

Rational Numbers 9: Decimal Form of Rational Numbers

Objectives

- To use base ten blocks to model operations on decimal numbers
- To review the algorithms for addition, subtraction, multiplication and division of decimal numbers
- To understand why the algorithms for operations on decimal numbers work

Activity 1: Using base ten blocks to model operations on decimals

Base ten blocks consist of: single units, longs (10 units), flats (100 units) and cubes (1000 units). There are often several different ways to model a given decimal number, depending on which type of block is selected to represent 1. For example, we could model the decimal number 1.34 in either of the following ways:

- If we choose the large cube to represent 1 then:

a flat represents $\frac{1}{10}$ of 1 or 0.1
a long represents $\frac{1}{100}$ of 1 or 0.01 and
a single unit represents $\frac{1}{1000}$ of 1 or 0.001.

Using this scheme, the number 1.34 is represented by 1 large cube, 3 flats and 4 longs.

- If instead we choose a flat to represent 1 then:

a long represents $\frac{1}{10}$ of 1 or 0.1 and
a single cube represents $\frac{1}{100}$ of 1 or 0.01 .

Using this scheme, 1. 34 would be modeled by 1 flat, 3 longs and 4 single cubes.

Model each of the following calculations with base ten blocks. Draw a sketch of your model and *be sure to specify the unit you chose to represent one* . Several groups will be chosen to present their models to the class.

1. $2.346 + 1.27$

2. $837/1000 + 2424/1000$

3. $3.3 - 2.875$

4. $1437/1000 - 582/1000$

Activity 2: Modeling products and quotients of decimal numbers

Model each of the following calculations with base ten blocks. Draw a sketch of your model and *be sure to specify the unit you chose to represent one*. Several groups will be chosen to present their models to the class.

1. 2×2.35

2. $4.84 \div 4$

3. $9.6 \div 3$ (hint: choose a long to represent 1)

4. $0.2 \times \frac{1}{3}$ (hint: choose 3 longs to represent 1)

5. $0.6 \div 0.2$

6. $0.2 \div 0.6$

7. 1.25×0.4

Activity 3: Placing the decimal point when operating on decimals

Recall that when we do operations on decimals, we use the familiar algorithms that are used for integers. The only difference arises when dealing with the decimal points. The following questions require you to summarize the decimal point rules and explain where they come from. Several groups will be selected to present their responses to the class at the next meeting.

1. What are the rules regarding decimal points when adding or subtracting decimal numbers? Make up several examples to illustrate them. Explain why these rules are valid.

2. What is the rule for placing the decimal point when multiplying decimal numbers? Make up several examples to illustrate it. Explain why this rule is valid. (Hint: you might want to rewrite the numbers in improper fraction form in your explanation. It can also be explained if you write the numbers in scientific notation.)

3. How do you deal with the decimal points when dividing a number by a decimal number? Give several examples to illustrate it. Explain why this rule is valid. (Hint: what are we really doing when we move a decimal point to the right?)

Rational Numbers 10: Conversions Between Fractions and Decimals

Objectives

- To learn to rewrite fractions in decimal form
- To learn to rewrite terminating decimals in fraction form
- To learn to rewrite repeating decimals in fraction form

Activity 1: Converting fractions to decimal form

1. Convert each of the following fractions to decimal form. DO NOT USE CALCULATORS!!!
 - a. $\frac{1}{2}$
 - b. $\frac{1}{3}$
 - c. $\frac{1}{4}$
 - d. $\frac{1}{5}$
 - e. $\frac{1}{6}$
 - f. $\frac{1}{7}$
 - g. $\frac{1}{8}$
 - h. $\frac{1}{9}$
 - i. $\frac{1}{110}$
 - j. $\frac{1}{11}$
 - l. $\frac{1}{15}$
 - m. $\frac{1}{16}$
2. Classify the decimals above into two groups.

3. Suppose that you are given a fraction $1/n$. Is it possible to tell if its decimal representation will terminate? If so, how can you tell? (Test your hypothesis on several additional examples)

4. Suppose that the fraction $1/n$ is represented by a repeating decimal. What, if anything, can you say about the fractions $2/n$, $3/n$, $4/n$, etc?

5. Suppose that you are given a fraction m/n , and you begin to divide in order to convert it to decimal form. Is it possible that your division will go on forever without terminating or repeating? Why or why not?

Activity 2: Converting terminating decimals to fraction form

1. Change each of the following terminating decimals to fraction form. (Hint: think *place value*) Make sure that your answer is completely reduced.
 - a. 0.345
 - b. 4.12
 - c. - 1.2359

2. Give a general algorithm for converting any terminating decimal to fraction form.

Activity 3: Converting repeating decimals to fraction form

There are several methods for converting repeating decimals to fraction form. The one that is most accessible to students in the upper elementary grades is illustrated below:

Example: Convert $.0\overline{9}$ to fraction form .

Solution: Let $n = .090909\dots = .0\overline{9}$

Then $100n = 09.0909\dots = 09.\overline{09}$

We have two equations:

$$\begin{array}{r} 100n = 9.\overline{09} \\ n = .\overline{09} \end{array}$$

If we subtract the second from the first we get:

$$99n = 9 \quad \text{or} \quad n = 9/99 = 1/11 .$$

1.
 - a. Review the example above carefully. Why did we decide to multiply each side of the original decimal by 100?

 - b. What was the purpose of subtracting the two equations?

2. Convert each of the following to reduced fraction form:
 - a. $.3\overline{}$

2. Convert each of the following to reduced fraction form:

b. $4.\overline{452}$

c. $1.\overline{532}$ (Hint: you will need to multiply twice.)

3. Can you use the method above to convert the following decimals to fractions ?

a. $.121121112 \dots$

b. $3.5252252225 \dots$

4. Are the above numbers equivalent to any fractions? If not, are they real numbers? What category do they fall into? List five other numbers in the same category.