# 6.1 The Set of Rational Numbers

In the grade 3 *Focal Points*, students develop an understanding of the meaning and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line (p. 15).

In the rational number $\frac{a}{b}$, $a$ is the **numerator** and $b$ is the **denominator**.

Research indicates that the area model is more flexible and helps develop deeper understanding than the set model of fractions.

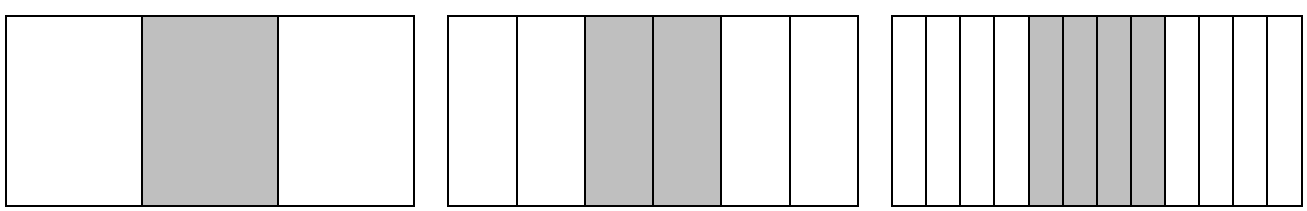
In the parts-to-whole model for the fraction $\frac{a}{b}$, consider 1) the whole being considered, 2) the number $b$ of equal-size parts into which the whole is divided, and 3) the number $a$ of parts of the whole that are being selected.

Every fraction has a place on the real number line.

A fraction $\frac{a}{b}$, where $0 \le a < b$, is a **proper fraction**. A fraction $\frac{a}{b}$, where $a \ge b > 0$ is an **improper fraction**.

**Equivalent fractions** represent the same rational number.

<u>Example</u>:



$\frac{1}{2}$ $\frac{2}{6}$ $\frac{4}{12}$

<u>Theorem 6-1</u>: Fundamental Law of Fractions
Let $\frac{a}{b}$ be any fraction and $n$ a nonzero integer. Then, $\frac{a}{b} = \frac{am}{bm}$.

A rational number $\frac{a}{b}$ is in **simplest form** if $b > 0$ and GCD($a$, $b$) = 1; that is, if $a$ and $b$ have no common factor greater than 1, and $b > 0$.

Example: Simplify $\frac{21}{30}$.

Example: Simplify $\frac{715}{1794}$.

Example: Simplify $\frac{x^2+5x+6}{x^2-9}$

Theorem 6-2: Equality of Fractions
Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal if, and only if, $ad = bc$.

Definition of Greater Than for Rational Numbers with Like Denominators
If $a$, $b$, and $c$ are integers and $b > 0$, then $\frac{a}{b} > \frac{c}{b}$ if, and only if, $a > c$.

Example: Order the fractions from least to greatest: $\frac{2}{3}, \frac{4}{7}, \frac{5}{9}$.

Theorem 6-3

If $a$, $b$, $c$, and $d$ are integers with $b > 0$ and $d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if, and only if, $ad > bc$.

Theorem 6-4: Denseness Property for Rational Numbers

Given two different rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, there is another rational number between these two rational numbers.

Example: $\frac{5}{11}$ and $\frac{6}{11}$

Theorem 6-5

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any rational numbers with positive denominators, where $\frac{a}{b} < \frac{c}{d}$. Then, $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.