Lecture #8 **Chapter 8: Hypothesis Testing**

This chapter introduces another major topic of inferential statistics: testing claims (or hypothesis) made about population parameters.

8-2 Basics of hypothesis testing

In this section, 1^{st} we introduce the language of hypothesis testing, then we discuss the formal process of testing a hypothesis.

A **hypothesis** is a statement or claim regarding a characteristic of one or more population

Hypothesis testing (or test of significance) is a procedure, based on a sample evidence and probability, used to test claims regarding a characteristic of one or more populations. To test a hypothesis, you should carefully state a pair of hypothesis—on that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true.

The **null hypothesis** (denoted by H_0) is a hypothesis that contains a statement of equality, =.

The alternative hypothesis (denoted by H_1or H_a) is the statement that contains a statement of inequality, such as >, <, $\neq$.

If the claim value is k and the population parameter is p, then some possible pairs of null and alternative hypothesis are

H_0: $p = k$ H_0: $p = k$ H_0: $p = k$

H_1: $p > k$ H_1: $p < k$ H_1: $p \neq k$

Identifying the null and alternative hypotheses:

Example 1: Write the claim as a mathematical sentence. State the null and alternative hypotheses, and identify which represents the claim.

a) A water faucet manufacturer claims that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

b) A cereal company claims that the mean weight of the contents of its 20-ounce size cereal boxes is more than 20 ounces.
c) The standard deviation of IQ scores of actors is equal to 15.

Test Statistic

The **test statistic** is a value computed from the sample data that is used in making the decision about the rejection of the null hypothesis.

One way to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the rejection region of the sampling distribution.

The **critical region** (or **rejection region**) is the set of all values of the test statistic that cause us to reject the null hypothesis.

A **critical value** separates the rejection region from the non-rejection region.

The **significance level** (denoted by α) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true.

I) **Left tailed test**: if the alternative hypothesis H_1 contains the less-than inequality symbol (<), the hypothesis test is a left-tailed test.
II) **Right-tailed test:** If the alternative hypothesis H_1 contains the greater-than inequality symbol (>), the hypothesis test is a right-tailed test.
III) **Two-tailed test:** If the alternative hypothesis H_1contains the not-equal-to symbol ($\neq$), the hypothesis test is a two-tailed test. In a two-tailed test, each tail has an area of $\frac{1}{2}\alpha$.

Example 2: Find the critical z values. In each case, assume that the normal distribution applies.

a) Left-tailed test with $\alpha = 0.01$
b) Two-tailed test with $\alpha = 0.05$
c) Right-tailed test with $\alpha = 0.04$

To conclude a hypothesis test, you make a decision and interpret that decision. There are only two possible outcomes to a hypothesis test: (1) reject the null hypothesis, and (2) fail to reject the null hypothesis.

Decision Rule Based on Rejection (Critical) Region:

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic (z or t). If the standardized test statistic

1. Is in the rejection region, then reject H_0.
2. Is not in the rejection region, then fail to reject H_0.

If we fail to reject the null hypothesis, it does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis.

A **type I error** occurs if the null hypothesis is rejected when it is actually true. α = probability of type I error (the probability of rejection the null hypothesis when it is true)

A **type II error** occurs if the null hypothesis is not rejected when it is actually false. β = probability of type II error

	Actual Truth of H_0	
Decision	H_0 is true	H_0 is false
Fail to reject H_0	Correct decision	Type II error
Reject H_0	Type I error	correct decision

Example 3: Provide statements explaining what it would mean to make a type I error and type II error if a researcher for the FDA whishes to test the claim that

the percentage of children taking the new antibiotic who experience headaches as a side effect is more than 2%.

The choice of the level of significance depends on the consequences of making type I error. If the consequences are severe, the level of significance should be small.

Example 4: In example 3, a) suppose the sample evidence indicates that the null hypothesis is rejected. State the conclusion. B) Suppose the sample evidence indicated that the null hypothesis is not rejected. State the conclusion.

8-3 Testing a claim about a proportion

In this section, we will learn how to test a population proportion. The following are examples of the types of claims we will be able to test.

Less than ¼ of all college graduates smoke.

The percentage of late-night television viewers who watch The Late Show with David Letterman is equal to 18%.

If a fatal car crash occurs, there is a 0.44 probability that it involves a driver who had been drinking.

Notation:

n= number of trails

$\hat{p} = \frac{x}{n}$ (sample proportion) = sample proportion of x successes in a sample of size n

p =population proportion (used in the null hypothesis)

q = 1 – p

If n p≥ 5 and n q$\geq 5$for a binomial distribution, then the sampling distribution for p is normal

With $\mu = np \ and \ \sigma = \sqrt{npq}$

Test statistic for testing a claim about a **proportion** $z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$

Guidelines: The traditional method for a proportion p $(np \geq 5 \ and \ nq \geq 5)$

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   (State H_0 and H_1).
2. Specify the level of significance. α
3. Sketch the sampling distribution.
4. Determine any critical values. (Use table A-2)
5. Determine any critical regions.
6. Find the test statistic. $z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$
7. Make a decision to reject or fail to reject the hypothesis.
   (If z is in the critical region, reject H_0. Otherwise, fail to reject H_0.)
8. Interpret the decision in the context o the original claim.

Example 5: A medical researcher claims that less than 20% of American adults are allergic to a medication. In a random sample of 100 adults, 15% say they have such an allergy. Test the researcher's claim at α = 0.01

Example 6: Harper's Index claims that 23% of Americans are in favor of outlawing cigarettes. You decide to test this claim and ask a random sample of 200 Americans whether they are in favor of outlawing cigarettes. Of the 200 Americans, 27% are in favor. At $\alpha = 0.05$, is there enough evidence to reject the claim?

The P-value method of testing hypotheses:

A **P-value** (or probability value) is the probability of getting a value of the sample test statistic that is at least as extreme as the one found from the sample data, assuming that null hypothesis is true. The H_0 is rejected if P-value is very small, such as 0.05 or less.

(the P-value is the likelihood that a sample, such as the one obtained, will occur when the null hypothesis is actually true.)

Left-tailed test: P is the area to the left of z (test statistic)

Right-tailed test: P is the area to the right of z (test statistic)

Two-tailed test: P is twice the area to the left of a negative z (test statistic)or P is twice the area to the right of positive z(test statistic).

Decision Rule Based on P-value: To use a P-value to make conclusion in a hypothesis test, compare the P-value to α

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Example 6: The p-value for a hypothesis test is P-value = 0.0237. What is your decision if the level of significance is a) $\alpha = 0.05$ and b) $\alpha = 0.01$?

Guidelines: P-value method for proportion p: ($np \geq 5$ *and* $nq \geq 5$)

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   (State H_0 and H_1).
2. Specify the level of significance. α
3. Sketch the sampling distribution.
4. Find the test statistic. $z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$
5. Find the area that corresponds to z. (the test statistic)
6. Find the P-value.
   Left-tailed test: P is the area to the left of z (test statistic)
   Right-tailed test: P is the area to the right of z (test statistic)

Two-tailed test: P is twice the area to the left of a negative z (test statistic)or P is twice the area to the right of positive z(test statistic).

7. Make a decision to reject or fail to reject the hypothesis.
   If P$\leq \alpha$, then reject H_0.
   If P> α, then fail to reject H_0.

8. Interpret the decision in the context o the original claim.

Example 7: Technology is dramatically changing the way we communicate. In 1997, a survey of 880 U>S. households showed that 149 of them use e-mail. Use those sample results to test the claim that more than 15% of U.S. households use e-mail. Use a 0.05 significance level.

Example 8: In a study of store checkout-scanners, 1234 items were checked and 20 of them were found to be overcharges. Use a 0.05 significance level to test the claim that with scanners, 1% of sales are overcharges.

8-4 Testing a claim about a mean: (standard deviation of the population is known)

Guidelines: The traditional method for a population mean

(standard deviation of the population is known and The population is normally distributed or n > 30)

1. State the claim mathematically and verbally. (State H_0 and H_1).
   Identify the null and alternative hypotheses.
2. Specify the level of significance. α
3. Sketch the sampling distribution.
4. Determine the critical value(s). (Use table A-2)
5. Determine any critical regions.
6. Find the test statistic. $z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$
7. Make a decision to reject or fail to reject the hypothesis.

(If z is in the critical region, reject H_0. Otherwise, fail to reject H_0.)

8. Interpret the decision in the context o the original claim.

Example 9: Claim: the mean IQ score of statistics professors is greater than 118. Sample data: n = 50

Sample mean = 120. Assume that the standard deviation of the population is 12 and the significance level is 0.05.

Example 10: Claim: The mean time between uses of a TV remote control by males during commercials equals 5.00 sec. Sample data: n = 80, sample mean = 5.25 sec. Assume the standard deviation of the population is 2.50 sec and the significance level is 0.01.

Guideline: P-value method for a population mean (standard deviation of the population is known)

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   (State H_0 and H_1).
2. Specify the level of significance. α
3. Sketch the sampling distribution.
4. Find the test statistic. . $z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$
5. Find the area that corresponds to z.
6. Find the P-value.
   Left-tailed test: P is the area to the left of z (test statistic)
   Right-tailed test: P is the area to the right of z (test statistic)
   Two-tailed test: P is twice the area to the left of a negative z (test statistic)or P is twice the area to the right of positive z(test statistic).

7. Make a decision to reject or fail to reject the hypothesis.
   If P$\leq \alpha$, then reject H_0.
   If P> α, then fail to reject H_0.

8. Interpret the decision in the context o the original claim.

Example 11: In order to monitor the ecological health of the Florida Everglades, various measurements are recorded at different times. The bottom temperatures are recorded at the Garfield Bight station and the mean of 30.4^0C is obtained for 61 temperatures recorded on 61 different days. Assuming that the standard deviation of the population is 1.7^0C, test the claim that the population mean is greater than 30^0C. Use a 0.05 significance level.

Example 12: When people smoke, the nicotine they absorb is converted to cotinine, which can be measured. A sample of 40 smokers has a mean cotinine level of 172.5. Assuming that the standard deviation of the population is known to be 119.5, use a 0.01 significance level to test the claim that the mean cotinine level of all smokers is equal to 200.0.

8-5 Testing a claim about a mean (standard deviation of the population is not known)

The main objective of this section is to develop the ability to test claims made about population means when the population standard deviation is not known. The methods of this section are much more practical and realistic because they assume that the standard deviation of the population is not known, as is usually the case.

Test statistic for testing a claim about a mean (standard deviation of the population is not known and the population is normally distributed or n > 30.) $t = \frac{\bar{x}-\mu_{\bar{x}}}{\left(\frac{s}{\sqrt{n}}\right)}$

Guidelines: Finding critical values in a t-distribution:

1. Identify the level of significance.
2. Identify the degrees of freedom, d.f. = n -1
3. Find the critical value using Table A-3 in the row with n-1 degrees of freedom. If the hypothesis test is
   a. Left-tailed, use "α one tail" column with a negative sign.

b. Right-tailed, use "α one tail" column with a positive sign.

c. Two-tailed, Use "α two tails" column with a negative and a positive sign.

Example 13:

a) Find the critical t value for a left-tailed test with α = 0.05 and n =21.

b) Find the critical t value for a right-tailed test with α=0.01 and n = 17.

c) Find the critical t values for a two-tailed test with α=0.05 and n =26.

Guidelines: The traditional method for a population mean

(standard deviation of the population is not known and the population is normally distributed or n > 30)

1. State H_0 and H_1.
2. Identify the α.
3. Identify the d.f. = n -1
4. Determine any critical values. Use table A-3
5. Determine any critical regions.
6. Find the test statistic. . $t = \frac{\bar{x}-\mu_{\bar{x}}}{\left(\frac{s}{\sqrt{n}}\right)}$
7. Make a decision to reject or fail to reject the null hypothesis. If t is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.

Example 14: Forty subjects followed the weight watchers diet for a year. Their weight changes are summarized by these statistics: sample mean = -6.6 Ib, standard deviation of the sample = 10.8 Ib. Use a 0.01 significance level to test the claim that the diet has no effect. Based on the results, does the diet appear to be effective?

Example 15: A used car dealer says the mean price of a 2000 Ford F-150 Super Cab is more than \$16,500. You suspect this claim is incorrect and find that a random sample of 14 similar vehicles has a mean price of \$15,700 and a standard deviation of \$1250. Is there enough evidence to reject the dealer's claim at 0.05 significance level? Assume the population is normally distributed.

Example 16: An Industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 19 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.24, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$? Assume the population is normally distributed.

Example 17: Try it yourself: #23 on section 8-5.