Chapter-1 Rational Numbers

Learning Objective: Everybody should be able to apply the properties of rational numbers for performing various operations on rational numbers.

The chapter has been divided into four modules:

Module I: We will learn the properties of the rational numbers (Closure and Commutativity).

Module II: We will learn the properties of the rational numbers (Associativity and role of zero and 1).

Module III: We will learn Negative of number Reciprocal and Distributivity of multiplication over addition for rational numbers.

Module IV: Representation of Rational Numbers on the Numbers Line and Rational Numbers between two rational numbers.

1.1 Introduction

In Mathematics, we come across simple equation to be solved. E.g $x + 7 = 12$

The solution $x = 5$ is a **natural number**.

On the other hand for the equation $x + 3 = 3$ the solution $x = 0$ is a **whole number**.

Let us take some equations and try to find what solutions we are getting.

For example: The equation $x + 5 = 2$, the solution $x = -3$ is not a whole number but is an **integer** **(Integers are positive and negative).** Note that the positive integers correspond to natural numbers.

And the equations $7x + 5 = 8$ and $11x + 5 = 2$ have the solutions

$x = \frac{3}{7}$ $\quad and \quad x = \frac{-3}{11}$ are **rational numbers.**

In earlier classes you have learnt that:

- Counting numbers are known as Natural numbers. E.g : $1, 2, 3, \ldots$
- All natural numbers together with 0 form whole numbers. E. g: 0,1,2,3,…

- All natural numbers, 0 and the negatives of natural numbers form the collection of integers. E. g: ... $-4, -3, -2, -1, 0, 1, 2, 3$ ...
- The numbers in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are known as **Rational numbers**. E. g: $-3, -2$, 0,1 ,2 ,$\frac{1}{2}, \frac{3}{7}, \frac{-5}{11}$ ...
- Rational numbers include natural numbers, whole numbers and integers.

You have already done basic operations on rational numbers in class VII. Let us explore some properties of operations on different types of numbers seen so far.

1.2 Properties of Rational Numbers

1.2.1 Closure

i) Whole numbers:

Addition

$0 + 7 = 7$ is a whole number

$6 + 8 = 14$ is a whole number

So, we can say that whole numbers are closed under addition.

In general, for any two whole numbers $a \text{ and } b$, $(a + b)$ is a whole number.

Subtraction

$5 - 11 = -6$, is not a whole number. So, we can say that whole numbers are **not** closed under subtraction.

Multiplication

$0 \times 9 = 0$, is a whole number

$3 \times 4 = 12$, is a whole number

So, we can say that whole numbers are closed under multiplication.

In general, for any two whole numbers $a \text{ and } b$, $(a \times b)$ is a whole number.

Division

$5 \div 7 = \frac{5}{7}$, is not a whole number, so we can say that whole numbers are **not** closed under division.

ii) Integers:

Addition

$9 + 7 = 16$, is an integer

$-6 + (-8) = -14$, is an integer

$3 + (-5) = -2$, is an integer

So, we can say that integers are closed under addition.

In general, for any two integers $a \text{ and } b$, $(a + b)$ is an integer.

Subtraction

$5 - 11 = -6$, is an integer.

$-6 - (-8) = 2$, is an integer

$-8 - (-6) = -2$, is an integer

So, we can say that integers are closed under subtraction.

In general, for any two integers $a \text{ and } b$, $(a - b)$ is an integer.

- $(b - a)$ is also an integer.

Multiplication

$5 \times 9 = 45$, is an integer

$-3 \times 4 = -12$, is an integer

$-3 \times (-5) = 15$, is an integer

So, we can say that integers are closed under multiplication.

In general, for any two integers $a \text{ and } b$, $(a \times b)$ is an integer.

Division

$5 \div 7 = \frac{5}{7}$, is not an integers. So, we can say that integers are **not** closed under division.

iii) Rational numbers:

Addition

$\frac{3}{5} + \frac{-2}{7} = \frac{21+(-10)}{35} = \frac{11}{35}$, is a rational number

$\frac{-3}{8} + \frac{-2}{9} = \frac{-27+(-16)}{72} = \frac{-43}{72}$, is rational number

So, we say that rational numbers are closed under addition.

In general, for any two rational numbers $a\ and\ b$, $a + b$ is also a rational number.

Subtraction

$\frac{3}{5} - \frac{-2}{7} = \frac{21-(-10)}{35} = \frac{31}{35}$, is a rational number

$\frac{-3}{8} - \frac{-2}{9} = \frac{-27-(-16)}{72} = \frac{-11}{72}$, is rational number

So, we say that rational numbers are closed under subtraction.

In general, for any two rational numbers $a\ and\ b$, $a - b$ is also a rational number.

Multiplication

$\frac{3}{5} \times \frac{-2}{7} = \frac{3\times(-2)}{35} = \frac{-6}{35}$, is a rational number

$\frac{-3}{8} \times \frac{-2}{9} = \frac{-3\times(-2)}{72} = \frac{6}{72} = \frac{1}{6}$, is rational number

So, we say that rational numbers are closed under multiplication.

In general, for any two rational numbers $a\ and\ b$, $a \times b$ is also a rational number.

Division

$\frac{3}{5} \div \frac{-2}{7} = \frac{3}{5} \times \frac{7}{-2} = -\frac{21}{10}$, is a rational number

$\frac{-3}{8} \div 0 = not\ defined.$ [As you know that for any rational number a , $a \div 0$ $is\ not\ defined$]

So, rational numbers are not closed under division.

- However, if we exclude zero then the collection of all other rational numbers is closed under division.

To sum up-

Numbers	Closed under			
	addition	subtraction	multiplication	division
Rational Number	√	√	√	×
Integers	√	√	√	×
Whole numbers	√	×	√	×
Natural Numbers	√	×	√	×

1.2.2 Commutativity

i) Whole number

Addition: $0 + 5 = 5 + 0 = 5$, $3 + 8 = 8 + 3 = 11$

In general, for any two whole numbers $a\ and\ b$, $a + b = b + a$

Subtraction: $11 - 7 = 4$, $7 - 11 = -4$ hence $11 - 4 \neq 4 - 11$

In general, for any two whole numbers $a\ and\ b$, $a - b \neq b - a$

Multiplication: $3 \times 5 = 5 \times 3 = 15$, $7 \times 4 = 4 \times 7 = 28$

In general, for any two whole numbers $a\ and\ b$, $a \times b = b \times a$

Division: $15 \div 3 = 5$ and $3 \div 15 = \frac{1}{5}$ hence $15 \div 3 \neq 3 \div 15$

In general, for any two whole numbers $a\ and\ b$, $a \div b \neq b \div a$

So, we can say that whole numbers are commutative under addition and multiplication but not commutative under subtraction and division.

ii) Integers

Addition: $-7+5=5+(-7)=-2$, $3+8=8+3=11$

In general, for any two integers $a \text{ and } b$, $a+b=b+a$

Subtraction: $12-7=5$, $7-12=-5$ hence $12-7 \neq 7-12$

In general, for any two integers $a \text{ and } b$, $a-b \neq b-a$

Multiplication: $3\times5=5\times3=15$, $7\times4=4\times7=28$

In general, for any two integers $a \text{ and } b$, $a\times b=b\times a$

Division: $15\div3=5$ and $3\div15=\frac{1}{5}$ hence $15\div3 \neq 3\div15$

In general, for any two integers $a \text{ and } b$, $a\div b \neq b\div a$

So, we can say that integers are commutative under addition and multiplication but not commutative under subtraction and division.

iii) Rational numbers

Addition – Let us add two rational numbers.

$\frac{2}{3}+\frac{5}{7}=\frac{29}{21} \quad and \ \frac{5}{7}+\frac{2}{3}=\frac{29}{21}$

Hence $\frac{2}{3}+\frac{5}{7}=\frac{5}{7}+\frac{2}{3}$

$\frac{-3}{8}+\frac{5}{9}=\frac{-27+40}{72}=\frac{13}{72}$ and $\frac{5}{9}+\frac{-3}{8}=\frac{40+(-27)}{72}=\frac{13}{72}$

Hence $\frac{-3}{8}+\frac{5}{9}=\frac{5}{9}+\frac{-3}{8}$

In general, for any two rational numbers $a \text{ and } b, a+b=b+a$

Thus, we can that addition is commutative for rational numbers.

Subtraction – $\frac{2}{3}-\frac{1}{5}=\frac{10-3}{15}=\frac{7}{15}$ and $\frac{1}{5}-\frac{2}{3}=\frac{3-10}{15}=-\frac{7}{15}$

Hence, $\frac{2}{3}-\frac{1}{5} \neq \frac{1}{5}-\frac{2}{3}$

In general, for any two rational numbers $a \; and \; b$, $a - b \neq b - a$

Thus, we can say that subtraction is **not** commutative for rational numbers.

Multiplication – Let us take two rational numbers and find their product

$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21} \quad and \; \frac{5}{7} \times \frac{2}{3} = \frac{10}{21}$

Hence $\frac{2}{3} \times \frac{5}{7} = \frac{5}{7} \times \frac{2}{3}$

And $\frac{-3}{8} \times \frac{5}{9} = \frac{-15}{72}$ and $\frac{5}{9} \times \frac{-3}{8} = \frac{-15}{72}$

Hence $\frac{-3}{8} \times \frac{5}{9} = \frac{5}{9} \times \frac{-3}{8}$

In general, for any two rational numbers $a \; and \; b$, $a \times b = b \times a$

Thus, we can say that multiplication is commutative for rational numbers.

Division-

$\frac{4}{7} \div \frac{3}{5} = \frac{4}{7} \times \frac{5}{3} = \frac{20}{21}$ and $\frac{3}{5} \div \frac{4}{7} = \frac{3}{5} \times \frac{7}{4} = \frac{21}{20}$

Hence, $\frac{4}{7} \div \frac{3}{5} \neq \frac{3}{5} \div \frac{4}{7}$

So, we can say that division is **not** commutative for rational numbers.

Now try to fill in the boxes with (√ OR ×)

Numbers	Commutative for			
	addition	subtraction	multiplication	division
Rational Number	√			
Integers				
Whole numbers		x		
Natural Numbers				

Thank you.