 years.

18. The following loans were taken to finance the planning, design and construction of a project:

Loan	*Annual interest rate*	*date*
€ 1,000,000	10 %	31st Dec.2002
€ 2,000,000	8 %	1st Jan. 2004
€ 5,000,000	6 %	1st Jan. 2005
€ 5,000,000	4 %	31st Dec.2005

An additional loan will be needed for the final payment of **€ 1,000,000** due on the 1st of January 2007. All previous and future loans are consolidated ("refinanced") at an interest rate of **4 %** per year on the 1st of January 2006, the day the project is put into operation. The expected annual equivalent operation and maintenance cost are **€ 1,000,000**. The expected annual revenue (gross benefit) is **€ 3,250,000**. The net salvage value after 20 years is expected to be **€ 1,500,000**.
Questions:
   a. What is the first cost of this project and what is the total depreciation ?
   b. What is the internal rate of return ?
   c. What is the equivalent annual surplus (profit) of this project ?
   d. What is the marginal rate of return of a proposed extension which will cost an additional **€ 1,500,000** , which will not raise the O & M costs and net salvage value but which will raise the annual revenue to **€ 3,400,000** ?
   e. Will it be justified from an economic point of view to invest these **€ 1,500,000** in the proposed extension if this money can also be invested in another project which will have an internal rate of return of **12 %** ?

19. A project according plan A requires an investment of **€ 4,000,000**. Its useful service life is 15 years. The annual costs for maintenance and operation are **€ 200,000**. The annual benefits are estimated at **€ 624,000**. It is being considered to extend the project by an additional investment of **€ 1,000,000**. This plan B (the extended version of plan A) requires a total investment of **€ 5,000,000**. The total annual costs for maintenance and operation will increase to **€ 240,000** , whereas the total annual benefits are now estimated at **€ 722,000**.

Questions:

a. Determine the Benefit-Cost ratios of the plans A and B with interest at **6 %.**

b. Determine the rate of return of the plans A and B, as well as the marginal rate of return of plan B with respect to plan A.

c. Will it be worthwhile to execute plan A or plan B if unused funds can be invested in other projects having a rate of return of **5 %** ?

20. The useful life of an **€ 10 million** bridge depends on how often it is repaired and painted. Use the formula: $y = x^2 + 20$, in which y is the useful life in years, and x is the number of times per decade that the bridge gets a repair and paint job at a cost of **€ 250,000** each time. The interest rate is **5 %**. Determine the most economic frequency (in times per decade) of giving the bridge a repair and paint job.

21. In a country a new coal mine will be put into production; the total output will be exported. There are 2 options for the transportation of the coal to the port of export:

a. Water transport

The river on which the mine is situated has to be improved for navigation:

- Length 400 km
- Construction capacity 50 km/year
- Start of construction 1st January 2002
- Construction costs LC 40×10^6 per 100 km, spread evenly over the construction period, payable at the end of each year
- Maintenance costs LC 2×10^6 per 100 km per year
- Transportation costs LC 0.05 per ton per 100 km

b. Rail transport .

A new railway line has to be constructed:

- Length 375 km
- Construction capacity 75 km/ year
- Start of construction 1st January 2005
- Construction costs LC 32×10^6 per 100 km, spread evenly over the construction period, payable at the end of each year
- Maintenance costs LC 2.5×10^6 per 100 km per year
- Transportation costs LC 0.07 per ton per 100 km.

Other relevant data are:

- **LC** is one unit of Local Currency
- Total production 5×10^6 ton per year
- For water transport start of construction: 1st January 2002
- For rail transport start of construction: 1st January 2005
- Both options have a life time of 50 years, without any residual value.
- All costs and benefits occur at the end of the year.
- Discount rate **10 %.**

Questions :

a. If the project is financed from internal resources (local currency = L.C.), which of the two options is to be preferred?

b. Investments will be provided partly from external resources (foreign currency **F.C**.), but all costs for maintenance and transportation will be financed from internal resources (L.C.). The local currency (L.C.) is overvalued by a factor 2; meaning foreign component costs (F.C.) is 2x expressed in local currency (L.C.).
Proportion of foreign currency in total investment costs:
- Water transport 20% F.C. (and 80 % L.C.)
- Rail transport 80% F.C. (and 20 % L.C.)

Which option is to be preferred now?

c. Transportation time for the railway line is 5 hours less than for water transport, against a value of LC 0.02 per ton per hour.
Which option is to be preferred for each of the cases 1. and 2. above?

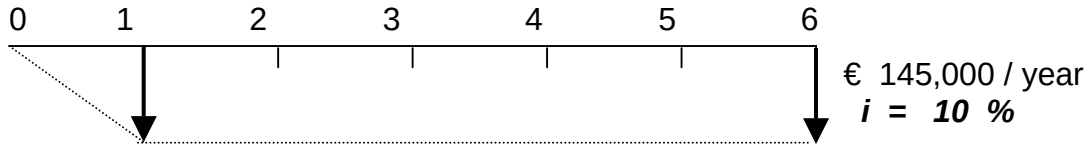
22. The purchase price for a piece of construction equipment is **€ 20,000** . The operating costs based on the annual average estimated hours of operation are: **€ 800** in the first year; **€ 1200** in the second year, **€ 1500** in the third year, **€ 1800** in the fourth year and **€ 2100** in the fifth year.
The resale value of the plant can be assumed as follows: **€ 15,000** after 3 years, **€ 12,000** after 4 years and **€ 8,000** after 5 years .
The cost of capital is **8 %** per year.
Question: Calculate the optimum replacement age.

23. A reinforced concrete road pavement, including the base, is laid for **€ 100.00 per m^2**.
A flexible pavement to give the same service is laid for **€ 90.00 per m^2**.
The flexible pavement has major maintenance every 5 years, which costs the equivalent of **€ 3.25 per m^2 per year**. The concrete pavement has a first lifetime of 40 years, after which it is resurfaced with asphalt costing **€ 31.00 per m^2**.
Thereafter it is maintained at the same cost as a flexible pavement. In addition, both types of road require annual maintenance estimated to amount to **€ 0.67 per m^2.**
On the basis of both roads giving perpetual service, compare the capitalized costs of 2000 m^2 of road at an interest rate of **12 %.**

4.7. Answers exercises

Problem 1.

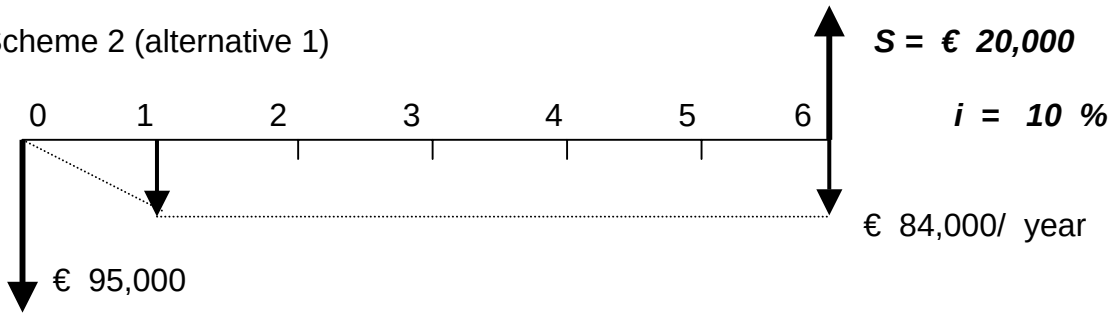
a. equivalent annual cost method. Scheme 1 (original situation)



Annual cost of labour = € 145,000

This is the sole annual outgoing and requires no conversion to annual payments.

Scheme 2 (alternative 1)



Annual maintenance costs of grader =	€	4,000
Annual cost of labour =	€	80,000
Subtotal =	€	84,000

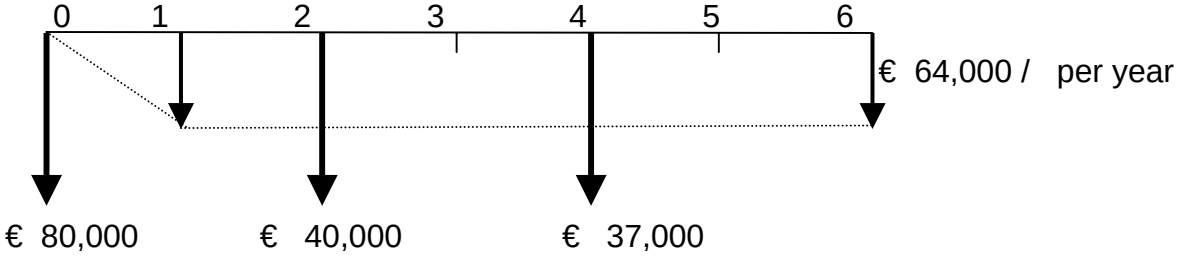
Annual capital recovery cost of the motor grader (where S is the salvage value of the grader):

(P – S) *(A/P, 10 %, 6 years)* + S . i

(95,000 – 20,000) (0.2296) + 20,000 (0.10) = € 19,220

Total equivalent annual cost = **€ 103,220**

Scheme 3 (alternative 2)



Annual capital recovery of initial cost:

80,000 *(A/P, 10 %, 6 years)* = 80,000 (0.2296) = € 18,368

Annual capital recovery for capital cost at end of 2 years:

40,000 *(P/F, 10 %, 2) (A/P, 10 %, 6)* = 40,000 (0.8265) (0.2296) = € 7,591

Annual capital recovery for capital cost at end of 4 years:

40,000 *(P/F, 10 %, 4) (A/P, 10 %, 6)* = 40,000 (0.6830) (0.2296) = € 5,802

Annual labour costs = € 64,000

Total equivalent annual cost = **€ 95,761**

Scheme 3 is therefore the most economic on the basis of this evaluation because its equivalent annual cost is lower than those of the other two schemes.

There are a number of points to be noted. The first concerns the treatment of salvage values when computing annual capital recovery costs. The salvage value (€ 20,000) will become available from the sale of the grader at the end of 6 years. Therefore, the part of the cost which is invested over the 6 years of the grader's useful life, and which will not be recoverable as salvage, is the initial cost *less* the salvage value (€ 95,000 – € 20,000 = € 75,000). Since the salvage value will become available again at the end of 6 years it is only necessary to *charge* to each equivalent annual cost the interest on that amount. Treating each year separately, the salvage value can be looked on as being locked up or loaned for the initial purpose of the grader during each year and it is therefore not possible to earn interest or profit by investing the money elsewhere. Account is taken of this in the calculation.

In Scheme 3, each of the payments is converted to present value before being converted to an equivalent uniform series of payments over the 6 years of the comparison.

Finally, the only overriding assumption is that each of the three schemes considered will either give equally good service if put into operation and/or at least will provide the minimum service required. In making an economic choise between the alternatives, it is assumed that the technical merit of each alternative has been examined and found to be satisfactory. The only considerations that may now affect the ultimate decision are the *irreducible* factors.

One example of an irreducible factor might be that there is an ample supply of skilled labour in an area where unemployment is high. It therefore becomes a social obligation of the contractor to act beneficially as he is able towards the local community. There may, for the contractor, be other spinoffs in doing that, which though irreducible in themselves, create a better climate in which to work – a benefit that may well outweigh some of the other considerations.

In the above problem, the comparison between the schemes was made on the basis that each of them represented the annual cost for 6 years. The equivalent annual costs were therefore comparable because the lives of the alternatives were assumed to be the same. This may not always be the case, particularly where the construction of more permanent installations is under consideration.

b. Present value method

Scheme 1

Present value of annual labour cost over 6 years:

€ 145,000 *(P/A, 10 %, 6 years)* = € 145,000 . (4.3552) =	**€ 631,504**

Scheme 2

Initial cost of motor grader =	€ 95,000
Present value of maintenance cost and labour cost	
€ 84,000 . *(P/A, 10 %, 6 years)* = € 84,000 . (4.3552) =	€ 365,837
Subtotal =	€ 460,837
Less: Present value of salvage value	
€ 20,000 . *(P/F, 10 %, 6 years)* = € 20,000 . (0.56448) =	€ 11,290
Present value of total costs =	**€ 449,547**

Scheme 3

Initial cost of first section of road = € 80,000

Present value of second investment:

€ 40,000 . *(P/F, 10 %, 2 years)* = € 40,000 . (0.82645) = € 33,058

Present value of third investment:

€ 37,000 . *(P/F, 10 %, 4 years)* = € 37,000 . (0.68302) = € 25,272

Present value of annual labour cost over 6 years

€ 64,000 . *(P/A, 10 %, 6 years)* = € 64,000 . (4.3552) = € 278,733

Present value of total costs = **€ 417,063**

Therefore, on the basis of the above present value evaluation the economic appraisal comes out in favour of Scheme 3, since, in effect, with the given interest rates, the whole scheme can be financed with a smaller lump sum than the other two.

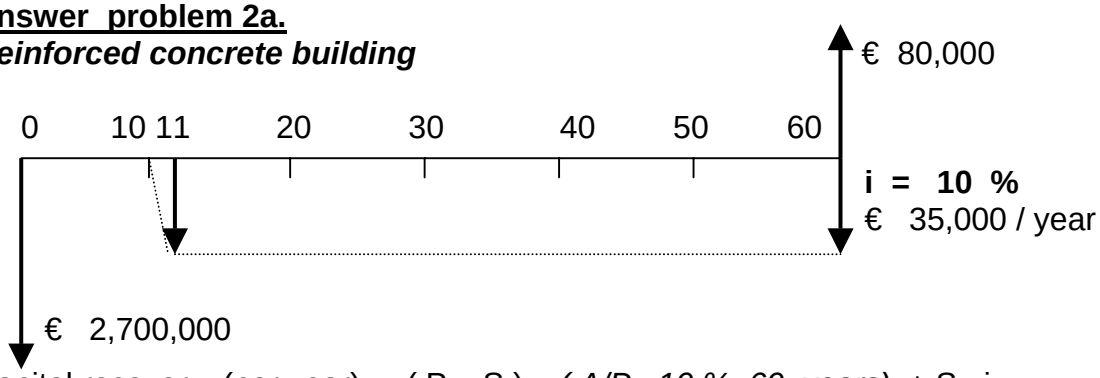
In the case of scheme 3, where there are several staged investments over the period under consideration, it will be noted that one step in the computation has been saved in considering present value rather than equivalent annual cost method for comparison purposes. On the other hand, all the payments for labour, for example, that are already convenient form for annual costs, need to be converted to a lump-sum present value.

Check

Scheme	Equivalent annual cost	%	Present value	%
1	€ 145,000	100 %	€ 631,504	100 %
2	€ 103,220	71.2 %	€ 449,547	71.2 %
3	€ 95,761	66.0 %	€ 417,063	66.0 %

Answer problem 2a.

Reinforced concrete building



Capital recovery (per year) = (P – S) . *(A/P, 10 %, 60 years)* + S . i =

(€ 2,700,000 – € 80,000) . (0.1003) + € 80,000 (0.10) = € 270,786

The sum of money at the end of year 10 equivalent to € 35,000 per year from years 11 to 60:

€ 35,000 . *(P/A, 10 %, 50 years)* = € 35,000 . (9.9148) = € 347,018

Present value of ε 347,018 at year 0:

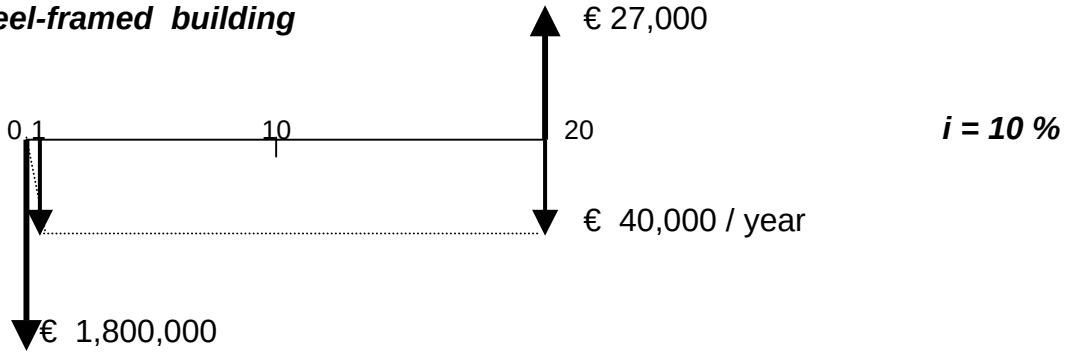
€ 347,018 *(P/F, 10 %, 10 years)* = € 347,018 (0.3856) = € 133, 810

Therefore, equivalent annual cost over 60 years of ε 35,000 a year from years 11 to 60:

€ 133,810 . *(A/P, 10 %, 60 years)* = € 133,810 (0.1003) = € 13,421

Therefore, **total equivalent annual cost** = 270,786 + 13,431 = **€ 284,207**

Steel-framed building



Capital recovery = (€ 1,800,000 – € 27,000) . *(A/P, 10 %, 20 years)* + 27,000 . i
= 1,773,000 . (0.1175) + 27,000 . (0.10) = € 211.028 / per year
Therefore, **total equivalent annual cost** = € 211,028 + € 27,000 = **€ 238,073**

The steel-framed building is therefore cheaper when the comparison is made on basis of annual costs.

This problem raises a number of points. A comparison has been made on the basis of annual cost and it is therefore implicit in the calculation that after 20 years the steel-framed building can be replaced at the same cost as the initial installation and that the repla-cement will continue at this cost at intervals of 20 years. Rising costs are inevitable in this context, though it is not unreasonable that such a method of comparison should be used because, in the majority of cases, the future cost increases, when discounted to the present time, quickly become a relatively small proportion of present costs.

In the case of the reinforced concrete building, the capital investment is being made now, and therefore no question of increased cost in the replacement situation arises.

If the replacement cost of the steel-framed building in 20 years' time is increased by 50 % over the present-day cost, that is, it becomes € 2,700,000, then the present value of the increase in cost under similar conditions of interest amounts to :

€ 900,000 . (0.1486) = € 133,779

If the second replacement cost in 40 years' time increases by 50 % over the first replacement value, that is, it becomes € 4,050,000, then the present value of the total increase amounts to:

€ 2,250,000 . (0.02209) = € 49,703

The two sums produce an equivalent uniform annual cost of
(€ 133,779 + € 49,703) . (0.10032) = € 18,407
over the total life of 60 years under consideration.

The **total equivalent annual cost** now becomes: **€ 256,480**

The steel-framed building remains therefore to be cheaper when the comparison is made on basis of annual costs.

Quite apart from the financial aspects of the economical appraisal, there may be considerable advantages within many businesses from constructing buildings with a shorter life.
New developments in products and building materials may enable such a company to replace the building in 20 years' time with one that gives improved performance.

Replacement may well take place at a cost comparable to that of the original building or investment because of technical improvements. With the long-life building in such a situation it may be difficult to make good use of it in changed circumstances unless money is spent on its rehabilitation. This aspect becomes an irreducible factor in such a situation.

Alternatively, future costs can be estimated only by the interpretation of historic trends. Since, historically, costs have always risen continuously and steadily (with a few exceptions), it seems likely that they will continue to do so. A longer-life investment is clearly advantageous in this circumstance.

Answer Problem 2b. **Present value method**

Reinforced concrete building

Initial cost of building =	€ 2,700,000
Less: Present value of salvage value +	
€ 80,000 . (*P/F, 10%, 60 years)* = € 80,000 . (0.0033) =	€ 264
Subtotal =	€ 2,699,736

Equivalent capital value at the end of year 10 of annual maintenance of € 35,000 per year from years 11 to 60:

€ 35,000 . *(P/A, 10 %, 50 years)* = € 35,000 . (9.9148) = € 347,018

Present value of € 347,018 at year 0:

€ 347,018 . *(P/F, 10 %, 10 years)* = € 347,018 . (0.3856) = € 133, 810

Present value of total payments over 60 years = € 2,833,546

Steel-framed building

Initial cost of building =	€ 1,800,000
Present value of maintenance cost:	
€ 40,000 . *(P/A, 10 %, 60 years)* = € 40,000 . (9.9671) = € 398,684	
Present value of renewal cost less salvage cost at the end of 20 years:	
(€ 1,800,000 – € 27,000) . *(P/F, 10 %, 20 years)* = € 1,773,000 . (0.14865) =	€ 263,556
Present value of renewal cost less salvage cost at the end of 40 years:	
(€ 1,800,000 – € 27,000) . *(P/F, 10 %, 40 years*) = € 1,773,000 . (0.02210) =	€ 39,183
Subtotal =	€ 2,501,423
Less:	
Present value of € 27,000 . *(P/F, 10 %, 60 years):*	
27,000 . (0.00328) =	€ 89
Present value of total payments over 60 years =	**€ 2,500,334**

This confirms the result of the analysis made by the equivalent uniform annual cost method .

In the above problem, using the present value method where the buildings have different lives, it should be noted that the comparison has to be made over a period of time that is the lowest common multiplier of the lives of the alternatives.

It is therefore necessary in the case of the steel building to consider the replacement costs at the end of 20 and 40 years, together with salvage values at the end of 20, 40, and 60 years.

The present value of the series of maintenance payments for the concrete building could have been calculated in a different way. The payments did not commence until year 11 and they continue until the end of year 60. If the factor for conversion of an annual payment to present value for the first 10 years is subtracted from the similar factor over a 60-year period and is then multiplied by the annual amount, the same result will be obtained (note small arithmetical error due to the rounding of the factors).

Present value of payments for years 11 – 60:
€ 35,000 . [*(P/A, 10 %, 60 years)* - *(P/A, 10 %, 10 years)*] =
€ 35,000 . (9.9671 - 6.1445) = € 35,000 . (3.8226) = **€ 133,791**

Having obtained either total equivalent annual costs or total present values, then either of these amounts can readily be converted into the other. For example, the total payments at total present value of the concrete building can be converted to total annual costs as follows:
Equivalent annual cost:
€ 2,833,546 . *(A/P, 10 %, 60 years)* = € 2,833,546 . (0.1003) = **€ 284,205**

Answer problem 3
Annual capital recovery cost of the piling rig
(€ 75,000 - € 37,500) . *(A/P, 10 %, 2 years)* + € 37,500 . i =
= € 37,500 . (0.5762) + € 3,750 = € 21,608 + € 3,750 = **€ 25,358**

Answer problem 4
Calculate the present value of each scheme using 10 %
Scheme A
Present value of installation cost = € 24,000
Present value of maintenance costs:
€ 9,500 . *(P/A, 10 %, 20 years)* = € 9,500 . (8.5135) = € 80,878
total present value = **€ 104,878**
Scheme B
Present value of installation cost = € 26,000
Present value of maintenance costs:
€ 6,000 . *(P/A, 10 %, 20 years)* = € 6,000 . (8.5135) = € 51,081
total present value = **€ 77,081**
Scheme C
Present value of installation cost = € 31,000
Present value of maintenance costs:
6,000 . *(P/A, 10 %, 20 years)* = € 5,200 . (8.5135) = € 44,270
total present value = **€ 75,270**
At **10 %** Scheme C is the most economical because it has the smallest present value.

Repeating the calculations at **6 %**

Scheme A

Present value of installation cost =	€	4,000
Present value of maintenance costs:		
€ 9,500 . *(P/A, 6 %, 20 years)* = € 9,500 . (11.4679) =	€	108,945
total present value =	**€**	**132,945**

Scheme B

Present value of installation cost =	€	26,000
Present value of maintenance costs:		
€ 6,000 . *(P/A, 6 %, 20 years)* = € 6,000 . (11.4679) =	€	68,807
total present value =	**€**	**94,807**

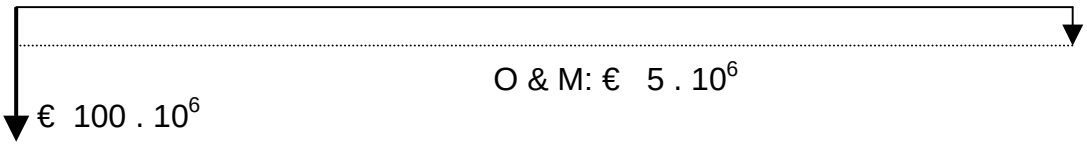
Scheme C

Present value of installation cost =	€	31,000
Present value of maintenance costs:		
€ 6,000 . *(P/A, 6 %, 20 years)* = € 5,200 . (11.4679) =	€	59,633
total present value =	**€**	**90,633**

Scheme C is at 6 % the most economical; the difference has become larger due to the lower interest rate . Only for a certain interest rate higher then 10 % there will be a certain interest rate whereby Scheme B becomes more economical as the difference in maintenance costs has less weight.

Answer problem 5

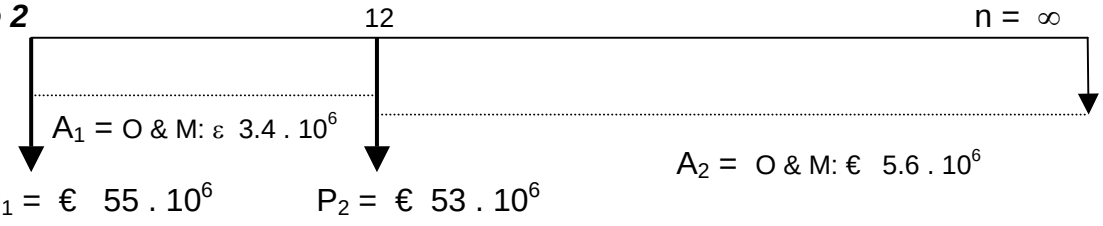
Alternative 1 ***i = 12 %*** n = ∞



Annuity A = P . i (for n = ∞) = € 100 . 10^6 . 0.12 =	€	12 . 10^6
Operation and Maintenance (O & M)	€	5 . 10^6
Total equivalent annual costs:	**€**	**17 . 106**

Or capitalized costs: $P + \frac{5.10^6}{0.12}$ = (€ 100 + € 41.67) . 10^6 = **€ 141.67 .10^6**

Alternative 2
i = 12 %



Present value (Capitalized costs):

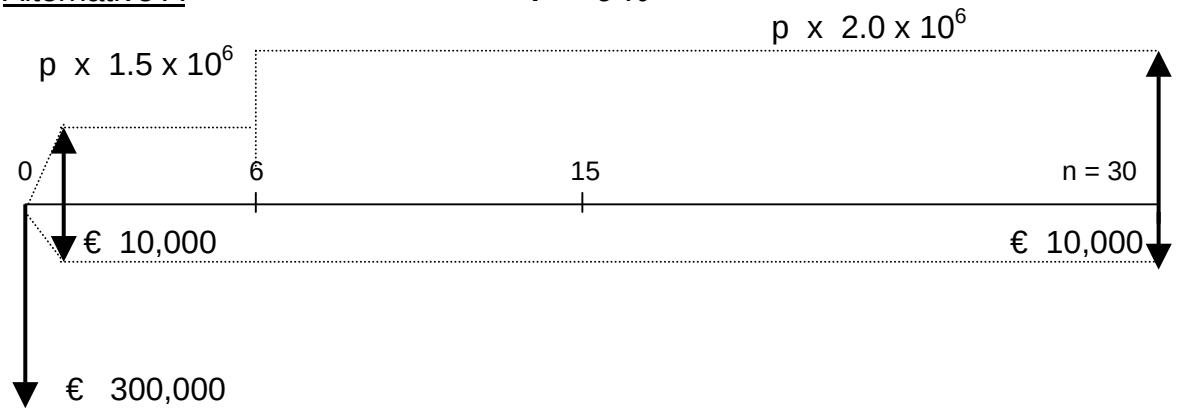
$$P_1 + \frac{A_1}{i} + [\,P_2 + \frac{A_2}{i}\,] \,.\; (P/F, i, 12) =$$

$$\{\,55 + \frac{3.4}{0.12} + [\,53 + \frac{2.2}{0.12}\,]\,\} \,.\, 10^6 \,.\, \frac{1}{1.12^{12}} =$$

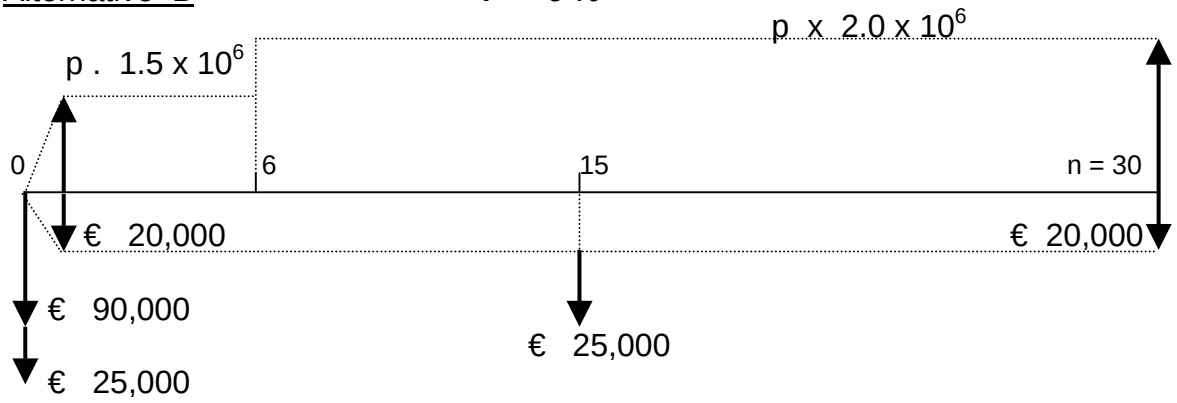
$$\{\,55 + 28.33 + 71.33 \,.\, 0.2567\,\} \,.\, 10^6 = \qquad \underline{\mathbf{€\ \ 101.64\ .10^6}}$$

The second alternative is much cheaper.
The total equivalent annual costs of alternative 2: $101.64 \,.\, 10^6 \,.\, 0.12 =$ € $12.2 \,.\, 10^6$

<u>Answer problem 6</u>
Question a (i = 6 %)
<u>Alternative A</u> ***i = 6 %***



<u>Alternative B</u> ***i = 6 %***



Equivalent annual costs
<u>Alternative A</u>
€ 300,000 . *[A/P, 6 %, 30]* + € 10,000 = € 300,000 . 0.07265 + € 10,000
= **<u>€ 31,795</u>** (P.V. = € 437.6 x 10^3)

Alternative B

{ € 115,000 + € 25,000 . *[P/ F, 6 %, 15]* } . *[A/ P, 6 %, 30]* + € 20,000 =
{ € 115,000 + € 25,000 x 0.4172 } . 0.07265 + € 20,000 =
€ 125,430 x 0.07265 + € 20,000 = **€ 29,112**
(P.V. = € 400.7 x 10^3)

Conclusion: **Alternative B is the most economic alternative**.

Or: € 90,000 . *[A/ P, 6 %, 30]* + € 25,000 *[A/ P, 6 %, 15]* + € 20,000 =
€ 90,000 . 0.0727 + € 25,000 . 0.103 + € 20,000 =
€ 6,543 + € 2,575 + € 20,000 = **€ 29,118**

Answer Question 6b. (**i = 4 %**)

Equivalent annual costs

Alternative A

€ 300,000 . *[A/P, 4 %, 30]* + € 10,000 = € 300,000 . 0.05783 + € 10,000 =
€ 27,349 (P.V. = € 472.9 x 10^3)

Alternative B

[€ 115,000 + € 25,000 . *(P/F, 4 %, 15)]* . *(A/P, 4 %, 30)* + € 20,000 =
[€ 115,000 + € 25,000 . 0.5552] . 0.05783 + € 20,000 =
128,881 . 0.05783 + 20,000 = **€ 27,453**
(P.V. = € 474.7 x 10^3)

Conclusion: **Alternative A is the most economic alternative (just).**

Question c.

P.V. annual benefit (for both alternatives) for **p** = unit cost per m^3 :
(p . 1.5 . 10^6) . *(P/A, 6 %, 6)* + (p . 2.0 .10^6) . *(P/A, 6 %, 24)* . *(P/F, 6%, 6)*
[(1.5 . 4.9164) + (2.0 . 12.5502 . 0.7050)] . p. 10^6 =
[7.3746 + 17.6958] . **p** . 10^6 = 25.07 . **p** . 10^6
Cost per m^3 (alternative B):
€ 400.7 x 10^3 = 25.07 x p x 10^6 ⟶ **p = € 0.016 / m^3**

Answer problem 7

i = 15 %

	Investment	Maintenance (per year)	Lifetime
Bridge	€ 1,350,000	€ 70.000	n = 60 years
Embankment	€ 215,000	€ 30,000 for the first 4 years ; € 14,000 thereafter	n = ∞
Other earthworks	€ 38,000		

Embankment & other earthworks:

Present value: : 215,000 + 38,000 + 30,000 . *(P/A, i, 4)* + $\frac{14{,}000}{i}$. *(P/F, i, 4)* =

$$215{,}000 + 38{,}000 + 30{,}000 \,.\, \frac{1.15^4 - 1}{0.15x1.15^4} + \frac{14{,}000}{0.15} \,.\, \frac{1}{1.15^4} =$$

263,000 + 30,000 . 2.855 + 14,000 . 3.8117 =
263,000 + 85,649 + 53,364 = € 402,013

Equivalent annual costs: P . i = € 402,013 . 0.15 = **€ 60,302**

Bridge:

Annuity (Capital recovery): € 1,350,000 . *(A/P, i, 60)* =

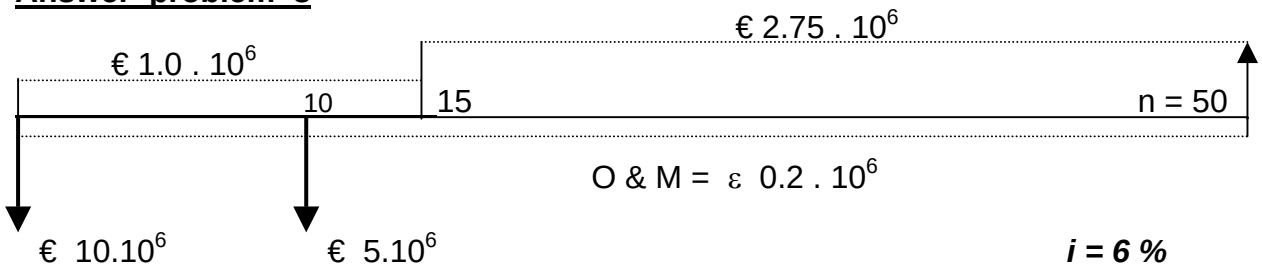
$$€\ 1.350{,}000 \,.\, \frac{0.15x1.15^{60}}{1.15^{60} - 1} = €\ 1{,}350{,}000 \,.\, 0.15003 = €\ 202{,}546$$

Annual maintenance :	€ 70,000
Total equivalent uniform annual costs	**€ 332,848**

Summary:

Capital recovery for the bridge: € 1,350,000 . 0,15003 =	€ 202,546
Annual maintenance of bridge:	€ 70,000
Interest for embankment and earthworks: € 263,000 . 0.15 =	€ 39,450
Basic annual maintenance on embankment and earthworks	€ 14,000
Equivalent annual cost of extra maintenance during first 4 years: € 16,000 . *(P/A, i, 4)* . 0.15 =	€ 6,852
Total equivalent uniform annual costs:	**€ 332,848**

Answer problem 8



Σ Costs : € 10. 10^6 + € 5. 10^6 *(P/F, i, 10)* + € 0.2 . 10^6 . *(P/A, i, 50)* =

$$€\ 10.\ 10^6 + €\ 5.\ 10^6 \,.\, \frac{1}{1.06^{10}} + €\ 0.2 \,.\, 10^6 \,.\, \frac{1.06^{50} - 1}{0.06.1.06^{50}} =$$

€ 10. 10^6 + € 5. 10^6 . 0.5584 + € 0.2 . 10^6 . 15.76 =
(10 + 2.792 + 3.152) . 10^6 = **€ 15.944 . 10^6**

Σ Benefits : € 1.0. 10^6 . *(P/A, i, 15)* + € 2.75. 10^6 *(P/A, i, 35) . (P/F, i, 15)* =

$$(€\ 1.0 \cdot \frac{1.06^{15}-1}{0.06.1.06^{15}} + €\ 2.75.\ 10^6 \cdot \frac{1.06^{35}-1}{0.06.1.06^{35}} \cdot \frac{1}{1.06^{15}}) \cdot 10^6 =$$

(€ 1.0 . 9.7122 + € 2.75 . 14.4925 . 0.417) . 10^6 =
(€ 9.7122 + € 16.6498) . 10^6 = **€ 26.342 . 10^6**

B/C ratio: Σ Benefits / Σ Costs **=** € 26.342 / € 15.944 = **1.65**
NPV Σ Benefits - Σ Costs **=** (€ 26.342 - € 15.944) . 10^6 =
€ 10.399 . 10^6

IRR Try i = 10 % (as 6 % gives a positive NPV)
Σ Costs : € 10. 10^6 + € 5. 10^6 *(P/F, i, 10)* + € 0.2 . 10^6 *(P/A, i, 50)* =

$$€\ 10.\ 10^6 + €\ 5.\ 10^6 \cdot \frac{1}{1.10^{10}} + €\ 0.2 \cdot 10^6 \cdot \frac{1.10^{50}-1}{0.10x1.10^{50}} =$$

€ 10. 10^6 + € 5. 10^6 . 0.3855 + € 0.2 . 10^6 . 9.92 =
(€ 10 + € 1.93 + € 1.98) . 10^6 = **€ 13.91 . 10^6**

Σ Benefits : € 1.0. 10^6 . *(P/A, i, 15)* + € 2.75. 10^6 *(P/A, i, 35) . (P/F, i, 15)* =

$$(€\ 1.0 \cdot \frac{1.10^{15}-1}{0.10x1.10^{15}} + €\ 2.75.\ 10^6 \cdot \frac{1.10^{35}-1}{0.10x1.10^{35}} \cdot \frac{1}{1.06^{15}}) \cdot 10^6 =$$

(€ 1.0 . 7.606 + € 2.75 . 9.644 . 0.239) . 10^6 =
(€ 7.606 + € 6.35) . 10^6 = **€ 13.95 . 10^6**
The Internal rate of return is 10 %.

<u>Answer problem 9</u>
Scheme 1
Capitalized cost:
€ 2,500,000 + € 40,000 / i = € 2,500,000 + € 333,333 = € **2.833 million**
Scheme 2
Capitalized cost
€ 1,750,000 + € 80,000 / i + € 120,000 *(P/F, 12 %, 10 years)* + € 120,000 .
(P/F, 12 %, 20 years) + € 120,000 . (*P/F, 12 %, 30 years)* + € 120,000 .
(P/F, 12 %, 40 years) + etc. =
€ 1,750,000 + € 666,667 + € 38,637 + € 12,440 + € 4,005 + € 1,290 + …
= **€ 2.474 million**

<u>Remark</u>: The replacement cost of € 120,000 every 10 years can be considered as an equivalent 'annual' cost, whereby annual is now 10 years and the compounded interest rate for 10 years is 1.12^{10} = 3.10585 - 1 = 2.10585

Scheme 2
Capitalized cost

€ 1,750,000 + € 80,000 / *0.12* + € 120,000 / 2.10585 =
€ 1,750,000 + € 666,667 + € 56,984 = **€ 2.474 million**

<u>Answer problem 10</u>
<u>Question a</u>
Annual costs

Plan A Capital recovery cost: € 10,000,000 . *[A/P, i, 20]* = € 1,175,000
Operation and maintenance : € 2,000,000
total **€ 3,175,000**

Plan B Capital recovery cost:€ 20,000,000 . *[A/P, i, 20]* = € 2,350,000
Operation and maintenance : € 400,000
total **€ 2,750,000**
Plan B is less costly then Plan A.

<u>Question b</u>
Present value

Plan A Initial costs € 10,000,000
Dredging € 100,000 . *[P/A, i, 20]* + € 200,000 *[P/C, i, 20]*
= € 100,000 . 8.5136 + 200,000 . 55.41 = € 11,953,000
total **€ 21,953,000**
or **€ 2,580,000 / year**

Plan B Initial costs € 20,000,00
Operation and maintenance:
€ 400,000 . *[P/A, i, 20]* = € 400,000 . 8.5136 = € 3,405,000
total **€ 23,405,000**
or **€ 2,750,000 / year**
Plan A is less costly then Plan B.

<u>Answer problem 11</u>
i = 5 %, n = 50, annuity factor = 0.0548

Slope	Capital recovery costs of excavation (€)	Annual cost slope maintenance	Total annual cost (€)
1 : 1 (n = 1)	350.000 . € 3.00 . 0.0548 = 57,540	€ 80,000	€ 137,540
1 : 2 (n = 2)	450.000 . € 3.00 . 0.0548 = 73,980	€ 50,000	€ 123,980
1 : 3 (n = 3)	550.000 . € 3.00 . 0.0548 = 90,420	€ 34,000	€ 124,000
1 : 4 (n = 4)	650.000 . € 3.00 . 0.0548 = 106,860	€ 24,000	€ 130,860

The most economical slope will be around 1 : 2.5 (n = 2.5).

Answer problem 12

i = 10 %

	Project A	Project B	Project C
n (life) [P / A] P.V. benefits Initial cost P.V. of scrap value NPV Recommen-Dation	8 years 5.335 18,400 . 5.335 = € 98,163 € 100,000 nil **- € 1.837** return < 10 % **rejected**	8 years 5.335 30,600 . 5.335 = € 163,251 € 160,000 nil + **€ 3,251** return > 10 % **acceptable**	10 years 6.145 42,300 . 6.145 = € 259,915 € 280,000 € 40,000 . 0.3856 = € 15,422 **- € 4,663** return < 10 % **rejected**

Answer problem 13

i = 10 %, n = 3,5 years

Costs:	Capital recovery cost: (€ 300,000 – € 30,000) . *(A/P, i, n)* = € 270,000 . (*A/P, 10 %, n)*	
	Annuity for 3.5 years: 0.353; capital recovery: € 270,000 . 0.353 =	€ 95,200
	Interest: € 30,000 . 0.10 =	€ 3,000
	Extra maintenance costs:	€ 6,000
	Total costs	€ 104,200
Benefits:	Nett annual cost savings: (€ 150,000 – € 45,000) =	€ 105,000

As benefits exceeds costs (just) the investment is justified (for i = 10 %)

Answer problem 14

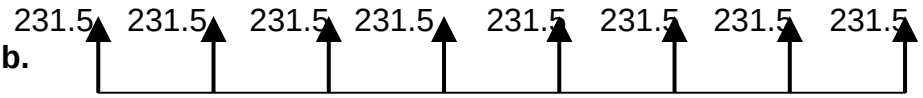
n = ∞

	Alternative A	Alternative B	Alternative C	Alternative D
Investment Annual benefits PV benefits NPV IRR Conclusion:	100 20 20 / i 20 / i - 100 20 / 100 = 0.2 **(20 %)** **Highest rate Of return**	200 30 30 / i 30 / i - 200 30 / 200 = 0.15 (15 %)	300 50 50 / i 50 / i - 300 50 / 300 = 0.167 (16.7 %)	500 75 75 / i 75 / i - 500 75 / 500 = 0.15 (15 %)
Unused funds: (**i = 10 %**)				
NPV or Unused funds Annual benefits Conclusion:	20 / 0.10 – 100 = 200 – 100 = 100 500 – 100 = 400 400 . 0.10 + 20 = 40 + 20 = 60	30 / 0.10 – 200 = 300 – 200 = 100 500 – 200 = 300 300 . 0.10 + 30 = 30 + 30 = 60	50 / 0.10 – 300 = 500 – 300 = 200 500 – 300 = 200 200 . 0.10 + 50 = 20 + 50 = 70	75 / 0.10 – 500 = 750 – 500 = **250** nil 0 + 75 = **75** **Preferred Alternative**
Unused funds: (**i = 14 %**)				
NPV Conclusion:	400 . 0.14 + 20 = 56 + 20 = 76	300 . 0.14 + 30 = 42 + 30 = 72	200 . 0.14 + 50 = 28 + 50 = **78** **Preferred Alternative**	0 + 75 = 75

<u>Answer problem 15</u>
Payment schedule a.

Annual payment for i = 12 %: € 1150 . *(A/ P, 12 %, 8)* = 1150 . 0.2013 =
€ 231.50. This payment is done at the end of the year !

Payment schedule b.



Present Value (PV) of payment schedule B **(i = 12 %)**:
€ 231.5 + € 231.5 . *(P / A, 12 %, 7 years)* = 231.5 + 231.5 . 4.564 =
€ 231.5 (1 + 4.564) = € 231.5 . 5.564 = € 1,288

The same payment of **€ 231.50** is done at the beginning of the year.

Schedule A is the better schedule for the party that is paying; schedule B is the better schedule for the receiving part; the difference is € 231.50 . 0.12 = € 27.78 per year. For payment schedule b one only needs to borrow
€ 1,150 - € 231,5 = € 918.50.
n = 7 years:
€ 918.50 . *(A / P, i, 7)* = € 231, 50
(A / P, i, 7) = 231,50 / 918.50 = **0.2520**
i = 16 %: Annuity = 0.2476; i = 18 %: Annuity = 0.2624
Interpolation gives an effective annual interest of 16.6 %.

<u>Answer problem 16</u>
Question a
Compare annual costs against annual benefits (NPV = 0)
Capital recovery: € 100,000 . *(A/P, i, 25)*
Annuity factors: i = 10 % : 0.1102; i = 12 % : 0.1275

NPV = 0: € 100,000 . *(A/P, i, 25)* + € 15,000 = € 26,500
(A/P, i, 25) = (26,500 – 15,000) / 100,000 = **0.1150**
By interpolation one finds the **IRR = approx. 10.5 %**

Question b : i = 4 %
Yes, investments can be made economically because the cost of money at 4 % will result in a positive NPV (NPV = 0 for i = 10.5 %).

Question c
Annual surplus: € 26,500 - € 15,000 = € 11,500
This surplus is being used to repay the loan, which carries an annual interest of 4 %.
So € 100,000 . (*A / P, 4 %, n)* = 11,500 (n = ?)
(A / P, 4 %, n) = 0.1150
n = 10 years : annuity = 0.1233
n = 12 years: annuity = 0.1066
By interpolation one finds **n = 11 years.**

Answer problem 17

Question a , i = 10 %

Annual capital recovery cost:

€ 1,000,000 . *(A / P, I, 20)* = € 1,000,000 . 0.1175 = € 117,500

Annual equivalent maintenance & operation costs: € 100,000

Total annual costs: € 217,500

Annual benefit:

$15 . 10^6$ x unit cost

Transportation cost (unit cost) per m^3: 217,000 / 15,000,000 = **€ 0.0145**

Question b

Present value of all costs: PV = 217,500 . (*P / A, 10%, 20 years)* = € 1,851,000

Present value of benefits, whereby X = unit cost:

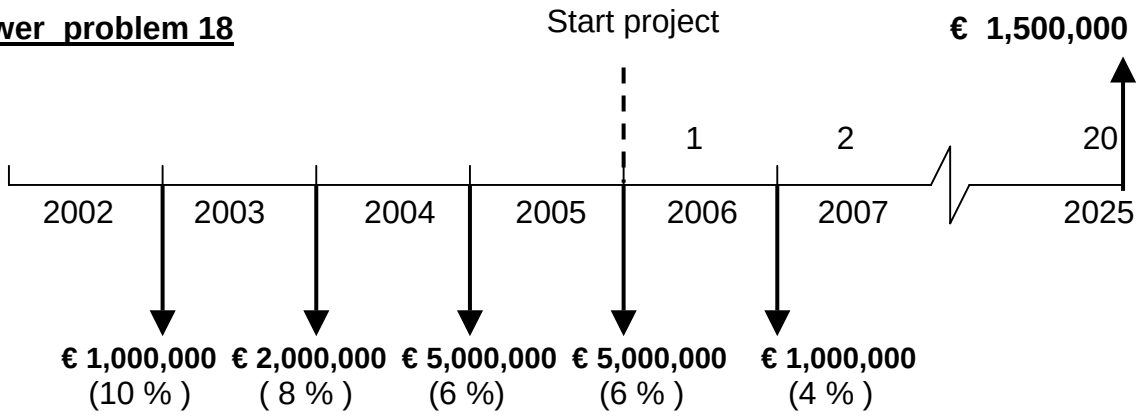
$13 . 10^6 . X$. *(P/ A, 10%, 10)* + $17 . 10^6 . X$. *(P/ A, 10%, 10) . (P/ F, 10%, 10)* =

$13 . 10^6 . X . 6.1446 + 17 . 10^6 . X . 6.1446 . 0.3855 =$

$(79.88 + 40.27). 10^6 . X = 120.15 . 10^6 . X$

Transportation cost **X** (unit cost) per m^3: 1,851,000 / 120,150,000 = **€ 0.0154**

Answer problem 18



a. First costs

Is compunded costs on the day the project is put into operation (1-1-2006) =
1,000,000 . (*F / P, 10 %, 3)* + 2,000,000 . (*F / P, 8 %, 2)* + 5,000,000 . *(F / P, 6 %, 1)* + 5,000,000 + 1,000,000 . *(P / F, 4 %, 1)* =
1,000,000 . 1.331 + 2,000,000 . 1.1411 + 5,000,000 . 1.06 + 5,000,000 + 1,000,000 / 1.04 =
1,331,000 + 2,282,200 + 5,300,000 + 5,000,000 + 961,500 =
€ 14,874,700 Say **€ 14.9 million.**

b. Total depreciation

First cost – salvage value = € 14.9 million - € 1.5 million =
€ 13.4 million

c. Internal rate of return

NPV = 0 or Σ all costs = Σ all benefits
€ 13,374,000 . *(A / P, i %, 20)* + € 1,500,000 . i + € 1,000,000 = € 3,250,000
Find i by trail and error.

For i = 10 %, annuity factor = 0.1175
13,374,000 . 0.1175 + 1,500,000 . 0.10 + 1,000,000 = € 2,725,000
For i = 12 %, annuity factor = 0.1339
13,374,000 . 0.1339 + 1,500,000 . 0.12 + 1,000,000 = € 2,975,000
For i = 14 %, annuity factor = 0.1510
13,374,000 . 0.1510 + 1,500,000 . 0.14 + 1,000,000 = € 3,235,000
IRR = 14 % (slightly more).

d. Equivalent annual surplus
Actual 'costs' of money is **4 %**.
Annual surplus = annual revenue - annual costs =
€ 3,250,000 - € 1,000,000 - € 1,500,000 . 0.04 - € 13,374,000 . *(A / P, 4 %, 20)* = € 3,250,000 - € 1,000,000 - € 60,000 - € 13,374,000 . 0.0736
= € 3,250,000 - € 1,000,000 - € 60,000 - € 986,000 =
€ 1,204,000

e. Marginal rate of return
Annual capital recovery cost of additional initial cost of € 1,500,000 =
€ 1,500,000 . *(A / P, i, 20)*
Additional revenues: € 3,400,000 - € 3,250,000 = € 150,000 .
(A / P, i, 20) = 150,000 / 1,500,000 = 0.100
for i = 6 % annuity = 0.0872; for i = 8 % annuity = 0.1019; **so i = 7.9 %** (approx.)

f. Justification
The investment of € 1,500,000 is not justified because these amount can yield 12 % in another project against about 8 % in this project.

Answer problem 19
Question a (i = 6 %, n = 15 years)
Plan A
Σ Present Value Benefits: € 624,000 . *(P / A, 6 %, 15)* = 624,000 . 9.7122 = € 6,060,500
Σ Present Value All Costs:
€ 4,000,000 + € 200,000 . (*P / A, 6 %, 15)* =
€ 4,000,000 + € 200,000 . 9.7122 = € 5,942,000
B / C – factor : € 6,060,500 / € 5,942,000 = **1.02**

Plan B
Σ Present Value Benefits: € 722,000 . *(P / A, 6 %, 15)* = € 722,000 . 9.7122
= € 7,012,000
Σ Present Value All Costs:
€ 5,000,000 + € 240,000 . *(P / A, 6 %, 15)* = € 5,000,000 + € 240,000 . 9.7122 =
€ 7,330,900
B / C – factor : € 7,012,000 / € 7,330,900 = **0.96**

Question b

Plan A

The IRR > 6 % as B / C- factor > 1 for i = 6 %.

Try 7 %: discounting factor : 9.1079 and B / C = 0.98 ⟶ **IRR = 6.5 %**

Plan B

The IRR < 6 % as B / C – factor < 1 for i = 6 %.

Try 5 %: discounting factor : 10.38 and B / C = 1.02

Try 5.5 %: discounting factor : 10.0376 and B / C = 0.98 ⟶ **IRR = 5.25 %**

Marginal rate of return of Plan B with respect to Plan A:

Plan B – Plan A (= actual extension)

Σ Present Value Benefits:

(€ 722,000 - € 624,000) . *(P / A, i, 15)* = € 98.000 . *(P / A, i, 15)*

Σ Present Value All Costs:

(€ 5,000,000 - € 4,000,000) + (€ 240,000 - € 200,000) . *(P / A, i, 15)*

For i = 5 % B / C – factor = (98,000 . 10.38) / (1,000,000 + 40,000 . 10.38) = 0.72

For i = 2 % B / C – factor = (98,000 . 12.85) / (1,000,000 + 40,000 . 12.85) = 0.83

For i = 0 % B / C – factor = (98,000 . 15) / (1,000,000 + 40,000 . 15) = 0.92

the marginal rate of return of the extension is negative !

Question c

Unused funds are defined as the difference in investment of Plan A and Plan B: € 1,000,000 (the additional investment). As the marginal rate of return of the additional investment is lower than 5 % (and even negative) the unused funds should be invested in other projects (with a rate of return of 5 %).

Answer problem 20 i = 5 %

Frequency x (times/ decade)	Useful life y $y = x^2 + 20$ (in years)	Annual capital recovery cost (depreciation) $= 10.\ 10^6$. annuity	Annual repair & maintenance cost at € 250,000 / time	Total annual costs
x = 0 (no painting)	y = 20 years	$10.\ 10^6.\ 0.0802$ = 820,000	0	820,000
x = 5 / decade (every 2 years)	y = 45 years	$10.\ 10^6.\ 0.0563$ = 563,000	5 x 250,000 / 10 = 125,000	688,000
x = 6 / decade	y = 56 years	$10.\ 10^6.\ 0.0535$ = 535,000	6 x 250,000 / 10 = 150,000	**685,000**
x = 7 / decade	y = 69 years	$10.\ 10^6.\ 0.0518$ = 518,000	7 x 250,000 / 10 = 175,000	693,000
x = 10 (every year)	y = 120 years	$10.\ 10^6.\ 0.05$ = 500,000	= 250,000	750,000

Most economic frequency: **6 times / decade**

Answer problem 21

Question a

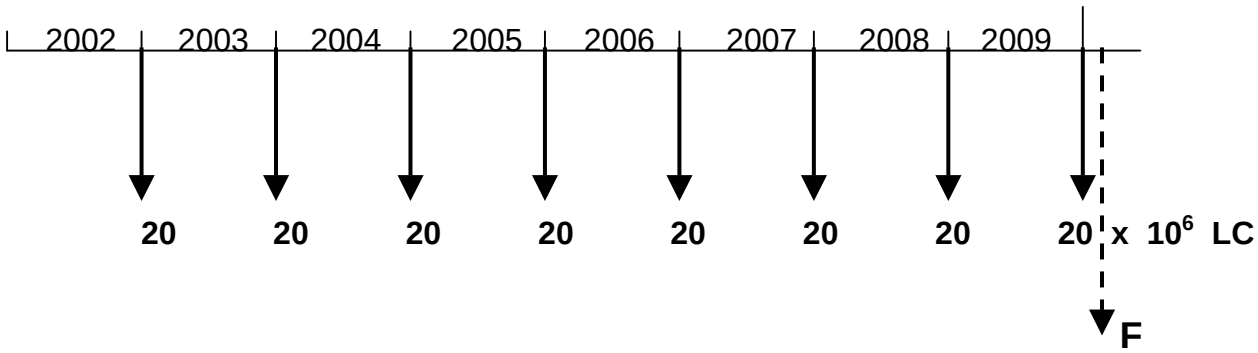
Water transport

Construction time: $\frac{400}{8}$ = 8 years

Start of construction: 1st January 2002

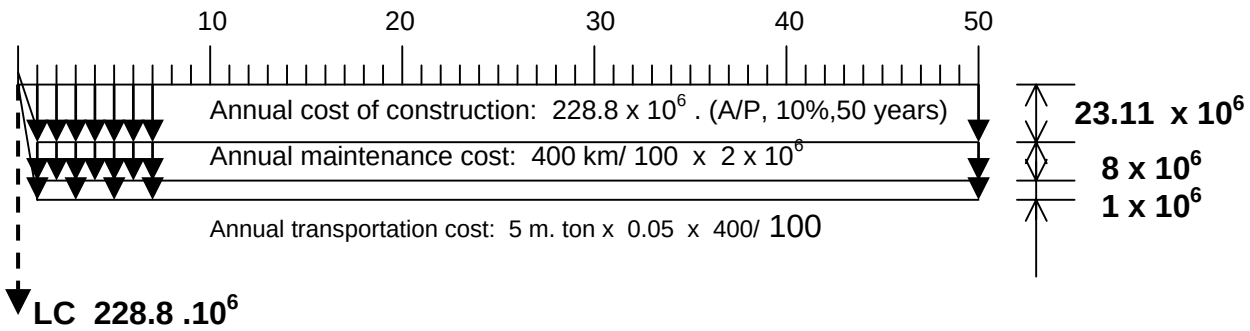
Construction cost/ year: $\frac{(40x10^6)}{8}$ x $\frac{400}{100}$ = LC 20 . 10^6 / year

End of construction: 31st December 2009



Compounding factor: *(F/ A, 10 %, 8 years)* = $\frac{(1+0.10)^8 - 1}{0.10}$ = **11.44**

Construction cost at the <u>end</u> of the project: F = 11.44 . (20 . 10^6) = **LC 228.8 .10^6**



Annuity (10 %, 50 years): $\frac{0.10x(1+0.10)^{50} - 1}{(1+0.10)^{50} - 1}$ = **0.101**

Annual cost of construction costs: LC (228.8 .10^6) . 0.101 = LC 23.11 .10^6

Annual maintenance costs: LC (2 .10^6) . $\frac{400}{100}$ = LC 8.00 .10^6

Transportation costs: LC 0.05. $\frac{400}{100}$. 5 .10^6 = <u>LC 1.00 .10^6</u>

Total annual costs **<u>LC 32.11 .10^6</u>**

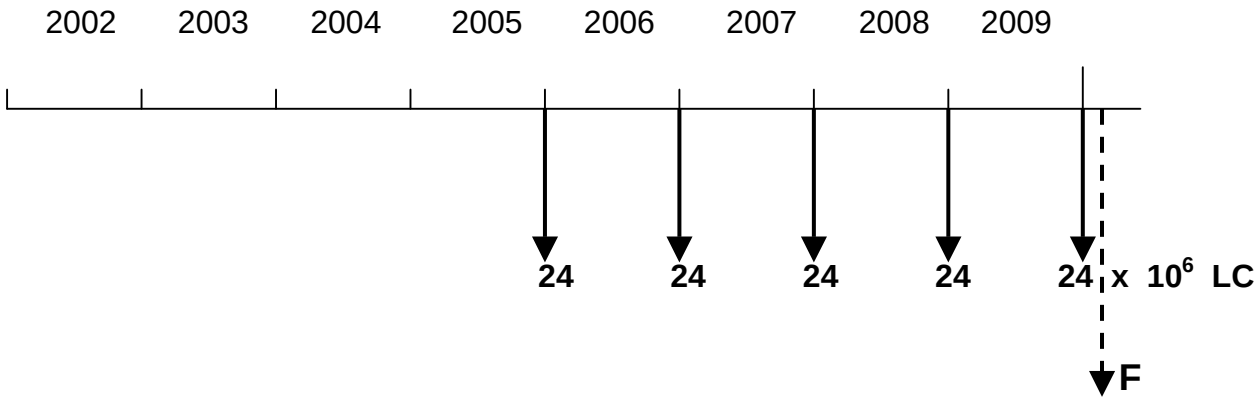
Rail transport

Construction time: $\frac{375}{5}$ = **5 years**

Start of construction: 1st January 2005

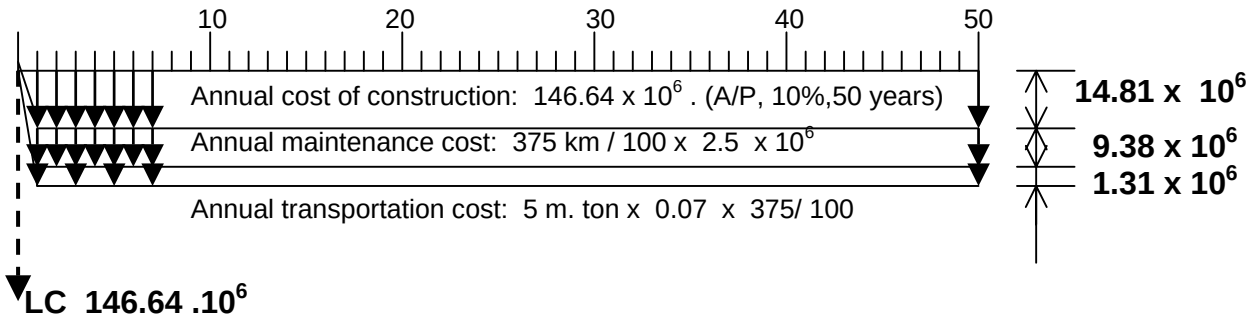
Construction cost/ year: $\frac{(32x10^6)}{5}$ x $\frac{375}{100}$ = LC 24 . 10^6 / year

End of construction: 31st December 2009



Compounding factor: *(F/ A, 10 %, 5 years)* = $\frac{(1+0.10)^5 - 1}{0.10}$ = **6.11**

Construction cost at the <u>end</u> of the project: F = 6.11 . (24 . 10^6) = **LC 146.64 .10^6**



Annuity (10 %, 50 years): $\frac{0.10x(1+0.10)^{50} - 1}{(1+0.10)^{50} - 1}$ = **0.101**

Annual cost of construction costs:	LC (146.64 .10^6) . 0.101 =	LC	14.81 .10^6
Annual maintenance costs:	LC (2.5 .10^6) . $\frac{375}{100}$ =	LC	9.38 .10^6
Transportation costs:	LC (0.07. $\frac{375}{100}$) . 5 .10^6 =	LC	1.31 .10^6
	Total annual costs	**LC**	**25.50 .10^6**

So, **Project B is preferred**.

Question b

Water transport (20 % FC)

Construction cost / year: + 20 %

Annual cost of construction costs: + 20 % = 1.2 . LC 23.11 .10^6 =	LC	27.73 .10^6
Annual maintenance costs (same as question a):	LC	8.00 .10^6
Transportation costs (same as question a):	LC	1.00 .10^6
Total annual costs	**LC**	**36.73 .10^6**

Rail transport (80 % FC)

Construction cost / year: + 80 %

Annual cost of construction costs: + 80 % = 1.8 . LC 14.81 .10^6 =	LC	26.66 10^6
Annual maintenance costs costs (same as question a):	LC	9.38 .10^6
Transportation costs (same as question a):	LC	1.31 .10^6
Total annual costs	**LC**	**37.35 .10^6**

In this case, **Project A is prefererred**.

Question c

Compare the difference between the two projects.

The 'costs' of the transportation time are relative:

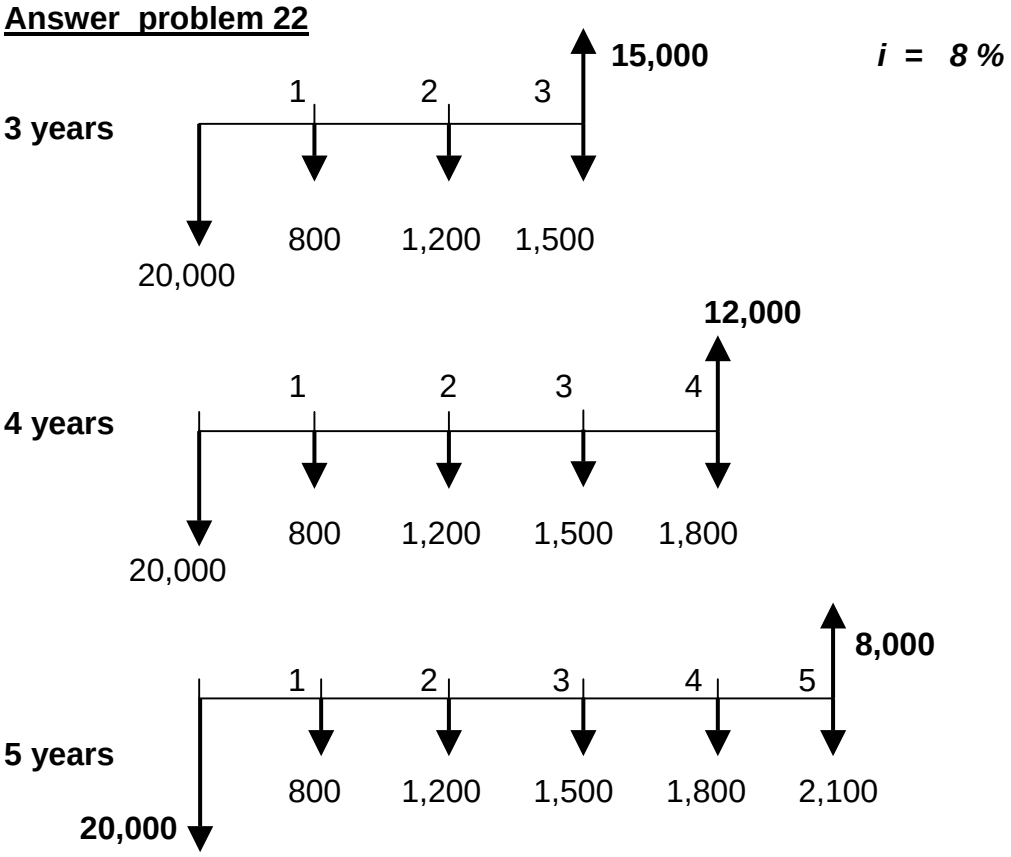
the railway time is faster and therefore cheaper, by

5 hours x LC 0.02 / ton x 5 x 10^6 ton per year = **LC 0.5 . 10^6**

For the first case the difference between the two alternatives becomes larger.

For the second case the difference between the two alternatives becomes smaller and the two alternatives are about the same.

Answer problem 22



	Depreciation (annual)	Interest (8 %) on resale value	Equivalent annual cost of maintenance costs	Total annual costs
3 years	5,000 . *(A/P, i, 3)* = 5,000 . 0.3880 = 1,940	15,000 . 0.08 = 1,200	(800/ 1.08 + 1200/ 1.08^2 + 1500 / 1.08^3) . (A/P, i, 3) = 2,960 . 0.3880 = 1,149	**4,289**
4 years	8,000 . *(A/P, i, 4)* = 8,000 . 0.3019 = 2,415	12,000 . 0.08 = 960	(800/ 1.08 + 1200/ 1.08^2 + 1500 / 1.08^3 + 1800/ 1.08^4) . (A/P, i, 4) = 4,283 . 0.3019 = 1,293	4,668
5 years	12,000 . *(A/P, i, 5)* = 12,000 . 0.2505 = 3,006	8,000 . 0.08 = 640	{ 800/ 1.08 + 1200/ 1.08^2 + 1500 / 1.08^3 + 1800/ 1.08^4 + 2100/1.08^5 } . [A/P, i, 5] = 5,712 . 0.2505 = 1,496	5,142

Sell the equipment after 3 years !

<u>Answer problem 23</u>

i = 12 %, n = ∞

Reinforced concrete road pavement per m^2

Σ Present value costs:

€ 100 + 0.67 / 0.12 + 31 . *(P/ F, 12%, 40 years)* +
3.25 / 0.12 . *(P/ F, 12%, 40 years)* =
100 + 5.583 + 31 . 0.0107 + 27.08 . 0.0107 =
100 + 5.583 + 0.624 = € 106.207 / m^2
per 2,000 m^2 : 2,000 . € 106.207 / m^2 = **<u>€ 212,414</u>**

Flexible pavement per m^2

Σ Present value costs:

90 + 3.25 / 0.12 + 0.67 / 0.12 =
90 + 3.92 / 0.12 = € 122.667 / m^2
per 2,000 m^2 : 2,000 . € 122.667 / m^2 = **<u>€ 245,333</u>**