

Lecture Notes 13

Bond Portfolio Management

- I. Readings and Suggested Practice Problems**
- II. Risks Associated with Default-Free Bonds**
- III. Duration: Details and Examples**
- IV. Immunization**

Buzz Words: *Interest Rate Risk, Reinvestment Risk, Liquidation Risk, Macaulay Duration, Modified Duration, Convexity, Target-Date Immunization, Net-Worth Immunization, Duration Gap*

I. Readings and Suggested Practice Problems

BKM, Chapter 16, Sections 16.1-16.3

Suggested Problems, Chapter 16: 16-18.

E-mail: Open the *Bond Immunization* program to generate the examples in Section IV, and to construct your own examples of target-date immunization.

II. Risks Associated with Default-Free Bonds

A. *Reinvestment Risk*

If an individual has a particular time horizon T and holds an instrument with a fixed cash flow received prior to T , then the investor faces uncertainty about what yields will prevail at the time of the cash flow. This uncertainty is known as *reinvestment risk*.

Example

Suppose an investor has to meet an obligation of \$5M in two years time. If she buys a two year coupon bond to meet this obligation, there is uncertainty about the rate at which the coupons on the bond can be invested. This uncertainty is an example of reinvestment risk.

B. Liquidation (Price) Risk

If an individual has a particular time horizon T and holds an instrument which generates cash flows that are received after T , then the investor faces uncertainty about the price of the instrument at time T . This uncertainty is known as liquidation risk.

Example

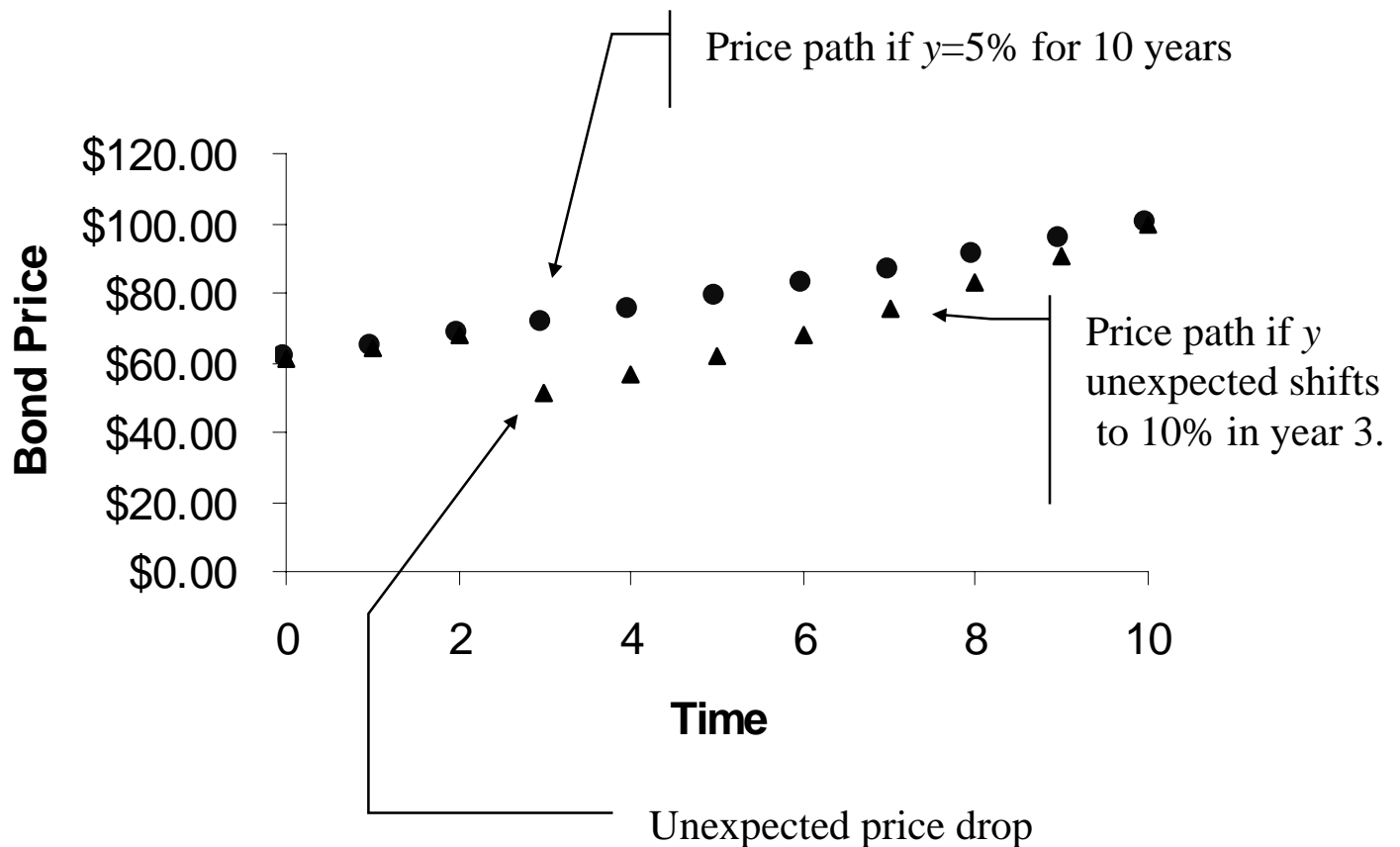
Suppose an investor has to meet an obligation of \$5M in two years time. If she buys a five year discount bond to meet this obligation, there is uncertainty about the price at which this bond will sell in two years time. This uncertainty is an example of liquidation risk.

C. Yield Changes and Price Changes

1. If a bond sells at a premium or discount, its price will converge to par, even if the YTM y stays constant. (This price change is *expected*, and is not normally considered risk.)
2. Unexpected bond price changes will occur if market interest rates changes unexpectedly. This is *interest rate risk*, which causes the reinvestment risk and liquidation risk; It affects the rate at which coupon payments can be reinvested, and affects the price at which a bond can be sold (prior to maturity).

Example

Price path of a \$100-par zero that matures in year 10



- Can examine the plots of price P of any bond as a function of the YTM, y , using the following functional relation:

$$P = \sum_{t=1}^T \frac{CF_t}{(1+y)^t}$$

If you know how Δy affects ΔP , you can tell how Δy affects portfolios of bonds, and can decide how to manage your bond portfolio given your objectives.

$$\Delta P = \frac{dP}{dy} \Delta y + \frac{1}{2} \frac{d^2 P}{dy^2} (\Delta y)^2 + \dots$$

Divide by P :

$$\begin{aligned} \frac{\Delta P}{P} &= \frac{1}{P} \frac{dP}{dy} \Delta y + \frac{1}{2} \frac{1}{P} \frac{d^2 P}{dy^2} (\Delta y)^2 + \dots \\ &= -D^* \Delta y + \frac{1}{2} (\text{Convexity}) (\Delta y)^2 + \dots \\ &= -\frac{D}{1+y} \Delta y + \frac{1}{2} (\text{Convexity}) (\Delta y)^2 + \dots \end{aligned}$$

A second order approximation for the impact of yield changes is therefore (using the definitions for D^* and *Convexity* given below):

$$\frac{\Delta P}{P} \approx -D^* \Delta y + \frac{1}{2} (\text{Convexity}) (\Delta y)^2$$

A first order approximation (a linear, and less precise one) is:

$$\frac{\Delta P}{P} \approx -D^* \Delta y = -\frac{D}{1+y} \Delta y = -D \frac{\Delta(1+y)}{1+y}$$

with the *Duration* (D), the *Modified Duration* (D^*), and *Convexity* defined as:

$$D = D^* (1+y) = -\frac{1}{P} \frac{dP}{dy} (1+y) = \sum_{t=1}^T t \times \frac{CF_t / (1+y)^t}{P}$$

$$D^* = D / (1+y)$$

$$\text{Convexity} = \frac{1}{P} \frac{d^2 P}{dy^2} = \frac{1}{P} \frac{1}{(1+y)^2} \sum_{t=1}^T \frac{CF_t (t^2 + t)}{(1+y)^t}$$

III. Duration: Details and Examples

To understand the important features of managing fixed income portfolios, we will focus on the simpler, *first-order approximation* to the impact of yield changes on prices.

A. *Duration*

Duration (the D defined above is Macaulay's first measure of duration) is *used* to measure the price risk of a bond (i.e., interest rate sensitivity).

Duration relates the change in a bond price (ΔP) to the associated change in the bond's YTM (Δy).

Duration is *computed* as the effective (weighted average) maturity of the bond.
(This is distinct from the nominal maturity of the bond.)

B. *The price change caused by a given change in YTM: Δp vs. Δy*

The link between maturity and price risk is easiest to demonstrate for a zero:

Example

15-year zero, \$100 par, $y = 8\%$

$$\Rightarrow p = PV(100, 8\%, 15) = 31.5242$$

Suppose we have a small change in y : $\Delta y = -.01\% = -0.0001$ (“ y drops by one basis point”).

$$p = PV(100, 7.99\%, 15) = 31.5680$$

$$\Delta p = 31.5680 - 31.5242 = 0.0438$$

One way to relate Δp and Δy is to compute

$$-\frac{\text{Proportional change in } p}{\text{Proportional change in } (1+y)} = -\frac{\frac{\Delta p}{p}}{\frac{\Delta(1+y)}{(1+y)}} = -\frac{\frac{0.0438}{31.5242}}{\frac{-0.0001}{1.08}} = \frac{0.001389}{0.0000926} = 15 = M \text{ (the maturity)}$$

The example illustrates that for a zero coupon bond, we can estimate its relative price change (for a given change in y) by:

$$\frac{\Delta p}{p} \approx -M \times \frac{\Delta(1+y)}{(1+y)}$$

For any bond :

$$\frac{\Delta p}{p} \approx -D \times \frac{\Delta(1+y)}{(1+y)}$$

where D is the duration of the bond, and y is the YTM.

C. Computing the Duration for Zeros

For a zero-coupon bond: $D = M$ (the stated maturity)

For a portfolio of two zeros: D = the weighted average maturity of the two zeros:

$$D_P = w_1 D_1 + w_2 D_2$$

where the weights, w 's, are market-value weights.

Example

$y = 10\%$

Bond 1 is a 5-yr, 100 par zero, $\Rightarrow P_1=62.09$

Bond 2 is a 10-yr, 100 par zero, $\Rightarrow P_2=38.55$

\Rightarrow The total value of the portfolio is 100.64

$$\begin{aligned} D_P &= \left(\frac{62.09}{100.64} \right) 5 + \left(\frac{38.55}{100.64} \right) 10 \\ &= 0.62 \times 5 + 0.38 \times 10 = 6.9 \text{ years} \end{aligned}$$

Exercise: verify that if $1+y$ goes to 1.101, the value of the portfolio changes by $-6.9\Delta(1+y)/(1+y)$

D. The Duration of a Coupon Bond

A coupon bond is simply a portfolio of zeros.

Let CF_t = coupon and principal payments in year t , for $t = 1, \dots, T$ (the bond's maturity).

Then

$$D = w_1 \times 1 + w_2 \times 2 + \dots + w_T \times T$$

where the w 's are the market value weights:

$$w_t = \frac{PV(CF_t, y, t)}{PV(CF_1, y, 1) + \dots + PV(CF_T, y, T)}$$

$$= \frac{PV(CF_t, y, t)}{\text{Bond Price}} = \frac{\left(\frac{CF_t}{(1+y)^t} \right)}{\text{Bond Price}}$$

Duration of a coupon bond: the "Seesaw"

Figure 1
Cash Flows and Present Values of a Seven-Year 7 Percent Bond

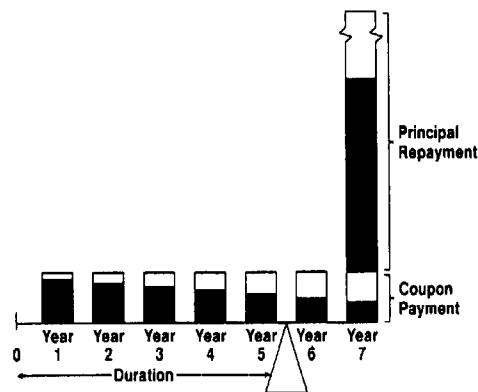
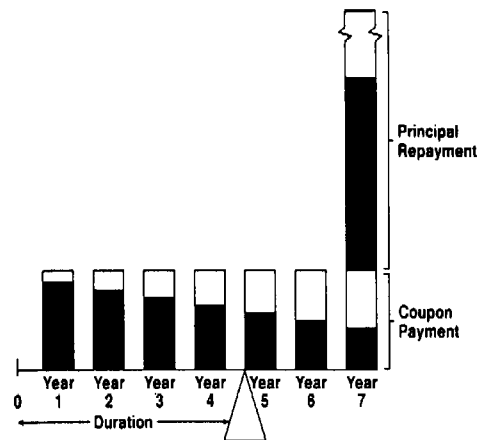


Figure 2
Cash Flows and Present Values of a Seven-Year 14 Percent Bond



$y = 8\%$. D of the 7% bond = 5.6 years; for the 14%, $D=5.0$ years.

E. Shortcut Duration Formulas

For a perpetuity, $D = \frac{1+y}{y}$

For a level annuity with end date T , $D = \frac{1+y}{y} - \frac{T}{(1+y)^T - 1}$

For a coupon bond with coupon rate c , $D = \frac{1+y}{y} - \frac{(1+y) + T(c-y)}{c[(1+y)^T - 1] + y}$

If $c = y$, (bond sells at par), $D = \frac{1+y}{y} \left[1 - \frac{1}{(1+y)^T} \right]$

F. Related Measures

1. Above defined the Macaulay's Duration Measure:

$$\Delta P/P = -D \times \Delta y / (1+y) = -D \times \Delta(1+y) / (1+y)$$

2. $D/(1+y)$ is called *modified duration*:

$$\Delta P/P = -(\text{modified duration}) \times \Delta y$$

3. $(P \times D)/(1+y)$ is called *dollar duration*:

$$\Delta P = -(\text{dollar duration}) \times \Delta y$$

IV. Immunization

A portfolio is immunized when it is “unaffected” by interest rate changes.

A. Target Date Immunization

Consider an institution or investor that needs to meet future fixed obligations.

Target date immunization ensures that a stream of fixed outflows can be met from available assets.

This is **achieved** by holding a portfolio of fixed income assets,
with the same current value as the liability stream,
and
with the same (modified) duration.

1. Immunization when a liability is a single cash flow

For illustration assume

- the yield curve is flat at y
- any shift in the curve keeps it flat.

Given the flat yield curve, all bonds have the same YTM, y .

- a. For this reason, a small shift in the yield curve will have the same effect on the current value of the immunizing assets and on the current value of the liabilities (using the definition of duration), and so the assets will still be sufficient to meet the stream of fixed outflows at the target date(s).
- b. Equating the (modified) durations of the assets and the liabilities is the same as equating the durations.

Example 1

Consider the value of a bond portfolio consisting of one 7-year annual coupon bond:

$$\text{the portfolio value} = \text{value of the reinvested coupons} + \text{the market price of the bond.}$$

Consider the case of $y = 10\%$ (Scenario I), and also consider an *unexpected increase in y* (Scenario II), where y jumps from 10% to 11% immediately after purchase:

Bond portfolio value over time.

Bond Characteristics: Coupon: 15.30%

Par: 1,000.00 Maturity: 7

Time	0	1	2	3	4	5	6	7
Maturity	7	6	5	4	3	2	1	0
Coupon		153.00	153.00	153.00	153.00	153.00	153.00	153.00
Principal								1,000.00
Scenario I:								
YTM		10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Duration	5.00	4.50	3.94	3.33	2.64	1.87	1.00	
Bond Price	1,258.03	1,230.83	1,200.91	1,168.00	1,131.80	1,091.98	1,048.18	1,000.00
Reinv Cpn		153.00	321.30	506.43	710.07	934.08	1,180.49	1,451.54
Total	1,258.03	1,383.83	1,522.21	1,674.43	1,841.88	2,026.06	2,228.67	2,451.54
Scenario II:								
YTM		11.00%	11.00%	11.00%	11.00%	11.00%	11.00%	11.00%
Duration	4.95	4.47	3.92	3.32	2.64	1.87	1.00	
Bond Price	1,202.62	1,181.91	1,158.92	1,133.41	1,105.08	1,073.64	1,038.74	1,000.00
Reinv Cpn		153.00	322.83	511.34	720.59	952.85	1,210.67	1,496.84
Total	1,202.62	1,334.91	1,481.75	1,644.75	1,825.67	2,026.49	2,249.41	2,496.84
$\Delta: \text{II-I}$	-55.40	-48.92	-40.46	-29.69	-16.21	0.43	20.74	45.30

Does the shift help us or hurt us? It depends on our investment horizon!

An unexpected decrease in y: Here, y goes from 10% to 9%

Bond portfolio value over time.

Bond Characteristics: **Coupon: 15.30%** **Par: 1,000.00 Maturity: 7**

Time	0	1	2	3	4	5	6	7
Maturity	7	6	5	4	3	2	1	0
Coupon		153.00	153.00	153.00	153.00	153.00	153.00	153.00
Principal								1,000.00
Scenario I:								
YTM		10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Duration	5.00	4.50	3.94	3.33	2.64	1.87	1.00	
Bond Price	1,258.03	1,230.83	1,200.91	1,168.00	1,131.80	1,091.98	1,048.18	1,000.00
Reinv Cpbs		153.00	321.30	506.43	710.07	934.08	1,180.49	1,451.54
Total	1,258.03	1,383.83	1,522.21	1,674.43	1,841.88	2,026.06	2,228.67	2,451.54
Scenario II:								
YTM		9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%
Duration	5.05	4.53	3.96	3.34	2.65	1.87	1.00	
Bond Price	1,317.08	1,282.61	1,245.05	1,204.10	1,159.47	1,110.82	1,057.80	1,000.00
Reinv Cpbs		153.00	319.77	501.55	699.69	915.66	1,151.07	1,407.67
Total	1,317.08	1,435.61	1,564.82	1,705.65	1,859.16	2,026.48	2,208.87	2,407.67
Δ: II-I	59.05	51.78	42.61	31.22	17.28	0.42	-19.80	-43.87

From the tables we see that:

- 5 years is the duration of the bond at time 0.
- At year 5, whether y goes up or down, our portfolio value suffers only a minor change in value.

How can we use this for immunization?

- At time zero, if we set the duration equal to the target date (a point at which we must fund some known obligation, e.g., \$2,026 in 5 years), *and* if we set the bond's future value at the target date equal to the amount of the obligation (i.e., the bond's current value equal to the current value of the obligation), then we are not greatly affected by changes in y .
- In general, as long as target date = duration, *and* current value of assets = current value of liabilities, we are said to be ***immunized***.
- In the example, the portfolio value at time zero has the same value as a 5 year zero with par 2,026.06 *and* has the same (modified) duration as a 5 year zero. Hence, changes in y affect the portfolio value similarly to their effect on the above zero, and therefore the portfolio's 5 year future value remains similar to zero's par (2,026).
- Target date immunization is frequently practiced by pension funds and insurance companies.
- When the assets' duration is not matched to the target duration of the liabilities, we have a "Duration Gap."

Large changes in y: Immunization is only approximate. As the changes in y get larger, the quality of the approximation deteriorates.

Here's a large decrease:

Bond portfolio value over time.								
Bond Characteristics:		Coupon: 15.30%		Par: 1,000.00		Maturity: 7		
Time	0	1	2	3	4	5	6	7
Maturity	7	6	5	4	3	2	1	0
Coupon		153.00	153.00	153.00	153.00	153.00	153.00	153.00
Principal								1,000.00
Scenario I:								
YTM		10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Duration	5.00	4.50	3.94	3.33	2.64	1.87	1.00	
Bond Price	1,258.03	1,230.83	1,200.91	1,168.00	1,131.80	1,091.98	1,048.18	1,000.00
Reinv Cpbs		153.00	321.30	506.43	710.07	934.08	1,180.49	1,451.54
Total	1,258.03	1,383.83	1,522.21	1,674.43	1,841.88	2,026.06	2,228.67	2,451.54
Scenario II:								
YTM		1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Duration	5.41	4.77	4.11	3.42	2.68	1.88	1.00	
Bond Price	1,962.13	1,828.75	1,694.04	1,557.98	1,420.56	1,281.77	1,141.58	1,000.00
Reinv Cpbs		153.00	307.53	463.61	621.24	780.45	941.26	1,103.67
Total	1,962.13	1,981.75	2,001.57	2,021.59	2,041.80	2,062.22	2,082.84	2,103.67
Δ : II-I	704.11	597.92	479.36	347.15	199.93	36.16	-145.83	-347.87

Here's a large increase:

Bond portfolio value over time.								
Bond Characteristics:		Coupon: 15.30%		Par: 1,000.00		Maturity: 7		
Time	0	1	2	3	4	5	6	7
Maturity	7	6	5	4	3	2	1	0
Coupon		153.00	153.00	153.00	153.00	153.00	153.00	153.00
Principal								1,000.00
Scenario I:								
YTM		10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Duration	5.00	4.50	3.94	3.33	2.64	1.87	1.00	
Bond Price	1,258.03	1,230.83	1,200.91	1,168.00	1,131.80	1,091.98	1,048.18	1,000.00
Reinv Cpbs		153.00	321.30	506.43	710.07	934.08	1,180.49	1,451.54
Total	1,258.03	1,383.83	1,522.21	1,674.43	1,841.88	2,026.06	2,228.67	2,451.54
Scenario II:								
YTM		20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%
Duration	4.54	4.18	3.74	3.22	2.60	1.86	1.00	
Bond Price	830.58	843.70	859.44	878.33	901.00	928.19	960.83	1,000.00
Reinv Cpbs		153.00	336.60	556.92	821.30	1,138.56	1,519.28	1,976.13
Total	830.58	996.70	1,196.04	1,435.25	1,722.30	2,066.76	2,480.11	2,976.13
Δ : II-I	-427.44	-387.13	-326.17	-239.18	-119.58	40.70	251.44	524.60

2. *Immunization of a single cash flow over time*

To now, we've assumed that the Δy occurred immediately.

Suppose that y goes to 11% at the end of year 3 (i.e., it is applicable for year 4 and beyond).

Bond portfolio value over time.

Bond Characteristics: Coupon: 15.30%

Par: 1,000.00 Maturity: 7

Time	0	1	2	3	4	5	6	7
Maturity	7	6	5	4	3	2	1	0
Coupon		153.00	153.00	153.00	153.00	153.00	153.00	153.00
Principal								1,000.00
Scenario I:								
YTM		10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Duration	5.00	4.50	3.94	3.33	2.64	1.87	1.00	
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Total	1,258.03	1,383.83	1,522.21	1,674.43	1,841.88	2,026.06	2,228.67	2,451.54
Scenario II:								
YTM		10.00%	10.00%	10.00%	11.00%	11.00%	11.00%	11.00%
Duration	5.00	4.50	3.94	3.32	2.64	1.87	1.00	
Bond Price	1,258.03	1,230.83	1,200.91	1,133.41	1,105.08	1,073.64	1,038.74	1,000.00
Reinv Cpbs		153.00	321.30	506.43	715.14	946.80	1,203.95	1,489.39
Total	1,258.03	1,383.83	1,522.21	1,639.84	1,820.22	2,020.44	2,242.69	2,489.39
Δ: II-I	0.00	0.00	0.00	-34.60	-21.66	-5.62	14.02	37.85

What happened?

Moral: Immunization is not a set-and-forget strategy. You need to rebalance (“readjust”) your portfolio duration whenever.

- interest rates change
- time elapses

You don’t have to turn over your entire portfolio to rebalance. You might simply shift a small part of the portfolio from one end of the maturity spectrum to the other.

3. *Immunization when the liability is a series of cash flows:*

- Target-date immunization above assumes that the target liability is at a single point (like a zero coupon bond).
- For a zero coupon bond, $D = T$.
- If the liability is a series of cash flows, compute the duration of the liability portfolio.

Immunization then requires $D_{Assets} = D_{Liabilities}$

Example 2

Firm XYZ is required to make a \$5M payment in 1 year and a \$4M payment in 3 years. The yield curve is flat at 10% APR with semi-annual compounding. Firm XYZ wants to form a portfolio using 1-year and 4-year U.S. strips to fund the payments. How much of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve?

The *value* of the liabilities is given by:

$$5M/[1+0.1/2]^2 + 4M/[1+0.1/2]^6 = 4.5351M + 2.9849M = 7.5200M.$$

The *duration* of the liabilities is given by:

$$1 \times [4.5351/7.5200] + 3 \times [2.9849/7.5200] = 1.7938 \text{ years.}$$

Let A_1 be the portfolio's dollar investment in the 1-year strips and A_4 be the portfolio's dollar investment in the 4-year strips.

The dollar value of the portfolio must equal the value of the liabilities. So

$$A_1 + A_4 = 7.5200M.$$

The duration of the portfolio equals

$$w_1 D_1 + (1 - w_1) D_4$$

where $w_1 = A_1 / 7.5200M$. The duration of the 1-year strip is 1 and the duration of the 4-year strip is 4.

Setting the duration of the portfolio equal to the duration of the liabilities gives:

$$1.7938 = w_1 D_1 + (1 - w_1) D_4 = w_1 1 + (1 - w_1) 4 \quad \Rightarrow \quad w_1 = 0.7354.$$

Thus,

$$\begin{aligned} A_1 &= 0.7354 \times 7.5200M = 5.5302M \\ A_4 &= 7.5200M - 5.5302M = 1.9898M. \end{aligned}$$

B. Net Worth Immunization

Unlike e.g. pension funds, firms and banks care about their net worth as opposed to future commitments.

Suppose that a firm has a balance sheet:

<u>Assets</u>	<u>Liabilities</u>
100 ($D_{Assets}=10$)	60 ($D_{Liabilities}=10$)
	40 Net Worth

The firm's shareholders are not immunized:

If y goes from 10% to 10.5%, a mark-to-market balance sheet will show (approximately):

<u>Assets</u>	<u>Liabilities</u>
95	57
	38 NW

To immunize the *net worth*, need

$$A \times D_{Assets} = L \times D_{Liabilities} .$$

To achieve this, the duration of the assets should be set to $(60/100) \times 10 = 6$ years.

C. Cash Flow Matching (Dedication Strategy)

Cash flow matching is the strategy of funding every component of a liability with a zero-coupon bond. The bond is said to be dedicated to a particular obligation.

Limitation: availability of zeros.

D. Limitations of Immunization

- We have assumed that the yield curve is flat (the same y for all maturities).

If the curve is not flat, a simple modification of D will serve as the basis for immunization *as long as shifts in the yield curve are parallel* (all y 's go up or down by the same amount).

- Should you immunize?

Immunization is appropriate when the target liability is a fixed nominal (\$) amount.

But many common motives for saving (e.g., retirement or a child's education) involve liabilities that are not for fixed \$ amounts. In these cases, we need an investment that has inflation hedging properties.