3.3 Truth Tables for Negation, Conjunction, and Disjunction Notes

1. Use the definition of negation, conjunction, and disjunction.

Negation (not): Opposite truth value from the statement

Negation	
p	$\sim p$
T	F
F	T

Conjunction (and): Only true when both statements are true.

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (or): Only false when both statements are false.

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example: Using the Definitions of Negation, Conjunction, and Disjunction.

Let p and q represent the following statements:

p: $10 > 4$

q: $3 < 5$

Determine the truth value for each statement:

a. $p \wedge q$ __Since both are true, the conjunction is true.

b. $\sim p \wedge q$ ___Since p is true, $\sim p$ is false, so the conjunction is false.

c. $p \vee \sim q$ 10 > 4 is true or is false. A disjunction is false only when both components are false. Only one component is false, this is a true statement.

d. $\sim p \vee \sim q$ _Since q is true, $\sim q$ is false. So, the disjunction is false.

2. Construct Truth Tables
Example: Constructing Truth Tables

Construct a truth table for $\sim(p \wedge q)$

Step 1: First list the simple statements on top and show all the possible truth values.

Step 2: Make a column for $p \wedge q$ and fill in the truth values.

p q	$p \wedge q$
T T	T
T F	F
F T	F
F F	F

Step 3: Construct one more column for $\sim(p \wedge q)$. The final column tells us that the statement is false only when both p and q are true.

p q	$p \wedge q$	$\sim(p \wedge q)$
T T	T	F
T F	F	T
F T	F	T
F F	F	T

For example:

***p*: Harvard is a college. (true)**

***q*: Yale is a college. (true)**

~(*p* ∧ *q*): It is not true that Harvard and Yale are colleges.

A compound statement that is always true is called a tautology. Is this a tautology? NO

Example: Constructing a Truth Table

Construct a truth table for $(\sim p \vee q) \wedge \sim q$.

p q	$\sim p$	$\sim p \vee q$	$\sim q$	$(\sim p \vee q) \wedge \sim q$
T T	F	T	F	F
T F	F	F	T	F
F T	T	T	F	F
F F	T	T	T	T

3. Determine the Truth Value of a Compound Statement for a Specific Case.

Constructing a Truth Table with Eight Cases

There are eight different true-false combinations for compound statements consisting of three simple statements.

p	*q*	*r*
T	**T**	**T**
T	**T**	**F**
T	**F**	**T**
T	**F**	**F**
F	**T**	**T**
F	**T**	**F**
F	**F**	**T**
F	**F**	**F**

Construct a truth table for the following statement:

a. I study hard and ace the final, or I fail the course.

b. Suppose that you study hard, you do not ace the final and you fail the course. Under these conditions, is this compound statement true or false?

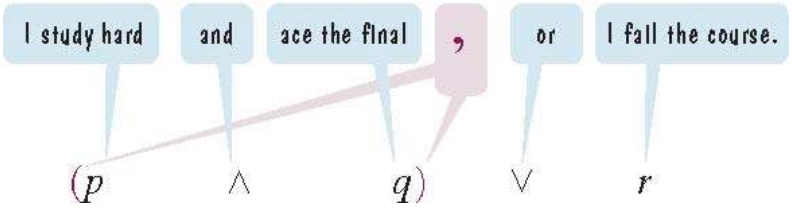
Solution:

We represent our statements as follows:

p: I study hard.

q: I ace the final.
r: I fail the course

Writing the given statement in symbolic form:



The completed table is:

The conjunction is true only when p, the first column, is true and q, the second column, is true.

The disjunction is false only when $p \wedge q$, column 4, is false and r, column 3, is false.

Show eight possible cases.

These are the conditions in part (b).

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

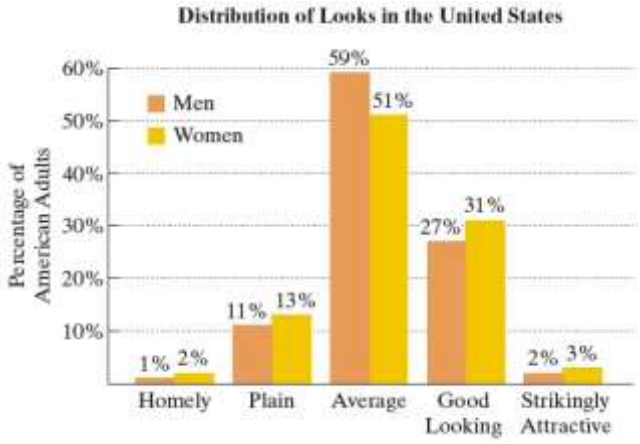
b. The statement is True.

We can determine the truth value of a compound statement for a specific case in which the truth values of the simple statements are known, without constructing an entire truth table.

Substitute the truth values of the simple statements into the symbolic form of the compound statement and use the appropriate definitions to determine the truth value of the compound statement.

Example: Determining the Truth Value of a Compound Statement

The bar graph shows the distribution of looks for American men and women, ranging from homely to strikingly attractive.



Use the information in the bar graph to determine the truth value of the following statement:

It is not true that 1% of American men are homely and more than half are average, or it is not true that 5% of American women are strikingly attractive.

We begin by assigning letters to the simple statements, using the graph to determine whether each simple statement is true or false. As always, we let these letters represent statements that are not negated.

p: One percent of American men are homely.

This statement is true.

q: More than half of American men are average looking.

This statement is true. 59% of American men are average, which is more than half: $\frac{1}{2} = 50\%$.

r: Five percent of American women are strikingly attractive.

This statement is false. 3% of American women are strikingly attractive.

Substitute the truth values for *p*, *q*, and *r* that we obtained from the bar graph to determine the truth value.

~(*p* ∧ *q*) ∨ ~*r* This is the given compound statement in symbolic form.

~(T ∧ T) ∨ ~F Substitute the truth value obtained from the graph.

~T ∨ ~F Replace T ∧ T with T. Conjunction is true when both parts are true.

F ∨ T Replace ~T with F and ~F with T. Negation gives the opposite truth value.

T Replace F ∨ T with T. Disjunction is true when at least one part is true.

We conclude that the given statement is true.