THE ROLE OF RATIONAL NUMBERS IN MATHEMATICAL ACHIEVEMENT AND DECISION MAKING

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

SASHANK VARMA, ADVISER

December, 2016

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Acknowledgments

This dissertation could not have been completed without the assistance of my advisor, colleagues, committee, family, and friends. First, I would like to thank my advisor, Sashank Varma, for providing the feedback and advice on my early plans that eventually converged on this topic, the financial support to compensate the undergraduates who took part in these experiments, and the large amount of time in editing drafts. Also, I am appreciative of the high intellectual and professional standards he has insisted upon throughout my graduate training.

I also am indebted my committee members, Keisha Varma, Robert delMas, and Michele Mazzocco, for agreeing to take time to consult, advise, and ultimately evaluate this dissertation. Their assistance was very, very helpful during the prospectus stage.

Next, I have to thank many of my fellow graduate students for their assistance. Jean-Baptiste Quillien, Purav Patel, and Soo-hyun Im proved invaluable during the editing process. This kind of technical writing, especially across more than 100 pages, requires more than grammatical and sentence-level precision. Without the assistance of Jean-Baptiste, Jay, and Soo-hyun, it would have been an insurmountable task to identify where the reader was lost or a better explanation could be provided.

Finally, I also must thank my family and friends for always supporting my academic pursuits. Many steps along the way would have been much more difficult were it not for their support and encouragement. In particular, I must thank my parents for always strongly encouraging intellectual curiosity and academic pursuits, even when direct pay-offs were unclear.

Abstract

Understanding rational numbers requires reorganizing our initial understanding of numbers as whole numbers. Coordinating the relationship between the different symbolic formats for expressing rational numbers (i.e., as fractions and as decimals) and their underlying non-symbolic magnitudes is an important component of mathematical development in children (Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Pyke, 2013; Mazzocco et al., 2013). It is also an important component of decision making in everyday life (Simon, Fagley, & Halleran, 2004; Peters et al., 2006). The goal of the present experiments was to investigate the relationship between rational numbers, expressed in various formats, on one hand and general mathematical achievement and decision-making on the other. Two experiments demonstrated that the format of rational numbers impacts processing: the fraction format hinders magnitude processing compared to the decimal format. Experiment 1 additionally demonstrated that the precision of rational number magnitudes is related to general mathematical achievement. This is evidence that a better understanding of rational numbers is important for more abstract mathematics in adults. Experiment 2 showed that individual differences in rational number ability are also associated with individual differences in bias in decision-making. These findings have practical implications. Educationally, these results suggest that using number lines and intermixing decimal and fraction formats might improve rational number ability and therefore better prepare children for later, more abstract mathematics. Pragmatically, the results of this study suggest numerical ability alone is not a sufficient guard against biased decision making when probabilities are involved. Instead it appears other, non-numerical task features cause bias and need to be identified to make decision making more normative.

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Chapter 1: Overview

While basic numerical ability with natural numbers is clearly a prerequisite for basic math literacy, rational numbers, especially those expressed as fractions, remain a novel area of research and difficult concept to master (Schneider & Siegler, 2010). One reason for this difficulty is that what is true for natural numbers is often conflated with what is true for all numbers. Consider the statement: “When we multiply numbers, the numbers get bigger”. This statement is inconsistently true, only applying to a subset of real numbers; yet many people are inclined to agree with it (Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015). Rational numbers challenge such oversimplified generalizations based on the integers alone. This difficulty is compounded by the fact that rational numbers are formatted in different ways, unlike natural numbers which are in only one format.

Current research on mathematical ability suggests that a robust understanding of fractions as magnitudes is a strong predictor of later, more complex mathematical problem solving abilities in children and adolescents. Researchers have paired traditional magnitude comparison tasks (i.e., comparing which of two fractional numbers is greater or lesser) with number line estimation (NLE) tasks (i.e., locating fractional numbers on an unmarked number line). Both tasks together are significant predictors of standardized measures of mathematical achievement (Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Pyke, 2013). These results highlight the importance of fractions in developing mathematical competency, particularly the importance of understanding them as symbols representing magnitudes that are both similar to and distinct from integers.

Rational numbers are not just important as abstractions in the realm of mathematics but also can model the real world in ways important for everyday decision-making. Probabilities are typically expressed as rational numbers formatted as fractions, decimal, and percentages. Thus, assessing the likelihood of outcomes expressed as probabilities requires an understanding of the magnitudes of these rational numbers in decision-making contexts. These rational numbers and contexts can interact, sometimes causing bias. One such bias is *framing effect*. The framing effect obtains in decision-making tasks that deal with probabilistic information. Positive valence elicits more decisions that avoid risk, while negative valence elicits more decisions that seek risk (Tversky & Kahneman, 1981).

This current study consisted of two experiments. The goal of Experiment 1 was to investigate the magnitude representations of rational numbers and the relationship between the precision of these representations and mathematical achievement. Magnitude representations were measured using two standard tasks, comparison and number line estimation. The goal of Experiment 2 was to build on the results of Experiment 1 and demonstrate how numerical cognition literature can inform the study of probabilistic decision-making when rational numbers become models of different outcomes. It investigated whether people with better magnitude representations of rational numbers reason more consistently about probabilities, and in particular were less susceptible to the framing effect.

The motivation for this research is twofold. First, I aim to extend the current research on rational numbers (fractions and decimals) to an adult population with two measures of mathematical ability—the Rational Ability test of procedural knowledge of rational numbers developed for these experiments, and ACT-math sub-score, a standardized test of general mathematical ability. Second, I aim to explore how numerical processing of rational numbers drives two measures of the framing effect, a new probabilistic number line estimation task and a classic decision-making task.

The results of these experiments have implications for mathematics education, suggesting the use of tasks that mix different rational number formats and the employment of number lines. They also have practical applications, suggesting that in some situations, rational number ability may not be enough to guard against biases such as the framing effect.

Chapter 2: Literature Review

Rational Numbers

Competency with rational numbers, especially fractions, is difficult for many to obtain. For example, on the National Assessment of Education Progress (NAEP), 50% of 8th graders failed to order three fractions from small to large: 2/7, 5/9, and 1/12 (Martin, Strutchens, & Elliot, 2007). This rational number illiteracy continues into adulthood as demonstrated by the fact that community college students only attained 70% accuracy in a magnitude comparison task involving fractions compared to the 50% rate one can achieve by guessing (Schneider & Siegler, 2010).

Unfortunately, we would be unwise to leave rational number ability to the competent few because this ability is a prerequisite for higher mathematical domains (Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013). Mazzocco et al. (2013) found competency with fractions in children was a predictor of a broader range of mathematical abilities including addition, subtraction, multiplication, and division. This study measured 4^{th} to 8^{th} grade children's ability to judge which of two fractions is larger (i.e., a magnitude comparison task). Some fractions had the same numerator while the rest had the same denominator. Fractions also varied in presentation format between Arabic numerals vs. a visual block display (rectangles and squares divided into equal squares to represent the denominator with shaded squares representing the numerators). They found that participants across all grades and ability levels performed worse for symbolic versus visual depictions of fractions, showing the format of rational numbers can influence processing. More importantly, difficulties with basic fraction comparisons, even when assisted by the same denominator or numerator persist even into 8th grade. These difficulties can distinguish between low achieving and math disabled students: children's status as typically achieving, low achieving, or math learning disabled predicted their accuracy on a magnitude comparison task (after controlling for full scale IQ). These results show that individual difference in rational number processing are related to a broader range of mathematical abilities.

Why might rational numbers, which are typically taught later as formal symbolic representations, be related to other mathematical abilities? First, we should consider how a ratio sense may develop independently of formal education. Then, we should consider the link between abstract, symbolic representations of rational numbers and their concrete, non-symbolic magnitudes emphasized by formal education.

Humans may be predisposed to understanding rational numbers even before receiving formal instruction. Employing a preferential looking experimental paradigm, Xu and Garcia (2008) found that infants attend to sample and base-rate information, an analogous activity. In one experiment, they showed infants a box and drew 5 balls from the box (4 red and 1 white), then removed a panel on the front of the box so the infant could see inside clearly. In one condition, the balls inside matched the sample initially drawn (in this case the balls were mostly red) and in another case, the balls inside did not match the sample drawn (mostly white). Infants showed increased looking time in the mismatched condition, indicating they were sensitive to proportional (a out of b) information. Therefore, rational number processing may be linked to non-symbolic, informal brain systems formal education eventually exploits.

Some researchers have gone as far as to propose humans have a *ratio processing system* (RPS), an informal brain system that is sensitive to nonsymbolic ratio that underlies symbolic rational number processing. This system is analogous to the approximate number system (ANS) that has been well supported in underlying symbolic systems for natural numbers (Feigenson, Dehaene, & Spelke, 2004). Heretofore, due to their difficulty and resistance to education, many researchers and educators have thought symbolic fraction representations were too disconnected from natural brain systems to have such connections: somewhat like quantum theory in physics is disconnected from the everyday physics of our world, and is therefore purely a symbolic system humans developed by language, logic, and education. This assumption appears to be wrong. Matthews, Lewis, and Hubbard (2016) had participants compare non-symbolic dot arrays in which the ratio between dots (or size) varied, deciding which ratio is greater. They also had participants represent a single ratio of non-symbolic dots on a number line between 0 and 1. These two measures created a proxy for the RPS. Next, participants completed the same tasks using fractions, the symbolic representations of the same ratios, and completed a fraction knowledge assessment. After controlling for both ANS acuity and a measure of executive function, RPS performance accounted for 11%, 4%, and 15% in variance on the symbolic comparison task, symbolic number line estimation task, and fraction knowledge assessment respectively. These results suggest a non-symbolic system, such as the RPS, does underlie symbolic fraction performance.

Further evidence for the link between rational numbers and informal brain systems also come from employing a classic finding on natural numbers to the study of rational numbers: the *distance effect*. The distance effect (Moyer & Landauer, 1967) is the finding that people more quickly judge which of two numbers is greater the farther the distance between the two numbers. For example, people judge 9 to be greater than 2 more quickly than they judge 6 to be greater than 5. This finding is considered to be evidence that there must be some correspondence between the symbols for these numbers and their underlying non-symbolic magnitudes. Distance effects have been found with decimals and fractions (Varma & Karl, 2013; Matthews & Chesney, 2015), suggesting a similar correspondence. However, there is evidence this effect may disappear for fractions in certain conditions, suggesting fraction representations may be more difficult to link to magnitudes (Zhang, Fang, Gabriel, & Szűcs, 2016; DeWolf & Vosniadou, 2015).

Since rational numbers do appear to have some foundations in non-symbolic systems, the links between symbolic notations and these non-symbolic magnitudes is crucial. This link is built primarily via formal instruction. Mathematics expresses rational numbers in various notations and this may have consequences for integrating them into a unified number system. Differences in how rational numbers are presented may be related to contextual congruencies with the non-symbolic objects or concepts being represented. Rapp, Bassock, DeWolf, and Holyoak (2015) explored how mathematics textbooks and college students choose to represent rational numbers in word problems (as decimals or fractions) and found strong preferences based on the nature of the quantities being represented. When the problem context referred to continuous variables (such as mL of a solution), both math educators and college students choose to use decimals to express the problem solution. However, when the problem context referred to discrete variable (such as colored marbles), both groups chose to use fractions to express the problem solution. This representational congruence has also been demonstrated to affect performance outcomes on tasks with different applications of rational numbers: while decimals, as mentioned above, facilitate magnitude comparisons, fractions appear to be advantageous in relational reasoning between discrete concepts (DeWolf, Bassok, & Holyoak, 2015).

These findings represent a logical, but potentially problematic trend in our education and comprehension of rational numbers. Decimals are a more natural fit for continuous variables as the range of values is unbounded, while fractions naturally fit discrete variables as the range of values is bounded (by a part and a whole), thus these representational preferences reflect a natural congruency with problem context. However, understanding the connection between ¼ and .25 beyond just *symbols*, but as congruent representations of magnitudes, whether of continuous or discrete variables, may be an important developmental benchmark, and our educational system may not emphasize this connection enough. This connection between symbolic representations of rational numbers and their non-symbolic magnitudes may be an indicator of an *integrated number theory*, a more flexible and complete understanding of numbers (Siegler & Lortie-Forgues, 2014).

Siegler and Lortie-Forgues (2014) propose that there are four trends in developing an Integrated Theory of Number Development. First, we increase the precision of our representations of the magnitudes expressed nonsymbolically. Next, we link nonsymbolic to symbolic representations of small numerical magnitudes. Next, we extend the range of whole numbers whose magnitudes can be represented accurately. Finally, we accurately represent the magnitudes of numbers other than whole numbers, in particular fractions, decimals, and negatives. This theory assumes that the link between these symbolic and nonsymbolic magnitudes takes place on a mental number line and that understanding rational numbers requires reorganizing the structure of this number line. The capacity to flexibly connect different representations of rational numbers (decimal and fractions) to their nonsymbolic magnitudes may be a particular challenge to this integrated number theory and the use of number lines to measure this ability may be a strong indicator of more holistic numerical abilities.

In summary, humans may be predisposed to processing ratios without formal education. Different symbolic representations of rational numbers may build on this capacity differently. Education with rational numbers then may challenge overly simplified connections between abstract mathematical symbols and their connection with actual magnitudes. Thus, performance on tasks requiring processing of rational numbers may relate to other mathematical abilities not directly involving rational numbers. As an individual connects the abstract symbols of rational numbers, particularly fractions, with their non-symbolic magnitudes, they gain a more complete and flexible understanding of a broad range of concepts and skills needed for mathematical success. Next, we consider more evidence of the connection between rational numbers and broader mathematical ability and how two different tasks capture this connection by requiring participants to access some magnitude representation of these symbols.

Magnitude Comparison, Number Line Estimation and Rational Number Processing

Recently, some researchers have begun using number line estimation tasks in parallel with magnitude comparison tasks, and have found both to be strong predictors of general mathematics ability (Fazio, Bailey, Thompson, & Siegler, 2014). During magnitude comparison, participants are given two rational numbers and are asked to indicate which one is larger or smaller. Response times or accuracy rates have both been used as a measures of performance on these tasks. During number line estimation tasks, participants are given a rational number and are asked to mark that number on a number line with polls marked 0 and 1, or sometimes extending between 0 and 5. For the Number Line Task, average absolute error (how far off these markings are from the correct markings) is the measure of performance. Siegler and Pyke (2013) measured fraction comprehension in children (6^{th} and 8^{th} graders) with both a magnitude comparison task and number line estimation task involving fractions. They found that the magnitude comparison and number line estimation tasks combined accounted for 9-10% of variance in state mathematics achievement exam, using both reading scores on the same state test and executive function tasks as controls.

The same researchers also examined whether participants who based their number line markings on actual fraction magnitudes did better on the mathematical achievement test. To explore this, they computed gamma correlations between each child's 0–1 number line estimates and the order of the fractions if ranked by numerator, denominator, or fraction magnitudes. The largest of these three correlations was interpreted as the child's predominant approach, unless all relations were weak (< .25), which resulted in a strategy classification of *unknown*. This strategy essentially places individuals relying on non-magnitude based strategies that can lead to accurate markings into a different category from those relying more on the actual magnitudes involved, a tendency overall performance can mask. For example, performance could be comparable between a participant relying mostly on a numerator-focused approach and a participant relying on a magnitude based approach since sometimes the larger numerator does indicate a larger number. Both 6th and 8th graders who relied on the magnitude strategy as opposed to a numerator or denominator strategy had higher state math achievement scores (d = 1.26 and 1.86 respectively). This finding highlights both the use of number lines and the importance of comprehending fractions as magnitudes as an important skill in mathematics. It suggests that number lines may be superior in revealing people's actual magnitude understanding of rational numbers, because performance relies more on actual magnitudes while magnitude comparisons are more difficult to decouple from other strategies.

Number lines have also been explored as an instructional tool and found to be effective at post instructional performance with rational numbers (Saxe, Diakow, & Gearhart, 2013; Maertens, De Smedt, Sasanguie, Elen, & Reynvoet, 2016). Saxe et al. (2013) compared instruction using an Everyday Mathematics standard lesson (control) to a Learning Mathematics through Representations (LMR) lesson on fractions. The LMR lesson emphasized the connection between integers and fractions via number lines with the aim of building a more “coherent integration of integers and fractions.” Results showed that the LMR lessons increased performance on test items involving a strong understanding of the concepts of integers and fraction, regardless of whether they were presented with a number line or not, suggesting number lines may also have instructional value.

Further support for this instructional potential comes from a large scale intervention on 4th graders at-risk in the domain of mathematics by Fuchs, Schumacher, Long, Namkung, Hamlett, Cirino, Jordan, Siegler, Gersten, and Changas (2013). The intervention focused on the use of number lines to highlight a measurement conceptualization of fractions while the control group focused on the more typical part-whole concept. The number-line based intervention increased post-test performance on both conceptual and procedural measures (d = 0.29 and 2.50) compared to the typical instruction, and reduced the gap between at-risk and low-risk students.

Rational Number Ability and Algebra

Next, I explore what connections there may be between rational numbers and basic algebra. This connection is of particular interest as Experiment 1 employed an outcome measure of mathematical achievement with many items requiring algebraic manipulation. Van Hoof, Vandewalle, Vershaffel, and Van Dooren (2015) posit the difficulty with rational numbers stems from four important differences they have as a class from natural numbers. The first difference is *density*. The natural numbers are discrete, countable, (you can point to what comes next), while rational numbers are infinitely dense. The second difference is *representation*. Natural numbers have a single symbolic notation, while rational numbers have many. The third difference is *determining size*. Size with natural numbers is related to visual cues (e.g. 42 is greater than 8) which is not true for rational numbers (e.g. 0.42 is *not* greater than 0.8, and 11/99 is *not* greater than 8/9). The fourth difference is *the effect of arithmetic operations*. The consequence of basic arithmetic operations on natural numbers is straightforward (multiplying reliably increases magnitudes), while more complicated for rational numbers (sometimes multiplying decreases magnitudes). Van Hoof, et al (2015) refer to cases when people err due to these difference in favor of the what work for natural numbers *natural number bias*.

Natural number bias has been shown to impede algebraic learning. Van Hoof, et al. (2015) tested 8^{th}, 10^{th} and 12^{th} graders on a verification task in which participants decided whether algebraic statements that were either congruent with or incongruent with natural number bias could or could not be true. For example, $3 / x < x$ can be true and is congruent with natural number bias, while 3 / x > 3 can also be true, but is incongruent with natural number bias. The difference between the two statements is that the second requires comprehending the role rational numbers can play in evaluating algebraic statements. The experiment also contained true incongruent statements that required considering negative numbers, such as 4 + x < 4. Students showed higher performance on congruent items overall across all grades. The difference in performance between congruent and incongruent items was almost entirely due to the difference between items that involved multiplication and division, the items that require considering rational numbers. These results demonstrate a plausible reason rational number ability extends into later mathematical concepts, in this case, algebra. Students with a more flexible and complete concept of number, particularly including rational numbers, are likely to be able to more flexibly manipulate more abstract mathematical concepts.

Summary

Symbolic rational numbers pose a challenge to people when initially taught and continue to be challenging into adulthood. The fact that a rational number magnitude can be represented in various formats poses a unique challenge to rational number ability and crossing representations fluently may be an important benchmark in mathematics development. Using both magnitude comparison and number line estimation tasks with rational number has been shown to be related to general math achievement. There are plausible procedural reasons individuals who are more fluent with rational numbers may be better able to engage in other mathematics, such as manipulations in algebra requiring multiplying or dividing with unknown quantities that can or must be rational numbers.

Probabilities, The Framing Effect, and Loss Aversion

Rational numbers are not just important as abstract symbols in the realm of mathematics but also can model the real world in ways important for everyday decision-making. In particular, probabilities also depend on rational number representations of fractions, decimal, and percentages. Thus, assessing the likelihood of outcomes expressed as probabilities requires both an understanding of the magnitudes of these rational numbers and what these magnitudes mean in contexts involving other, sometimes irrelevant, information. One such source of irrelevant information is information that is framed positively or negatively. For example, whether a person got 80% correct or 20% incorrect on an exam. Note the information is exactly the same, but research shows that people evaluate information differently based on frame.

Probabilities and Rational Numbers

Probabilities are expressed as rational numbers and those numbers can be mapped to their meanings as magnitudes. For example, a 50% chance of rain may come from records showing that on 15 out of 30 similar days, it rained. These numbers can be reduced to 1 out of 2 and therefore represents the same magnitude as ½ and .5. The difference is that this estimate can lead to decisions and may have contextual information that can change how one perceive that magnitude. Thus, it makes sense to explore the relationship between people's rational number processing ability and decision-making when probabilities are involved.

Prior studies suggest that individual differences in numerical ability are related to individual differences in probabilistic reasoning. For example, people who are low in numeracy express a desire to be less involved in medical decision-making regarding their own health compared to those who are high in numeracy (Galesic & Garcia-Retamero, 2011) and such decisions are almost always related to outcome expectations that can be expressed as a probability. In a large-scale survey, Galesic and Garcia-Retamero measured participants' numeracy using the Lipkus (2001) scale, a measure of the ability to translate between percent, ratio, and fraction representations of rational numbers. They also asked participants to rate the degree to which they (1) felt competent to make and (2) wished to be involved in making decisions regarding their healthcare. Those with low numeracy scores had lower ratings of their own competence and desire to take part in medical decision-making regarding their health.

A plausible intermediary ability linking numerical processing and decision-making is the structure of one's mental number line and the mapping of symbolic numbers to it. Probabilities are often presented in decimal or percent formats, which can be understood similarly to the numbers 0-100. It is therefore important to consider whether probabilities are linearly scaled, as natural numbers are in older children (Booth & Siegler, 2004) and adults, and what contexts may affect this mental representation. Siegler and Booth (2004) had children mark the position of a subset of the numbers 1-100 on a number line that was unlabeled except for its endpoints. 6-year-olds made marks reflecting a logarithmic number line representation, with relatively far distances between those corresponding to smaller numbers and relatively near distances between those corresponding to larger numbers. By contrast, 8-year-olds made marks reflecting a strikingly linear number line representation. Their data are reproduced in Figure 1.

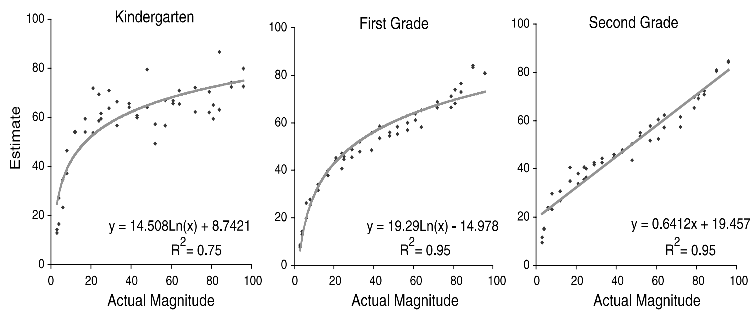

Figure. 1. Children's number line estimates progress from logarithmic to linear throughout early elementary school (Siegler & Booth, 2004).

Regarding probabilities expressed as rational numbers, Tverksy and Fox (1995) found that people do not treat risk in a linear fashion. In one experiment, they asked participants to choose between a 25% chance to win $150 and a sure prize that varied from $40 on down. If people behave rationally, they would calculate that 25% of $150 is $37.5, the *expected value* of the bet, reflecting the long run expectation of make similar decisions. Then, they would compare this to the sure bet of $40 and conclude the risky bet is less attractive. The key prediction of this normative analysis is that the attractiveness of the risker option should increase as the sure bet is reduced. In this experiment, that would be seen in the percent of participants choosing the riskier option over the sure bet increasing as a linear function of the difference between the sure bet and expected value. However, participants displayed a non-linear decision pattern. This suggests that incorporating the magnitude of risk (i.e., probability) into a decision is not a matter of a linearly mapping between 0-100% as a normative analysis expects.

It is important to note here that some people are more likely to choose the sure bet regardless of the numerical information because they are just more conservative in how they approach chance. The design of the experiment described above and others (including part of Experiment 2 of this dissertation) allow for the detection of how changes in numerical information or context affect people's decisions above and beyond their *a priori* comfort with risk. In the case above, this control is accomplished by looking at the proportion of people choosing each option, thereby eliminating individual differences in general risk seeking.

Now, consider again the example above with a little more context. Some proportion of people would choose to take the sure bet of $40 and some portion (a smaller proportion if the group is actually considering the expected value) would choose the riskier option. If that sure bet reduces to $30, those proportions should change in favor of the risker option and those proportions should also further change in favor of the risker option if the sure bet is reduced to $20. If people are considering the options numerically with some consideration, formally or informally, of the expected value of the risky option, these shifts in the proportions of people preferring the sure bet vs. a risker choice should be a linear function of the difference between the expected value and the sure bet. They are not, suggesting there is an aspect of risk seeking and risk aversion that is not rational and not linear.

However, people with high numerical ability are more sensitive to the magnitude of a probability in decision-making, responding more normatively when risk increases, than people with low numeracy (Peters, 2012). For example, Cokely and Kelley (2009) presented participants with 40 hypothetical choices between certain and risky options of the same nature as the Tverksy and Fox (1995) experiment described above. One trial might ask participants to choose the sure bet of winning $200 or a 20% chance to win $900, while another might be a choice between a sure bet of $200 or a 30% chance to win $900. Participants with higher numeracy scores made more high-expected-value choices—meaning, they were more likely to take the $200 in the first case, when the expected value was $180 (.2 x $900) and more likely to gamble in the second case, when the expected value was $270 (.3 x $900). Thus, rational number competence appears to modulate how sensitive they are to the numerical information relevant to that decision. This finding suggest rational number ability can improve decision making, or at least improve consistency in applying one's desired level of risk-seeking behavior, when probabilities are involved by improving attention to the actual magnitude of probability.

The Framing Effect

The previous example is one in which people must combine numerical processing (of rational numbers) with decision making. Researchers have also added non-numerical and irrelevant contexts to such decisions to see what affect this has on the outcome of such decisions. One example of this comes from Tversky and Kahneman (1981) in which they asked participants to choose between two options in a between groups design called the “Asian Disease” problem. All participants read the following introduction:

> *Imagine that the United States is preparing for an outbreak of an unusual Asian disease that is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Scientific estimates of the consequences of the programs are as follows:*

Then, half the participants read and choose between the following two options (the positive frame):

> *If Program A is adopted, exactly 200 people will be saved.*
>
> *If Program B is adopted, there is a 1 in 3 probability that all 600 people will be saved and a 2 in 3 probability that no people will be saved.*

The other half read and choose between the following two choices (the negative frame):

> *If Program C is adopted, exactly 400 people will die.*
>
> *If Program D is adopted, there is a 1 in 3 probability that nobody will die and a 2 in 3 probability that all 600 will die.*

It is important to note that choices A and C and choices B and D have the same expected values. A and C represent the same sure bet of saving 200 people. B and D represents the same risky chance of saving or losing everyone based on an expected value calculation of (1/3 x 600= 200). Therefore, participants' decisions should not vary between the two framings if they are based on a normative analysis of the numerical information presented. However, Tverksy and Kahneman (1981) found participants were more likely to make the risk-averse choice A in the positive framing condition than the risk-averse choice C in the negative framing condition. More generally, people are more risk averse when choices are framed positively and more risk seeking when choices are framed negatively. This finding is called the *framing effect.*

The framing effect may be the consequence of a more general bias toward avoiding losses more than seeking gains, sometimes referred to as *loss aversion* (Kahneman & Tversky, 1979). If we assume a person's current net state is 0, the idea of adding 2 to that state is appealing and the idea of subtracting 2 from that state is unappealing. Loss aversion is a bias to avoid the loss of 2 more than to approach the gain of 2. If this is a tendency, those who can better process numerical information may make more consistent decisions since they are not just relying on a subjective desire to avoid loss, but on a more normative assessment of the options. This fuller assessment includes evaluations of relevant information such as expected value and information related to the actual magnitude of the option or information provided. A central premise of Experiment 2 is that how a problem is framed may modify people's perception of a number representing a probability. In particular, do people's representation of the magnitude of a chance framed as a gain or a loss differ, and are people with higher ability with rational numbers more consistent in their representation of these magnitudes?

Numerical Ability Modulates the Framing Effect

Many studies have explored the framing effect since Tversky and Kahneman's original paper. Numerical ability has been explored as a potential mediating factor, and indeed evidence suggest those with higher numeracy are less susceptible to the effect of positive or negative valence. For example, Simon, Fagley, and Halleran (2004) found individuals who self-reported higher math skills were less influenced by the framing of risky choices as those who reported low math skills. More direct evidence of the relationship between numeracy and framing effects comes from Peters, Västfjäll, Slovic, Mertz, Mazzocco, and Dickert (2006). They measured participants' numeracy using the Lipkus scale (2001). Participants were also presented with exam scores and asked to rate them on a 7-point range from -3 (very poor) to + 3 (very good). The frame was manipulated by altering whether the scores were presented as percent *correct* or percent *incorrect*. For example, 74% correct or 26% incorrect represented the same test performance but only varied in positive or negative frame. The difference in ratings between conditions (correct vs. incorrect) was significantly greater among the people with lower numerical ability and more comparable among those with higher numerical ability. The same researchers also employed A (sure bet) vs. B (risk) paradigm, mirroring the original Tversky and Kahneman (1981) framing condition. In this experiment, they also asked participants to rate the attractiveness of each option (on a -3 to 3 scale), thus combining preference for a riskier option with a measure of more subjective attractiveness. The choices of participants with higher numeracy matched their ratings: People with higher numeracy were more likely to pick the option which they rated as more attractive, regardless of frame. People with low numeracy, however, exhibited framing effects regardless of their attractiveness ratings. In particular, they were drawn to the riskier choice in the negatively framed condition. These results imply people with higher numerical ability are integrating the numerical information in their decisions, while those with lower numerical ability are more susceptible to more superficial cues which bias their decisions.

Garcia-Retamero and Galesic, 2010 replicated these results with consequential scenarios and included the use of visual aids to decrease the impact of general cognitive ability on decision making. They also used the Lipkus scale as a measure of numeracy. All participants were asked to rate the risk of medical procedures on a scale from 1 (not risky at all) to 4 (very risky). The frame was manipulated by altering whether outcomes were presented as *chances of surviving after surgery* or *chances of dying after surgery*. Again, people with lower numeracy were more susceptible to framing as they differed more in their risk ratings when considering the cases of 20% of dying after surgery vs. 80% chance of surviving after surgery compared to those with higher numeracy. However, the use of visual aids (pie charts and vertical and horizontal bars) eliminated the framing effect regardless of numerical ability. These results suggest although individuals low in numerical ability may not be predisposed to focus on the numerical information or to fully comprehend the magnitude of risk expressed as numerical symbols, they can with the aid of visual aids. This may be because individuals with lower numerical ability may not attend to the numerical information provided as much as those with higher numerical ability. Also, they may have trouble translating the numerical symbols systematically into a magnitude representation of the risk of each decision.

The Framing Effect and General Cognition

Research has shown that people who exert more cognitive effort are less likely to be influenced by frames. For example, a neuroimaging study found increased amygdala activity in the negatively framed condition, a region associated with emotional discomfort and displeasure (De Martino, Kumaran, Seymour, & Dolan, 2006), supporting a loss aversion explanation. However, this study also found a suggestive individual difference: individuals who were least affected by the framing effect (i.e., made more normative judgments) had significantly more prefrontal activity, a region associated with more effortful, controlled, and cognitive processing. Further, higher SAT scores, a broad measure of cognitive ability, are associated with a reduction in the framing effect (Stanovich & West, 1998). These findings suggest some people do exert cognitive effort to override this loss aversion tendency or may be more prone to attend to and process the numerical information available.

It appears the framing effect can also be reduced by encouraging or giving more time for people to exert such effort before making their decisions. Takemura (1994) encouraged roughly half the participants to “think about the justification of their decision and ….to write down the content of justification" in a framing task. This manipulation removed the framing effect completely within that group. Similarly, Miu and Crisan (2011) found a cognitive reappraisal task – asking participants to "think about your decision in a way that makes you stay calm" – reduced the framing effect.

Taken as a whole, these studies suggest that increased cognitive effort, whether from internal individual differences or external encouragement, reduces or eliminates the framing effect. Given these non-numerical cognitive abilities have been shown to influence the framing effect, it makes sense to design stimuli in a way to highlight the effect of the framing effect on magnitude representations that does not requires manipulations of numbers, like calculating expected value. In this manner, we can see the effect of frame more independently of cognitive ability.

Summary

Numerical ability appears to modulate the biases in problem solving with probabilities. Superior numerical ability is associated with more consistent and normative answers. These findings may be related to people's ability to accurately represent rational numbers, for example as analog magnitudes on a mental number line. The framing effect also appears to be related to numerical ability. The framing effect appears to be caused by some interruption of rational mathematical thinking due to non-numerical factors, such as loss aversion, that frame introduces. It may particularly impact individuals with already lower numerical ability, i.e., those least likely to attend to the numerical information systematically. This relationship is complicated by affective components of the contexts provided during problem solving and the way in which numerical ability is measured. Cognitive effort alone seems to reduce the framing effect.

Chapter 3: Research Questions

The goal of Experiment 1 was to investigate the magnitude representations of rational numbers and the relationship between the precision of these representations and mathematical achievement. Symbolic rational numbers pose a challenge to many children and adults. A comprehensive understanding of their magnitudes may be an important precursory skill along the way to more general math achievement. One unique challenge to understanding rational numbers is the fact they can be represented using different symbolic formats. It is unclear how well people reason about rational numbers in various formats. In particular, there has been little to no work on which format is easier to reason about and whether there are costs to reasoning about both decimal and fractions simultaneously.

Another question concerns the relationship between rational number processing and mathematical achievement. Prior research has been limited to work on children and middle school aged students. However, the relationship in adults remains unexplored. In addition, the question of how processing different formats of rational numbers across different tasks relates to mathematical achievement has also not been systematically explored. There are two main formats of rational numbers of interest: decimal and fractions. These have been studied using two types of laboratory tasks that probe magnitude processing of rational numbers: magnitude comparison and number line estimation tasks. Parsing out which symbolic representation of rational numbers *and* which type of magnitude processing task is most predictive of mathematical achievement would better inform our understanding of this relationship. In addition to laboratory measures of rational number magnitudes, we can also ask whether procedural knowledge (i.e., solving paper-and-pencil problems) involving rational numbers and mathematical achievement are associated.

To close these gaps, Experiment 1 addressed two main questions:

1. Do rational number formats differ in the fluency with which people can access the underlying magnitudes?
    a) In addition, is there a cost when performing a magnitude task that includes both fractions and decimals?
2. What is the relationship between rational number magnitude processing and mathematical achievement in adults?
    a) Does either representation format (decimal or fraction or both) of rational numbers predict mathematical achievement?
    b) Does performance on the magnitude comparison task or the number line estimation task (or both) predict mathematical achievement?
    c) Do individual differences in procedural problem solving with rational numbers predict general mathematical achievement?

Experiment 2 applied the methods used in Experiment 1 to bridge between mathematical cognition and behavioral economics. More specifically, it explored the relationship between people's magnitude representations of rational numbers and their understanding of the framing of probabilistic information as gains vs. losses as revealed by the *framing effect*. The framing effect occurs in tasks when equivalent numerical information is presented either positively or negatively, such as reporting a test result as 80% correct vs. 20% incorrect. People evaluate these numerical equivalent outcomes differently due to frame, interpreting positively framed outcomes more conservatively than negatively framed outcomes.

Despite a rather large literature on the framing effect, there are limits in our knowledge of whether this effect is related to numerical processing, and in the broader question of whether such biases can be reduced. One limit is that it is unclear at which level the framing effect occurs. Heretofore, this effect has been tested at the *decision level*, based on the macroscopic choice between two options for solving a complex problem. However, it is possible that the framing effect originates at the lower level of probability magnitudes. Specifically, the frame might modulate how a person perceives the magnitude of a probability, which in turn biases their decision-making. For this reason, Experiment 2 measured the framing effect in two ways. First, it utilized a novel variant of the number line estimation where participants estimated the position of a risk framed as a gain or a loss on a number line with poles 0 and 1. This task allowed evaluation of the frame effect at the lower magnitude level. Second, Experiment 2 measured the framing effect at the higher decision level by having participants solve the original Tversky and Kahneman (1981) Asian Disease problem. The inclusion of this standard measure of the framing effect additionally allowed me to evaluate whether participants behaved as in previous studies.

Another limit of prior research on the framing effect has been that studies that have also measured the numerical ability of participants have used the Lipkus Scale – a general test of the ability to translate between percentages, ratios, and fractions. Performance on this scale predicts resilience against framing biases. However, its focus on learned arithmetic procedures leaves open the question of the relation between more fundamental rational number capacities and biases like the framing effect. For this reason, Experiment 2 utilizes the Experiment 1 measures of the precision of rational number magnitudes. It investigates whether this more fundamental ability modulates susceptibility to the framing effect. Again, the goal is to investigate whether bias when making decisions involving probabilities is rooted in more fundamental numerical processing abilities than have been previously explored in the literature.

Experiment 2 addressed two main questions:

3. Does the framing of probabilistic information affect people's magnitude representations of rational numbers?
4. Are individuals with stronger rational number ability less susceptible to exhibiting the framing effect?

Chapter 4: Experiment 1

The goal of Experiment 1 was to investigate the magnitude representations of rational numbers and the relationship between the precision of these representations and mathematical achievement. Magnitude representations were measured using two standard tasks, comparison and number line estimation. The first goal of Experiment 1 was to explore the effect of format (decimal vs. fraction) on magnitude processing of rational numbers. The second goal was to extend the current research on rational numbers (fractions and decimals) to an adult population with two measures of mathematical ability—the Rational Ability Test, a test of procedural knowledge of rational numbers developed for these experiments, and ACT Math sub-scores, a standardized test of general mathematical ability.

Experiment 1 addressed two main questions:

1. Do rational number formats differ in the fluency with which people can access the underlying magnitudes?
    a) In addition, is there a cost when performing a magnitude task that includes both fractions and decimals?
2. What is the relationship between rational number magnitude processing and mathematical achievement in adults?
    a) Does either representation format (decimal or fraction or both) of rational numbers predict mathematical achievement?

b) Does performance on the magnitude comparison task or the number line estimation task (or both) predict mathematical achievement?

c) Do individual differences in procedural problem solving with rational numbers predict general mathematical achievement?

Although not the central focus of this experiment, the tasks employed enabled replication of several classic effects related to natural numbers.

Participants

The participants were 64 undergraduates (48 female, 16 male) from a large, Midwest university. The average age of the participants was 20.7 years (*SD* = 2.5). They were recruited via personal appeals at the beginning or end of courses, email messages to a list of undergraduates interested in completing mathematical cognition studies, and fliers posted around campus. The criteria for inclusion for this experiment were that participants had to be university undergraduates and between the ages of 18 and 22. They were compensated $12 for approximately one hour of their time. This study was approved by the local Institutional Review Board.

Design

Experiment 1 was comprised of three tasks. The Magnitude Comparison task had one within-subjects factor, Type, with the levels of *Decimal, Fraction, and Mixed.* Response time and accuracy were the dependent variables. A repeated measured MANOVA was used to test the effect of Type as the same variables, response time and accuracy, were measured for each participant three times, varying by Type. This test is sensitive to differences due to a factor within the same participant measured repeatedly.

The Number Line Estimation (NLE) task had one within-subjects factor, Type, with levels of *Decimal and Fraction*. Time and absolute error were the dependent variables. A repeated measures MANOVA was also employed to analyze this task.

Finally, mean response times from the Magnitude Comparison task for each type and mean absolute errors from the NLE task for each type were used as independent predictors with the Rational Number Ability test and ACT math sub-scores as dependent variables in regression analyses. Hierarchical linear regression modeling was used here to test how much of the variance in each dependent variable can be predicted by the relevant levels of the Magnitude Comparison and NLE tasks, controlling for general academic ability.

Measures

Magnitude Comparison Task

The three blocks tested each of the three conditions – Decimals, Fractions, and Mixed – defined by the Type factor. Participants were randomly assigned to make either greater judgments or lesser judgments across all three blocks. Each block consisted of 48 trials. On each trial, participants compared two numbers. All numbers were pseudo-randomly generated to create fractional values between 0-1 with denominators and numerators between 1 and 10 (for the fractional versions) and converted to decimals rounded to the nearest tenths or hundredths along with the other restrictions mentioned here (see Table 1 for a sample of stimulus used).

Table 1
Sample Mixed Magnitude Comparison Stimuli

Left Column	Right Column	Left Column	Right Column
0.21	9/10	0.62	1/3
0.24	7/10	8/9	0.74
2/5	0.82	0.33	1/6
0.18	3/7	8/9	0.74
5/8	0.83	5/6	0.68
0.13	3/10	0.47	3/10
1/3	0.48	0.32	1/7
0.16	3/10	4/7	0.38
3/7	0.49	0.42	1/10
5/7	0.74	3/4	0.39
0.11	1/8	0.67	1/7
8/9	0.92	9/10	0.16

Note. Sample stimuli of half of the Mixed Magnitude Comparison Block (24 trials). A listing of the full stimulus set is available in Appendix A.

The numbers were counterbalanced in two ways to guard against response bias. Within each block, the larger number appeared on the left half of the time and the right half of the time. Also, within each block, the numbers appearing in the left were chose so that the average of the number used across all trials was close to 0.5; the same was true of numbers appearing on the right.

The stimuli were structured to enable testing classic magnitude comparison effects in the mathematical cognition literature. Within each block, half of the stimuli were far comparisons (i.e., the difference between the two numbers was greater than .2) and half were near comparisons (i.e., the difference between the two was less than or .2). This design allowed me to test the distance effect (Moyer & Landauer, 1967).

In the Decimal block, the stimuli were additionally structured to enable testing of unit-decade compatibility effect. This is the finding that the tens and ones digits can interfere when people compare two-digit numbers (Nuerk, Weger, & Willmes, 2001). For example, people compare 26 and 37 quickly because both the ones and tens digits lead to the same judgment (i.e., 2 < 3 and 6 < 7), whereas they compare 16 and 73 slowly because they lead to conflicting judgments (i.e., 1 < 7 but 6 > 3). This effect has been extended to decimal comparisons (Varma & Karl, 2013). Of the stimuli, 24 were congruent, i.e., the larger number had larger digits in both the tenths and hundredths places (e.g., .26 vs. .37). The remaining 24 were incongruent, i.e., the larger number had a larger digit in the tenths place but a smaller digit in the hundredths place (e.g., .16 vs. .73). In addition, six of the incongruent trials include one number that had a tenths place but no hundredths place (e.g., .23 vs. .9). These trials enabled testing of the semantic interference effect for the decimal numbers (Varma & Karl, 2013). This is the finding that people are slower when the decimal proportion and natural number interpretations lead to conflicting judgments (e.g., .23 < .9 but 23 > 9).

In the Fraction block, six of the stimuli had the same denominator (e.g., 2/5 vs. 3/5) and six had the same numerator (e.g., 1/3 vs. 1/4). These trials were included to ensure the overall task was not too difficult for participants since comparisons with the same numerator or same denominator have been shown to be easier for children to process when initially learning fractions (Behr, Wachsmuth, Post, & Lesh, 1984). The remaining comparisons varied both numerators and denominators (e.g., 2/3 vs. 3/10), ensuring that participants could not rely on any single strategy.

In the Mixed block, participants compared fractions to decimals (e.g. 1/3 vs. .42). The numbers for the Mixed condition were randomly selected using the same criteria as those for the Decimal and Fraction conditions. The numbers were counterbalanced to guard against response bias in two ways. First, the decimal number appeared on the left half of the time and the right half of the time. Second, the decimal number was the greater number half of the time and the lesser number half of the time.

The Decimal and Fraction blocks were the first or second blocks as determined by the Order factor. The Mixed condition was always last. Thus, there were two orders: <Fraction, Decimal, Mixed> and <Decimal, Fraction, Mixed>. This ordering allowed for the testing of whether processing one format of rational number facilitates processing of the other format as a secondary analysis.

Each block began with a set of 12 practice trials of the same type as the 48 experimental trials that followed. The practice trials were representative of the experimental trials.

Number Line Estimation Task

The four blocks tested each of two conditions-decimals and fractions-defined by the Type factor. Participants were randomly assigned to either complete the two decimal or the two fraction blocks first. Each block consisted of 38 trials. On each trial, participants placed a number on a number line between 0-1 with no landmarks marked (except for the ends at 0 and 1). See Figure 2 below for a screen shot of this task. All numbers were between 0 and 1 and none were .5. The numbers selected for this task were adapted from Siegler (2014) and are shown in Table 2.

Table 2
Number Line Estimation Stimuli

Fractions		Decimals	
1/19	6/11	0.05	0.55
1/11	5/9	0.09	0.56
1/10	4/7	0.10	0.57
2/17	5/8	0.12	0.63
1/8	9/14	0.13	0.64
2/13	7/10	0.15	0.70
1/6	5/7	0.17	0.71
3/16	11/15	0.19	0.73
3/14	3/4	0.21	0.75
2/9	7/9	0.22	0.78
3/13	5/6	0.23	0.83
4/15	11/13	0.27	0.85
2/7	7/8	0.29	0.88
3/10	8/9	0.30	0.89
1/3	9/10	0.33	0.90
6/17	11/12	0.35	0.92
3/8	13/14	0.38	0.93
5/12	17/18	0.42	0.94
7/16	18/19	0.44	0.95

The numbers were counterbalanced such that half were above .5 and half below, with a broad range between 0 and 1. Further, the mean of all numbers in each block was approximately .5.

Each block began with a set of 10 practice trials of that were representative of the 38 experimental trials that followed but with different numbers. Figure 2 shows a screen shot of the NLE task. Participants responded by using the computer mouse to click on the number line below within 5 seconds.

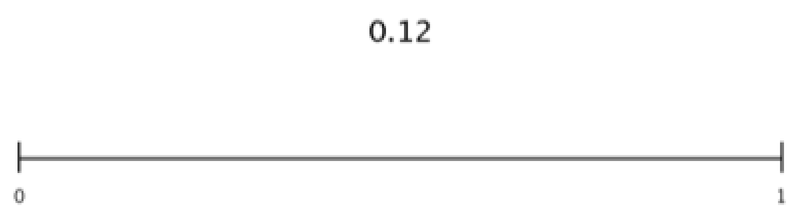

Figure 2. Screen shot of the image participants viewed as they completed the Number Line Estimation task in Experiments 1 and 2.

Strategy Self-Reports

After both the Magnitude Comparison task and the NLE task, participants completed a paper-and-pencil questionnaire where they self-reported the strategies they believe they used to perform each condition of each task. For the Magnitude Comparison task, the questionnaire was a single page, beginning with the instructions, “Please list all the strategies you can remember that you employed during this task to determine which value was larger or smaller for comparing:” followed by an example of each type of comparison (e.g. 1. Decimals to Decimals .15 vs .24). For the NLE task, the questionnaire was also a single page, with instructions to “Please list all the strategies you can remember that you employed during this task to determine where you placed each number for:” followed by an example of each type of marking (e.g. Decimals .24).

Rational Ability Test

Participants then completed a paper-and-pencil test of basic skills related to rational numbers; see Appendix C. It was developed by considering items and sections from a pre-algebra text book (Yang, 2003). The test includes 3 sections. The first section includes six items relating to conversions, such as converting 3/5 to a decimal representation. The second section includes eight items relating to simple computations, such as 3/4 – 2/3 = _____. The third section includes 6 items relating to computations similar to the second section, but of a more complex and verbal nature, such as "What number is 1/4 of 150?". The test included 20 items total.

Mathematical Achievement

Generalized math ability was measured by the participants' ACT-Math (or SAT-Quant, converted to ACT-Math) scores. This is a standard mathematical achievement measure for high school and college-age students. Because this test covers high school mathematics (i.e., through Algebra II and pre-calculus), it is unlikely that participants' knowledge of these topics will have changed much because of college mathematics experiences. Participants' standardized test scores were obtained from the Office of Student Records. Participants granted written permission to access these scores through the consent process.

Procedure

After consenting to the study, participants completed the three tasks on a Dell PC running Windows 7 Enterprise in an isolated room. The Magnitude Comparison task was employed via the software program E-prime, version 2.0 and the NLE task was employed via Java.

Participants completed the two tasks in one of 8 orders. Participants either started with the Magnitude Comparison or NLE task (2 options). Within the Magnitude Comparison task, participants were randomly assigned to complete either greater than or less comparisons. Also, within the Magnitude Comparison task, participants either started with the Fraction or Decimal condition first (2 options), and completed the other second, with the Mixed condition was always last; this was the Order factor. Finally, within the NLE task, participants either completed the two Decimal blocks together first and then completed the two Fraction blocks, or the opposite (2 options). Thus, these combinations produce 2x2x2 = 8 orders, with half the participants completing greater than judgements and the other half completing less than judgements in the Magnitude Comparison task. These 8 orders were included to counterbalance across any possible order effects.

The Magnitude Comparison task was implemented via E-Prime 2.0 on a PC with an extended keyboard and a display measuring 55.6 cm diagonally. This program allows for the recording of response times (RT) at the millisecond (ms) level. Each trial began with a blank screen for 1000 ms, followed by ‘‘Ready’’ for 1000 ms, followed by a fixation cross (‘‘+’’) for 1000 ms, followed by the stimulus. The numbers appeared 2 spaces to the left and right of the fixation cross. Participants responded by pressing either the ‘‘Z’’ or ‘‘M’’ key, whichever was below the number representing the correct judgment (greater or lesser). After a response or a 3000 ms deadline, feedback was presented for 2000 ms (‘‘Correct’’, ‘‘Incorrect’’, or ‘‘No response detected’’). Participant performance on this task was measured by mean response time and accuracy (percent correct).

The NLE task was implemented on the same PC via a Java program adapted from a previous study of fraction comprehension in children (Siegler, 2014).[1] This program allows for the recording of response times at the millisecond level. For each trial, the number appears on the screen above the number line (approximately 20 cm in length) and the participant used the computer mouse to click on his or her estimate on the number line. Time was limit to 5000 ms on each trial at which time participants received a “times up” message and the next trial began. Participant performance on this task was measured by the mean response time and the mean absolute error. The error on each trial is the millimeter difference between the actual magnitude placement of the fraction on the number line and the location of participant’s mouse click. The absolute values of these differences will be averaged across all trials. Thus, the average will always be positive with larger averages indicate larger overall error.

After completing the Magnitude Comparison and NLE tasks (in whichever order they appear), participants completed the Strategy Self-Report for that task by pencil untimed.

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[1] I thank Robert Siegler for providing the source code for the NLE tasks.

Finally, they completed the Rational Number Ability test. Participants' time on this test was limited to 10 minutes. Performance was measured by the total number correct out of 20, with blanks and incorrect answers scored 0.

After completing the study tasks, participants were debriefed, compensated, and dismissed.

Results

61 participants were included in this analysis due to missing data due to program failures (2 participants) or overall low accuracy (1 participant whose mean accuracy on the binary choice Magnitude Comparison task was 0.34, well below the chance level of 0.50). Only 51 participants were included in analysis that involved ACT scores due to missing scores.

Data Trimming

For the Magnitude Comparison task, the RT data were trimmed using a procedure similar to that used in other studies (Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2011; Nuerk, Weger, & Willmes, 2001; Varma & Karl, 2013). First, trials with incorrect responses were excluded. Then, trials that were not within the interval 200-2000 milliseconds were removed. Finally, any remaining trials more than 3 standard deviations from each participant's mean were removed. This procedure removed 2.6% of correct trials from subsequent analyses of the RT data. Analyses of the accuracy were on the full, untrimmed data set. For accuracy analysis, all trials were included.

For the NLE task, trials that included absolute errors exceeding 3 standard deviations from each participant's mean were excluded. Next, RTs exceeding 3 standard deviations from each participant's mean were excluded from analysis. This procedure removed 3.6% of trials respectively from all subsequent analysis with both extreme errors and RTs excluded.

Differences in Representations of Rational Numbers

The first research question guiding this analysis was whether different rational number formats recruit the same mental representation or different mental representations. I addressed this question using both the Magnitude Comparison task and the NLE data.

I analyzed the Magnitude Comparison task data in a repeated measures MANOVA with within-subjects factor Type (Decimal, Fraction, and Mixed) and with dependent variable RT. The test showed a significant effect for type ($F(1, 60) = 207.2$, $p < .001$, $\eta^2 = .872$). Post-hoc testing for contrasts using Pillai's test revealed significant differences between all three types of comparisons: Decimals to Fractions, ($F(1, 60) = 344.6$, $p < .001$, Cohen's $d = 2.40$), Decimals to Mixed ($F(1, 60) = 259.5$, $p < .001$, Cohen's $d = 2.08$) and Fractions to Mixed ($F(1, 60) = 7.63$, $p < .001$, Cohen's $d = 0.36$). Mean RTs for Decimal, Fractions, and Mixed comparisons were 716 (*SD* = 107), 1062 (*SD* = 162), and 1011 (*SD* = 169) ms, respectively; see Figure 3.

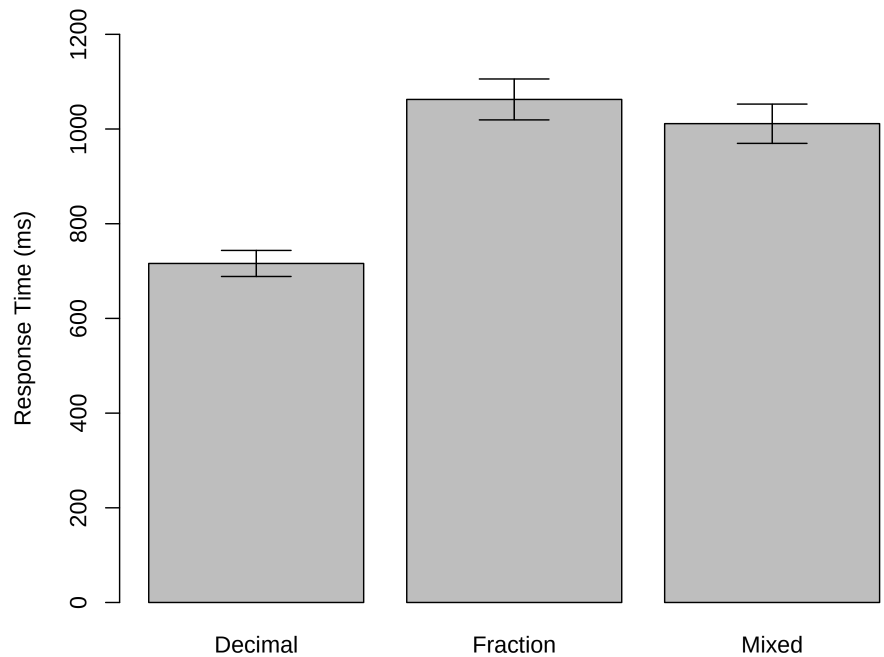

Figure 3. Mean response times by type for the Magnitude Comparison task. Error bars represent standard errors.

I conduced a parallel analysis of the accuracy data. The test showed a significant effect for type ($F(1, 60) = 65.3$, $p < .001$, $\eta^2 = .682$). Post-hoc testing for contrasts (using Pillai's test) revealed significant differences between only decimals and fractions ($F(1, 60) = 126$, $p < .001$, Cohen's $d = 1.45$) and between decimals and mixed ($F(1, 60) = 27.6$, $p < .001$, Cohen's $d = .68$). Mean accuracy rates for Decimal, Fractions, and Mixed comparisons were .97 ($SD = .03$), .86 ($SD = .13$), and .88 ($SD = .08$), respectively; see Figure 4.

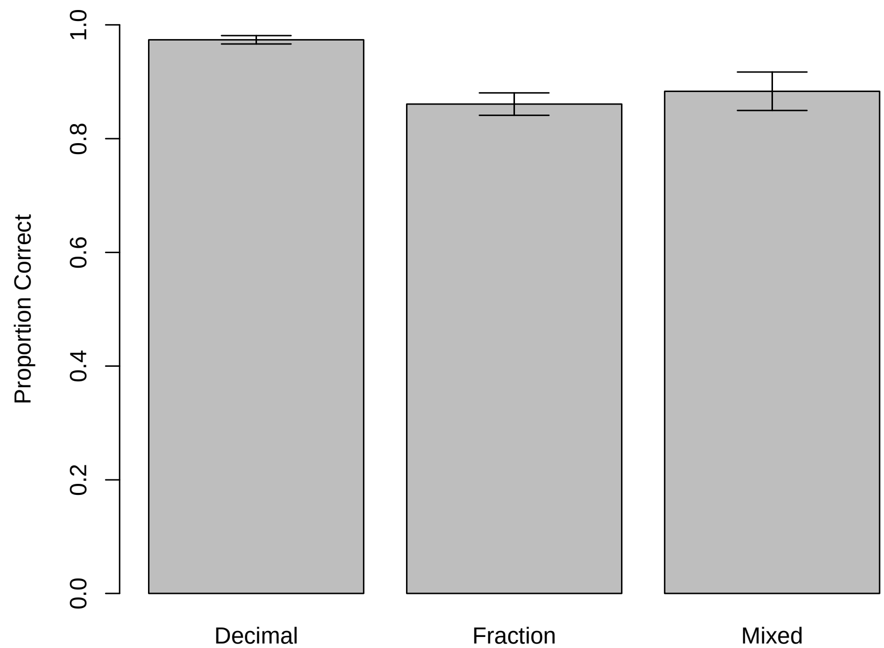

Figure 4. Mean accuracy by type for the Magnitude Comparison task. Error bars represent standard errors.

I also analyzed the NLE task data in a repeated measures MANOVA with within-subjects factor Type (Decimal and Fraction) and dependent variable response time. The test showed a significant effect for type ($F(1, 64) = 63.53$, $p < .001$, $\eta^2 = .494$). Cohen's d was .98 for the effect of type on response times. Mean RTs for Decimal and Fractions markings were 2284 (SD = 402), and 2510 (SD = 439), respectively; see Figure 5.

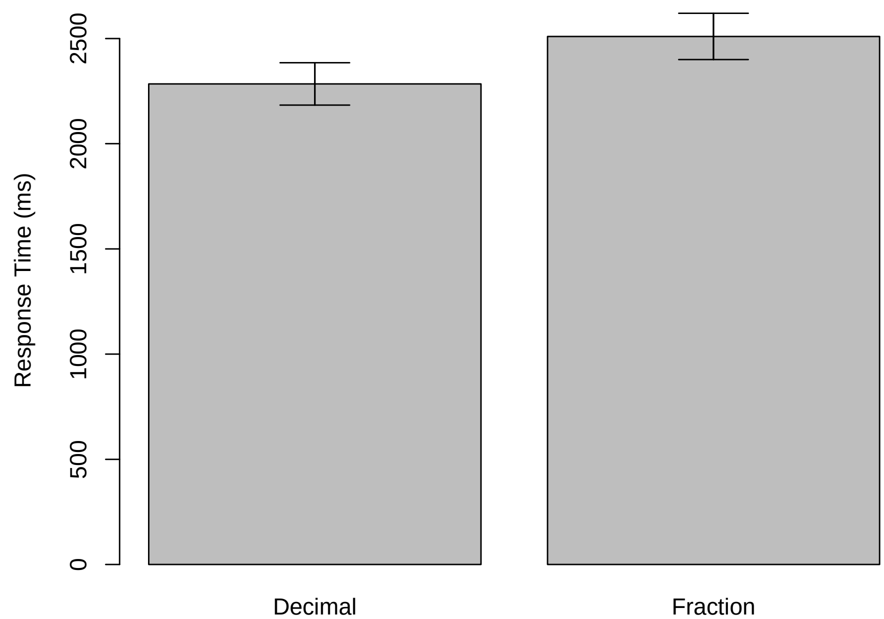

Figure 5. Mean response times by type for the NLE task. Error bars represent standard errors.

I conducted a parallel analysis of the absolute error data. Each participant's absolute error is the average across trials of the absolute difference between the actual magnitude placement of the fraction on the number line and the location of participant's mouse click in millimeters. The test showed a significant effect for type (F(1, 64) = 10.12, $p < .01$, $\eta^2 = .135$). Cohen's d was .39. Mean absolute error for Decimal and Fractions markings were 4.68 (SD = 1.53) and 5.05 (SD = 1.62) mm, respectively; see Figure 6.

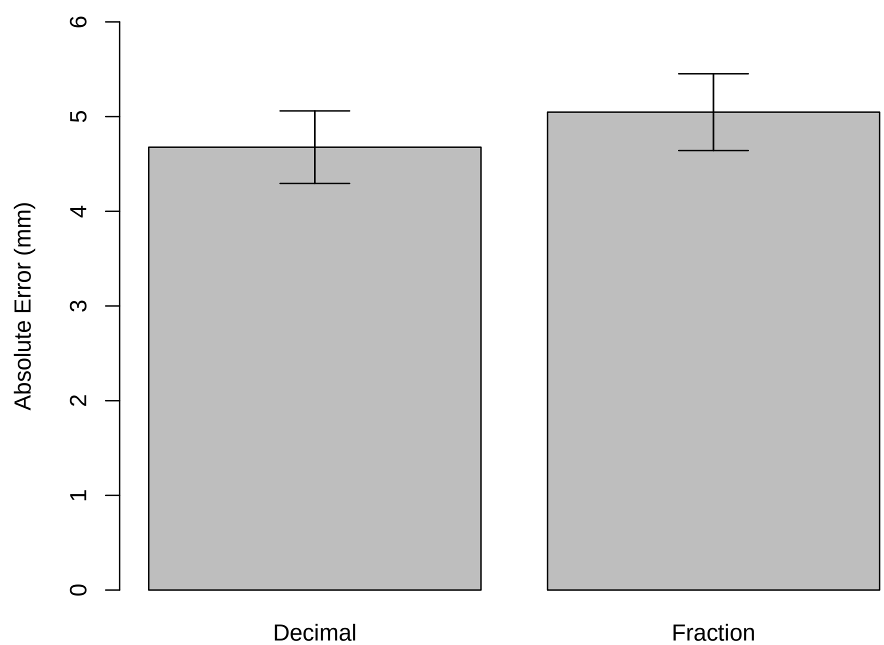

Figure 6. Mean absolute errors by type for the NLE task. Error bars represent standard errors.

Order Effect. There was one ordering effect of interest in Experiment 1: the potential difference between completing the Decimals or Fraction comparison first, before the Mixed comparisons. This ordering was intended to assess whether one facilitated the other. If Decimals facilitate Fractions or Fractions facilitate Decimals, then this is evidence for a common underlying mental representation of rational numbers, whereas if they appear to be independent, this suggest different representation depending on representational type. To test this Order effect, an independent-samples t-test was conducted to compare response times for those who completed Decimal comparisons 1st and those who completed them second. There was a not a significant difference in response times for those completing their decimal comparisons first (*M*=716 ms, *SD*=107 ms) and those completing them second (*M*=716 ms, *SD*= 110 ms); $t(59) = -.009$, $p = .993$. The same test was repeated for Fraction response times. There was a trending, but not a significant, difference in response times for those completing their Fraction comparisons first (*M* = 1019 ms, *SD* = 180 ms) and those completing them second (*M* = 1102 ms, *SD* = 150 ms); $t(59) = -1.963$, $p = .054$. Participants who completed Fraction comparisons first were slightly quicker than those completing them second. These results do not suggest a facilitation effect: warming up on decimals or fractions does not appear to make either subsequent comparisons quicker. In fact, it appears there may be a small cost to shifting from decimal to fraction comparisons for fraction processing that does not happen in reverse. This trend suggests some representational or strategy shift in mental representations of rational numbers when fractions are initially introduced that is more pronounced after getting into a routine on decimals.

Rational Ability Test

Performance on the Rational Ability test is summarized in Table 3 below and Table 4 shows correlations among overall performance, sub-sections, and items left blank.

Table 3
Summary of Performance on the Rational Number Ability Test

	Mean	SD	Max
Overall	16.04	4.18	20
Conversion	4.99	1.30	6
Computations	6.35	1.85	8
Computations (w)	4.69	1.81	6

Note. The Computations (w) refer to computational word problems.

Table 4
Correlations between Total Performance, Number Blank, and Three Sub-sections of the Rational Ability Test

	Overall	Conversions	Computations	Computations (w)
Overall				
Conversions	.864**			
Computations	.830**	.611**		
Computations (w)	.835**	.649**	.451**	
Blank	-.532**	-.365**	-.331**	-.626**

Note. ** $p < .001$; Items left blank were scored as incorrect. The Computations (w) refer to computational word problems.

These results suggest performance was relatively consistent within all sections of this test as all three correlated similarly with the overall score. Since the overall score is composed of the sub-scores, the correlations between each subsection and overall score may be masking important difference between sub-sections. To explore this possibility and to further support the use of the overall score as a single performance score for this range of questions, a principle components analysis was conducted with the three sub-scores entered as variables. A single factor was extracted with an eigenvalue above 1 explaining 71% of variance, with the three sub-scores loading on this single factor at .900, .806, and .827 respectively.

Individual Difference Measure Correlations

I initially computed the correlations between all of the following individual difference measures: average RT and accuracy for the three conditions (Decimal, Fraction, and Mixed) of the Magnitude Comparison task (6 variables), average RT and average absolute error for both conditions (Decimal and Fraction) of the NLE task (4 variables), slopes (distance/RT betas) for the three conditions of the Magnitude Comparison task (3 variables), and overall score on the Rational Ability test. I also included scores on the ACT-Math.

Slope (distance/RT betas) refers to a measure of how much the distance between the two numbers being compared affects reaction times. For each participant, for each comparison Type (Decimal, Fraction, and Mixed), a linear model was fitted predicting response times from the absolute distance between the two numbers being compared. These slopes were largely negative, as expected, indicated a distance effect. The distance effect (Moyer & Landauer, 1967) is the finding that people more quickly judge which number is greater (or lesser) the larger the distance between the two numbers being compared (e.g., comparing 2 vs. 9 is quicker than comparing 5 vs. 6). In this case, a distance effect is evidenced by a negative slope as this indicates as the distance between the two numbers increases, the time it takes to compare them decreases. This effect will be discussed later in different manner. But, here it was considered as a potential predictor of generalized math performance with the reasoning that individuals least affected by distance have superior mental representations of rational numbers. Informally speaking, they are slowed less by the more difficult, near-distance comparisons.

The final correlation matrix focused on the variables that prior studies that have used the Magnitude Comparison and NLE tasks have found most informative: RT (both raw and distance betas) for the former and accuracy for the latter (see Table 5). Correlational analysis revealed that the Rational Ability test, RTs for both the Fraction and Mixed conditions of the Magnitude Comparison Task, and Absolute Error on both Decimal and Fraction conditions of the NLE task correlated significantly with ACT Math scores. Of particular note is how strongly overall performance on the Rational Ability test correlated at .706 with ACT Math scores. A positive correlation is not surprising, but a correlation this high is surprising given the ACT covers a broader range of mathematics, including geometry and trigonometry, elementary and intermediate algebra, and a handful of miscellaneous concepts such as logarithms and matrices. Finally, the NLE task, for both the Decimal and Fraction condition, appears to be the most related to the two outcome measures of Experiment 1. In both cases, less error on this task predicted better overall performance than did the Magnitude Comparison task.

Table 5

Correlations between the Rational Number Ability Test, ACT Math Scores, and Individual Difference Measures

	1	2	3	4	5	6	7	8	9
1 ACT Math									
2 Rational Test	**.706****								
3 MC RT Dec	-.269	-.155							
4 MC RT Fract	**-.299***	**-.261***	.519**						
5 MC RT Mixed	**-.380****	-.202	.497**	.616**					
6 MC RT Beta Dec	.108	.089	.021	.054	.015				
7 MC RT Beta Fract	-.077	.066	.180	.051	-.144	-.054			

8 MC RT Beta Mix	.016	-.078	.084	-.066	-.339*	.136	.238		
9 NLE Error Dec	**-.548****	**-.412****	.324*	.203	.056	.014	.213	.095	
10 NLE Error Fract	**-.500****	**-.427****	.327*	.141	-.020	-.036	.258*	.229	.822**

Note. * $p < .05$, ** $p < .001$, MC = Magnitude Comparison task, NLE = Number Line Estimation task, RT = response times, Beta = slope of RT modeled by distance. Significant correlations with outcome measures are in bold.

Predicting Generalized Math Ability

To assess how well the individual difference measures collected in this experiment predicted more generalized rational number ability, a three stage hierarchical linear regression was conducted with scores on the Rational Number Ability test as the dependent variable. ACT English was entered at stage one to control for general academic ability. Then, in stage two, the Magnitude Comparison RT scores (for Fraction and Mixed comparisons only) were entered. Finally, in stage three, the Number Line absolute errors were entered. The results can be seen in Table 6.

Table 6
Hierarchical Linear Regression Results for Rational Ability Test as Dependent Variable

Model		Beta	t	p	Model R^2	ΔR^2
1	(Constant)		1.196	.237	.179	.179*
	ACT English	.423	3.299	.002		
2	(Constant)		1.808	.077	.219	.040
	ACT English	.383	2.908	.005		
	MC RT Fract	-.190	-1.102	.276		
	MC RT Mixed	-.022	-.126	.900		
3	(Constant)		3.525	.001	.393	.174*
	ACT English	.182	1.377	.175		
	MC RT Fract	-.138	-.885	.381		
	MC RT Mixed	-.047	-.298	.767		
	NLE Error Dec	-.059	-.283	.778		
	NLE Error Fract	-.417	-2.041	.047		

Note. * Change in F statistic significant at $p < .05$. All models were significant at $p < .001$.

As shown in Table 6, the addition of the Magnitude Comparison response times explains an additional 4% of variance in Rational Ability test scores, while the NLE absolute errors explain an additional 17% of variance. Also, it is worth noting errors on NLE fraction task are the only individually significant predictor of overall Rational Number Ability test scores in the final model. It is also important to note the correlation between the absolute errors on the two conditions of NLE task was .822, so collinearity is likely a factor in this model (and the next) that should temper over-interpretation of this finding.

Next, to assess how well the individual difference measures collected in this experiment predicted more generalized math abilities, a three-stage hierarchical linear regression was conducted with ACT Math as the dependent variable. The distribution of

ACT Math scores is shown in Figure 7 below. As can be seen the distribution has a slight negative skew, but is largely normal. ACT English was entered at stage one to control for general academic ability. Then, in stage two, the Magnitude Comparison RT scores (for Fraction and Mixed comparisons only) were entered. Finally, in stage three, the Number Line absolute errors were entered. The results can be seen in Table 7.

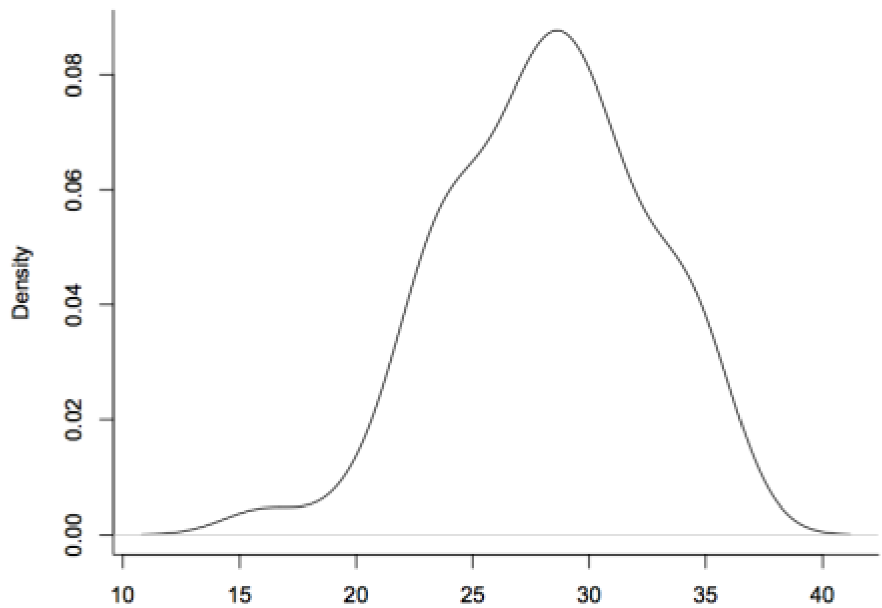

Figure 7. Distribution of ACT Math Scores used in regression analysis.

Table 7
Hierarchical Linear Regression Results for ACT Math as Dependent Variable

Model		Beta	*t*	*p*	Model R^2	ΔR^2
1	(Constant)		4.021	.000	.306	.306**
	ACT English	.553	4.691	.000		
2	(Constant)		4.202	.000	.370	.064
	ACT English	.487	4.116	.000		
	MC RT Fract	-.072	-.467	.642		
	MC RT Mixed	-.208	-1.326	.191		
3	(Constant)		5.369	.000	.476	.106*
	ACT English	.325	2.643	.011		
	MC RT Fract	-.018	-.127	.899		
	MC RT Mixed	-.232	-1.584	.120		
	NLE Error Dec	-.312	-1.624	.111		
	NLE Error Fract	-.066	-.347	.730		

Note. ** Change in *F* statistic significant at $p < .001$. * Change in *F* statistic significant at $p < .05$. All models were significant at $p < .001$.

As shown in Table 7, the addition of the Magnitude Comparison response times explains an additional 6% of variance in ACT Math scores, while the NLE errors explains an additional 11% of variance.

In both cases, we see the rational number tasks explaining variance in two general measures of math ability. And in both the rational number based measure and the much more generalized measure of mathematical achievement, errors on the NLE task are the more robust predictor of success.

Classic Magnitude Comparison Effects

After considering the primary research questions, this section turns to secondary analysis of classic effects found in previous studies on magnitude comparisons and rational numbers. These findings add to the continuing study of how rational number processing is similar to or differs from the processing of positive integers and serve as evidence that the current participants performed similarly to those of prior studies

First, I consider effects related to the distance between the two numbers being compared. These results are informative for the debate over whether people process fractions as they do integers, via an analog number line, which is considered *holistic* processing, or whether they process the numerator and denominator separately, which is considered *componential* processing. Note that the distance effect is considered an indicator of holistic (i.e., magnitude) processing.

Distance Effect. The distance effect (Moyer & Landauer, 1967) is the finding that people more quickly judge which number is greater (or lesser) the larger the distance between the two numbers being compared (e.g., comparing 2 vs. 9 is quicker than comparing 5 vs. 6). It is considered an indicator of holistic processing via an analog number line of numbers. I tested for this effect by first calculating average response times for each participant for *near* comparisons where the distance between the numbers was less than 0.2 (roughly half the trials) and for the remaining *far* comparisons. I did this separately for each of three comparison types (decimal, fraction, mixed). There was a distance effect for decimal comparisons ($t(60) = 12.191$, $p < .001$, $d = 1.561$), fraction comparisons ($t(60) = 7.906$, $p < .001$, $d = 1.012$), and mixed comparisons ($t(59) = 11.817$, $p < .001$, $d = 1.526$). These results replicate prior findings of distance effects for decimal comparisons (Varma & Karl, 2013) and fraction comparisons (Jacob & Nieder, 2009; Schneider & Siegler, 2010).

Distance Effect Slopes. I also tested for the distance effect in a more continuous fashion. For each participant, for each comparison Type, a linear model was fitted predicting response times from the absolute distance between the two numbers being compared. These slopes are largely negative, as expected. A distance effect is evidenced by a negative slope as this indicates as the distance between the two numbers increases, the time it takes to compare them decreases. The question is whether the slopes are comparable across the three types of rational number comparison, consistent with the use of common magnitude representations, or whether they differ. A repeated measures MANOVA was conducted with within-subjects factor Type (Decimal, Fraction, and Mixed) and dependent measure average RT/distance slope. There was a significant main effect of Type ($F(1, 61) = 4.90$, $p < .05$, $\eta^2 = .138$). Post-hoc contrasts using Pillai's test revealed significant differences between Fraction and Decimal comparisons ($F(1, 60) = 6.06$, $p < .05$, Cohen's $d = .32$) and between Fraction and Mixed comparisons ($F(1, 60) = 8.41$, $p < .01$, Cohen's $d = .37$); see Figure 8.

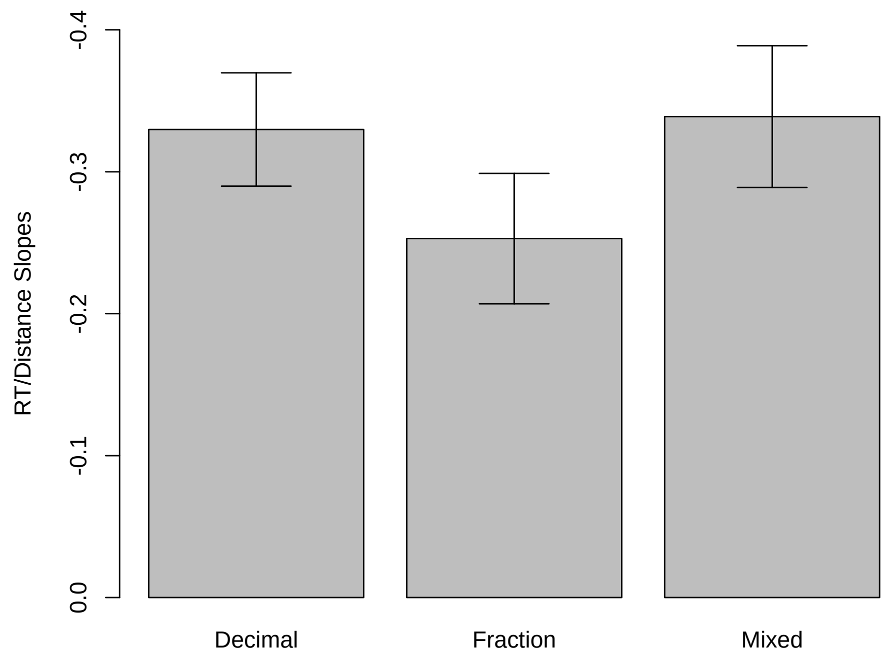

Figure 8. Mean distance effect slope by type for the Magnitude Comparison task. Error bars represent standard errors.

Next, I explored these distance effect slopes on a subset of *near* comparisons. Again, near comparisons were defined as trials in which the difference between the two numbers was less than .2, and amount to roughly half of the total trials participants completed. I focused on this subset because they require the most refined magnitude processing of rational numbers. The goal was to replicate and expand upon (by including the Mixed condition) the prior finding that the distance effect disappears for comparisons of near-distance fractions (Zhang, Fang, Gabriel, & Szűcs, 2016; DeWolf & Vosniadou, 2015). A repeated measures MANOVA was conducted with within-subjects factor Type (Decimal, Fraction, and Mixed) and dependent measure average RT/distance slope on near trials. There was a significant main effect for type ($F(1, 59) = 7.509$, $p < .01$, $\eta^2 = .206$). Post-hoc contrasts using Pillai's test revealed significant differences between Fraction and Decimal comparisons ($F(1, 59) = 12.36$, $p < .001$, $d = 2.377$) and between Fraction and Mixed comparisons ($F(1, 59) = 11.36$, $p < .01$, $d = 2.062$). Thus, the effect of representational difference is stronger when this more difficult set of trials is considered. More importantly, participants' average distance slopes are near 0 for the Fraction conditions on these near trials, while the Decimal and Fraction conditions retain negative distance slopes replicating previous findings; see Figure 9.

Thus, there is evidence that fractions are less likely to be processed like the natural numbers for particularly near comparisons, while decimal appear to still be processed similarly to natural numbers, replicating previous findings (Zhang, Fang, Gabriel, & Szűcs, 2016; DeWolf & Vosniadou, 2015). Also, by including the Mixed condition, both experiments suggest that while the inclusion of the fraction in those comparisons makes processing more difficult than the Decimal condition, the inclusion of the decimal provides an anchor that facilitates magnitude processing. In this case, it appears that anchor grounds near comparisons in a way that produces a distance effect that does not occur when only fractions are involved.

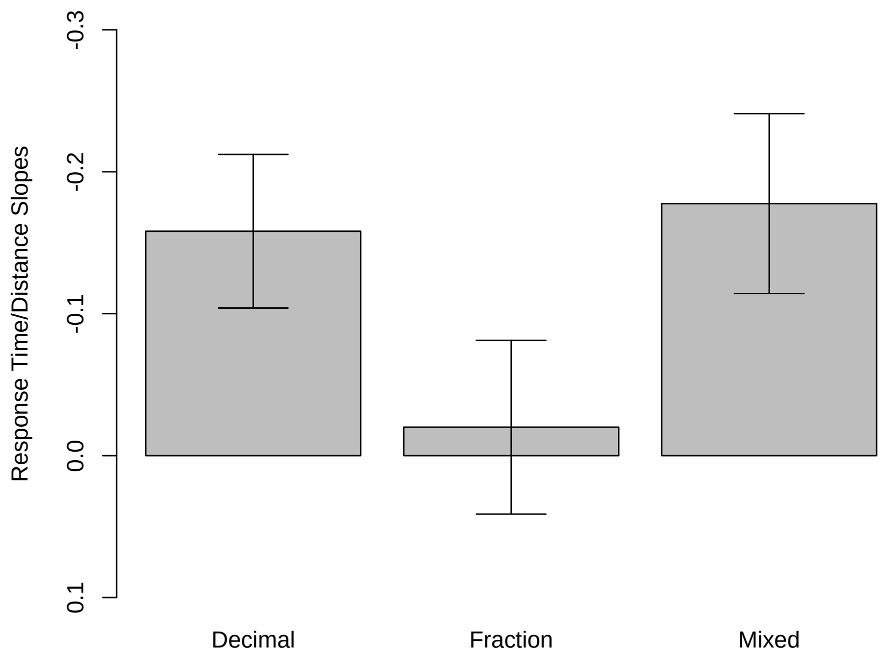

Figure 9. Mean distance effect slope by type for the Magnitude Comparison task for near comparisons only. Error bars represent standard errors.

The results of the distance effect overall in this experiment inform the debate over holistic versus componential processing of fractions, and thus whether rational numbers as a class are processed at the magnitude level. Largely, they suggest all three types of rational numbers are processed in a way that shares similarities with processing natural numbers, which supports holistic processing. However, there is evidence fractions may be the least amenable representational type to common magnitude processing based on the analysis of near comparisons where the distance effect disappeared, which suggest they require more componential processing. Further, by including the Mixed conditions, these results largely suggest that including decimals alongside fractions can encourage more holistic processing of rational numbers than do fractions by themselves.

The Size Effect. The size effect (Parkman, 1971) is the finding that people more quickly judge the greater (or lesser) of two numbers the smaller their average size, when the distance between them is held constant (e.g., comparing 3 and 4 is quicker than comparing 8 and 9). I tested for this effect by first calculating average response times for each participant for "large" comparisons, where the sum of the two numbers compared was above .5 (roughly half the trials), and also for the remaining "small" comparisons. I did this separately for each of three comparison types (decimal, fraction, mixed). None of these tests indicated a size effect for these rational number comparisons: $t(60) = -.561$, $p = .577$), $t(60) = -1.412$, $p = .163$), $t(59) = -.183$, $p = .855$), respectively. Given the numbers in these comparisons were bounded between 0 and 1, it may be this effect is unlikely to be found unless a broader range of numbers are used or a larger number of participants are analyzed to find what may be a small effect.

Unit-Decade Compatibility Effect. The unit-decade compatibility effect is the finding that the tens and ones digits can interfere when people compare two-digit numbers (Nuerk, Weger, & Willmes, 2001). For example, people compare 21 and 87 quickly because both the ones and tens digits lead to the same judgment, whereas they compare 24 and 82 slowly because they lead to conflicting judgments. This effect has been extended to decimal comparisons (Varma & Karl, 2013). This effect was tested by sorting the decimal trials into congruent and incongruent trials and comparing participants' average response times using a paired *t*-test. This experiment failed to replicate this effect ($t(60) = .405$, $p = .687$).

This null finding was likely due to a low number of truly difficult interference trials. Previous research (Nuerk, & Willmes, 2005) has found that the unit-decade compatibility effect is very small overall. Moreover, it is larger in extreme cases, when the difference in the conflicting place is large (e.g. 0.41 vs. 0.29), and absent when this difference is small (e.g. 0.41 vs 0.23); it is likely the trials used here did not include enough of these larger difference trials (in terms of the interfering hundredths digit) to replicate this effect.

Speed-Accuracy Trade-Off. To determine whether there was a speed accuracy trade-off for the magnitude comparison task, for each comparison Type (Decimal, Fraction, and Mixed), bivariate correlations were computed between each participants' average RT and accuracy. There was evidence of a speed-accuracy trade-off for Decimal comparisons ($r(59) = .335$, $p < .01$) indicating participants who took longer on average had higher accuracy rates. However, for Fraction and Mixed comparisons, there was no evidence of a speed accuracy trade-off ($r(59) = .196$, $p = 131$; $r(59) = -.121$, $p = .355$). The fact there was no speed-accuracy trade-off for Fraction and Mixed comparisons supports only using response times in the regression models predicting rational number ability and mathematical achievement. (Recall Decimal comparisons were also dropped in the final models because they demonstrated a ceiling effect.)

Strategy Self Report Questionnaires

After completing each of the Magnitude Comparison and NLE tasks, participants self-reported which strategies they believe they employed during the task. For the Magnitude Comparison task, participants reported an average of 3.03 strategies (roughly 1 per type of comparison). Table 8 lists the top 10 most reported strategies.

Table 8

Top Ten Strategies Reported on the Magnitude Comparison Task

Strategy	Percent Reporting
Fractions to decimals (f and m)	63
Consider tenths number first (d)	50
Holistic/treat as integer (d)	42
Consider relationship between numerator and denominator (f)	31
Noticed if .5 was in the middle of two numbers (f and m)	20
Took advantage of same denominator/numerator (f)	14
Convert to commonly known (e.g. close to 2/5 or .4) (f and m)	14
Denominator-based approach/consider base (f)	9
Visualize circle representation (f)	8
Base 10 strategy (f and m)	3

Note. Lower case d, f, and m refer to under which comparison type these strategies were reported. d = decimals, f = fractions, and m = mixed.

The most commonly reported strategy was converting fractions to decimals during the Fraction and Mixed comparisons with 63% of participants reporting this strategy. This finding supports many of the results found in this analysis in which the Mixed condition is more similar to the consistently quicker and more integer-like

Decimal condition: in the Mixed condition there appears to be a cost to translating one fraction to a decimal that is more complicated and difficult in the Fraction condition when there are two fractions to convert (or deal with componentially).

For the NLE task, participants reported an average of 2.86 strategies, nearly 1.5 per condition. Table 9 lists the top 10 most reported strategies.

Table 9
Top Ten Strategies Reported on the Number Line Estimation Task

Strategy	Percent Reporting
Visualize .5, consider if number is more or less than .5 (d and f)	83
Visualize break line by denominator (f)	31
Break line into quarters/thirds to visualized placement (d and f)	23
Holistic/treat as integer (d)	22
Convert to decimals or percentages (f)	16
Decimals to Fractions (d)	13
Look for nearest common (d and f)	13
Consider distance from 0 and 1 (d and f)	8
Break by .1 (d)	8
Consider tenth 1^{st}/primarily (d)	6

In contrast to the Magnitude Comparisons, the most commonly reported strategy for both types of markings was to use .5 as an anchor as a strong starting point in deciding where to mark the numbers. 83% participants reported using this strategy on Fraction and Decimal comparisons or both. From there, participants seem to be breaking the number line by the denominator or into third or quarters, thirds, or the denominator (if a fraction) to place the values to the left or right of .5.

Discussion

Processing Different Representations of Rational Numbers

Research question (1) was which representation of rational numbers make it more difficult for people to access their underlying magnitudes. The results of this experiment largely support the idea that processing of rational numbers comes at a cost when at least one of the numbers involved is formatted as a fraction. This conclusion was supported across two tasks with measures of speed, accuracy, and error. Results from the Magnitude Comparison task showed Decimal comparisons were completed more quickly and more accurately than either the Fraction or Mixed comparisons. The same pattern of results was found for the NLE task with the Decimal estimations being done more quickly and with less error. These results support an advantage for rational numbers formatted as decimals.

Next, I consider more specifically the cost of crossing representations when magnitude processing of rational numbers is required. The Mixed condition of the Magnitude Comparison task in which participants had to compare a fraction to a decimal allowed me to test if such a cost exists. While the Mixed comparisons did take longer and were less accurate than the Decimal comparisons, the Mixed comparison were completed more quickly and accurately than the Fraction comparisons – though only the difference in response times was significant. The fact the Mixed comparisons were generally no more difficult to complete and slightly easier than the Fraction comparisons suggest crossing formats is not particularly costly for adults. It appears then that instead of a cost, there is a slight advantage when one number is a decimal as opposed to processing two fractions. Based on participant self-reports, converting fraction to decimals was the most common strategy during the Magnitude Comparison task. Therefore, it appears the inclusion of a decimal in the Mixed condition provide an anchor for participants that the Fraction condition did not have that slightly speeded processing at no cost to accuracy.

Rational Number Processing and Mathematical Achievement

Research question (2) explored the relationship between rational number magnitude processing and mathematical achievement in adults. The results of Experiment 1 demonstrated for the first time that magnitude tasks involving rational numbers can indeed predict mathematical achievement in adults, as was previously demonstrated with middle school students (Siegler & Pyke, 2013). This experiment used the Magnitude Comparison and the NLE tasks. These tasks which require magnitude processing of rational numbers combined accounted for 21% and 17% of variance in the two measures of mathematical achievement used in this experiment. The Rational Ability test measures procedural knowledge, including conversion and computations with rational numbers. The second measure of mathematical achievement was ACT math, a standardized test covering a broad range of topics including Algebra II and some pre-calculus. It is perhaps not surprising that magnitude tasks predicted the Rational Ability test given that both use symbolic representations of rational numbers. However, it is striking that ACT math scores were predicted by these rational number magnitudes tasks (after controlling for general academic/verbal ability). The ACT math section does include fractions and decimals throughout. However, few questions, if any, rely solely on conceptual or procedural knowledge of rational numbers, and even fewer, if any, require direct access of magnitude representations of rational numbers as a primary task feature. (That calculators are allowed is another reason such processing is not required.)

Next, I considered more specifically whether either or both representational format (decimal or fraction) of rational numbers predicts mathematical achievement.

While the answer to the other questions in this experiment were relatively clear, the answer to this question was not. First, the decimal condition of the Magnitude Comparison task does not appear related to either outcome measure in this experiment. Since there was a ceiling effect present with this condition (accuracy on decimal comparisons was 97%), it is reasonable to assume there was not enough variability in performance to predict other, related abilities. By contrast, based on correlational and regression analysis, performance on both fraction and mixed conditions of the Magnitude Comparison task was related to both mathematical achievement measures. Further, errors on the fraction condition of the NLE task was the single significant predictor of performance on the Rational Ability test in the final full model (see Table 6).

However, in the final model predicting ACT Math scores, errors on the decimal condition of the NLE task were closer to approaching significance and had a beta weight much larger in magnitude than did the errors on the fraction condition (see Table 7). Since the two conditions of the NLE task correlated at .822, collinearity was an issue. Thus, it remains unclear if the findings regarding the decimal versus fraction conditions of the NLE task can be interpreted absolutely in either model. Overall, these findings suggest that either representation (or both) may be related to mathematical achievement.

Next, I considered whether either or both type of magnitude *task* (Magnitude Comparisons or Number Line Estimation) predicts mathematical achievement. The NLE task, for both the decimal and fraction conditions, appears to be the most related to the two outcome measures. In both cases, less error on this task predicted better overall performance than did the Magnitude Comparison task. This is true based on correlational analysis and the regression models, particularly the significant contribution in variance explained in the final models. For the Rational Number Ability test and ACT Math scores, the two best predictors from the Magnitude Comparison task added an additional 4% and 6% of variance explained, while absolute errors the NLE task added 17% and 11% respectively.

Finally, I briefly consider whether individual differences in *procedural problem solving* specifically with rational numbers predict mathematical achievement, as measured by the ACT. Procedural problem solving was one of the outcome measures used in the primary analysis of this experiment: the Rational Ability test. While the focus of this experiment was on the tasks that require comparing or representing on a number line the actual magnitude of rational numbers on two different outcome measures, the relationship between the two outcomes measures themselves is of interest. The correlation between the Rational Ability test and ACT math scores was .706. This relationship is rather large. On the one hand, performance on the Rational Ability test, distinct from the lower level magnitude processing of the other tasks, requires more knowledge of formulas and concepts. Therefore, a significant correlation between the two outcome measures is not surprising. On the other hand, just as was mentioned previously regarding the relationship between the magnitude tasks and ACT Math, good performance on ACT math cannot be attributed directly to knowledge, procedural or otherwise, of rational numbers. Therefore, the *magnitude* of the relationship between the two outcome measures is surprising. This correlation provides additional evidence there is something transferable about rational number ability to other skills.

Rational Numbers and Classic Effects

Finally, Magnitude Comparison task employed in this experiment was designed to allow for detecting classic effects found in previous studies on magnitude comparisons and rational numbers. Thus, they serve as evidence that the current participants performed similarly to those of prior studies. More importantly, these effects were originally found with natural numbers. Thus, these findings add to the continuing study of how rational number processing is similar to or differs from the processing of positive integers and would serve as evidence of an integrated number theory proposed by Siegler and Lortie-Forgues (2014).

For the purposes of this discussion and study overall, I focus on the finding regarding the *distance effect*. The distance effect (Moyer & Landauer, 1967) is the finding that people more quickly judge which number is greater (or lesser) the larger the distance between the two numbers being compared (e.g., comparing 2 vs. 9 is quicker than comparing 5 vs. 6). This experiment found a distance effect for all three types of comparisons. These results suggest all three types of rational numbers are processed in a way that shares similarities with processing natural numbers.

However, there is evidence fractions may be the least amenable representational type to common magnitude processing. First, while the distance effect was present for fraction to fraction comparisons, in comparative analysis it was significantly weaker than the distance effect for decimal to decimal and mixed comparisons. Further, when analyzing particularly near comparisons, a subset that took longer for all participants regardless of representational type, the distance effect disappeared for fraction comparisons (was near 0), but remained for the other two types. Finally, since the distance effect for the Mixed condition was similar to that for the Decimal condition, and remained in the analysis of near comparisons, these results suggest that including decimals alongside fractions may encourage more holistic processing of rational numbers than do fractions by themselves.

Chapter 5: Experiment 2

Experiment 2 applied the methods used in Experiment 1 to bridge between mathematical cognition and behavioral economics. Assessing the likelihood of outcomes expressed as probabilities requires both an understanding of the magnitudes of these rational numbers and what these magnitudes mean in contexts involving other, sometimes irrelevant, information. One such source of irrelevant information is information that is framed positively or negatively. For example, whether a person got 80% correct or 20% incorrect on an exam. Note the information is exactly the same, but research shows that people evaluate information differently based on frame. Experiment 2 explored the relationship between people's magnitude representations of rational numbers and their understanding of the framing of probabilistic information as gains vs. losses as revealed by the *framing effect*.

The key difference between Experiment 1 and 2 was modifying the Number Line Estimation task to test the framing effect, by referring to probabilities as gain or losses and asking participants to mark their estimates on the number line. This modification allowed testing of the effect of frame on the precision and direction of people's magnitude estimates. I also include the standard framing effect task designed by Tversky and Kahneman (1981), the "Asian disease" problem, to confirm participants were behaving as in previous studies and to compare and contrast this task to the new Probability Number Line task. Experiment 2 addressed two main questions:

3. Does the framing of probabilistic information affect people's magnitude representations of rational numbers?
4. Are individuals with stronger rational number ability less susceptible to exhibiting the framing effect?

Thus, in Experiment 2 the framing effect was explored at two levels. First, it was explored at the magnitude level, based on how people mark their estimates of probabilities framed as either Gains or Losses on a number line. Next, it was explored at the decision level, based on their answers to the Asian Disease problem. Finally, by creating a composite rational number ability score using the same tasks employed in Experiment 1, Experiment 2 explored how the framing effect interacts with rational number ability.

Participants

The participants were 68 undergraduates (48 females, 20 males) from a large Midwestern university. The average age of participants was 20.6 years (*SD* = 2.4). They were recruited via personal appeals at the beginning or end of courses, email messages to a list of undergraduates interested in completing mathematical cognition studies, and fliers posted around campus. The criteria for inclusion for this experiment were that participants had to be university undergraduates and between the ages of 18 and 22. They were compensated $12 for approximately one hour of their time. This study was approved by the local Institutional Review Board.

Design

Experiment 2 was comprised of four tasks. The first was the Magnitude Comparison task used in Experiment 1, with the within-subjects factor Type again having three levels, Decimal, Fraction, and Mixed. Response time and accuracy were again the dependent variables. A repeated measured MANOVA was used to test the effect of Type as the same variables, response time and accuracy, were measured for each participant three times, varying by Type. This test is sensitive to differences due to a factor or factors within the same participant measured repeatedly.

The second task was the Number Line Estimation task also used in Experiment 1. Once again, Type (Fraction and Decimal) was varied as a within-subjects factor with time and absolute error as dependent variables. The same repeated measured MANOVA was used to analyze this task as well.

The third task was new. The Probability Number Line Estimation task had one within-subjects factor, Frame, with levels of *Gain* and *Loss*. *Gain* was designed to elicit positive framing effects while *Loss* was designed to elicit negative framing effects. The dependent variables were the summed absolute error and the summed raw (i.e., directional) error of participant's markings on the number line. These measures are defined in more detail below in the Measures subsection. The same repeated measured MANOVA was used to analyze this task as well.

The fourth task was also new. Participants solved the Asian Disease problem (Tversky & Kahneman, 1981). This problem had one between-subjects factor, Frame, with levels of *positive* and *negative* valence. The dependent variable was the proportion of participants within each group who choose the sure bet option or risky-choice option. A *z*-test of proportions was used to analyze the difference in proportions between two groups as this test compares proportions, considering the size of each group in computing significance.

Measures

Magnitude Comparison Task

Experiment 2 used the same Magnitude Comparison task as described in Experiment 1.

Number Line Estimation Task

Experiment 2 used the same NLE task as described in Experiment 1.

The Probability Number Line Estimation Task

This task was implemented via the same NLE Java program used in Experiment 1. This program allowed for the recording of response times at the millisecond level. Participants completed 4 blocks of 38 trials each. Each trial consisted of placing a number (expressed as a percent) on a number line with poles 0 and 1 marked. No internal landmarks (e.g., the midpoint) were marked. All numbers were between 0% and 100%, and excluded 50%. All of the numbers used in this task are listed in Appendix B.

The numbers selected for this task were adapted from Siegler (2014) to ensure that no numbers were repeated when fractions were converted and rounded to decimals and ultimately percentages. These were the same numbers used in Experiment 1 for the

Number Line Task, but with each number converted to a percent. In addition, each percent was subtracted from 100 to form its complement.

Two conditions were tested in order to assess the framing effect. Two blocks referred to these probabilities positively, as *gains*, and two blocks referred to these probabilities negatively, as *losses*. Each number was tested both as a gain and a loss. Further, these numbers were classified as small (0-30%; 14 numbers), medium (30-70%; 10 numbers), and large (70-100%; 14 numbers) to support analyses of whether the effects of Frame vary depending on the size of the probability.

Rational Ability Test

Experiment 2 used the same Rational Ability test as Experiment 1.

The Asian Disease Problem

Participants also completed a single-item questionnaire as in Tversky and Kahneman's (1981) original framing effect study. For this task, all participants read the following introduction:

Imagine that the United States is preparing for an outbreak of an unusual Asian disease that is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Scientific estimates of the consequences of the programs are as follows:

Then, half the participants were randomly assigned to read and choose between the following two options (the positive frame):

If Program A is adopted, exactly 200 people will be saved.

If Program B is adopted, there is a 1 in 3 probability that all 600 people will be saved and a 2 in 3 probability that no people will be saved.

The other half read and choose between the following two choices (the negative frame):

If Program C is adopted, exactly 400 people will die.

If Program D is adopted, there is a 1 in 3 probability that nobody will die and a 2 in 3 probability that all 600 will die.

It is important to note that choices A and C and choices B and D are numerical equivalents. A and C represent the same sure bet of saving 200 people. B and D represent the same risky chance of saving or losing everyone, with an expected value calculation of saving (1/3 x 600 =) 200 people. Therefore, participants' decisions should not vary between the two framings if they are based on a rational judgment of the numerical information presented.

Procedure

After consenting to the study, participants completed the four tasks on a Dell PC running Windows 7 Enterprise in an isolated room. The Magnitude Comparison task was employed via the software program E-prime version 2.0 and the NLE tasks were employed via Java.

They completed the five tasks in one of 16 possible orders. Participants either started with the NLE task or the Probability NLE task (2 options). Within the NLE task, participants either completed the two Decimal blocks together first and then the two Fraction blocks, or vice versa (2 options). Within the Probability NLE task, participants either completed the two Gain blocks together first and then completed the two Loss blocks, or vice versa (2 options). Between the two number line tasks, participants completed the Magnitude Comparison task. Within this task, participants were randomly assigned to complete either greater than or less comparisons. Also, they either started with the Fraction or Decimal Condition first (2 options), and completed the other second; the Mixed condition always came last. Next, all participants completed the Rational Ability test. Finally, all participants completed the Asian Disease problem last. These combinations produced 2 x 2x 2 x 2 = 16 orders, with half completing greater than comparisons in the Magnitude Comparison task while the other half completed less than comparisons. Participants were randomly assigned to one of these 16 orders to counterbalance across order effects.

The Magnitude Comparison task was implemented as in Experiment 1.

The NLE task was implemented as in Experiment 1.

The Probability NLE task was implemented using the same program as the Number Line Estimation task. On each trial, the number and framing appeared on the screen above the number line, which was 20 cm long. The participant used the computer mouse to estimate its position on the number line; they had up to 5 second to respond. In the Gain condition, the number X was framed as “the chance of a X% gain”. The phrase “the chance of” was chosen to prime participants to interpret the number as a probability, and not merely as a percent. In the Loss condition, the number was framed as “the chance of a X% loss”. Figure 10 shows a screen shot of this task.

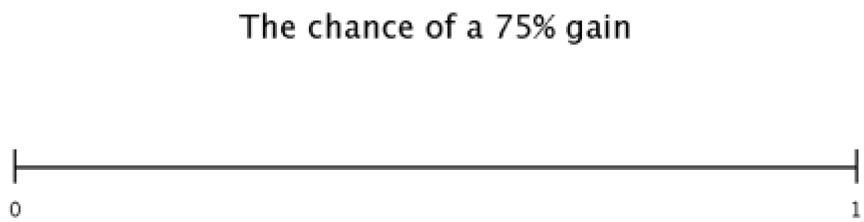

Figure 10. Screen shot of the image participants viewed as they completed the Probability NLE task.

Before beginning this task, participants read a brief statement about probabilities and gains and losses:

> *In this task, you are going to represent the chance of something happening. Chances can be expressed in different ways and represent different outcomes, like a coin flip—the chance is ½ or 50% and you can gain (win) or lose. Another name for these kinds of chances is probability—how likely is something to happen.*
>
> *In this task, you are going to see chances expressed as percents, like 50% in the example above. You will do a set in which you are going to consider what these percents mean about the chance of gaining something and a set in which you consider what these percents mean about the chance of losing something. You will not know exactly what it is that can be gained or lost, but that is okay because we want you to think generally about it.*

> *You will mark these percents on a number line between 0 and 1 to represent these chances. You will do some practice trials first to have an opportunity to get an idea of what we are asking you to do.*

Participants then proceeded to the task. At the beginning of the first Gain block and the first Loss block, participants completed a brief practice round of 10 trials to familiarize themselves with the interface and to ensure they did not misinterpret the "loss" wording and mistakenly invert the probabilities (e.g., place a 62% chance of a loss on the left side of the number line, suggesting an interpretation of a 38% chance of a positive outcome).

The dependent variables for this task were mean absolute error and mean raw error, computed separately for the Gain and Loss conditions. The mean absolute error was computed as in the NLE task. The error on each trial is the millimeter difference between the actual magnitude placement of the fraction on the number line and the location of participant's mouse click. The absolute values of these differences were averaged across all trials. Thus, the average was always positive, with larger averages indicating larger overall error. It was used for analyses where the *precision* of people's markings was important. The mean raw error was computed similarly, with the differences that the *raw* errors (i.e., the differences between the actual positions of the numbers and a participant's estimates) were averaged, not their absolute values. As a result, this measure can be positive, negative, or zero. It was used for analyses where the *direction* of errors was potentially important.

Two additional dependent measures were created for analysis based on summed and raw absolute errors. An Absolute Framing Effect Score was calculated for each participant by taking the difference between their average absolute errors on Gain trials and their average absolute errors on Loss trials. Thus, positive Absolute Framing Effort Scores indicate more error in Gain condition, scores of 0 indicate no difference between the conditions, and negative scores indicate more error in Loss condition.

Similarly, a Directional Framing Effect Score was calculated for each participant by taking the difference between their average raw errors on Gain trials and their average raw errors on Loss trials. Thus, a positive Directional Framing Effort score indicates probabilities framed as Gains were on average marked further to the right (indicating more positive error) compared to probabilities framed as Losses. A score of 0 indicates no difference between the conditions in terms of directional bias. A negative score indicates probabilities framed as Losses were on average marked further to the right (indicating more positive error) compared to probabilities framed as Gains.

To better understand what this score means, consider two hypothetical participants: Participant 1 and Participant 2. Participant 1 has raw errors on the Gain trials that average -1.2 and raw errors on Loss trials that average 3.1 (note these values represent average mm differences from the correct markings with overall direction captured). Figure 11 below shows these averages in reference to 0.5. For Gain trials, note the participant is biased to the left or negatively of .5. For Loss trials, note the participant is biased to the right or positively of .5.

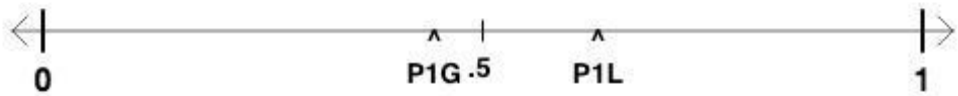

Figure 11. Depiction of Participant 1's raw average errors by condition for the Probability NLE task. Figure not drawn to scale. G = average raw errors for Gain markings. L = average raw errors for Loss markings.

To calculate Participant 1's Directional Framing Effect Score, we take the difference between -1.2 and 3.1, which is -4.3. Since this score is negative, this means there was a directional difference across trials between Gain and Loss markings for this participant, with Losses on average being further to the right compared to Gains. Note that if we shifted Participant 1's average markings for *both* Gain and Losses to the right by 2 mm, he or she would still have a Directional Framing Effect Score of -4.3 because this participant would still have a pattern of markings that demonstrate a bias to mark losses further to the right compared to gains.

Now, consider Participant 2. Participant 2 has raw errors on the Gain trials that average 2.15 and raw errors on Loss trials that average -2.15. Figure 12 below shows these averages in reference to .5. For Gain trials, note the participant is biased to the right or positively of .5. For Loss trials, note the participant is biased to the left or negatively of .5.

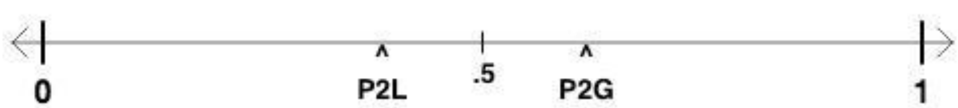

Figure 12. Depiction of Participant 2's raw average errors by condition for the Probability NLE task. Figure not drawn to scale. G = average raw errors for Gain markings. L = average raw errors for Loss markings.

To calculate Participant 2's Directional Framing Effect Score, we take the difference between 2.15 and -2.15, which is 4.3. Since this score is positive, this means there was a difference across trials between Gain and Loss markings for this participant with Gains on average being further to the right compared to Losses. Note that if we shifted Participant 2's average markings for both Gain and Losses to the left by 3 mm, he or she would still have a Directional Framing Effect Score of 3.4 because this participant would still have a pattern of marking that demonstrates a bias to mark Gains further to the right compared to Losses.

The key idea here is that a negative, 0, or positive Directional Framing Effect score is an indication of a participant's bias in the Gain condition *compared to* their bias in the Loss condition, not his or her overall tendency left or right. Also note this score is independent of overall precision. Since this score begins with averaging errors for each condition, large positive errors can be canceled out by large negative errors. Therefore, this score is an indication of how frame affects each participant's directional tendency independent of both his or her overall directional tendency and precision on marking probabilities.

Next, participants completed the Rational Number Ability Test. Participants' time on this test was limited to 10 minutes. The dependent variable was the total number correct out of 20, with blanks and incorrect answers scored 0.

Finally, participants completed the Asian Disease problem. There was no time limit. Performance was measured by dummy coding 0 for all sure bet—choosing an exact value to be saved or lost—responses and 1 for all risky choice options—choosing the option to save or lose all based on odds. The dependent variable for this task was the difference between the proportion of participants choosing the sure bet option versus the risky-choice option, computed separately for the two framing groups. Thus, this was the only measure analyzed as a between-subjects factor.

After completing the study tasks, participants were debriefed, compensated, and dismissed.

Results

For the Magnitude Comparison task, the RT data were trimmed using a procedure similar to that used in other studies (Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2011; Nuerk, Weger, & Willmes, 2001; Varma & Karl, 2013). First, trials with incorrect responses were excluded. Then, trials that were faster than 200 ms or slower than 2000 ms were removed. Finally, any remaining trials more than 3 standard deviations from each participant's mean were removed. This procedure removed 9.74% of the RT data. Analyses of the accuracy were on the full, untrimmed data set.

For the NLE task, trials where the absolute error was more than 3 standard deviations from each participant's mean were excluded. Next, RTs that were farther than 3 standard deviations from each participant's mean were excluded. This procedure removed 4.82% of trials from all subsequent analyses of both the RT and accuracy measures.

For the Probability NLE task, the same trimming procedures were used as for the NLE task. This procedure removed 3.8% of trials from all subsequent analyses of both the RT and accuracy measures.

As a reminder, the framing effect was measured in two ways. First, it was measured at the magnitude level, based on number line estimates of probabilities framed as either Gains or Losses. Second, it was measured at the decision level, based on a participants' answers to the Asian Disease problem.

The Framing Effect and the Probability Number Line

The first research question was whether the framing (as gains vs. losses) affects people's magnitude representations of probabilities. To address this question, I first analyzed the Probability NLE task in a repeated measures MANOVA with within-subjects factor Frame (Gain and Loss) and dependent variable absolute error. There was no effect of Frame ($F(1, 65) = 1.16$, $p = .286$, $\eta^2 = .017$). Mean absolute errors for the Gain and Loss markings were 3.612 ($SD = .866$) and 3.528 ($SD = .843$) mm, respectively; see Figure 13.

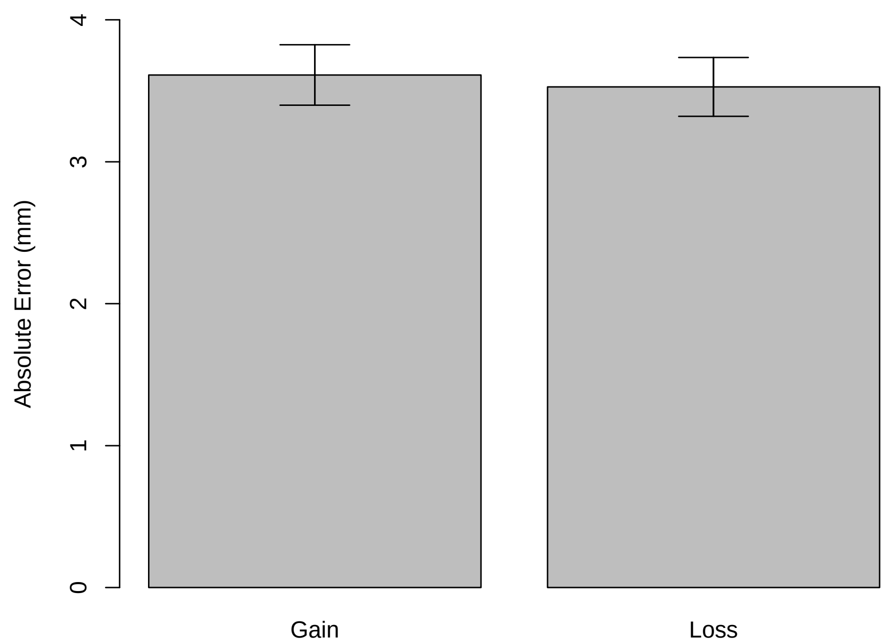

Figure 13. Mean absolute errors by condition for the Probability NLE task. Error bars represent standard errors.

Next, I conducted an analogous analysis in a repeated measures MANOVA with within-subjects factor Frame (Gain and Loss), but where the dependent variable was raw error, not absolute error. Again, there was no effect of Frame ($F(1, 65) = 0.029$, $p = .886$, $\eta^2 = .000$). Mean raw errors for Gain and Loss markings were -1.73 (*SD* =1.113) and -1.747 (*SD* = 1.062) mm, respectively; see Figure 14.

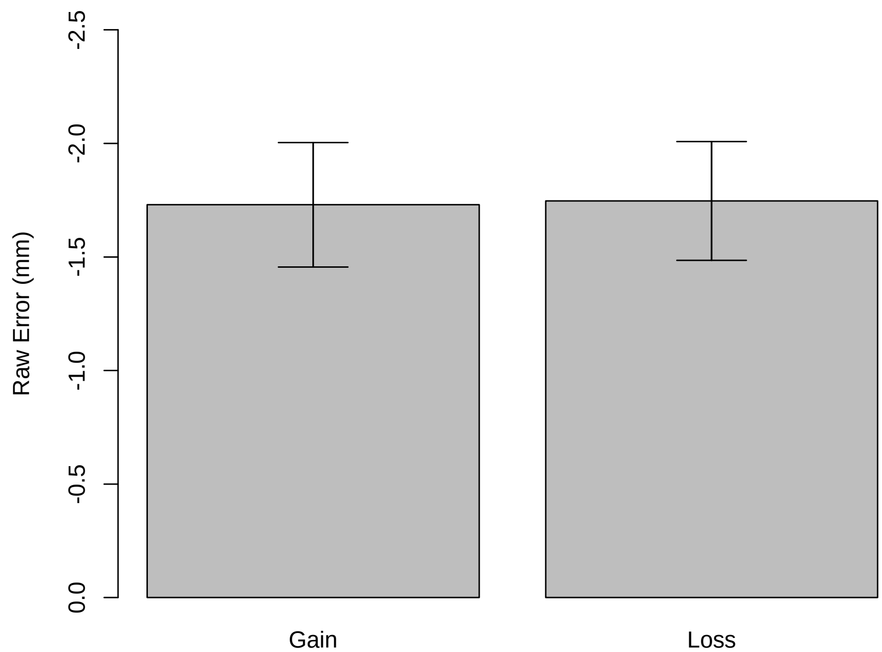

Figure 14. Mean raw errors by condition for the Probability NLE task. Error bars represent standard errors.

To refine these analyses, I investigated whether the framing effect differs based on the size of probabilities. I computed an Absolute Framing Effect Score for each participant by taking the difference between their average absolute errors on Gain versus Loss trials. Thus, a positive score indicates more error in the Gain condition, a score of 0 indicates no difference between the conditions, and a negative score indicates more error in Loss condition. I conducted a repeated measures MANOVA with within-subjects factor Size (Small, Medium, and Large) and dependent variable Absolute Framing Effect

Score. There was no effect of Size ($F(1, 64) = 0.573$, $p = .573$, $\eta^2 = .02$). Mean Absolute Framing effect for Small, Medium, and Large markings were 0.120 ($SD = 0.858$), 0.056 ($SD = 1.012$), and -0.034 ($SD = 0.937$) mm, respectively.

I also investigated whether the framing effect differs by the size of probabilities for raw errors. I computed a Directional Framing Effect Score for each participant by taking the difference between their average raw errors on Gain versus Loss trials. Thus, a positive Directional Framing Effort score indicates probabilities framed as Gains were on average marked further to the right (indicating more positive error) compared to probabilities framed as Losses. A score of 0 indicates no difference between the conditions in terms of directional bias. A negative score indicates probabilities framed as Losses were on average marked further to the right (indicating more positive error) compared to probabilities framed as Gains. I conducted a repeated measures MANOVA with within-subjects factor Size (Small, Medium, and Large) and dependent variable Directional Framing Effect Score. There was no effect of Size ($F(1, 64) = 0.055$, $p = .946$, $\eta^2 = .001$). Mean Directional Framing effect for Small, Medium, and Large markings were 0.084 ($SD = 1.422$), 0.030 ($SD = 1.272$), and -0.009 ($SD = 1.395$) mm, respectively.

Individual Differences in the Framing Effect and Rational Number Ability

The second research question was whether individuals with stronger rational number ability are less susceptible to the framing effect. To address this question, I first computed an overall Rational Number Ability Score for each participant. This score was based on *z*-scores for their performance on the three tasks associated with rational numbers. First, a *z*-score was calculated for each participant's average response time for the Decimal, Fraction, and Mixed conditions of the Magnitude Comparison task, and these *z*-scores were averaged together. Next, a *z*-score was calculated for each participant's average absolute error for the Decimal and Fraction conditions on the NLE task, and these scores were averaged. Next, a *z*-score was calculated for the Rational Ability Test. Note that the directionality of these three measures differs: for the Magnitude Comparison and Number Line Estimation tasks, positive *z*-scores represent worse performance, whereas for the Rational Ability Test, they represent better performance. Thus, the overall Rational Number Ability Score was computed by subtracting the *z*-scores from the Magnitude Comparison task and the Number Line Estimation task from that of the Rational Ability test:

$$\text{Rational Number Ability} = z_{\text{rational ability test}} - z_{\text{magnitude comparison}} - z_{\text{number line estimation}}$$

Thus higher scores indicate better rational number performance.

I first assessed whether individual differences in rational number ability are associated with the size of the framing effect. Rational Number Ability scores were entered in a linear regression model as predictor of Absolute Framing Effect scores. Rational Number Ability did not predict the size of the framing effect, $b = -.041$, $t(63) = -.995$, $p = .324$.

Next, I conducted an analogous analysis using the Directional Framing Effect variable. I estimated a regression model with Rational Number Ability as the independent variable and Directional Framing Effect as the dependent variable. There was a marginal effect, *b* = .088, *t*(63) = 1.77, *p* = .080, η^2= .048. The mean directional framing effect for the low ability group was -.119 (*SD* = .772) mm and for the high group was .186 (*SD* = .774) mm. This means individuals with lower rational number ability tended to mark probabilities framed as Losses further to the right compared to those marked as Gains. Conversely, individuals with higher rational ability tended to mark probabilities framed as Gains further to the right compared to those framed as Losses; see Figure 15.

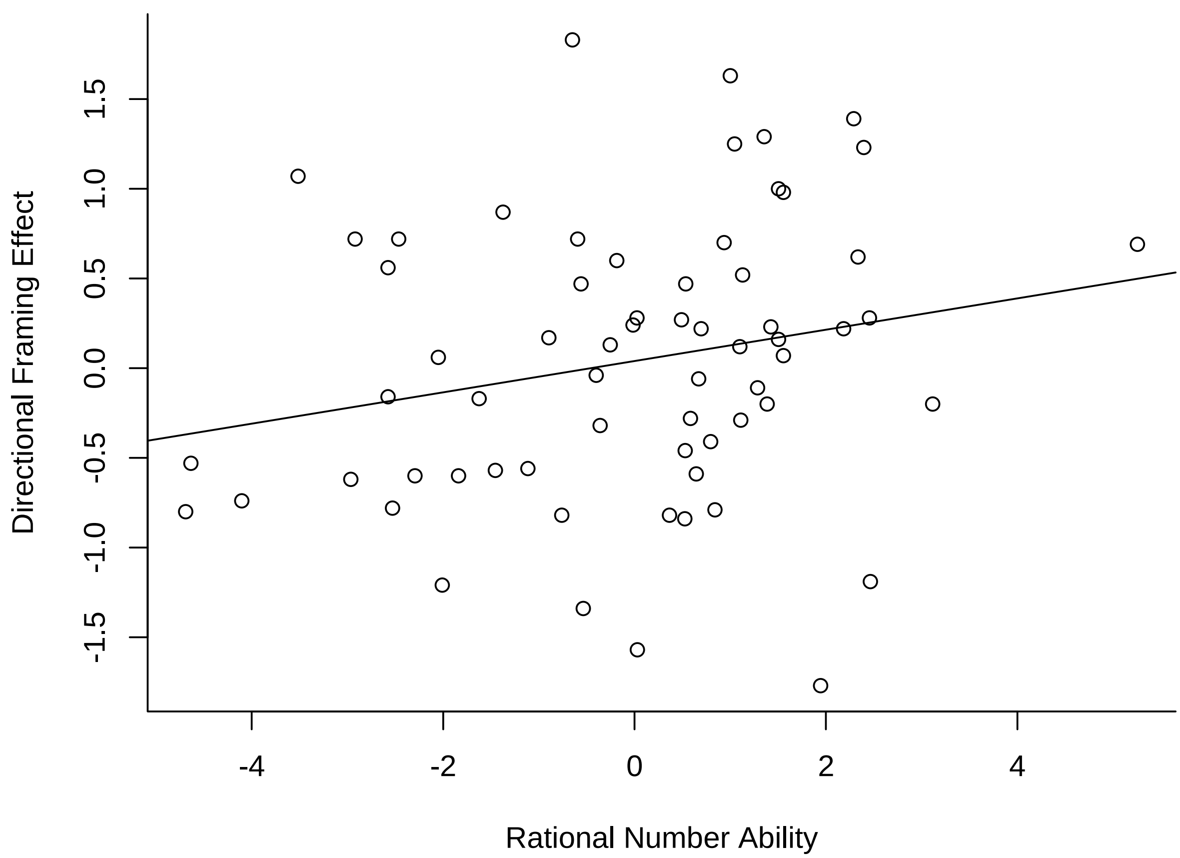

Figure 15. Association between the directional framing effect and rational number ability. The regression line represents the best-fitting linear model.

Individual Differences in the Asian Disease Problem and Rational Number Ability

Recall that each participant was randomly assigned to receive either the positively or negatively framed version of the Asian Disease problem. For this analysis, each participant either received a score of 0 if they chose the sure bet and a score of 1 if they chose the riskier option. The dependent variable was the proportion of participants in each group who chose the riskier option. Recall that Tversky and Kahneman (1981) found that participants in the negative framing condition were more likely to choose the riskier option.

A z-test of proportions revealed a difference between the groups, $z = 2.074$, $p < .05$. In the negative framing group ($n = 35$), 0.629 of participants choose the risk seeking option, whereas in the positive framing group ($n = 32$), 0.375 made this choice; see Figure 16. This replicated the classic Tversky and Kahneman (1981) finding.

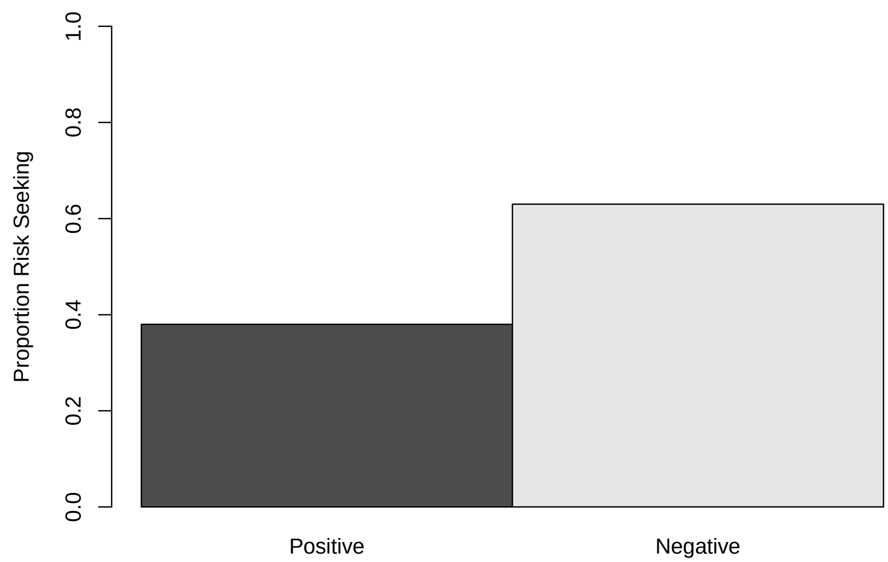

Figure 16. Overall framing effect shown by proportion choosing the risk-seeking option within the positive and negative framing groups.

To examine individual differences, I dichotomized the Rational Number Ability variable defined above by performing a median split to form a high group (n = 34) and a low group (n = 33). I then considered the framing effect separately for each group. For the low group, there was no framing effect, z = .354, p = .726. The proportion of participants choosing the risk-seeking option was comparable in the negative framing (0.53, n = 15) and positive framing (0.47, n = 17) conditions. By contrast, for the high group, there was a framing effect, z = 2.382, p = .05. The proportion of participants choosing the risk-seeking option was greater in the negative framing (0.70, n = 20) than the positive framing (0.286, n = 14) condition. Thus, the overall framing effect is driven by the high rational number ability group; see Figure 17.

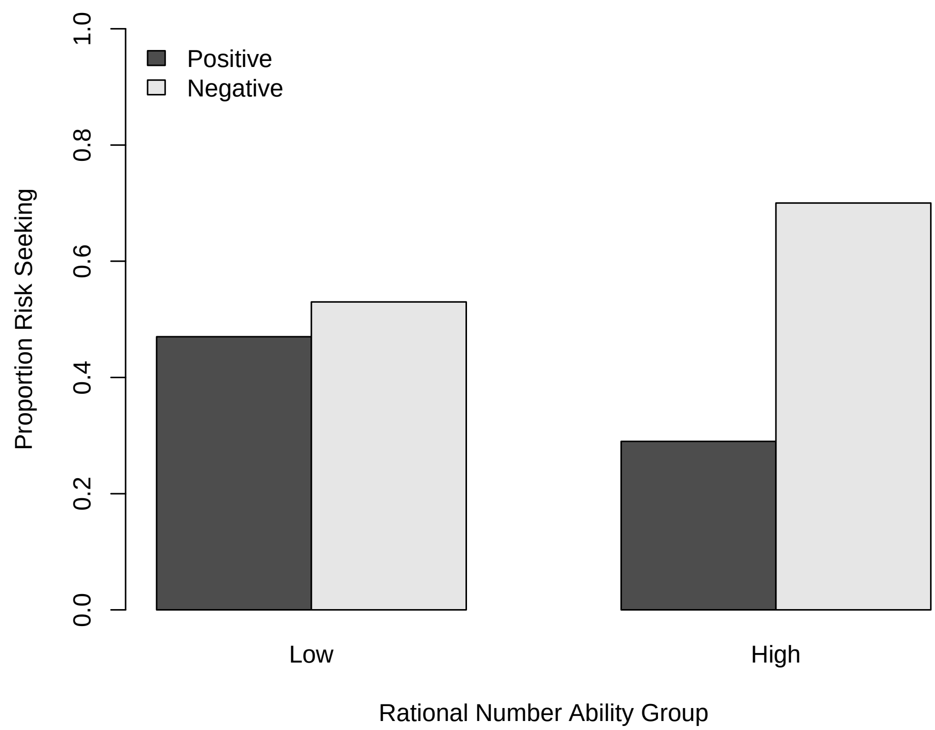

Figure 17. Effect of frame for each rational number ability group shown by proportion choosing the risk-seeking option within the positive and negative framing groups.

Replicating Experiment 1: Representation of Rational Numbers

To replicate some of the findings of Experiment 1, I analyzed the Magnitude Comparison task and the NLE task data. Again, the goal of these analyses was to assess whether different rational number formats (decimals vs. fractions) recruit the same mental representation or different mental representations.

Magnitude Comparison Task. I analyzed the Magnitude Comparison task data in a repeated measures MANOVA with within-subjects factor Type (Decimal, Fraction, and Mixed) and dependent variable RT. The test showed a significant effect for Type ($F(1, 65) = 122.85$, $p < .001$, $\eta^2 = .791$). Post-hoc testing using Pillai's test revealed significant differences between Decimals and Fractions ($F(1, 66) = 170.69$, $p < .001$, Cohen's $d = 1.596$) and Decimal and Mixed comparisons ($F(1, 66) = 231.04$, $p < .001$, Cohen's $d = 1.857$). Mean RTs for Decimal, Fractions, and Mixed comparisons were 722 ($SD = 121$), 1136 ($SD = 292$), and 1106 ($SD = 245$) ms, respectively; see Figure 18. These results largely replicate the pattern found in Experiment 1, with the exception that the significant difference between the Fraction and Mixed conditions in Experiment 1 was only marginally significant in Experiment 2, with Mixed being slightly quicker.

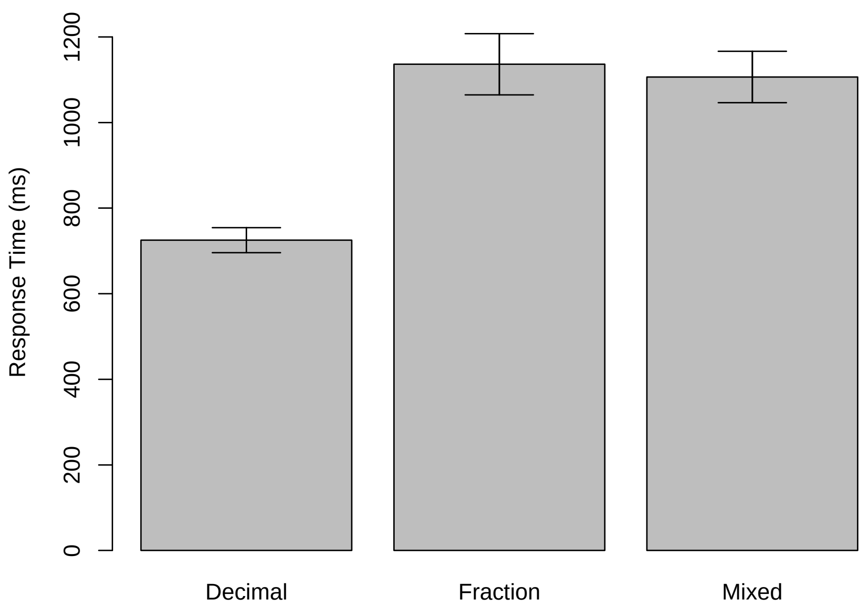

Figure 18. Mean response times by type for the Magnitude Comparison task. Error bars represent standard errors.

I conducted a parallel analysis of the accuracy data. The test showed a significant effect for Type ($F(1, 65) = 61.2$, $p < .001$, $\eta^2 = .653$). Post-hoc testing using Pillai's test revealed significant differences between all three comparison types: Decimals to Fractions ($F(1, 66) = 120.8$, $p < .001$, Cohen's $d = 1.34$), Decimals to Mixed ($F(1, 66) = 60.2$, $p < .001$, Cohen's $d = .948$), and Fractions to Mixed ($F(1, 66) = 11.8$, $p < .01$, Cohen's $d = 0.419$). Mean accuracy rates for Decimal, Fractions, and Mixed comparisons were .960 (*SD* = .047), .856 (*SD* = .080), and .887 (*SD* = .070), respectively; see Figure 19. These results largely replicate those found in Experiment 1, with the exception that the significant difference between the Fraction and Mixed comparisons in Experiment 2 was only a trend in Experiment 1 (p = .19).

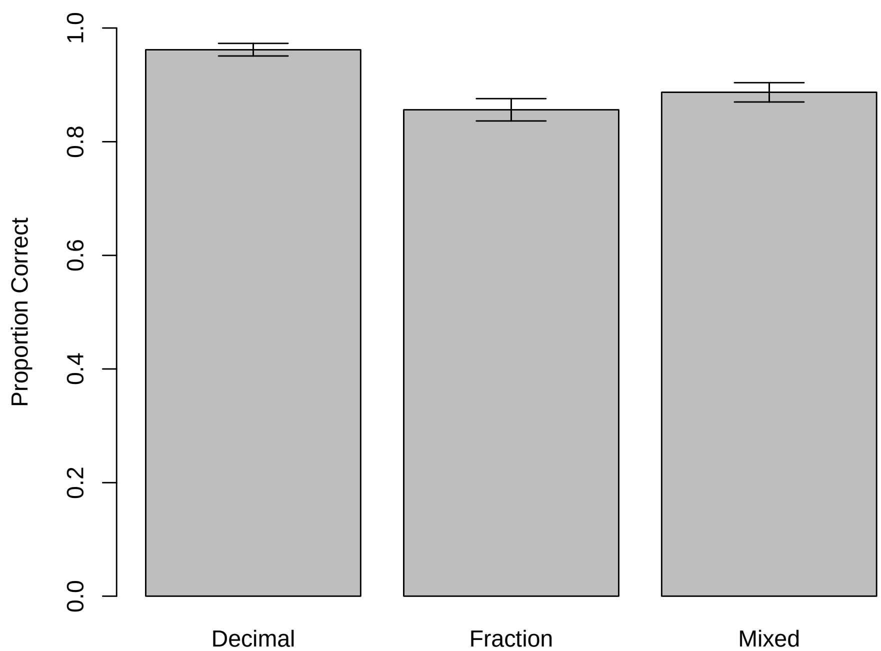

Figure 19. Mean accuracy by type for the Magnitude Comparison task. Error bars represent standard errors.

Number Line Estimation Task. I analyzed the NLE task data in a repeated measures MANOVA with within-subjects factor Type (Decimal and Fraction) and dependent variable RT. The test showed a significant effect for Type (F(1, 65) = 49.94.53, $p < .001$, $\eta^2 = .434$). Mean RTs for Decimal and Fractions markings were 2215 (SD = 460), and 2512 (SD = 432) ms, respectively; see Figure 20. These results are identical to what was found in Experiment 1.

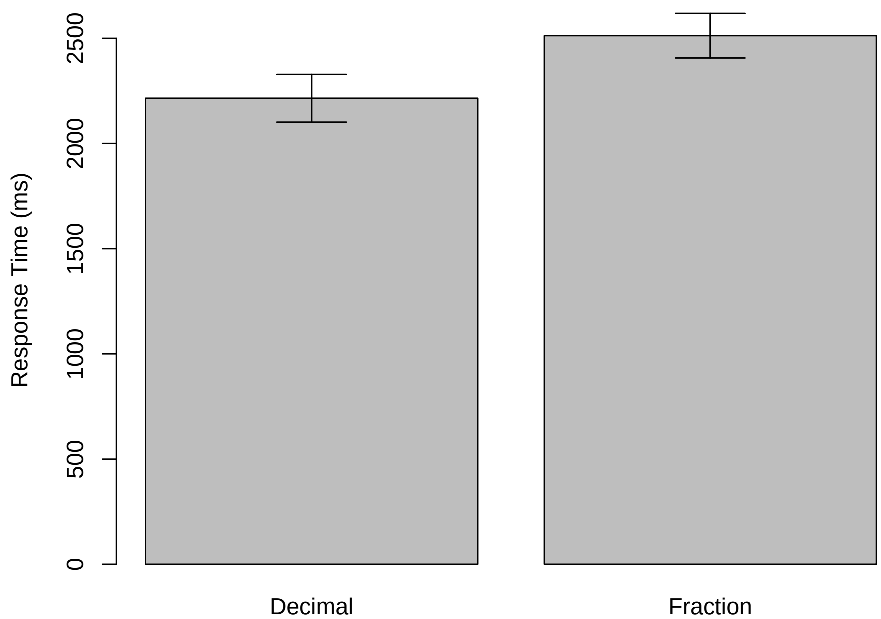

Figure 20. Mean response times by type for the NLE task. Error bars represent standard errors.

I conducted a parallel analysis of the absolute error data. The test did not show a significant effect for Type ($F(1, 65) = 2.98$, $p = .089$, $\eta^2 = .044$). Mean absolute error for Decimal estimates was 3.882 ($SD = 0.983$) mm and for Fractions estimates was 4.046 ($SD = 0.767$) mm; see Figure 21. This represents a failure to replicate the Experiment 1 of greater absolute error for Fraction estimates, although the trend was in the same direction.

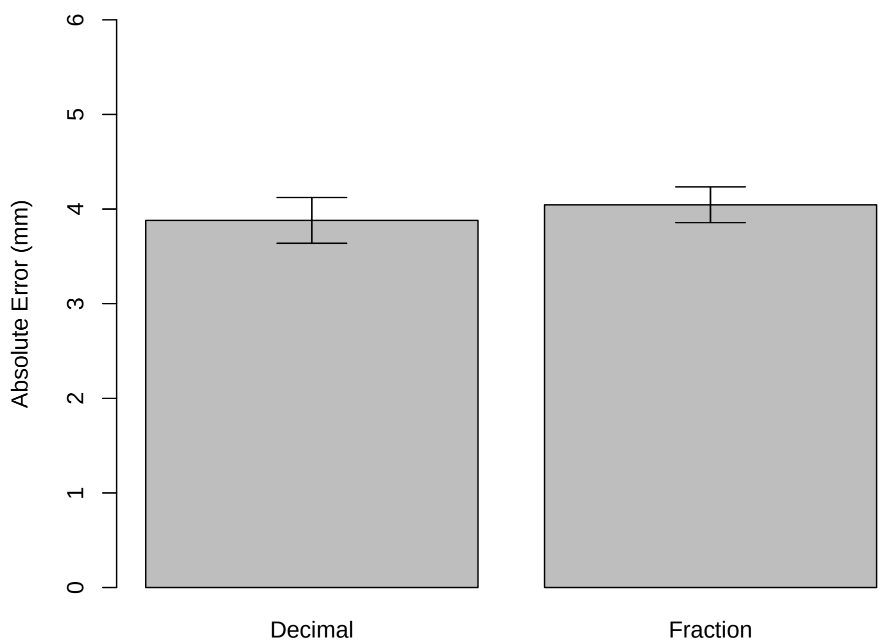

Figure 21. Mean absolute errors by type for the NLE task. Error bars represent standard errors.

Replicating Experiment 1 – Classic Effects

Next, I attempted to replicate the analyses of Experiment 1 intended to check for the same classic effects that are indicators that rational numbers are processed in a similar manner to natural numbers. These findings add to the continuing study of how rational number processing is similar to or differs from the processing of positive integers and serve as evidence that the current participants performed similarly to those of prior studies

First, I consider effects related to the distance between the two numbers being compared. These results are informative for the debate over whether people process fractions as they do integers, via an analog number line, which is considered *holistic* processing, or whether they process the numerator and denominator separately, which is considered *componential* processing. Note that the distance effect is considered an indicator of holistic (i.e., magnitude) processing.

Distance Effect. The distance effect (Moyer & Landauer, 1967) is the finding that people more quickly judge the greater (or lesser) of two numbers the larger the distance between them (e.g., comparing 2 vs. 9 is quicker than comparing 5 vs. 6). It is considered an indicator of holistic processing via an analog number line. I tested for this effect by first calculating average response times for each participant for "near" comparisons where the distance between the numbers was less than 0.2 (roughly half the trials) and also for the remaining "far" comparisons. I did this separately for each of three comparison types (decimal, fraction, mixed). There was a distance effect for decimal comparisons ($t(67) = 13.929$, $p < .001$, $d = 1.689$), fraction comparisons ($t(67) = 6.612$, $p < .001$, $d = .802$), and mixed comparisons ($t(67) = 10.365$, $p < .001$, $d = 1.257$). These results replicate the findings of Experiment 1 and prior findings of distance effects for decimal comparisons (Varma & Karl, 2013) and fraction comparisons (Jacob & Nieder, 2009; Schneider & Siegler, 2010).

Distance Effect Slopes. I also tested for the distance effect in a more continuous fashion. For each participant, for each comparison Type, a linear model was fit predicting response times from the absolute distance between the two numbers being compared. A distance effect is evidenced by a negative slope as this indicates as the distance between the two numbers increases, the time it takes to compare them decreases. The question is whether the slopes are comparable across the three types of rational number comparison, consistent with the use of common magnitude representations, or whether they differ. A repeated measures MANOVA was conducted with within-subjects factor Type (Decimal, Fraction, and Mixed) and dependent measure average RT/distance slope. Unlike Experiment 1, there was no effect of Type ($F(1, 65) = .76$, $p = .471$, $\eta^2 = .023$). The mean distance effect slopes for decimal, fractions, and mixed comparisons were -.301 (SD = .123), -.284 (SD = .158), and -.315 (SD = .175), respectively; see Figure 22.

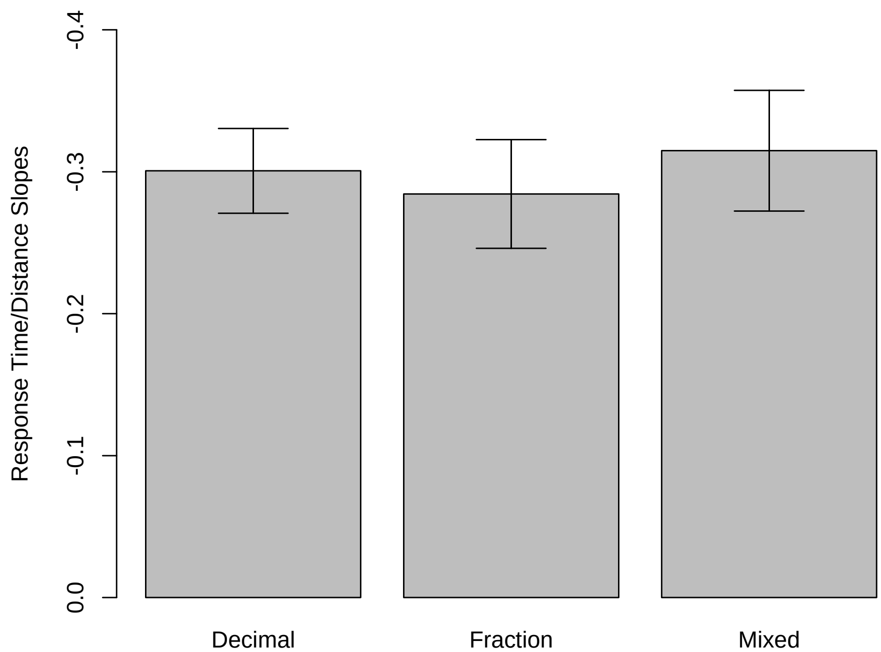

*Figure 22***.** Mean distance effect slope by type for the Magnitude Comparison task. Error bars represent standard errors.

Next, I explored these distance effect slopes on a subset of *near* comparisons. Again, near comparisons were defined as trials in which the difference between the two numbers was less than .2, and amount to roughly half of the total trials participants completed. I focused on this subset because they require the most refined magnitude processing of rational numbers. The goal was to replicate and expand upon (by including the Mixed condition) the prior finding that the distance effect disappears for comparisons of near-distance fractions (Zhang, Fang, Gabriel, & Szűcs, 2016; DeWolf & Vosniadou, 2015). A repeated measures MANOVA was conducted with within-subjects factor Type (Decimal, Fraction, and Mixed) and dependent measure average RT/distance slope on near trials. There was a significant main effect for type ($F(1, 67) = 7.649$, $p < .01$, $\eta^2 = .188$). Post-hoc contrasts using Pillai's test revealed significant differences between Fraction and Decimal comparisons ($F(1, 67) = 172.46$, $p < .001$, $d = 1.596$) only. Mean distance slopes for near comparisons for Decimal, Fractions, and Mixed comparisons were -.223 (*SD* = .162), -.013 (*SD* = .225), and -.180 (*SD* = .241), respectively; see Figure 23. These results differ from those of Experiment 1 in that the significant difference between Fraction and Mixed comparison failed to replicate; it was only marginal ($p = .059$). More importantly, these results are similar to those of Experiment 1 in that Fraction comparisons on these near trials have average distance slopes near 0, while the Decimal and Mixed conditions retain negative distance slopes.

Thus, in both experiments, there is evidence that fractions are less likely to be processed like the natural numbers for particularly near comparisons, while decimal appear to still be processed similarly to natural numbers, replicating previous findings (Zhang, Fang, Gabriel, & Szűcs, 2016; DeWolf & Vosniadou, 2015). Also, by including the Mixed condition, both experiments suggest that while the inclusion of a fraction in the comparison makes processing more difficult than in the Decimal condition, the decimal provides an anchor that facilitates magnitude processing. In this case, it appears that anchor grounds near comparisons in a way that produces a distance effect that does not occur when only fractions are involved.

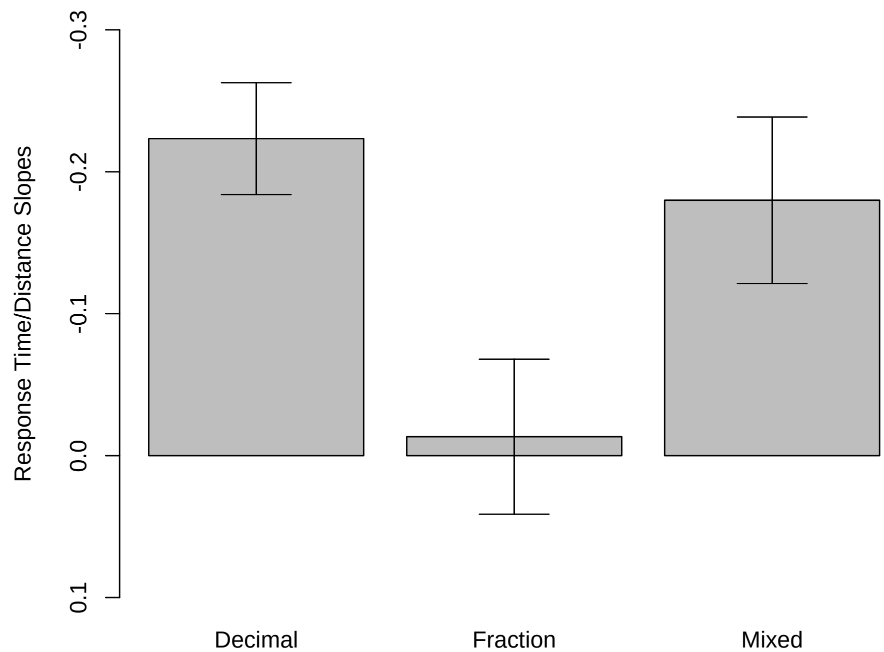

Figure 23. Mean distance effect slope by type for the Magnitude Comparison task for near comparisons only. Error bars represent standard errors.

The results of the distance effect overall across both experiments inform the debate over holistic versus componential processing of fractions, and thus whether rational numbers as a class are processed at the magnitude level. They suggest that all three types of rational numbers are processed in a way that shares similarities with processing natural numbers, i.e., using holistic processing. However, there is evidence fractions may be the format least amenable for magnitude processing. Specifically, the distance effect for difficult near-distance comparisons disappears, suggesting that they require more componential processing. Further, by including the Mixed conditions, these results largely suggest that including decimals alongside fractions can encourage more holistic processing of rational numbers than fractions alone.

The Size Effect. The size effect (Parkman, 1971) is the finding that people more quickly judge the greater (or lesser) of two numbers the smaller their average size, when the distance between them is held constant (e.g., comparing 3 and 4 is quicker than comparing 8 and 9). I tested for this effect by first calculating average response times for each participant for “large” comparisons, where the sum of the two numbers compared was above .5 (roughly half the trials), and also for the remaining “small” comparisons. I did this separately for each of the three comparison types (decimal, fraction, mixed). For decimal comparisons, there was a marginal effect of size ($t(67) = -1.937$, $p = .057$, $d = .235$). For fraction comparisons, there was a size effect ($t(67) = -3.174$, $p < .01$, $d = .385$). Finally, there was no effect of size for mixed comparisons ($t(67) = .250$, $p = .803$). These results differ from Experiment 1 where no size effect was found for any of the comparison types. Overall, the size effect may not appear with decimals or fractions limited to numbers between 0 and 1 due to range restriction.

Unit-Decade Compatibility Effect. The unit-decade compatibility effect is the finding that the tens and ones digits can interfere when people compare two-digit numbers (Nuerk, Weger, & Willmes, 2001). For example, people compare 21 and 87 quickly because both the ones and tens digits lead to the same judgment, whereas they compare 24 and 82 slowly because they lead to conflicting judgments. This effect has been extended to decimal comparisons (Varma & Karl, 2013). This effect was tested by sorting the decimal trials into congruent and incongruent trials and comparing participants' average response times using a paired *t*-test. Experiment 2 replicated the compatibility effect ($t(67) = 2.503$, $p < .05$, $d = .304$), with congruent trials ($M = 714$ ms, $SD = 118$) compared faster than incongruent trials ($M = 725$ ms, $SD = 122$). This effect was not found in Experiment 1, perhaps because it included 7 fewer participants.

Speed Accuracy Trade-off. To determine whether there was a speed accuracy trade-off, I computed bivariate correlations between each participants' average RT and accuracy separately for each comparison Type (Decimal, Fraction, and Mixed). A speed-accuracy trade-off would be indicated by a positive correlation, with slower (i.e., larger) response times associated with higher (i.e., larger) accuracies. There was no evidence of a speed-accuracy trade-off for Fraction comparisons ($r(65) = .076$, $p = .541$) or Mixed comparisons ($r(65) = -.113$, $p = .364$). However, there was such a trade-off for Decimal comparisons ($r(66) = .419$, $p < .001$). These findings replicate those of Experiment 1.

Discussion

Experiment 2 explored the relationship between people's magnitude representations of rational numbers and the framing of probabilistic information (gain vs. loss). The framing effect refers to the impact of positive and negative valences in decision-making tasks that deal with probabilistic information. Experiment 2 addressed the framing effect at two levels using three measures, as summarized in Table 10.

Table 10

Framing Effect Measures Analyzed in Experiment 2

	Magnitude Level	Decision Level
Task:	Probability Number Line Task	Asian Disease Problem
Outcome(s):	1) *Precision* of marking probabilities 2) *Direction* of marking probabilities	3) *Choice* between two options

The Effect of Frame on Magnitude Representations

First, I considered the effect of frame at the magnitude level, with the frame (as a gain vs. a loss) modulating the magnitude representations of the probability, using the Probability NLE task. The first question is whether the frame affects overall *precision* on the probability number line task. Precision is how far away on average participants were from the correct number; it was computed by averaging the *absolute* differences between each participant's markings from the actual probabilities separately for gains and losses. Experiment 2 found no effect of frame, with comparable mean absolute errors for gain vs. loss framings. I also investigated whether the effect of framing is different based on the size of probability magnitudes. This was not the case as there was no interaction between frame (gain vs. loss) and the size of the probability (small vs. medium vs. large). Overall, these results do not support the hypothesis that framing a probability as a gain vs. a loss affects the precision of people's representations of that probability as a magnitude.

Next, I considered that whether or not framing effect the precision of people's magnitude representations of probabilities, it may affect the *direction* of such representations. Direction was defined as the tendency to mark estimates to the right or to the left of the correct number. The general hypothesis is that framing probabilities as gains vs. losses may cause people to "shift" their estimates in a particular direction, for example to the left vs. right of the actual probability. Specifically, *loss aversion* may show up in people's magnitude representations. This refers to an affective bias toward avoiding losses more than seeking gains (Kahneman & Tversky, 1979). If we assume a person's current net state is 0, the idea of adding 2 to that state is appealing and the idea of subtracting 2 from that state is unappealing. Loss aversion is a bias to avoid the loss of 2 more than to approach the gain of 2. If this tendency does affect magnitude representations of probabilities, people may shift probabilities framed as losses more to the right, overestimating their magnitude in comparison to gains. The results of Experiment 2 did not support the hypothesis that frame would affect direction, with comparable mean raw errors for gain vs. loss framings. I also tested whether depending on the size of the probability magnitudes (small, medium, large), the framing effect may affect *direction* differently. This hypothesis was also not supported as there was no interaction between frame (gain vs. loss) and size (small vs. medium vs. large). Overall, these results do not support the hypothesis that framing a probability as gains or losses affects the direction of people's representation of probability magnitudes.

The absence of evidence that framing has its effect at the level of magnitude representations of probabilities suggests the framing effect may be due to other factors. It is also possible that the frame here, implemented by the phrasing "the ____% chance of a gain/loss" on each trial, was not strong enough to elicit the effect. Another possibility is that labeling the right pole as "1" may have caused problems. In particular, numbers were presented as percentages and the 1 was intended to represent 100% – a correspondence that some participants might have missed. However, during practice trials, when beginning the Loss condition, many participants asked whether should invert the probabilities, i.e., whether they should, for example, interpret a 20% loss as an 80% gain and mark it on the right side of the number line. In addition, some participants made that mistake spontaneously. These mistakes, which were corrected before the experimental blocks began, suggest that participants were cognizant of the difference between the conditions, at least when initially confronted with the loss framing, and also were able to map the right pole, labeled "1", to 100%.

The Effect of Frame on Decisions: The Classic Framing Effect

This experiment also included a commonly used framing effect task that operates at the decision level. Participants solved the Asian Disease problem from the original experiment on the framing effect (Tversky & Kahneman, 1981). This problem, a decision-based measure of the framing effect, was included to contrast with the Probability NLE task, a magnitude-based measure of the framing effect. Also, this task allows me to replicate previous findings of a framing effect. In this experiment, more participants chose the riskier option when the choices were negatively framed, replicating the classic Tversky and Kahneman (1981) finding. This finding supports the hypothesis that the framing effect largely manifests at the decision-making level. Further, since this has been a well-established finding in the literature, this can be taken as evidence that the participants involved in Experiment 2 were behaving similarly to those in past experiments.

The Interaction between the Framing Effect and Individual Numerical Ability

The second research question was whether individual differences in the framing effect interact with individual differences in rational number ability. If so, this would provide evidence of a relationship between numerical ability and bias in decision making when probabilities are involved. To test this hypothesis, each participant's overall rational number ability was estimated by the sum of the *z*-scores of their performance on the Magnitude Comparison and NLE tasks and the Rational Number Ability Test. These overall scores were then tested as predictors of the framing effect in the various ways it was measured in this experiment.

First, I considered whether rational number ability could predict the effect of frame in terms of how *precisely* participants marked their probabilities as measured by the difference in average absolute errors between the gain and loss conditions. The results of this analysis did not support this hypothesis, as there was no relationship between participants' overall rational number ability and their absolute framing effect.

Next, I considered whether rational number ability could predict the effect of frame in terms of *directionality* (i.e., the left or right bias) as measured by difference in average raw errors between the gain and loss conditions. There was a marginal relationship of rational number ability and the directional framing effect. Individuals with lower rational number ability tended to mark probabilities framed as Losses further to the right (indicating more positive error) compared to probabilities framed as Gains. Conversely, individuals with higher rational ability tended to mark probabilities framed as Gains (indicating more positive error) further to the right compared to those framed as Losses. See Figures 24 and 25 below for visual representations of this difference for each group. This marginal finding suggests that individuals with lower rational number ability may be more likely to exhibit *loss aversion* responses to negatively framed probabilities. Recall that *loss aversion* refers to an affective bias for avoiding losses more than seeking gains. The marginal results of this experiment suggest loss aversion may be partially due to people with lower rational number ability representing numbers framed negatively as higher magnitudes compared to the same numbers framed positively. Therefore, it may be that the loss aversion tendency has particularly influenced those with lower rational number ability in interpreting numerical information. As will be discussed below in more detail, there is reason to believe the current sample was particularly high performing when it comes to mathematical skill. Therefore, it is possible that with a more diverse group of participants, this relationship may be driven more by those with lower numerical ability exhibiting more bias to mark probabilities marked as losses to the right of their correct positions.

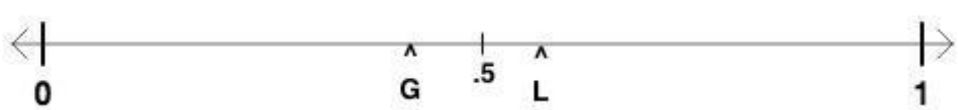

*Figure 24***.** Depiction of Low Rational Ability Group's raw average errors by condition for the Probability NLE task. Figure not drawn to scale. G = participants' average raw errors for Gain markings. L = participants' average raw errors for Loss markings.

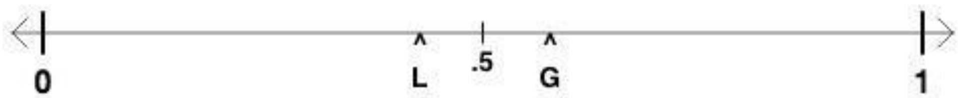

Figure 25. Depiction of High Rational Ability Group's raw average errors by condition for the Probability NLE task. Figure not drawn to scale. G = participants' average raw errors for Gain markings. L = participants' average raw errors for Loss markings.

Finally, there was an interaction between rational number ability and the framing effect at the decision level, as measured by the Asian Disease problem. The framing effect on this task was measured based on participants' choices between two options. Low and high ability groups were defined by a median split based on the same overall rational number ability score guiding the discussion in this section. The low ability group exhibited no framing effect, whereas the high ability group did. Thus, the overall framing effect at the decision level was driven by the high rational number ability participants.

This result was unexpected and runs counter to previous findings that the framing effect is driven by individuals with lower numerical ability (Simon, Fagley, & Halleran, 2004; Peters, et al., 2006). One explanation for this discrepancy is methodological differences between the experiments. First, in Simon et al. (2004), participants self-reported their mathematical ability on a scale of 1-7. In Peters et al. (2006), participants' scores were based on the 11-item Lipkus (2001) scale, a measure of the ability to translate between percent, ratio, and fraction representations of rational numbers. These are different measures of numerical ability than the ones used here. Furthermore, in the Simon et al. (2004) analysis, self-reported math ability was only related to the framing effect when included in interaction with another general cognitive measure, the Need for Cognition Scale.

Another explanation is that the "low ability" participants in Experiment 2 might not have been very low at all. Experiments 1 and 2 used similar methods of recruitment for this experiment, drawing from the same population of undergraduates. In Experiment 1, the mean ACT math score was 28, which approximately corresponds to the 90^{th} percentile nationally. It is likely that Experiment 2 included participants of similarly high mathematical achievement. Thus, the two groups might be better labeled "high" and "very high". (However, it is important to note that two groups *did* differ on five out of the six measures used to compute high and low groups used in this analysis, based on *t*-tests. Only reaction times for decimal comparisons on the magnitude comparison task were comparable between groups, likely due to a ceiling effect since performance was highest on that particular task.)

These results suggest that there are situations in which superior numerical ability can unexpectedly lead to less normative evaluations of situations involving probabilities with superficial differences, such as frame.

Chapter 6: General Discussion

Understanding rational numbers requires reorganizing of our initial understanding of numbers as whole numbers. Coordinating the relationship between the different symbolic names of rational numbers and their underlying non-symbolic magnitudes appears to be an important component of mathematical development in children, relating to more complex mathematical skills (Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Pyke, 2013; Mazzocco et al., 2013). It is also an important component of adult decision making in everyday life (Simon, Fagley, & Halleran, 2004; Peters, et al., 2006).

The mental representation and processing of rational numbers was explored across two experiment. The goal was to investigate whether these numbers, in various formats, underlie general mathematical achievement and decision making. Both experiments demonstrated that the format of rational numbers affects the ease of processing. In particular, the fraction format hinders magnitude processing compared to the decimal format. A related and novel finding is that the mixed format—where a fraction is compared to a decimal—facilitates magnitude processing compared to the fraction-only format. Experiment 1 additionally demonstrated that the precision of rational number magnitudes is related to general mathematical achievement. This is evidence that a better understanding of rational numbers is important for more abstract mathematics in adults. Experiment 2 showed that individual differences in rational number ability are associated with individual differences in bias in decision making. Individuals with lower rational number ability were marginally more likely to exhibit loss aversion in their representations of probabilities framed as losses, with the opposite pattern occurring in individuals with higher rational number ability. Also, individuals with higher rational number ability were more like to exhibit the framing effect in a classic decision-based problem.

These findings are important for theoretical, educational, and pragmatic reasons. Theoretically, they extend previous findings that fractions and decimal formats for rational numbers share common magnitude representations with natural numbers (Varma & Karl, 2013; Jacob & Nieder, 2009; Schneider & Siegler, 2010). However, these findings also confirm that in particularly difficult contexts, fraction processing is more detached from these common magnitude representations, and therefore slower (Zhang, Fang, Gabriel, & Szűcs, 2015; DeWolf & Vosniadou, 2015). Educationally, the observed relationship between individual differences in rational number magnitudes and individual differences in general mathematical achievement was extended from children (Fazio, Bailey, Thompson, & Siegler, 2014; Siegler & Pyke, 2013) to adults for the first time. This is an important continuity, one that maintains even after years of development and learning with a wide variety of mathematics instruction. To the best of my knowledge, this is the first known case of predicting performance on a college entrance exam using numerical magnitude tasks. Pragmatically, there was a marginal relationship between rational number ability and people's tendency towards loss aversion. Interestingly, one relationship found in Experiment 2 ran counter to previous findings and intuition: people with superior rational number ability were *more* likely to demonstrate a common decision-making bias.

Rational Number Formats

The first research question was whether the format of rational numbers – as decimals versus fractions – affects the ease of accessing and processing the underlying magnitude representations. Processing was measured using two standard tasks in the literature. In the magnitude comparison task, participants compare two rational numbers – both expressed as decimals, both expressed as fractions, or a mixture of one decimal and one fraction – and judge which one is greater or lesser. In the number line estimation task, participants estimate where a rational number – expressed as a decimal or fraction – should be placed on a number line with poles labeled 0 and 1. Both experiments used these tasks, and their results support the hypothesis that decimals are accessed more quickly and accurately than fractions. This suggests that decimals are more efficiently linked to their underlying magnitude representations. A secondary question was whether there is a cost (i.e., in response time or accuracy) of mixing the two rational number formats. This was not the case: Across the two experiments, the mixed comparisons were completed at least as quickly and accurately as the fraction comparisons.

Further support for an advantage for the decimal format comes from the observed pattern of distance effects (Moyer & Landauer, 1967). Distance effects were found for all three comparison types (decimal, fraction, and mixed). However, the subset of difficult near-distance trials, the distance effect for fraction comparisons disappeared, whereas it persisted for decimal and mixed comparisons. This suggests that fractions are less likely than other rational number formats (i.e., decimals) using magnitude representations (Zhang, Fang, Gabriel, & Szűcs, 2015; DeWolf & Vosniadou, 2015).

Rational Number Processing and Mathematical Achievement

The second research question of Experiment 1 was whether there is a relationship between rational number magnitude processing and mathematical achievement in an adult population. Both the magnitude comparison and number line estimations tasks predicted general mathematical achievement as measured by the ACT. This extends – for the first time – the previous finding of a predictive relationship in middle school students (Siegler & Pyke, 2013). Of the two magnitude tasks used in these experiments, number line estimation appears to be the most related to mathematical achievement. These findings suggest the consequences of rational number ability and instruction continue into adulthood. Further, representing rational numbers on a number line may be a better a way to assess this ability after initial instruction occurs.

Rational Number Ability and the Framing Effect

The third research question concerned the relationship between people's magnitude representations of rational numbers and their understanding of the framing of probabilities as gains vs. losses. Experiment 2 modified the number lines estimation task to require participants to mark the positions of probabilities (percentages) framed as gain and losses. The results demonstrated that the framing of probabilities in this way does not affect their processing as rational number magnitudes. Although there is reason to believe that this task was properly understood by participants, it is possible that the effect of frame was lost in the routine of making many quick markings without the time to consider the probabilistic gain/loss context. Thus, it remains unclear if the frame was properly primed.

The fourth research question was whether there is a relation between individual differences in rational number ability and individual differences in susceptibility to framing effects (Simon, Fagley, & Halleran, 2004; Peters, et al., 2006). This question was addressed in Experiment 2 in two ways. First, there was a marginally significant relationship between rational number ability and the direction of errors in the probability number line estimation task (i.e., a bias for people to mark a probability to the right or the left of the correct placement as a function of whether it is framed as a Gain or a Loss). People with lower rational number ability marked probabilities framed as Losses further to the right (i.e., made positively biased errors) compared to probabilities framed as Gains. This suggests loss aversion may impact individuals with lower rational number ability. Specifically, they may be more likely to interpret rational numbers expressed negatively as having larger magnitudes. However, given the potentially weak effect of frame on the probability number line task (mentioned above), and without a more diverse sample of participants in terms of mathematical ability, this claim merits further investigation.

Second, at the decision level, there was an interaction between rational number ability and the framing effect. The decision level refers to performance on participants' choices on the classic Asian Disease problem (Tversky and Kahneman, 1981). Surprisingly, the overall framing effect at the decision level was driven by the high rational number ability participants. They exhibited a framing effect, whereas the low rational ability group did not. This result stands in contrasts to the overall trends of prior studies finding that higher numerical ability was associated with *less* susceptibility to the framing effect (Simon, Fagley, & Halleran, 2004; Peters, et al., 2006). Most previous findings were interpreted as evidence that people with higher numerical ability are better able to attend to and process the problem-relevant numerical information. There are several plausible reasons for my different findings. First, numerical ability was measured with more tasks in this study than previous ones and those tasks were specifically related to broad abilities related to processing rational numbers. Second, the Asian Disease problem was always last for all participants. Therefore, those who did the best on the other tasks may have been the most exhausted (after 45 plus minutes of cognitive effort) and thereby behaved more like a lower performing group.

Limitations

There were a number of limitations of the current study that should be addressed in future research. One limitation of both experiments may have been power. Although many of the findings were clear in terms of statistical significance, a few were only marginal. It is unclear whether the marginal results are “real” or not. A replication with a larger sample would help answer this question.

Another limitation of the current experiments was the representativeness of the sample. These experiments would benefit from a more diverse sample in terms of mathematical ability. As a reminder, the group analyzed in Experiment 1 appears to be near the 90th percentile nationwide on one measure of mathematical achievement, and it is likely the group in Experiment 2 also contained a large proportion of high performers. The restricted range on the mathematical achievement dependent measure limited our ability to find significant predictors of individual differences.

Still another limitation is that the effect of the frame in the probability NLE task utilized in Experiment 2 may have worn off over time. In this task, participants marked decontextualized probabilities labeled either as a “gain” or a “loss”. The effect of this wording may have grown weaker across multiple trials as participants became habituated to the activity of marking the numbers.

A final limitation of the current experiments was the use of only one problem to test the framing effect at the decision level, the Asian disease problem. This limitation was due to time constraints and the primary interest in measuring the framing effect at using the probability number line task. However, it would be beneficial to measure this affect across multiple problems with different numbers and contexts.

Future Directions

The current study suggests a number of promising directions for future research and instructional design.

Strategy Selection

This study suggests that processing fractions is more difficult than processing decimals and *may* require selection from among a broader range of strategies. This suggests that differences in strategy selection between decimals and fractions should be explored. Fazio, DeWolf, and Siegler (2016) studied fraction comparisons only and found that better accuracy and speed were both associated with better strategy use, choice, and execution. Based on the results of this and previous studies, it is likely that this relationship is not as strong for decimal comparisons, which are easier than fraction comparisons. Strategy generation, selection, and execution should be more systematically compared between these two rational number formats to confirm this hypothesis. Moreover, if fractions rely more on strategic processing, then they are more likely to be related to general mathematical achievement.

Mathematical Achievement

The relationship between rational number ability and mathematical achievement should be investigated at a finer grain. In particular, research should better differentiate the component relationships to different domains of mathematics: algebra, geometry, and so on. In a study reviewed earlier Van Hoof et al. (2015) tested 8^{th}, 10^{th}, and 12^{th} graders on a task where participants verified whether algebraic statements could or could not be true. A majority of their mistakes were due to failures to consider how rational numbers could change their interpretation of these algebraic statements. These findings suggest that rational number processing may be more related to problems requiring basic algebraic manipulations than those requiring knowledge of geometry rules, for example. This hypothesis should be tested more formally.

Triangulating Framing Effects

Everyday decision making, especially in the realm of financial decisions, is full of cost/benefit analyses in which probabilities—rational numbers—play a critical role. In future studies though, the probabilistic number line estimation task could be combined with decision-level tasks like the Asian disease problem to better understand how Loss vs. Gain frames affect magnitude representations. For example, a future study could have participants complete multiple decision-level problems, choosing between two options that are either framed positively or negatively. Then, they could revisit each problem and estimate the position of each risky choice (e.g., 1/3 chance of saving all 600 people vs. a 2/3 of all dying) on a number line between 0-1. This would enable us to get both a fuller priming of the framing effect and to analyze difference between who exhibit the strongest and weakest framing effects.

Rational Number Training

The current findings suggest that training study interventions that intermix decimals and fractions tasks might be a fruitful way to improve rational number processing. Both experiments found that mixed comparisons of one fraction and one decimal are easier and more likely to elicit magnitude processing than comparisons of two fractions. In particular, mixed comparisons show distance effects on the most difficult (near) comparisons whereas fraction comparisons did not. Therefore, including decimals alongside fractions in comparison, number line estimation, or ranking tasks might encourage more magnitude processing of rational numbers than including fractions alone, which might encourage more symbolic processing. Intermixing different rational number representations might have the effect of improving the link between the symbolic names of fractions and their magnitudes representations. Further evidence for this suggestion comes from the Mazzocco and Devlin (2008) finding that the ability of middle-school students to rank-order intermixed fractions and decimals is related to their math disability status. In particular, students who were identified as having a math learning disability were least likely to intermix fractions with decimals. Instead, they were more likely to separate them into two homogeneous groups and rank-order each one separately. This finding supports the proposal that understanding rational numbers across formats, and mapping them all formats to a common magnitude representation, is important for mathematical achievement.

Conclusion

Across two experiments, the important role of rational number processing was considered in relation to mathematical achievement and everyday decision making. An important point reinforced by Experiment 1 was that rational number ability should not be ignored and efforts to understand this ability and improve it are worthwhile as they underlie more diverse mathematical ability. Considering the role of format with this key finding in mind strongly supports instructional designs that make connections between different rational number representations. Particularly, I advise using number lines and intermixing decimals and fractions to improve the connection between these numerical symbols and their non-symbolic magnitudes.

Decision making in everyday life was also explored. Research reports, news stories, political speeches, and many other information formats intended to both inform and persuade contain probabilistic information that can influence our perception of the world and the decisions we make (Jones and Thorton, 2005). Such probabilistic reasoning requires both numerical comprehension (use of fractions, decimals, percentages, and/or proportions) and decision-making to consider future outcomes that are unclear. The results of this study suggest numerical ability alone is not a sufficient guard against biased decision making when probabilities are involved, instead suggesting other task features that cause the bias may need to be more explicitly realized before numerical ability can make such decisions more normative. However, there were individual differences that could suggest bias can affect people differently depending on their facility with rational numbers. Given this side of rational number processing extends beyond academic pursuits, it is worthwhile to continue studying the relationship systematically and consider interventions that uncover relevant and irrelevant problem features. Maximizing people’s ability to apply what they have learned about rational numbers in a real-world decision making requires considering the role problem context has on such decisions.

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Appendix A

Magnitude Comparison task stimuli.

Decimals		Fractions		Mixed	
0.23	0.9	9/10	1/9	0.21	9/10
0.2	0.82	7/8	1/4	2/7	0.87
0.24	0.85	7/10	1/10	0.24	7/10
0.16	0.73	8/9	3/10	0.12	5/9
0.18	0.67	6/7	2/5	0.45	7/8
0.26	0.67	2/3	3/10	2/5	0.82
0.41	0.77	5/7	3/8	0.23	3/5
0.15	0.5	5/9	2/9	1/2	0.86
0.56	0.89	4/7	1/4	0.2	5/9
0.28	0.53	5/6	5/9	1/3	0.63
0.34	0.58	6/7	5/8	0.18	3/7
0.18	0.41	3/7	1/5	1/2	0.72
0.41	0.59	3/10	1/8	0.69	6/7
0.11	0.28	1/2	1/3	0.13	3/10
0.4	0.53	5/9	2/5	1/3	0.49
0.2	0.33	3/7	2/7	1/3	0.48
0.53	0.64	1/2	3/8	0.16	3/10
0.43	0.52	9/10	7/9	5/7	0.83
0.12	0.21	1/3	2/9	0.34	3/7
0.44	0.5	2/3	5/9	3/7	0.49
0.62	0.67	1/3	1/4	6/7	0.89
0.77	0.81	3/10	1/4	5/7	0.74
0.28	0.32	1/7	1/10	0.11	1/8
0.89	0.92	2/7	1/4	8/9	0.9
0.9	0.86	1/5	1/4	0.41	2/5
0.74	0.67	7/9	5/6	1/5	0.18
0.24	0.16	4/9	1/2	3/7	0.36
0.74	0.65	4/5	6/7	7/9	0.66
0.42	0.31	5/9	5/8	5/6	0.71
0.71	0.6	1/6	1/4	0.3	1/6
0.54	0.42	2/7	3/8	8/9	0.74
0.37	0.23	3/4	8/9	5/6	0.68
0.86	0.72	1/6	1/3	0.47	3/10

	0.47	0.3	3/7	3/5	0.32	1/7
	0.72	0.53	5/7	8/9	0.28	1/10
	0.48	0.29	3/5	7/9	4/7	0.38
	0.4	0.12	1/5	4/9	0.62	1/3
	0.87	0.59	1/2	3/4	0.4	1/10
	0.7	0.39	1/8	3/8	0.77	4/9
	0.68	0.25	2/5	4/5	3/4	0.4
	0.65	0.24	1/3	3/4	0.65	3/10
	0.63	0.2	3/7	7/8	2/3	0.29
	0.81	0.37	1/10	5/9	6/7	0.44
	0.61	0.13	2/5	8/9	2/3	0.24
	0.76	0.21	3/10	4/5	0.71	1/6
	0.82	0.24	1/7	7/10	0.7	1/7
	0.83	0.22	3/10	7/8	0.81	1/5
	0.85	0.23	1/10	5/6	9/10	0.16
Mean	0.50	0.49	0.49	0.49	0.50	0.49

Appendix B

Number Line Estimation task stimuli.

Fractions	Decimals	Percentages	Complements
1/19	0.05	5%	95%
1/11	0.09	9%	91%
1/10	0.10	10%	90%
2/17	0.12	12%	88%
1/8	0.13	13%	87%
2/13	0.15	15%	85%
1/6	0.17	17%	83%
3/16	0.19	19%	81%
3/14	0.21	21%	79%
2/9	0.22	22%	78%
3/13	0.23	23%	77%
4/15	0.27	27%	73%
2/7	0.29	29%	71%
3/10	0.30	30%	70%
1/3	0.33	33%	67%
6/17	0.35	35%	65%
3/8	0.38	38%	62%
5/12	0.42	42%	58%
7/16	0.44	44%	56%
6/11	0.55	55%	45%
5/9	0.56	56%	44%
4/7	0.57	57%	43%
5/8	0.63	63%	37%
9/14	0.64	64%	36%
7/10	0.70	70%	30%
5/7	0.71	71%	29%
11/15	0.73	73%	27%
3/4	0.75	75%	25%
7/9	0.78	78%	22%
5/6	0.83	83%	17%
11/13	0.85	85%	15%
7/8	0.88	88%	12%
8/9	0.89	89%	11%
9/10	0.90	90%	10%
11/12	0.92	92%	8%

13/14	0.93	93%	7%
17/18	0.94	94%	6%
18/19	0.95	95%	5%
Mean 1/2	0.50	50.37	49.63

Appendix C

Rational Number Fluency Test

Part I Complete the following chart by filling in equivalent numbers.

Please remember to reduce all fractions to their simplest form.

	Decimal	Fraction	Percent
a		3/5	
b	0.2		
c			16%
d		5/8	
e			85%
f	0.15		

Part II Compute the following solutions. Please show your work.

a. 0.16 + 0.05 ________________

b. 0.01 x 0.2 ________________

c. 0.5 ÷ 0.1 ________________

d. 0.8 – 0.12 ________________

e. 2/3 + 3/4 = ________________

f. 3/4 – 2/3 = ________________

g. 2/3 x 3/4 = ________________

h. 2/3 ÷ 3/4 = ________________

Part III Please answer the following 6 questions. Please show your work.

a. What is 2% of 60 ? ________________

b. What number is 1/4 of 150? ________________

c. 10 out of 25 is what percent? ________________%

d. What is 30% of 40? ________________

e. What is 120% of 15? _______________

f. A ratio is 4:3 is the same as a ratio of 16: ? _______________