

TEACHING PORTFOLIO

SHIRA VIEL

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1. INTRODUCTION

The organizing principle of my teaching is joy in mathematics: I inspire, challenge, and encourage my students with an inclusive classroom environment, an emphasis on active learning, and multiple avenues for engagement with the material. This portfolio provides evidence of my work and growth teaching undergraduate mathematics.

2. TEACHING EXPERIENCE

I have served as the instructor of record for the following courses at North Carolina State University. My responsibilities include writing the syllabus, preparing and delivering lectures and in-class activities, writing and grading quizzes, writing exams, holding office hours, and determining course grades. For MA 225, I also graded homework and exams.

- MA 105: Financial Math, Spring 2014 (64 students).
- MA 107: Precalculus I, Fall 2013 (57 students).
- MA 141: Calculus I (engineers), Fall 2014, 15 (90, 84 students).
- MA 225: Foundations of Adv. Math., Spring 2016 (22 students).
- MA 231: Calculus B (life science and business) hybrid, Spring 2017 (87 students).
- MA 231: Calculus B (life science and business) traditional, Fall 2017 (69 students).
- MA 241: Calculus II (engineers), Spring 2015 (50 students).

In Fall 2016, I served as a Teaching Assistant for MA 231 (90 students), assisting in the design, pilot implementation, and assessment of a hybrid version of the course incorporating both online and face-to-face instruction.

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3. TEACHING EVALUATIONS

3.1. Student Evaluations. Students at North Carolina State University fill out optional anonymous course evaluations at the end of each semester. Multiple choice questions are answered on a scale of 1 (Strongly Disagree) to 5 (Strongly Agree). The numerical data for these evaluations is provided below in reverse chronological order. In Spring 2017, I taught 3 hybrid sections of MA 231.

I make a particular effort in my teaching to serve and accommodate a diverse group of students, with a wide range of mathematical skills and experience. This involves not only designing and adapting class activities for different learning styles, but making myself available to students inside and outside of the classroom and fostering an environment of mutual respect. The class evaluation scores over my four years of instruction reflect this effort, as well as my improved ability to explain material and give feedback.

	231-4 S17	231-2 S17	231-0 S17	225 F16	141 F15
The instructors teaching aligned with the courses learning objectives/outcomes.	4.7	4.7	4.6	4.7	4.5
The instructor was receptive to students outside the classroom.	4.8	5.0	4.4	4.9	4.5
The instructor explained material well.	4.7	4.7	4.5	4.4	4.1
The instructor was enthusiastic about teaching the course.	5.0	5.0	4.6	5.0	4.8
The instructor was prepared for class.	5.0	4.9	4.5	4.8	4.6
The instructor gave useful feedback.	4.9	4.6	4.4	4.5	4.4
The instructor consistently treated students with respect.	5.0	4.9	4.5	5.0	4.7
Overall, the instructor was an effective teacher.	5.0	4.7	4.5	4.5	4.4

	241 S15	141 F14	105 S14	107 F13
The instructor stated course objectives/outcomes.	4.8	4.5	4.7	4.6
The instructor was receptive to students outside the classroom.	4.8	4.6	4.3	4.6
The instructor explained material well.	4.7	3.9	4.5	3.8
The instructor was enthusiastic about teaching the course.	4.9	4.8	4.7	4.8
The instructor was prepared for class.	4.8	4.4	4.6	4.7
The instructor gave useful feedback.	4.5	4.1	4.3	4.4
The instructor consistently treated students with respect.	4.8	4.4	4.5	4.6
Overall, the instructor was an effective teacher.	4.8	4.0	4.5	4.0

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A selection of student comments from the end of semester course evaluations are provided below. As reflected in the numerical data, students value my enthusiasm and investment in their learning, and appreciate the variety of opportunities for engaging with the material.

- “Fantastic at explaining concepts without giving too much to the student. Helps you based on your weaknesses and proposes potential thought paths that lead the student through their own deduction rather than ‘this is how it is done, and here is why’ approach.” (231 S17)
- “Shira was an excellent instructor who was always willing to go out of her way to help students. She was willing to reexplain material during practice sessions. She always had a warm presence and very positive attitude towards her students and the subject itself. She made students excited to learn about calculus.” (231 S17)
- “Shira deserves the outstanding teacher award because she has been the best teacher I have had since I’ve been in college. The material was challenging, but she explained things so well and enthusiastically that it made learning easy. She knows what she teaches and knows how to include everyone in the classroom.” (225 F16)
- “I have never had a teacher who was as helpful during office hours as Ms. [Viel]. Going to office hours gave me a better understanding of how to write better proofs and helped me better prepare for quizzes and exams. Ms. [Viel] was extremely respectful and friendly to her students and I could truly see she was passionate about math and teaching this subject.” (225 F16)
- “Shira is the perfect example of a good instructor. She explains the material very well and is patient with those who ask a lot of questions. She is also understanding of student struggles, but firm in pushing us to learn what we need to for the course. She is one of the best math teachers I have ever had.” (141 F15)
- “I love Ms. [Viel]! She really knows how to explain things and how to keep everything interesting. I love how she gives us so many opportunities to master the material and is always responsive when emailing or asking questions in person.” (141 F15)
- ““Shira was extremely receptive to me both in and outside the classroom. She ALWAYS answered emails within the hour and would not stop until I fully understood the subject. She was very understanding to our schedules as students, and she acted and treated us accordingly. She respected every question in the classroom, no matter how small or seemingly dumb, and answered them all evenly. Overall, I believe Shira [Viel] deserves the Outstanding Teacher Award because of her undying enthusiasm for math, respectfulness for ALL students and ALL questions, and her understanding of us as both humans and students.” (241 S15)
- “Very enthusiastic about mathematics which makes it easier to personally become excited about math. Very knowledgeable of the course material and explains it in a way that is easy to understand by using metaphors and analogies. Also explains why the material is useful to me as an engineer.” (241 S15)
- “She clearly demonstrated that she not only knew the material but could teach it. Furthermore she was always eager to help outside of class. In addition she always encouraged higher learning outside of the classroom. Meaning that if you wanted to explore a proof more or what the dx really means outside of class she was always more than willing to help.” (141 F14)

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- Shira [Viel] is an inspiring role model for all her students. Her bubbling enthusiasm for mathematics is simply contagious. Mrs. [Viel] has an ability of fostering an engaging learning environment for her students by promoting mastery of mathematical intuition and technique” (141 F14)
- “Shira is an incredible instructor, and is by far the best math instructor I have ever had. Just as an excellent teacher does, she makes class informative, interesting, and asks for feedback from students. On top of the countless hours she puts in to planning, she equally puts efforts into availability and personability. Even when she had a cold, she still showed that she loved teaching. Her bubbly personality made learning math unintimidating, which is a huge factor for me. Not only is Shira organized and posts lectures online for later review, but she details exactly how she got to where she did and how you can too.” (105 S14)
- “I would take another financial math class from Shira, even though I don’t need another for my degree, in a minute. The tools she taught me to help myself learn will be valuable for the rest of my life.” (105 S14)
- “Professor [Viel] is one of the most enthusiastic teachers at State that I’ve had the pleasure of being taught by. I not only felt encourage to keep trying in her classroom, but motivated to work hard for her too. I have no doubt that every one of my peers will leave that class with a full understanding of the material she taught, and an overwhelming sense of appreciation for her joyful spirit leading our class.” (107 F13)
- “Ms. [Viel] has an amazing enthusiasm for teaching! She thoroughly enjoys math and helping students, and she’s never not smiling! She is more than willing to help students, be that through extended office hours, extra study guides, email, or moodle. She truly went out of her way to help us. She also has a high standard when it comes to explaining things; she values clarity and will do everything she can to make sure her student’s understand.” (107 F13)

In the past two years, I have also started giving students opportunities to provide anonymous feedback mid-semester. This helps me get a sense of how students are feeling about the course at a time when I can still adapt my teaching to address their concerns, and has been a great help to me as an instructor. Students have responded positively as well: this is an end of semester comment from MA 241 S15:

- “Ms. [Viel] is so great! I love having her as a teacher. It appeared to be her first time teaching Calc II and her ability to teach the subject has grown exponentially since the start of the semester. She never taught the course badly, but her lectures in preparation for the first test were a bit disjointed and we did a lot of derivations, instead of examples. However, she gave out a survey after the first test to assess her strengths and weaknesses. She listened to what we suggested and the lectures became 50x more helpful. By taking our feedback to heart, she really showed that she cared about how successful we were in the class. She is an incredible teacher that has a great ability to connect with her students and make the class a fun environment for learning.”

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3.2. Faculty Evaluations. Evaluations from three mathematics faculty members at North Carolina State University are provided below in reverse chronological order.

Ernest Stitzinger, Professor and Administrator of the Graduate Program, observed me teaching MA 231: Calculus B for life science and business majors (traditional), in Fall 2017. He responded positively to my effort to make the material relevant and intuitive to my students as well as my use of technology: namely the SmartBoard and TopHat.

The Instructor

	Extremely well	Very well	Adequately	Inadequately	Very Poorly	Did not observe
Was prepared for class	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Presented the material effectively	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Was enthusiastic about the material	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Spoke clearly and audibly	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Made effective use of the board or other visual display	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Encouraged student involvement	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Treated students respectfully	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Please comment on the Instructors strengths or weaknesses.

She was teaching setting up differential equations and spent a lot of effort giving intuition on how to do it. She was very enthusiastic about the material. She used technology effectively and effortlessly throughout. She continually had the students involved.

Would you recommend the Instructor to teach this class again?

Yes, I would encourage it.

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Molly Fenn, Teaching Associate Professor and Assistant Department Head, observed me teaching MA 225 in Spring 2016. Dr. Fenn was my mentor during my participation in North Carolina State's Preparing the Professoriate Program, and I had observed and assisted her teaching of this course the during the previous semester. This course, essentially an introduction to proof writing, was my first time teaching a small class of mathematics majors and minors, and I experimented with a more active learning approach.

	Needs Improvement	Satisfactory	Well Done	Not Observed
Introduction: The lesson was introduced in an interesting and effective way, and the goal or purpose of the lesson was clear.			X	
Level/Audience: The lesson was presented at an appropriate level for the intended audience. The instructor seemed to know his/her audience.			X	
Content: The content was developed in an organized manner, emphasizing important points. Examples or applications helped students relate material to familiar concepts.			X	
Purpose of learning activities: The learning activities had an appropriate place in the lesson, and the purpose of each activity was clear.			X	
Student participation: The instructor effectively involved the students in the lesson.			X	
Climate: The instructor established a positive climate of approachability, setting an appropriate tone for the instructor-student relationship.			X	
Enthusiasm: The instructor showed enthusiasm for the subject and introduced interesting aspects of the content.			X	
Questions: The instructor checked for student understanding and invited student questions. When questions were asked, the instructor responded clearly.			X	
Visual aids: The instructor used visuals effectively. Visuals may include the board, handouts, PowerPoint presentations, content projected on a screen, etc.			X	
Time management: The pace of the lesson was appropriate, and the instructor managed the time well and brought the lesson to a logical conclusion.			X	

1. What were the strengths of this lesson?

Shira has created an excellent classroom atmosphere. Her students got right to work on the problems they were given and were very comfortable acknowledging when they were stuck or confused. They asked and answered questions freely. Throughout, Shira was cheerful, upbeat, and happy to be in class.

2. What suggestions do you recommend for this instructor?

I honestly have nothing to recommend about the lesson, it was excellent. Shira incorporated an appropriate amount of group work and also modeled good proof writing for the students. Longer term, as she continues teaching and has more experience with courses similar to this one I have no doubt that she will tweak things and try out new techniques and strategies. But be warned, it will probably never feel like you're doing it right!

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Elizabeth Dempster, Lecturer, observed me teaching MA 107 in in Fall 2013, the first course I taught at North Carolina State. To best serve the students in this precalculus course, several of whom were repeating, I posted note outlines in advance of each class.

Teaching characteristics – Comments	
1. Planning and start of session <small>Appropriateness of aims and outcomes (where it is possible to evaluate this). Communication of these to students. Continuity with other sessions and students' prior knowledge made explicit. Coping with any unexpected occurrences, e.g. latecomers, missing equipment.</small>	
<p>she had an outlined typed ahead of time. she gave a good overview of the lesson before she started.</p>	
2. Presentation <small>Structure. Relevance and organization of content. Attitude to subject matter. Clarity of presentation. Emphasis of key points. Pace of session (time management). Tone, volume, clarity of speech. Links made to other aspects of course (e.g. lectures, seminars or tutorials). Summary (end and/or interim).</small>	
<p>Wonderful presentation. You can tell she really likes math and enjoys teaching. Her voice is the perfect level. She is respectful of the students + their questions.</p>	
3. Student participation <small>Question and answer technique. Exercises/activities. Class management (appropriate level of control and authority). Instructions to students. General class atmosphere. Level of participation between students (excessive? lacking?). Attention and interest. Attitude to students. Awareness of individual needs. Student-teacher rapport.</small>	
<p>Good for a really large lecture hall.</p>	
4. Methods and approaches <small>Choice/variety of teaching/learning methods. Use and design of instructional materials (board, technology, handouts etc.) Use of appropriate reinforcement. Examples and analogies. References and links to research, other resources. Handling problems/disruptions.</small>	
<p>very good</p>	
5. General <small>Were the aims and outcomes achieved? Appropriateness of teaching/learning methods. Was effective communication achieved? Awareness of needs of learners and differences in approach.</small>	
<p>Shira does an excellent job in presenting the material and addressing students' questions. I think she communicates very well. She also seems to show different approaches when she can. Overall, she's one of the best I've ever observed.</p>	

4. REPRESENTATIVE SYLLABI, ASSIGNMENTS, AND EXAMINATIONS

Effective course design is a crucial component of effective teaching. With the exception of the hybrid version of MA 231 which I helped develop, the courses I have taught at North Carolina State have all had proscribed content, structure, and textbooks. However, I write my own syllabi and exams and prepare my own class content, and use this discretion to set a pace and level of difficulty appropriate to my students. I also develop lecture materials and activities to present, explain, and practice new concepts and skills in an enthusiastic and accessible manner, often incorporating technology and active learning. I have included representative samples of the teaching materials I have developed in the appendices:

- Syllabus for the hybrid version of MA 231 that I assisted in developing, including a description of the unique course setup and a detailed explanation of the different modes for course communication (see **Appendix A**, p.10).
- Syllabus for MA 141: Calculus I for engineers, including advice on online homework, suggestions for getting assistance, and a detailed calendar (see **Appendix B**, p.14).
- Proof-grading rubric for MA 225: Foundations of Advanced Mathematics, a course aimed at preparing students for success in upper-level mathematics. I designed a grading scheme for proofs based upon my reading of “Specifications Grading: Restoring Rigor, Motivating Students, and Saving Faculty Time” by Linda Nilson. The objective-specific proof rubric has clear expectations and examples, regularly reinforced with my modeling of proof writing in class and on our course Moodle site. Proofs were graded on a “Pass/No Pass/Resubmit” basis, with permission to rework a “Resubmit” proof, with assistance from the instructor and peers and the inclusion of a description of their changes, for possible full credit (see **Appendix C**, p.16).
- In-class review activity for MA 241: Calculus II for engineers. Homework for this course, completed online, was due the evening prior to each exam. To keep students accountable for the material and myself apprised of their understanding, I gave periodic “Recap quizzes” in class. These were open book and discussion, but were timed and graded individually. This quiz was given in the second week of class, after we had reviewed Calculus I material. The detailed solutions include hints and notes on where students can review the relevant content (see **Appendix D**, p.17).
- In-class applications activity for MA 231: Calculus B for life science and business majors. By using TopHat, students received immediate feedback on their answers. In addition to cementing the utility of exponential growth in modeling physical processes, this activity recalled earlier work on least-squares regression and gave students practice with Google sheets (see **Appendix E**, p.20).
- Test for MA 105: Financial Math. This 50 minute test was the third of three in-class exams. Many of the students in this course had informed me (on the notecards I collected on the first day of class) that they had negative math experiences in the past, so I made a particular effort to make the material accessible without diluting the content. Part of this effort included giving a theme to each in-class test: in this case, the Muppets (see **Appendix F**, p.28).
- Final for MA 225: Foundations of Advanced Mathematics. This three hour cumulative exam included both computational problems, testing knowledge and facility with definitions, and proofs of varying difficulty. In addition, I included two extra credit problems with more challenging proofs to give students the opportunity to demonstrate their abilities and understanding (see **Appendix G**, p.29).

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5. TEACHING IMPROVEMENT ACTIVITIES

I am constantly working to improve my teaching, and have taken every opportunity offered by North Carolina State to participate in professional development activities. Interested in further development, I often discuss teaching with my peers and faculty members, and organize an ongoing teaching seminar. In addition to my passion for undergraduate mathematics, I am also very interested in mathematical outreach, and have engaged in several programs to improve my teaching of elementary and middle school students.

Preparing the Professoriate, August 2015 - May 2016.

- A selective university-wide fellowship pairing students with faculty mentors to observe and then teach an upper-level course (MA 225: Foundations of Mathematics).
- Involved a monthly workshop series, including “Teaching with Technology” and “Leading with Care: Recognizing and Responding to Distress in Others.”

Certificate of Accomplishment in Teaching, December 2014 - May 2016.

- A two-year university-wide teaching certification program.
- Workshops included “Introduction to Teaching,” “Learning Styles,” and “Establishing Credibility and Authority in the Classroom.”

Graduate Instructor Support and Tools, September 2014 - Present.

- Founding member of student-led organization supporting graduate teaching.
- In addition to building and maintaining a Wiki for collaboration and resource sharing among graduate student and postdoctoral instructors, organized and facilitated regular faculty panel seminars, including “Learning Styles: identifying and addressing different student approaches,” “Research in Undergraduate Mathematics Education: understanding it, doing it, using it,” “Self Assessment: strategies and techniques for evaluating your teaching effectiveness,” and “Alternative Classroom Techniques.”

College for Kids at the University of Wisconsin, Madison, WI, Summer 2012.

- In preparation for serving as teacher facilitator in a summer program for advanced sixth grade students, participated in an intensive graduate-level education seminar.
- Received training on mentoring, instruction, and activity design for gifted students.

Circle on the Road Conference, Tempe, AZ, March 2012 and Houston, TX March 2011.

- To support my own position as a math circle instructor, attended this annual conference sponsored by the National Association of Math Circles and MSRI.
- Participated in workshops taught by expert math circle instructors and facilitators, and assisted in math circle demonstrations open to the public.

6. CONCLUSION

Teaching mathematics is my highest passion. I love the process of structuring a course to convey not only mathematical content but the process of logical reasoning, the importance of persistence in the face of a challenge, and the joy of rational thought and communication. I enjoy getting to know my class and learning new methods to adapt my instruction to best meet their needs. I am eager to continue my development as an educator, and serve many more students as an enthusiastic guide and mentor in their mathematical studies.

MA 231: Calculus for Life and Management Sciences B

Sections:	010, 012, 014	011, 013, 015
Instructor:	Shira Viel	Dr. Molly Fenn
Email:	spolste@ncsu.edu	mafenn2@ncsu.edu
Office:	SAS 3147	SAS 2108
Office Hours:	Wed 9:30-11:30 and by email appt.	By appointment. Sign up at http://go.ncsu.edu/FennAppointments

Textbook Calculus and Its Applications by Goldstein, Lay, Asmar, Schneider. 13th edition. 2013. Pearson. ISBN# 978-0321848901. Cost: \$150

Course Description

Functions of several variables - partial derivatives, optimization, least squares, Lagrange multiplier method; differential equations - population growth, finance and investment models, systems, numerical methods; MA 121 is not an accepted prerequisite for MA 231.

Course Design

This class is a hybrid course. This means 2/3 of the “lecture time” will occur online, via Moodle, and 1/3 of it will happen in a face-to-face (F2F) class. You will be responsible for using the online lessons to get comfortable with the new material before coming to your F2F class. During the F2F class, you will be actively working on the more difficult skills and applications of the material.

Course Learning Outcomes

After successfully completing this course, students will be able to:

1. Use the techniques of partial differentiation to explore the properties of a function of two or more variables
2. Set up and solve optimization problems in various contexts
3. Use least squares to fit linear and nonlinear functions to a given data set
4. Give examples of how and why different disciplines use differential equations and mathematical models
5. Create a mathematical model that describes a given problem from biology, economics, or business
6. Carry out numerical simulations and mathematical analyses of a model

For a schedule of topics to be covered and the detailed learning objectives for the course, see the course Moodle page.

Grades

This course uses standard NCSU letter grading, with no rounding.

$90 \leq A- < 93$	$93 \leq A < 97$	$97 \leq A+ \leq 100$
$80 \leq B- < 83$	$83 \leq B < 87$	$87 \leq B+ < 90$
$70 \leq C- < 73$	$73 \leq C < 77$	$77 \leq C+ < 80$
$60 \leq D- < 63$	$63 \leq D < 67$	$67 \leq D+ < 70$
$0 \leq F < 60$		

Grade Component	Weight	Details
Online Participation	5%	The online participation score is the percentage of Moodle lessons submitted. It is independent of the number of self-check questions answered correctly.
Before Class Homework	15%	Before class homework is done through WebAssign. An assignment corresponding to the online lesson is due each week on Thursdays at 5pm.
Classwork	10%	During class students will actively work with their peers to apply the concepts learned in the online lessons to more detailed problems. Many assignments will be done through WebAssign and some may be turned in on paper. Assignments will be due after the class meeting on Mondays at 5pm.
Attendance	5%	Attendance at F2F classes on Fridays is required. Up to 2 absences (excused or unexcused) are permitted.
Midterm Tests	40%	Two 60-minute tests, Feb 14-15 and April 4-5.
Final Exam	25%	Comprehensive, 180-minute exam, May 1-2.

Homework Policies

Part of your homework will be done through [WebAssign](http://webassign.ncsu.edu) (webassign.ncsu.edu). To use WebAssign you will need to purchase an access code online via credit card. The access code costs around \$30 per course.

Extensions are requested and granted automatically through WebAssign, so long as you have not viewed the answer key for an assignment. You may request one extension per assignment at any time **within a 5-day window** of the original due date at a **penalty of 40%** on unearned points. After the extension is granted, you have **24 hours** to complete the assignment within that 5-day window.

Exams

There are two 60 minute midterms and one 3-hour final. All three exams must be taken at a [DELTA testing center](#), which operates on a first-come, first-served basis. You may take each exam at any time during the designated testing window, listed below. Students are required to comply with the university policies on academic integrity and honesty.

- Midterm Test 1: Tues February 14 – Wed February 15
- Midterm Test 2: Tues April 4 – Wed April 5
- Final Exam: Mon May 1 – Tues May 2

Course Website

We will be using the *Moodle* learning management system (<http://wolfware.ncsu.edu>) for this course. You will log in using your Unity ID and password. (Refer to online information at <http://oit.ncsu.edu/unityid> or contact (919) 515-HELP or HELP@ncsu.edu for assistance with your Unity ID). After the beginning of the semester, you will see a link to our course site. Once in the site, you can Bookmark or add the site as a Favorite in your web browser so that you can return directly to that page.

Course Communications

Modes of communication in use for this course include email, office hours, and Moodle.

- Moodle discussion forums will be used to facilitate class discussion. Check these forums often and please feel free to reply to your fellow students' posts.
- We will do our best to respond to weekday e-mails and posts within 24 hours. Email messages or posts left after 4 pm Friday will be responded to by Monday evening.
- If you would like to speak with an instructor in person and you can't make it to the posted office hours, please email me to schedule a time that is convenient. Include several time slots that would work for you in your email.

Please be aware that ALL email communications for this course will be sent to your NCSU unity email. If you do not regularly use your ncsu.edu account, there are settings within Gmail that allow you to forward your e-mail to another account. For more information, please see <http://google.ncsu.edu/what-best-way-forward-my-nc-state-gmail-non-nc-state-e-mail-address>.

If you have a question that the whole class may benefit from hearing the answer to, please post on the "Course Content Q&A" forum. We will check this forum often to respond to open questions. You should also check frequently to answer or ask questions.

If you have a question that is very specific to the work you have done (i.e. if you nearly finished your work but got stuck towards the end), you can email your instructor with your question. Including a scan or photo of your work can help.

If an instructor receives an email with a question more appropriate to the forum, she may copy and paste the question there without identifying the student who sent it.

Academic Integrity

Students are required to comply with the university policy on academic integrity found in the Code of Student Conduct found at <http://policies.ncsu.edu/policy/pol-11-35-01>

The NCSU Student Code of Conduct covers all work done in this course.

Any suspected violations will be promptly reported. Academic dishonesty will result in an automatic failing grade for the course.

Course Evaluations

A formal evaluation is conducted by the University at the end of the semester and the goal is to achieve 100% class participation in this survey. Online class evaluations will be available for students to complete during the last two weeks of class. Students will receive an email message directing them to a website where they can login using their Unity ID and complete evaluations. All evaluations are confidential; instructors will never know how any one student

responded to any question, and students will never know the ratings for any particular instructor.

Accommodations for Disabilities

Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, student must register with the Disability Services Office (<http://www.ncsu.edu/dso>), 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation at <http://policies.ncsu.edu/regulation/reg-02-20-01>.

Non-Discrimination Policy

NC State University provides equality of opportunity in education and employment for all students and employees. Accordingly, NC State affirms its commitment to maintain a work environment for all employees and an academic environment for all students that is free from all forms of discrimination. Discrimination based on race, color, religion, creed, sex, national origin, age, disability, veteran status, or sexual orientation is a violation of state and federal law and/or NC State University policy and will not be tolerated. Harassment of any person (either in the form of quid pro quo or creation of a hostile environment) based on race, color, religion, creed, sex, national origin, age, disability, veteran status, or sexual orientation also is a violation of state and federal law and/or NC State University policy and will not be tolerated. Retaliation against any person who complains about discrimination is also prohibited. NC State's policies and regulations covering discrimination, harassment, and retaliation may be accessed at <http://policies.ncsu.edu/policy/pol-04-25-05> or http://www.ncsu.edu/equal_op/. Any person who feels that he or she has been the subject of prohibited discrimination, harassment, or retaliation should contact the Office for Equal Opportunity (OEO) at 919-515-3148.

Copyrighted Materials

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MA 141-006: Calculus I
Fall 2015, TR 8:05-9:55a, Cox Hall #206

Instructor: Shira Polster
Office: SAS #3147
Office Hours: F 9-11a and by appointment
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Course Description. Welcome to MA 141! This is the first course in a three-semester calculus sequence for science and engineering majors, and is designed to familiarize students with the basic concepts and tools of calculus, topics that are foundational to their fields of study. We will cover functions, graphs, limits, derivatives, rules of differentiation, definite and indefinite integrals, the Fundamental Theorem of Calculus, and applications thereof. MA 141 also introduces the use of computational software.

Required Materials.

- (1) *WebAssign access code.* You must purchase access to the course text and weekly homework via WebAssign at <http://www.webassign.net/ncsu/>. The total cost is \$32.95.
- (2) *Five small blue “Examination” booklets.* Available at the campus bookstore for around \$0.30 each. Submit *blank* books to TA by **Tuesday, Sep 8**: do not write anything on the books.

Structure and Grading. Letter grading follows the +/- 10 point scale below left. All other grading is in accordance with [University policy](#). Grades will be determined as shown below right.

Grading Scale

97-100	A+	93-96.99	A	90-92.99	A-
87-89.99	B+	83-86.99	B	80-82.99	B-
77-79.99	C+	73-76.99	C	70-72.99	C-
67-69.99	D+	63-66.99	D	60-62.99	D-

Component	Weight
WebAssign/Quizzes	12%
4 Maple Assignments	5%
4 In-class Tests	48%
Final Exam	35%

Attendance Policy. Attendance will be taken at all class sessions (TR lectures and W recitations) per NCSU policy. Active participation is expected for the entirety of the scheduled class time: please turn off your phones, tablets, and computers and give your full attention to the material. **If you have 3 or fewer total lecture absences (excused or unexcused), you have the option to replace your lowest test score with your final exam.** Check your attendance record at any time on Moodle, and address all attendance questions to the TA.

Course Text. Our textbook, accessible via [WebAssign](#), was written by three NCSU mathematics faculty members expressly for this course. This is a pilot semester for the text, so it is provided to you free-of-charge with your WebAssign access code. Please feel free to submit comments and suggestions to the authors at any point by emailing calculusbook@ncsu.edu or clicking on the link at the top of each chapter. I recommend that you read the text in its digital form, as it is hyperlinked for your convenience, but you are also welcome to download and print chapter pdfs for your personal use.

WebAssign. The course pack and WebAssign homework assignments are obtained, submitted, and graded online at <http://www.webassign.net/ncsu/> and are **due at 8a the morning of each test**. **No make-up work or extensions are available for missed assignments.** I encourage you complete assignments weekly to help you stay on top of the material and be prepared for quizzes. **I recommend you print assignments and work with pencil and paper before submitting: you will be required to show all work on exams, and HW is great practice.** There are two introductory WebAssign homeworks to acclimate you to the system: these are **due at 8a on Tuesday, Aug 25**.

Quizzes & Classwork. Throughout the semester I may give short quizzes and group work for completion in class to help assess your understanding. These assignments may be given both with and without advance notice, but will always be open-note: both personal lecture and HW notes are permitted.

Maple. Maple is a powerful mathematical software program that we will begin using in mid-September to complement our paper-and-pencil computation. Assignments are obtained, submitted, and graded online at <http://emarker.math.ncsu.edu/CalculusWithMaple/>, where help files can also be found. The software is available to all registered NCSU students on all campus computers, and online via the Virtual Computing Lab: <https://vcl.ncsu.edu/>. **Due dates are Sep 25, Oct 23, and Nov 6.**

Accommodations. Reasonable accommodations will be made for students with verifiable disabilities. To take advantage of available accommodations, students must register with Disability Services for Students at <http://www.ncsu.edu/dso/> and then meet with the instructor.

Code of Student Conduct. All students are expected to adhere to the University's regulations on academic integrity. Documentation of violations will be submitted to the Office of Student Conduct. http://www2.ncsu.edu/ncsu/stud_affairs/policies/code95.html.

Tests. The four in-class tests are closed-book, closed-note, with no graphing calculators permitted. Each is worth 12% of your final grade. The **test dates are Sep 10, Oct 6, Oct 29, and Nov 24.**

Test make-ups are administered in accordance with University policy. Make-up tests for anticipated absences must be arranged *before* the scheduled test date. Students with excused or legitimate emergency absences must **contact the instructor within 48 hours of the test date** to schedule a make-up. Make-ups for tests will be subject to instructor availability.

Answer keys for all tests will be posted on Moodle when the exams are returned in class. If after reviewing the answer key you believe an error has been made in the grading of your exam, **provide a written explanation of the error, attached to the original test, to the instructor within 1 week.** Do not alter the original work. The entire test may be re-graded and the test grade is *subject to remain the same, increase or decrease* at the discretion of the instructor.

Final Exam. The final exam is scheduled by the University for **Thurs, Dec 10, 8-11am** in our regular classroom (Cox #206) and is non-negotiable unless you have 3 exams within 24 hours. Determine as early as possible if that is the case, as you must go through the registrar and math department to reschedule.

Getting Help. Study groups are recommended for this course: your peers are excellent resources. In addition, office hours exist for the sole purpose of helping students, so come if you need help! Effectiveness is maximized if you arrive with specific questions in hand. For virtual assistance, a class Moodle site will be maintained with course announcements, lecture materials, and a discussion forum. **If emailing the instructor or TA directly, please include MA 141 in the subject line and your full name in the message.** Tutors for hire are listed at <http://www.math.ncsu.edu/mmc/tutorinfo.php>, and free drop-in tutoring and Maple help is available at the **MMC** in SAS 2105 weekdays 8a-5p.

Course Schedule. Test dates are final; lecture material and homework due dates are tentative based on class progress. Current due dates will be kept on Moodle and WebAssign.

WEEK	LECTURE MATERIAL	HOMEWORK DUE
1	Chapt 0 §2-4	
2	Chpt 1 §1-2	WA intro assmts
3	Chpt 1 §3-4, Chpt 2 §1	
4	Chpt 2 §1, Test #1 (Thurs Sep 10)	WA 0.2-2.1
5	Chpt 2 §2-4	
6	Chpt 2 §5-6	Maple #0, #1
7	Chpt 2 §7	
8	Test #2 (Tues Oct 6) Chpt 3 §1	WA 2.2-2.7
9	Chpt 3 §1-3	
10	Chpt 3 §4-5	Maple #2
11	Chpt 3 §6, Test #3 (Thurs Oct 29)	WA 3.1-3.6
12	Chpt 4 §1-3	Maple #3
13	Chpt 4 §3-5	
14	Chpt 4 §5-6	
15	Test #4 (Tues Nov 24)	WA 4.1-4.6
16	Review	
	Final Exam (Thurs Dec 10)	

The following contains a list of requirements that must be met in order to receive a **P** on a proof, signifying that it is valid, complete, and correct. While there are many different ways to structure a proof and write its details, clarity, precision, logical validity, and correctness are always necessary! **Examples are illustrated in blue**; **Non-examples are illustrated in red**. Each requirement has a symbol, like **(DV)**: when you receive a circle around something in your proof and this symbol, it means you forgot to define the given variable.

General Structure

(W): Writing is neat, grammatically correct, and in complete sentences.

“Assume $x - 2 = 0$. Then $x = 0$.”

“Assume $x - 2$. Then $x = 0$.”

(C): Claim is clearly stated before the proof is begun.

“**Claim**: Assume x represents a real number. If $x^2 - x - 2 \geq 0$, then $x \leq -1$ or $x \geq 2$.”

“**Claim**: $x \leq -1$ or $x \geq 2$ ”

(A): Assumption(s) is(are) clearly stated before they are used.

First sentence: “Assume x is a real number such that $x^2 - x - 2 \geq 0$.”

First sentence: “We know $(x - 2)(x + 1) \geq 0$.”

(P): Both the beginning and end of the proof are clearly indicated.

Logical Validity

(L1): Reasoning goes *from* what you know *to* what you want to show.

“Since we know $x^2 - x - 2 \geq 0$, we conclude that $(x - 2)(x + 1) \geq 0$.”

“Since we know $(x - 2)(x + 1) \geq 0$, we conclude that $x^2 - x - 2 \geq 0$.”

(L2): Logic and deduction are valid.

“If $x = 2$, then $x \leq 3$. Therefore, if $x > 3$, then $x \neq 2$.”

“If $x = 2$, then $x \leq 3$. Therefore, if $x \neq 2$, then $x > 3$.”

Mathematical Correctness

(M1): Reasoning and computation is mathematically correct.

“Since $x - 2 = 0$, we know that $x = 2$.”

“Since $x - 2 = 0$, we know that $x = -2$.”

(M2): Mathematical definitions and results from class are used correctly.

“An integer n is even if there is an integer k so that $n = 2k$.”

“An integer n is even if there is an integer k such that $2n = k$.”

(N): Notation is used properly.

“Since x is an element of both A and B , we write $x \in A \cap B$ ”

“Since x is an element of both A and B , we write $x \subset A \cap B$ ”

Completeness

(DV): All new notation and variables are clearly introduced and defined.

“Let ℓ and h denote the length and height of a rectangle, respectively. Then the rectangle’s area is given by ℓh .”

“We know area is ℓh .”

(J): ‘Non-obvious’ intermediate claims and implications are fully justified and explained.

Note: If you’re not sure if something is ‘obvious’ or not, err on the side of explanation!

“Assume $ab > 0$ and $bc < 0$. Since a and b have the same sign and b and c have opposite sign, we know a and c have opposite sign, so $ac < 0$.”

“Assume $ab > 0$ and $bc < 0$. Then $ac < 0$.”

(CS): All cases are addressed.

“Since $x + y$ is even, either (i) both x and y are even, or (ii) both x and y are odd.”

“Since $x + y$ is even, both x and y are even.”

MA241-009 Recap “Quiz” 1: §5.7-5.9

Instructions:

- Please take out a blank piece of paper and write your full name.
- Answer the following 5 questions, showing all work.
- You may use your notes and talk to one another, but please try each problem on your own first.
- This “quiz” will be graded and counted for extra credit.

(1) Evaluate $\int_1^2 \frac{x^2 - x + 6}{x^3 + 3x} dx$ using partial fraction expansion.

(2) Use trigonometric substitution to evaluate $\int \frac{x^3}{\sqrt{x^2 + 1}} dx$.

(3) Find $\int x\sqrt{x^2 + 2x + 4} dx$. Hint: complete the square! You may find the identity below helpful.

$$\mathbf{21} : \int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

(4) Use the Trapezoidal Rule and Simpson’s Rule with $n = 4$ to approximate $\int_1^5 \frac{1}{x^2} dx$, then compute associated errors E_T and E_S .

(5) Tell me a fun fact about yourself!

Wed 1/14 Recap "Quiz" Answer Key

MA 241-009 18

① $\int_1^2 \frac{x^2 - x + 6}{x^3 + 3x} dx$ Evaluate using partial fraction expansion.

$$\frac{x^2 - x + 6}{x^3 + 3x} = \frac{x^2 - x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3} = \frac{2}{x} + \frac{\overbrace{-x+1}^{-(x+1)}}{x^2 + 3}$$

$$\Rightarrow x^2 - x + 6 = A(x^2 + 3) + (Bx + C)x = (A + B)x^2 + Cx + 3A$$

$$\Rightarrow \left. \begin{array}{l} 1 = A + B \\ -1 = C \\ 6 = 3A \end{array} \right\} \Rightarrow \underline{2 = A} \Rightarrow B = 1 - 2 \Rightarrow \underline{B = -1}$$

$$\int_1^2 \frac{x^2 - x + 6}{x^3 + 3x} dx = \int_1^2 \left[\frac{2}{x} - \frac{x+1}{x^2+3} \right] dx$$

$$= 2 \int_1^2 \frac{1}{x} dx - \underbrace{\int_1^2 \frac{x}{x^2+3} dx}_{\text{use u-sub, } u=x^2+3, du=2x dx} - \underbrace{\int_1^2 \frac{1}{x^2+3} dx}_{\text{use identity } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C}$$

$$= 2 \ln|x| \Big|_1^2 - \frac{1}{2} \ln|x^2+3| \Big|_1^2 - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) \Big|_1^2$$

$$= 2(\ln 2 - \ln 1) - \frac{1}{2}(\ln 7 - \ln 4) - \frac{1}{\sqrt{3}} \left(\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \right)$$

$$= \boxed{2 \ln 2 - \frac{1}{2} \ln \frac{7}{4} - \frac{1}{\sqrt{3}} \left[\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \right]} \approx \underline{0.91396}$$

Having trouble?
Review notes from 1/9
§5.7 and Appendix G
"Partial Fractions"

② Evaluate $\int \frac{x^3}{\sqrt{x^2+1}} dx$ using trig substitution.

$$x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$$

$$\sqrt{x^2+1} = \sqrt{\tan^2 \theta + 1} = \sqrt{\sec^2 \theta} = \sec \theta$$

$$\int \frac{x^3}{\sqrt{x^2+1}} dx = \int \frac{\tan^3 \theta}{\sec \theta} \sec^2 \theta d\theta = \int \tan^3 \theta \sec \theta d\theta$$

$$= \int \tan^2 \theta \cdot \tan \theta \sec \theta d\theta$$

$$= \int (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta$$

$$\text{Use u-sub with } u = \sec \theta, du = \sec \theta \tan \theta d\theta$$

$$= \int (u^2 - 1) du$$

$$= \frac{1}{3} u^3 - u + C$$

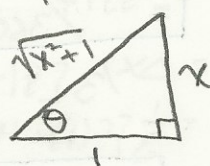
$$= \frac{1}{3} \sec^3 \theta - \sec \theta + C$$

$$= \frac{1}{3} (\sqrt{x^2+1})^3 - \sqrt{x^2+1} + C$$

$$= \frac{1}{3} (x^2+1)^{3/2} - \sqrt{x^2+1} + C$$

$$= \frac{1}{3} (x^2-2) \sqrt{x^2+1} + C$$

$$\frac{x}{1} = \tan \theta$$



$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+1}}{1}$$

Having trouble?
Review notes from
1/8: §5.7 "trig sub"
1/7: §5.7 "trig integrals"

③ Find $\int x\sqrt{x^2+2x+4} dx$ given formula
 $\int \sqrt{a^2+u^2} du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + C$

Complete the square:

$$x^2+2x+4 = x^2+2x+\frac{1^2}{1}+4-\frac{1^2}{1} = (x+1)^2+3$$

To find $\frac{a^2}{2}$, set $C = \frac{2}{2} = 1$

$$\int x\sqrt{x^2+2x+4} dx = \int x\sqrt{(x+1)^2+3} dx$$

Use u-sub with $u = (x+1) \Rightarrow x = u-1$
 $dx = du$
 $= \int (u-1)\sqrt{u^2+3} du$

$$= \underbrace{\int u\sqrt{u^2+3} du}_{\text{use "v"-sub with } v=u^2+3, dv=2u du} - \underbrace{\int \sqrt{u^2+3} du}_{\text{use given identity for } \int \sqrt{a^2+u^2} du}$$

$$= \frac{1}{2} \int \sqrt{v} dv - \int \sqrt{u^2+3} du$$

$$= \frac{1}{2} \left(\frac{2}{3} v^{3/2} \right) - \frac{u}{2} \sqrt{u^2+3} - \frac{3}{2} \ln(u+\sqrt{u^2+3}) + C$$

$$= \frac{1}{3} (u^2+3)^{3/2} - \frac{u}{2} \sqrt{u^2+3} - \frac{3}{2} \ln(u+\sqrt{u^2+3}) + C$$

$$= \frac{1}{3} (x^2+2x+4)^{3/2} - \frac{x+1}{2} \sqrt{x^2+2x+4} - \frac{3}{2} \ln(x+1+\sqrt{x^2+2x+4}) + C$$

Having trouble?

Review notes from 1/12
 §5.8 "Table of Integrals"

Trouble with #4?

Review notes from 1/13
 §5.9 "Approximate Integration"

④ Use the Trapezoidal Rule and Simpson's Rule to approximate $\int_1^5 \frac{1}{x^2} dx$ with $n=4$, then compute associated errors E_T and E_S .

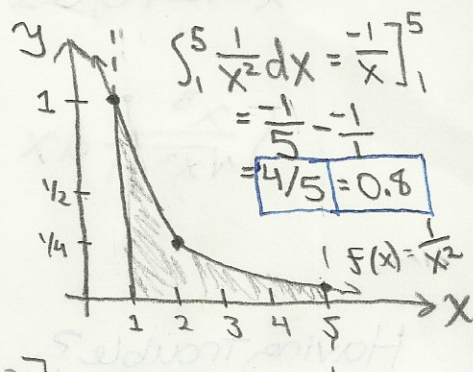
First, compute Δx and interval endpoints x_i

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = \frac{4}{4} = 1, \quad x_i = 0 +$$

$$x_i = a + i\Delta x \Rightarrow x_0 = 1 + 0 \cdot 1 = 1$$

$$x_1 = 1 + 1 \cdot 1 = 2$$

$$x_4 = 1 + 4 \cdot 1 = 5$$



$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$T_4 = \frac{1}{2} [f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)] = \frac{1}{2} \left[\frac{1}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{1}{5^2} \right]$$

$$= \frac{3397}{3600} \approx 0.94361, \quad E_T = 0.8 - 0.94361 = -0.14361$$

$$S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)]$$

$$S_4 = \frac{1}{3} [f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)] = \frac{1}{3} \left[\frac{1}{1^2} + 4\left(\frac{1}{2^2}\right) + 2\left(\frac{1}{3^2}\right) + 4\left(\frac{1}{4^2}\right) + \frac{1}{5^2} \right]$$

$$= \frac{2261}{2700} \approx 0.83741, \quad E_S = 0.8 - 0.83741 = -0.03741$$

Modeling the population of Raleigh

Solutions: https://docs.google.com/spreadsheets/d/1F1CEaFaFu6m0LclyTT2Ua6nrDIExeBNnvaM2Qjfu_l/edit?usp=sharing

According to the U.S. Census Bureau (by way of www.raleighnc.gov), the population of the city of Raleigh between 1920 and 2010 is given below. We'll use this data to model population growth and make predictions about the future population using Google Sheets.

Year	Population
1920	24,418
1930	37,379
1940	46,879
1950	65,679
1960	93,931
1970	122,830

1980	150,255
1990	212,092
2000	276,093
2010	406,432

PART I: ORGANIZE AND EXPLORE THE DATA

Step 1: Enter the data into Google Sheets. Copy (PC: ctrl-c, Mac: cmd-c) the two columns of data (including titles) given above,. Then, navigate to drive.google.com, click NEW and select Google Sheets. This will open a blank spreadsheet. Paste (ctrl-v, cmd-v) the data into columns A and B of the sheet and name it "MA 231 Exponential Growth" by typing in the Untitled Spreadsheet box at top. (Alternatively, you can manually enter the data by hand).

Step 2: Adjust the year data to begin at 0: call this "t". Select column B by clicking on the B. From the Insert menu, insert a column to the LEFT of column B. This column will list our years, shifted so that $t=0$ corresponds to the year 1920. Type "t" in the first cell of this new column (B1). In the second cell (B2), type "=", then click on the cell containing 1920, then type "-1920." Before you hit enter, the cell should read "= A2 - 1920". After you hit enter, the cell should have a "0" in it. (If it shows a "-" instead, go to the Format menu, select Number, then select Automatic.) The spreadsheet has performed the requested calculation. (Note: you can also type "A2", rather than clicking on it, but sometimes selecting which cell you want to use is easier.) Now we want the spreadsheet to calculate the given year minus 1920 for the remaining rows of data. There are two options for doing this automatically:

(a) Copy (ctrl-c/cmd-c) the cell with the formula, highlight the remaining cells in the column that you want to fill, then paste (ctrl-v/cmd-v). The program will automatically adjust the formula

(b) With the formula cell selected (it will appear with a blue box around it) click and drag the small box at the lower right corner of the cell to highlight the rest of the cells in the column you want to fill. Again, the formula will populate these selected cells automatically.

**Exponential growth activity #1**[Show Correct Answer](#)[Show Responses](#)

What number is displayed in cell B4?

Step 3: Plot the data. Now highlight all entries (including the titles) in the columns titled "t" and "Population" (Columns B and C). With those highlighted, open the Insert menu and select Chart: we want a scatter chart (or scatterplot). If this is not one of the recommendations, click over to the Chart types tab and scroll down until you see scatter chart. With this option selected, click Insert. You can then move and resize the chart however you like on your spreadsheet. You should have time (t) along the x-axis, and population along the y-axis.

Step 4: Fit a trendline to the data. Select the chart and right-click or ctrl-click to pull up the chart editing menu. Select Advanced edit and in the Customization tab scroll down to the Trendline category at the bottom. Try the different options and choose whichever one you think fits the data points best.

**Exponential growth activity #2**[Show Correct Answer](#)[Show Responses](#)

What type of function best fits these data points?

A	Linear
B	Exponential
C	Logarithmic

Our goal is to find the equation of the exponential curve which best models the data. We'll do this by first taking the natural log of our population data, then using least-squares regression to find the line of best fit, and finally "exponentiating" that line to find the exponential curve of best fit.

Step 5: Take the natural log of the population data. Currently, your spreadsheet has 3 columns of data: "Year", "t", and "Population". Now we'll add a fourth column. Type "ln(Population)" in the first cell of the this new column (D1), and in the next 10 cells compute the natural logs of the corresponding population sizes. For example, type "=ln(C2)" in cell D2, then auto-populate the rest of the column with the formula using either of the methods from Step 2.



Exponential growth activity #3

[Show Correct Answer](#)[Show Responses](#)

What does it read in cell D4? (Round to the nearest hundredth.)

Step 6: Plot the "log-scale" data. Highlight the "t" column and the new "ln(Population)" column and make a new chart as in Step 3. Again you'll have time along the x-axis, but now you'll have log-scale population along the y-axis. (Hint 1: you can select an entire column by clicking on the column header, say "B." Hint 2: to highlight two columns that aren't next to one another, first select one column, then hold down the ctrl/cmd key while selecting the second column.)

Step 7: Fit a trendline to the log-scale data. As in Step 4, find the best trendline for your new chart. You should find that the best option is a linear relationship between "t" and "ln(Population)."

Step 8: Find the line $y(t)$ of best-fit to the log-scale data. Google sheets will use least squares regression to find the equation $y = At + B$ of the line of best fit. (Recall "t" is our independent variable). Select an empty cell anywhere on your spreadsheet and type "=SLOPE(D2:D11, B2:B11)". (You may need to adjust the cell references: the first set should be the "ln(Population)" data and the second set should be the "t" data.) This will tell you the slope A of the line of best fit. To find the y-intercept B, use the same process as above but change "SLOPE"

to "INTERCEPT." (Note: Google is computing these quantities with the formulas we learned in Unit 1 that minimize the sum-squared-error!) 24



Exponential growth activity #4

[Show Correct Answer](#)[Show Responses](#)

What is your slope, A? Round to the nearest thousandth.



Exponential growth activity #5

[Show Correct Answer](#)[Show Responses](#)

What is your y-intercept, B? Round to the nearest thousandth.

Step 9: Exponentiate to find the curve $P(t)$ of best fit to the original data. Since we took the natural log of the population data, our line $y(t)$ represents

$$y(t) = \ln(P(t))$$

where $P(t)$ is the population of Raleigh in year t , with $t = 0$ corresponding to 1920. Exponentiating both sides of this equation yields

$$e^{y(t)} = e^{\ln(P(t))} = P(t)$$

So we have the following equation for the curve of best fit, $P(t)$, graphed in Step 4:

$$P(t) = e^{y(t)} = e^{At + B} = e^{At} e^B = (e^B) e^{At}$$

We know the population of Raleigh looked like it was growing exponentially. This means our function $P(t)$ should be an exponential growth function:

$$P(t) = P_0 e^{kt}$$

Comparing this standard exponential growth function to the equation for our curve of best fit:



Exponential growth activity #6

[Show Correct Answer](#)[Show Responses](#)

25

P_0 , the population in 1920 ($t=0$) given by the model, corresponds to

A

e^A

B

e^B

C

e^{At}

D

e^{A+B}



Exponential growth activity #7

[Show Correct Answer](#)[Show Responses](#)

k , the growth rate constant of the model, corresponds to

A

B

B $A+B$ **C**

e^B

D

A

E

e^A

Use the values for A and B you found in Step 8 to write down your model for the growth of Raleigh's population in $P(t) = P_0 e^{(kt)}$ form.

**Exponential growth activity #8**[Show Responses](#)**No correct answers:** No correct answer has been set for this question

Use the values for A and B you found in Step 8 to write down your model for the growth of Raleigh's population in the form $P(t) = P_0 e^{kt}$.

PART III: USE YOUR MODEL

**Exponential growth activity #9**[Show Correct Answer](#)[Show Responses](#)

According to your model, what was the population of Raleigh in 1925? Give your answer accurate to 5 decimal places.

You can, and should, use Google sheets for your computation in all of these questions! That way you can use the EXACT values of A and B by clicking on their cells rather than rounding when entering in your calculator. For example, say your value of B is stored in cell C15 and your value of A in cell 14. Then to find $y(5)$, find an empty cell and type `"=EXP(C15) * EXP(C14 * 5)"`.

**Exponential growth activity #10**[Show Correct Answer](#)[Show Responses](#)

According to your model, by what percentage does the population of the city increase every 5 years? Give your answer as a percentage, not a decimal, and round to the nearest hundredth.

**Exponential growth activity #11**[Show Correct Answer](#)[Show Responses](#)

According to your model, by what percentage does the population increase each year? Again, give your answer as a percentage, not a decimal, and round to the nearest hundredth.

Careful! I'm not asking for the growth constant k , I'm asking for a percentage increase in population. In one year, this will be very close to k , but will be slightly larger!



Exponential growth activity #12

[Show Correct Answer](#)[Show Responses](#)

What does your model predict as the current population of Raleigh (in 2017)?



Exponential growth activity #13

[Show Correct Answer](#)[Show Responses](#)

What does your model predict as the current growth rate (in 2017)? Round to the nearest person/year.

Note again, this is not asking for the growth constant k , it is asking for the growth rate. Perhaps a differential equation might be useful...



Exponential growth activity #14

[Show Correct Answer](#)[Show Responses](#)

According to your model, during what year will Raleigh's population reach 1 million?

NCSU MA 105-001, Spring 2014

Test #3

Friday, March 28

Please write your name and seat number on the front of the blue book. Start each problem on a new page, number questions, including subparts, box your final answers, and make sure to **SHOW ALL WORK** to receive full credit: **include intermediate steps and values**, list calculator inputs, and either **draw a time diagram OR explain your reasoning** in detail. You are welcome to either round intermediate answers to the **hundredths place** or carry forward the exact values in your calculator: both will be marked correct. No phones, notes, or other aids besides calculator(s) are allowed.

Good luck, and don't forget to show your work!

1. **(25 pts)** Kermit the Frog took out a \$400,000 loan at 11% compounded monthly for his new lily pad. If he makes end-of-month payments of \$12,000 towards the loan,
 - (a) How many full payments are required?
 - (b) What is the size of the smaller concluding payment at the end of the next month?
2. **(25 pts)** Aware of her expensive tastes, as a gift for Miss Piggy's 6th birthday her loving parents opened an account earning 5.3% compounded annually with an initial deposit. On her 18th birthday, she makes her first withdrawal of \$2000 from the account and continues to make withdrawals of the same amount on every birthday thereafter until the account is exhausted. If the withdrawal on her 29th birthday empties the account, what was the amount of her parents' initial deposit?
3. **(25 pts)** In anticipation of the day when the Muppet Show would one day go off air, Fozzie Bear made end-of-quarter deposits to a retirement fund earning 6.2% compounded quarterly. He made his first deposit on May 21, 1978 and his last deposit on November 21, 1985. He then let the money sit in the account earning interest until the return of The Muppets on February 21, 2012.
 - (a) If Fozzie has \$850,000 on February 21, 2012, what size were his quarterly deposits?
 - (b) What is the total amount of interest earned on the retirement account?
4. **(25 pts)** Animal has decided its time to be responsible, so he opens a savings account earning 7% compounded monthly and dutifully deposits \$50 semiannually with his first deposit made immediately. If he continues to make these regular deposits, how much will be in the account exactly 10 years after the day he opened it?

MATH 225

Final Exam

Spring 2016

Please answer each of the following questions on the blank paper provided, making sure to number problems and write your name on each page. Read and follow instructions carefully. Other than #3, you may use any proof methods you would like. You may use extra paper for scratch work and answer problems in any order you wish, but *please turn in problems in numerical order*.

Good Luck and have a great summer!

1. (2 points each) Determine whether each statement is true or false and justify your answers.

- (a) If $A \subseteq B \cup C$ and $B \cap C = \emptyset$ then $A \subseteq B$ or $A \subseteq C$.
- (b) If $f = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$, then f is a function.
- (c) If $A \subseteq B$ and $A \approx B$, then $A = B$.
- (d) $\exists M \in \mathbb{N}$ such that $\forall k \in \mathbb{N}$ with $k > M$, $\frac{1}{k+1} < 0.1$.
- (e) $\forall x \in \mathbb{N}$, $\exists y \in \mathbb{N}$ such that $y < x$.
- (f) $\exists! x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $xy = x$.

2. Let x be a real number and consider the following claim:

If $x < 1$ or $x > 6$, then $x^2 - 8x + 12 > 0$.

- (a) (6 points) Prove the claim.
 - (b) (6 points) Give each of the following and state whether the result is true or false:
 - i. the converse of the claim.
 - ii. the contrapositive of the claim.
 - iii. the negation of the claim.
3. (6 points) Use contraposition to prove that for any $x \in \mathbb{R}$, $|x - 1| > 4$, then $|x| > 3$.
(Hint: Recall that $|a| \leq |b| \iff -b \leq a \leq b$)
4. (3 points) Let $A = \{1, 2, 3, 4, 7\}$, $B = \{5, 6, 7\}$, $C = \{4, 5, 6\}$ be sets in the same universe $U = \{x \in \mathbb{Z} : 0 < x \leq 8\}$. Find $B \cap (A^C \cup (B - C))$.
5. (6 points) Let A , B , and C be sets. Prove that if $B \subseteq C$, then $A \times (B \cup C) \subseteq (A \times C)$.
6. (2 points each) Let A be the set of all rectangles in the plane. Define a relation R on A by xRy if and only if rectangles x and y have the same area or the same perimeter.
- (a) Is S reflexive? Justify your answer.
 - (b) Is S symmetric? Justify your answer.
 - (c) Is S transitive? Justify your answer.

7. Define a relation R on \mathbb{Z} as $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x - y \text{ is divisible by } 4\}$.
- (6 points) Prove R is an equivalence relation.
 - (2 points) Find three elements of \mathbb{Z}/R .
8. Let $A = \{\alpha, \beta, \gamma\}$ (where all 3 elements in A are distinct). Define a relation h from $\mathcal{P}(A)$ to \mathbb{Z} via $(B, n) \in h$ if and only if subset B has n elements in it (i.e., $|B| = n$).
- (2 points) Find $\mathcal{P}(A)$.
 - (2 points) Describe h using your favorite notation (i.e., table, arrow diagram, or set).
 - (2 points) Is h a function? Explain.
 - (4 points) Is h 1-1? Prove your claim.
 - (4 points) Is h onto? Prove your claim.
9. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} 1 + 2x & \text{if } x < 0 \\ 1 + x^2 & \text{if } x \geq 0 \end{cases}$.
- (8 points) Prove f is 1-1 and onto \mathbb{R} .
 - (5 points) Find $g \circ f$ where $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = \begin{cases} x - 1 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$.
10. (8 points) If $A = \{a, b\}$ (where $a \neq b$), prove that $A \times \mathbb{N}$ is denumerable. (You must both give a function explicitly and prove that it works. *Hint*: think about evens and odds.)
11. (6 points) Prove that for any natural number n , $\sum_{i=1}^n i(i+1) = \frac{n(n+1)(n+2)}{3}$.
12. For each natural number n , define $A_n = [\frac{1}{2n}, 2 + \frac{1}{n})$.
- (2 points) Find A_1 , A_2 , and A_3 and draw them on a number line. (You can draw them all on the same number line but make sure you label them clearly.)
 - (2 points) Find $\bigcup_{n=1}^{\infty} A_n$.
 - (2 points) Find $\bigcap_{n=1}^{\infty} A_n$.
13. (3 points extra credit) Prove one of the true claims from Question #1.
14. (5 points extra credit) We showed in class, using a *combinatorial* proof, that for all $n \in \mathbb{N}$, the cardinality of the power set of a finite set $X = \{x_1, x_2, \dots, x_n\}$ of size n is given by $|\mathcal{P}(X)| = 2^n$. Prove this claim is true using a proof by induction.