

Fixed Income Portfolio Management

- Interest rate sensitivity, duration, and convexity
- Passive bond portfolio management
- Active bond portfolio management
- Interest rate swaps

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Interest rate sensitivity, duration, and convexity

- bond price: $\sum_{t=1}^T \frac{C}{(1+y)^t} + \frac{F}{(1+y)^T}$, where $y = \text{YTM}$
- when yields change, bond prices will change; the percentage price differences will be larger when:
 - C is lower (given same T)
 - T is larger (given same C)
- example (text Tables 13.1-13.2):

YTM	$T = 1$ year	$T = 10$ years	$T = 20$ years
8% Coupon Bond			
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
% change in price	0.94%	6.50%	9.20%
Zero Coupon Bond			
8%	924.56	456.39	203.29
9%	915.73	414.64	171.93
% change in price	0.96%	9.15%	17.46%

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- *duration* measures how long, on average, a bondholder must wait to receive cash payments
- a zero maturing in T periods has a duration of T ; a coupon bond maturing in T periods has duration $< T$ since some payments are received before T
- duration is a weighted average where the weights are the proportion of total bond PV due to the payments, i.e.

$$D = \sum_{t=1}^T tW_t,$$

where

$$W_t = \frac{CF_t/(1+y)^t}{P},$$

where CF_t denotes cash flow at t and P is the price of the bond

- duration is related to the price sensitivity of a bond to changes in yields:

$$\begin{aligned} P &= \frac{C}{1+y} + \frac{C}{(1+y)^2} + \dots + \frac{C+F}{(1+y)^T} \\ \Rightarrow \frac{\partial P}{\partial y} &= -\frac{C}{(1+y)^2} - \frac{2C}{(1+y)^3} - \dots - \frac{T(C+F)}{(1+y)^{(T+1)}} \\ &= \frac{-1}{1+y} \left[\frac{C}{1+y} + \frac{2C}{(1+y)^2} + \frac{3C}{(1+y)^3} + \dots + \frac{T(C+F)}{(1+y)^T} \right] \end{aligned}$$

Dividing by P gives:

$$\begin{aligned} \frac{\frac{\partial P}{\partial y}}{P} &= \frac{\frac{-1}{1+y} \left[\frac{C}{1+y} + \frac{2C}{(1+y)^2} + \dots + \frac{T(C+F)}{(1+y)^T} \right]}{P} \\ &= \frac{-1}{1+y} \times D \end{aligned}$$

- switching to discrete changes:

$$\frac{\frac{\Delta P}{\Delta y}}{P} \approx \frac{-1}{1+y} \times D$$

$$\Rightarrow \frac{\Delta P}{P} \approx \frac{-\Delta y}{1+y} \times D$$

- a related measure is “modified duration”:

$$D^* = \frac{D}{1+y}$$

$$\Rightarrow \frac{\Delta P}{P} \approx -D^* \Delta y$$

- note that this expression for the percentage price change is an approximation based on a first-order Taylor’s series expansion:

$$f(x + \Delta x) \approx f(x) + \frac{\partial f}{\partial x} \Delta x$$

- letting $P = f(y)$, we have

$$P(y + \Delta y) \approx P(y) + \frac{\partial P}{\partial y} \Delta y$$

$$\Rightarrow \frac{P(y + \Delta y) - P(y)}{P(y)} \approx \frac{\frac{\partial P}{\partial y} \Delta y}{P(y)}$$

$$\Rightarrow \frac{\Delta P}{P} \approx \frac{\frac{\partial P}{\partial y} \Delta y}{P} = -D^* \Delta y = -\frac{D \Delta y}{1+y}$$

- note that we can express duration and modified duration as:

$$D = -\frac{(1+y) \frac{\partial P}{\partial y}}{P}$$

$$D^* = -\frac{\frac{\partial P}{\partial y}}{P}$$

- another related measure is just $-\frac{\partial P}{\partial y}$. For the special case where the change in yield is 1 basis point, this is called the *price value of a basis point* (PVBP).

- for U.S. T-bills, PVBP is very simple to calculate:

$$P_0 = \left[1 - r_{BDY} \left(\frac{n}{360} \right) \right] \times F$$

$$P_1 = \left[1 - (r_{BDY} + .0001) \left(\frac{n}{360} \right) \right] \times F$$

$$\Rightarrow \text{PVBP} = P_1 - P_0 = \left[.0001 \left(\frac{n}{360} \right) \right] \times F$$

- the basic idea underlying D , D^* , and PVBP is:



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- we can get a more accurate approximation by using a second-order expansion:

$$f(x + \Delta x) \approx f(x) + \frac{\partial f}{\partial x} \Delta x + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (\Delta x)^2$$

$$\Rightarrow \frac{\Delta P}{P} \approx \frac{\frac{\partial P}{\partial y} \Delta y}{P} + \frac{\frac{1}{2} \frac{\partial^2 P}{\partial y^2} (\Delta y)^2}{P}$$

$$= -D^* \Delta y + \frac{1}{2} \times \text{convexity} \times (\Delta y)^2$$

where

$$\text{convexity} = \frac{\frac{\partial^2 P}{\partial y^2}}{P} = \frac{\sum_{t=1}^T t(t+1)W_t}{(1+y)^2}$$

- note that convexity for a zero is $T(T+1)/(1+y)^2$
- example handout #1

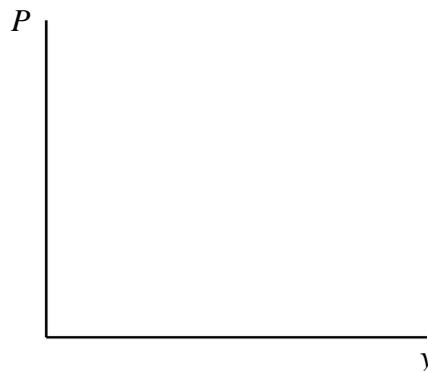
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- some properties of duration:
 1. Given y and T , duration is higher when C is lower.
 2. Given C and T , duration is lower when y is higher.
 3. Given C and y , duration is generally higher when T is higher (except for some deep discount coupon bonds).
 4. The duration of a portfolio P containing M bonds is $D_P = \sum_{i=1}^M w_i D_i$, where w_i is the percentage weight of bond i in P .
 5. Common formulas:

Contract	Duration
Zero coupon bond	T
Flat perpetuity	$\frac{1+y}{y}$
Flat annuity	$\frac{1+y}{y} - \frac{T}{(1+y)^T - 1}$
Coupon bond	$\frac{1+y}{y} - \frac{(1+y) + T(c-y)}{c[(1+y)^T - 1] + y}$
Coupon bond selling at par	$\frac{1+y}{y} \left[1 - \frac{1}{(1+y)^T} \right]$

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- some properties of convexity
 1. Convexity is always positive (for bonds without option features) (see text Figure 13.7).
 2. Given y and T , convexity is higher when C is lower.
 3. Given y and D , convexity is lower when C is lower.
- note that, given D , convexity is valuable:



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Passive bond portfolio management

- indexing
 - a bond index is intended to track broad movements over time in fixed income securities
 - Scotia McLeod provides four Canadian bond indexes: universe, short term, mid term, and long term
 - a bond index portfolio will have the same risk/return characteristics as the index it is based on
 - in practice it is hard to form bond index portfolios since:
 - * there can be a large number of securities involved
 - * many securities are thinly traded
 - * index composition changes frequently

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- the general approach which is usually adopted is that a portfolio comprising a representative set of securities (e.g. in terms of maturity/duration) is formed and rebalanced over time so as to track the index reasonably closely
- cash flow matching
 - for some purposes, it is desirable to reduce interest rate risk below the broad market risk that an indexing strategy assumes (e.g. hedging future cash flow obligations)
 - the simplest alternative is cash flow matching, in which the portfolio manager simply purchases zeros (or coupon bonds) of the appropriate maturity in sufficient amounts to meet the obligations
 - this is not always feasible due to liquidity/availability constraints

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- immunization

- consider a portfolio $V(y)$:

$$V(y + \Delta y) \approx V(y) + \frac{\partial V}{\partial y} \Delta y$$

$$\Rightarrow \text{if } \frac{\partial V}{\partial y} = 0, \quad V(y + \Delta y) \approx V(y)$$

and the portfolio is said to be immunized against interest rate risk

- immunization can be done on a *net worth* basis (e.g. banks try to mitigate effects of interest rates on their worth) or in terms of a *target date* (e.g. pension funds, any institution with a fixed future obligation)
- immunization is accomplished by equating the duration of assets and liabilities, i.e. set $D_A \times A = D_L \times L$
- example handout #2

- potential problems:

1. Use a second-order expansion:

$$V(y + \Delta y) \approx V(y) + \frac{\partial V}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 V}{\partial y^2} (\Delta y)^2$$

What happens if the portfolio is immunized? Since $\frac{\partial V}{\partial y} = 0$, $\frac{\partial^2 V}{\partial y^2} > 0$, $V(y + \Delta y) > V(y)$ and $V(y - \Delta y) > V(y)$, i.e. the strategy profits as long as yields move in either direction!

2. The strategy assumes there is no default risk or call risk for bonds in the portfolio.
3. The strategy assumes a flat term structure and any shifts in it are parallel.
4. Duration will change over time, so the portfolio may have to be rebalanced.

- some other points about duration/immunization:
 - if the term structure is not flat, then the formula for duration must be modified—in particular, the weights become:

$$W_t = \frac{CF_t / (1 + y_t)^t}{P}$$

- there are also extensions available to handle:
 - * non-parallel shifts in the term structure
 - * callability
 - * default risk
 - etc.
- hedging with non-parallel shifts in the yield curve
 - consider a portfolio $V = n_1 P_1 + n_2 P_2$. Using modified duration,

$$\Delta V = -n_1 D_1^* P_1 \Delta y_1 - n_2 D_2^* P_2 \Delta y_2.$$

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- initially, suppose the yield curve will only shift in a parallel manner ($\Rightarrow \Delta y_1 = \Delta y_2$). Then we can set $\Delta V = 0$ by choosing:

$$n_2 = -n_1 \frac{D_1^* P_1}{D_2^* P_2}$$

- if $\Delta y_1 \neq \Delta y_2$, then we need to specify a new objective for the hedging policy. A common objective is to minimize the variance of changes in V :

$$\begin{aligned} \text{Var}(\Delta V) &= (n_1 D_1^* P_1)^2 \text{Var}(\Delta y_1) + (n_2 D_2^* P_2)^2 \text{Var}(\Delta y_2) \\ &\quad + 2n_1 n_2 D_1^* D_2^* P_1 P_2 \text{Cov}(\Delta y_1, \Delta y_2) \end{aligned}$$

- take the derivative of this (e.g. with respect to n_2) and set equal to zero to get:

$$\begin{aligned} \frac{\partial \text{Var}(\Delta V)}{\partial n_2} &= 2n_2 (D_2^* P_2)^2 \text{Var}(\Delta y_2) \\ &\quad + 2n_1 D_1^* D_2^* P_1 P_2 \text{Cov}(\Delta y_1, \Delta y_2) = 0 \\ \Rightarrow n_2 &= -n_1 \frac{D_1^* P_1 \text{Cov}(\Delta y_1, \Delta y_2)}{D_2^* P_2 \text{Var}(\Delta y_2)} \end{aligned}$$

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- note that in the above one of the two bonds must be sold short. This is done using the repo market. For example, suppose a trader wishes to be long a 2 year T-note and short a 30 year T-bond:

Initial Transactions

1. Borrow cash from repo dealer, purchase 2 year note, post it as collateral
2. Borrow the 30 year bond from repo dealer, sell it, post cash as collateral

Closing Transactions

1. Collect 2 year note from repo dealer and sell it. Repay amount borrowed plus interest.
2. Collect cash plus interest from repo dealer and purchase 30 year bond to cover short position.

- the costs involved include:
 - * the bid/ask spread and accrued interest when buying/selling the Treasury issues
 - * the repo borrowing rate exceeds the repo lending rate
 - * margin (“haircut”)
 - * special repo rates (occasionally demand for particular issues becomes very high, e.g. due to deliverability in the futures market)

Active bond portfolio management

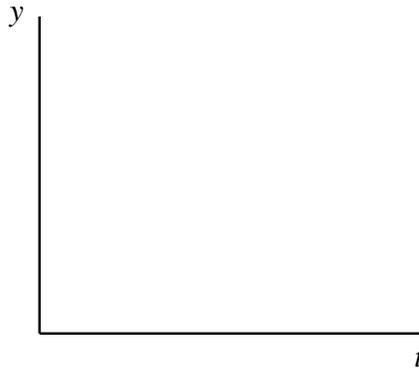
- aspects of active bond management include views about the level of interest rates, the shape of the term structure, and the pricing of individual bonds
- for portfolios with international holdings, exchange rates must be considered in addition to the above for each country
- a simple example: prices of long term bonds are more sensitive to changes in the general level of interest rates than prices of short term bonds; an investor who anticipates that rates will fall should lengthen portfolio maturity to generate capital gains, whereas an investor who believes rates will rise should switch to shorter maturity instruments to reduce potential capital losses

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- forecasting interest rates
 - one source of information is the current term structure (recall the expectations hypothesis)
 - if bond markets are (semi-strong form) efficient, all that is publicly known about future rates should be impounded into today's yield curve
 - other techniques used range from fairly simple time series forecasts to full scale macroeconomic models (recall links between term premia and credit spreads and economic cycles)
 - available evidence suggests that none of these methods works very well
 - in some cases a forecast of the direction of interest rates is sufficient, but even this is hard to do accurately on a consistent basis

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- riding the yield curve
 - if the current term structure slopes up and it is expected to remain unchanged, then buying longer term bonds produces higher returns over time due to additional capital gains as their yields drop over time



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- example: suppose a 5 year zero has a yield of 6% (so a price of \$747.26) and a 10 year zero has a yield of 7.5% (i.e. a price of \$485.19). Buying the 5 year zero locks in a return of 6%. If the yield curve remains unchanged, in 5 years the 10 year zero will sell for \$747.26. Buying the 10 year zero and holding it for 5 years generates a return of $(747.26/485.19)^{1/5} - 1 = 9.02\%$.
- note that if the yield curve shifts upward (even if its slope remains the same), investing in long term bonds will turn out to have been an inferior choice
- also note that this strategy implicitly assumes that the expectations hypothesis does not hold

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- the barbell strategy
 - basic idea is to duplicate the duration of an existing bond using a portfolio with one shorter maturity bond and one longer maturity bond in such a way as to increase convexity
 - underlying assumption is that yield curve shifts in a parallel way
 - example from November 20, 1987 using on the run Treasuries:

Issue	Yield	D^*	Convexity
2-year	7.71%	1.78	0.041
5-year	8.35	3.96	0.195
10-year	8.84	6.54	0.568

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- suppose an investor holds \$10M of the 5-year Treasury note. This could be sold, and used to purchase \$5.4M of the 2-year note and \$4.6M of the 10-year note. The modified duration of the portfolio is:

$$\frac{5.4 \times 1.78 + 4.6 \times 6.54}{10} = 3.96$$

and its convexity is:

$$\frac{5.4 \times 0.041 + 4.6 \times 0.568}{10} = 0.283$$

- assuming a parallel shift in the yield curve the barbell will outperform the 5-year note as long as the curve shifts up or down
- however, if the yield curve steepens the barbell can substantially underperform the 5-year note

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- security selection
 - the basic bond PV equation

$$PV = \sum_{t=1}^T \frac{C}{(1+y_t)^t} + \frac{F}{(1+y_T)^T}$$

can be used to estimate a value for any given bond; this can be compared to market prices to see if a bond is overvalued or undervalued

- many factors can complicate this analysis, however:
 - * tax effects: e.g. relatively low capital gains taxes increase the attractiveness of low-coupon bonds, which may increase their relative prices
 - * embedded options

- * liquidity: on-the-run Treasuries typically have more liquidity (i.e. a narrower bid-ask spread) and so sell at a slightly higher price than off-the-run issues \Rightarrow an investor without the need for liquidity may be better off selling on-the-run issues and purchasing off-the-run issues
 - suppose that a fund owns \$100M of an on-the-run 7 year T-note, selling at par with a yield of 8%. Suppose also that two 7 year off-the-run issues are available: (i) an 11.75% coupon with a yield of 8.4%; and (ii) a zero with a yield of 8.2%. If the fund manager were to sell the on-the-run issue and purchase a portfolio with \$79,974,052 of the 11.75% note and \$18,183,739 of the zero, this portfolio would have the same cash flows as the on-the-run note (if held for 7 years) but would only cost \$98,157,791.

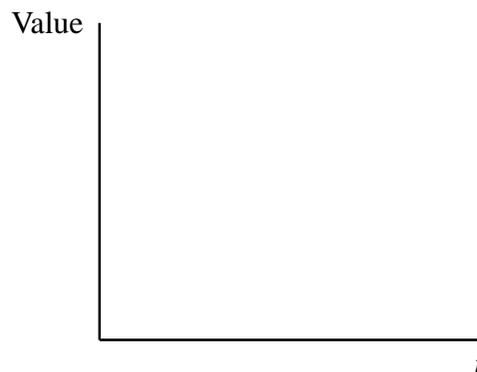
- bond swaps (portfolio rebalancing strategies)

1. Substitution swap: exchange one bond for another which is similar in terms of coupon, maturity, and credit quality but offers a higher yield
2. Intermarket spread swap: if an investor believes that spreads between two different types of bonds (e.g. corporates and Treasuries) are not currently at normal levels but will revert to them, he/she can switch into the market expected to perform relatively better
3. Rate anticipation swap: if an investor believes yields will fall (rise), he/she can switch to bonds of higher (lower) duration
4. Pure yield pickup swap: a simple switch to higher yield bonds (without regard to any expectations about changes in yields, spreads, etc.)
5. Tax swap: a switch undertaken for tax reasons, e.g. realize a capital loss and invest in a close substitute

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- contingent immunization

- similar to a “stop-loss” strategy for equities
- idea is to follow an active strategy, but ensure that the portfolio value never falls below a trigger point (which is designed to give minimum acceptable performance); if the trigger point is reached an immunization strategy is undertaken



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Interest rate swaps

- private agreements to exchange future cash flows according to a predetermined formula
- market size has increased from zero in 1980 to trillions of dollars today
- there are many different kinds of swaps, we will concentrate mostly on *plain vanilla interest rate swaps*
 - party *A* makes fixed rate payments to party *B*; in return *B* makes floating rate payments to *A*
 - payment size is based on notional principal
 - floating rate is usually 6 month LIBOR (London InterBank Offered Rate—interest rate offered by banks on Eurodollar deposits from other banks; a common reference rate in international financial markets)

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- example
 - notional principal is \$35M
 - *A* pays 7.19% per year semi-annually to *B*, receives 6 month LIBOR + 30 bps from *B*
 - amount of fixed payment:

$$\$35,000,000 \times \left(\frac{182}{365} \right) (.0719) = \$1,254,802.74$$

- if current 6 month LIBOR is 6.45%, floating payment is:

$$\$35,000,000 \times \left(\frac{182}{360} \right) (.0675) = \$1,194,375$$

- payments are netted, so *A* pays \$60,427.74 to *B*

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- normally parties do not negotiate directly with each other; a bank serves as an intermediary
- a typical pricing schedule looks like:

Maturity Years	Bank Pays Fixed	Bank Receives Fixed	Current TN Rate
2	2 yr TN + 31 bps	2 yr TN + 36 bps	6.79
5	5 yr TN + 41 bps	5 yr TN + 50 bps	7.06
7	7 yr TN + 48 bps	7 yr TN + 60 bps	7.10

(All rates are quoted against 6 month LIBOR)

- for a 7 year swap, the bank will pay 7.58% on the fixed side in return for 6 month LIBOR; the bank will pay 6 month LIBOR in return for 7.70% fixed
- bank's profits are 12 bps if it can negotiate offsetting transactions (if it cannot, the swap is "warehoused" and the interest rate risk is hedged, e.g. using futures contracts)

- motivation for using swaps:

1. Transform a liability: e.g. a fixed rate loan to floating rate loan

Suppose *B* has a \$35M loan on which it pays 7.5%. Its net position after the swap is:

Pays 7.5% to outside lenders
Pays LIBOR + 30 bps in swap
Receives 7.19% in swap
<hr/>
Pays LIBOR + 61 bps

2. Transform an asset:

Suppose *B* has a \$35M asset earning 6 month LIBOR - 20 bps. Its net position after the swap is:

Receives 6 month LIBOR - 20 bps on asset
Pays LIBOR + 30 bps in swap
Receives 7.19% in swap
<hr/>
Receives 6.69%

- early theories about why the swap market evolved relied on comparative advantage arguments
- in this case potential gains arise from relative differences in fixed and floating rates
- example:

	Fixed	Floating
<i>A</i>	8%	6 month LIBOR + 40 bps
<i>B</i>	9%	6 month LIBOR + 70 bps

- *B* is less credit worthy than *A*, but has a comparative advantage in floating
- let *A* borrow fixed, *B* borrow floating, and suppose they enter a swap where *A* pays 6 month LIBOR + 10 bps and receives 8.05%

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– *A*:

Pays 8% to outside lenders

Pays LIBOR + 10 bps to *B*

Receives 8.05% from *B*

Pays LIBOR + 5 bps

– *B*:

Pays LIBOR + 70 bps to outside lenders

Pays 8.05% to *A*

Receives LIBOR + 10 bps from *A*

Pays 8.65%

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- however, in floating rate markets lenders have the option to review terms every 6 months whereas fixed rates are usually on a 5-10 year term \Rightarrow the greater differential reflects a greater chance of a default by *B* over the longer term
- the apparent gain of 35 bps to *B* assumes that *B* can continue to pay LIBOR + 70 bps outside, if its credit rating worsens this amount could increase
- *A* does lock in LIBOR + 5 bps for the length of the swap, but it is also taking on the risk of default by a counterparty

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- a simple variation is a plain vanilla currency swap which involves exchanging fixed rate payments in different currencies and principal, e.g.

	\$	£
<i>A</i>	6%	8.7%
<i>B</i>	7.5%	9%

- suppose *A* borrows at 6% in \$ outside. It enters into a swap where it pays 8.25% in £ and receives 6% in \$ from a bank. *B* borrows at 9% in £ outside, receives 9% in £ from the bank and pays 7.05% in \$.

A trades a 6% \$ loan for an 8.25% £ loan

B trades a 9% £ loan for a 7.05% \$ loan

Bank gains 1.05% on \$, loses 0.75% on £

- the principal exchange is roughly of equal value at the start, e.g. £10M = \$15M
- the bank has FX risk, can hedge via forwards

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- other variations:
 - fixed in one currency, floating in another
 - floating in both currencies
 - amortizing/accreting swaps (N_p rises or falls over time, possibly depending on future interest rates)
 - constant yield swaps (both parts floating, different maturities)
 - rate capped swaps (floating rate is capped at some maximum level)
 - deferred swaps (rates set now, contract starts later)
 - extendable/puttable swaps (one party has the option to increase or reduce the maturity of the contract)
 - commodity swaps (e.g. a portfolio of forward contracts to buy a commodity such as oil)
 - equity swaps (many variations, e.g. floating payments depend on returns on stock index, both parts float (one S&P, the other Nikkei; one Chrysler, the other GM))
 - options on swaps (a.k.a. “swaptions”)