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Opportunity Cost Quantification for Ancillary Services Provided by Heating, Ventilating, and Air-Conditioning Systems

David H. Blum, *Student Member*, Tea Zakula, and Leslie K. Norford

***Abstract*— In ancillary service electricity markets, opportunity costs account for the profit forgone by a generator due to the use of generating capacity for ancillary service provision instead of energy sales. They are calculated for each generator providing ancillary services and are included in the determination of market-clearing prices by market operators throughout the U.S. Recent interest in the use of smart heating, ventilating, and air-conditioning systems for ancillary service provision prompts an investigation into their associated opportunity costs. This paper proposes a definition for these costs and method of accounting for them through time. This is done by recognizing the impacts intra-hour consumption modification associated with ancillary service provision have on daily energy efficiency and costs. A numeric example is presented to demonstrate the method. It is found that marginal ancillary service opportunity costs vary over time and that properly adjusting energy consumption at an hour of interest can reduce the total opportunity cost of providing ancillary services.**

***Index Terms*— ancillary services, demand response, electricity markets, smart buildings, smart grid.**

I. Introduction

BUILDINGS account for 74% of electricity consumption in the U.S. [1] and are often cited as a possible source of added grid-scale flexibility, commonly called demand response. Building heating, ventilating, and air-conditioning (HVAC) systems provide a particular advantage for demand response because they link the thermal energy storage inherent in construction materials to electricity consumption. Traditionally associated with consumption modification in response to grid emergencies or high-price signals [2-3], recent attention has been paid to their provision of ancillary services, such as reserves and frequency regulation, through modification of fan, compressor, and pump electricity consumption at the direction of regional electric grid operators [4-5].

While many studies have focused on direct load control of on-off heating and cooling systems for the use of ancillary services [6-8], many commercial building HVAC systems are more complex and may be difficult to control by the electric grid operator. They typically consist of multiple electrical components working in tandem, such as fans, chillers, pumps, and cooling towers, whose power consumption can be varied continuously through variable speed drives. These systems must maintain indoor air quality (IAQ) for each occupied zone in addition to maintaining thermal comfort and adhering to capacity and operational constraints. These equipment complexities and system limits warrant the use of smart building-level controllers to direct HVAC operations along with interface platforms, such as [9-10], for communication with electricity grid operators and demand-side aggregators.

Recent HVAC system research suggests these smart controllers use models to predict room temperatures and time-dependent heating or cooling efficiencies in order to plan daily operating strategies that obey system limits and minimize energy usage or cost. This technique is commonly called model predictive control (MPC) in HVAC system research. However, providing short-term, intra-hour modifications to energy consumption associated with ancillary services can require or lead to deviations from these energy efficient operating strategies and increase energy costs. With growing interest in both HVAC MPC and HVAC ancillary service provision, building owners and electric system operators must consider whether such modifications for the supply of ancillary services to the grid are worth the associated increase in energy costs to the building owner. This focus is a departure from the HVAC ancillary service research mentioned previously [4-8], which has mainly been concerned with developing methods of how buildings can provide ancillary services, rather than developing methods to address the proposed economic consideration.

This paper proposes defining the increased energy cost that results from providing an ancillary service as an opportunity cost. FERC Order 755 [11] already defines an opportunity cost for a generator as the lost energy profit that results from providing capacity for ancillary services instead of energy. Order 755 also requires the inclusion of these costs in ancillary service market clearing and remuneration practices. HVAC opportunity costs should be similarly considered, however, no previous research has presented a method to quantify them. Therefore, the primary objectives of this paper are to identify the sources of and develop a method for quantifying opportunity costs associated with HVAC systems providing ancillary services that is consistent with current practice.

The structure of this paper is as follows. Section II pro-

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vides a literature review of HVAC MPC and an overview of generator opportunity cost calculations. Section III develops the method for calculating HVAC opportunity costs. Section IV demonstrates the calculation and analysis of these costs using a numerical example. Lastly, Section V concludes and discusses future work.

II. Literature Review

A. HVAC Model Predictive Control

Due to room thermal capacity, system efficiency dependencies on such operating conditions as ambient air temperature and part-load ratio, and real-time energy prices, the times at which HVAC systems operate strongly influence energy use and cost. Beginning with [12], the last two decades of building research have seen a large focus on the use of model predictive control (MPC) for HVAC systems in order to minimize energy usage, costs, and/or peak demand subject to system limits. In this approach, zone heat transfer and system thermodynamic models, along with weather, internal load, and energy price forecasts, are used to optimize system operation, typically for the following 24 hours. A number of studies using MPC for both air-based and thermally active building structure (TABS) systems have shown HVAC energy usage savings of 5-70% and peak energy savings of 10-45% [13], compared to more conventional operation strategies. Studies that include energy prices have shown cost savings up to 75% [14-20].

More recent work has used MPC to optimize daily operating strategies when the building has a priori knowledge of energy and ancillary service market prices cleared in day-ahead or real-time wholesale markets [21-23]. The objective functions of these studies include energy cost and expected revenue from ancillary service provision. In [21-23], it is observed that HVAC systems can be incentivized to consume more energy during times of high ancillary service prices in order to provide more service, despite high energy prices or lower efficiency operating conditions. Considered another way, an HVAC system providing ancillary services can incur extra energy costs, relative to an economic baseline, for which the building owner should be compensated. As the penetration of HVAC-provided ancillary services grows, it will become important to include these incurred energy costs in the determination of ancillary service market prices and update demand-side remuneration practices.

Therefore, this paper defines these costs as opportunity costs, reviewed in the context of wholesale markets in the next section, and shows how solving a series of three HVAC MPC problems can be used to quantify them. The problems are similar in nature to the energy cost optimization problems reviewed above, however, have differences in the constraints that allow them, as a group, to determine HVAC opportunity costs. While [24] has very recently published a method to quantify the economic cost of HVAC flexibility using HVAC MPC, the authors acknowledge that the method is not suitable for dynamic operations, and is instead suitable for assessments of a building's ability to shift load. In contrast, the method proposed in this paper sequentially accounts for deviations from an economic baseline through time and is more appropriate for the production of bids along ancillary service market timelines.

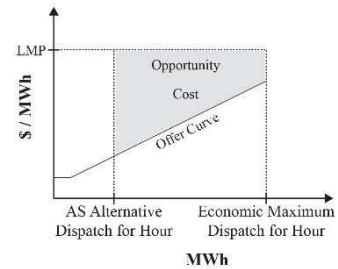

Fig. 1. Generator opportunity costs represent decreased energy profits as a result of potential energy capacity used instead for ancillary services [25-26, 28-29].

B. Comparison to Generator Opportunity Costs

FERC Order 755 [11] addressed the issue of fair and economically efficient compensation methods for all grid resources providing ancillary services, specifically frequency regulation. It did so by requiring two specific modifications to market rules. Note that while only one of them specifically refers to opportunity costs, both of these rules are applicable to the contents of this paper.

1) It required regulation payment to be split into two components: capacity and performance. The capacity payment reflects the capacity a resource must keep in reserve to provide regulation. The performance payment reflects the amount of work a resource does to provide regulation, including the accuracy with which it follows dispatch instructions. Each of the ISO/RTOs in the U.S. has implemented market rules that separately account for regulation capacity and performance costs.

2) It required the calculation and inclusion of opportunity costs within the capacity payment. These costs result from foregone profits that could have been gained in an energy market from capacity that, instead, is used for ancillary services.

Per Order 755 rule 2), all regional transmission organizations (RTO) and independent system operators (ISO) in the U.S. include generator opportunity costs in their ancillary service market marginal clearing prices. The costs are either calculated explicitly or abstracted implicitly from unit-commitment optimization shadow prices. For frequency regulation and operating reserves, the Pennsylvania-New Jersey-Maryland RTO (PJM) and ISO New England (ISO-NE) explicitly calculate opportunity costs, while the California ISO (CAISO), Midcontinent ISO (MISO), and New York ISO (NYISO) abstract them implicitly from shadow prices [25-30]. For voltage support services, NYISO calculates them explicitly. For clarity of demonstration, let us consider the explicit calculation of generator opportunity costs as defined by PJM, ISO-NE, and NYISO, which has been presented mathematically in Equation 1 and graphically in Figure 1.

$$\pi_{g,h} = \int_{x_{g,h}^{AS}}^{x_{g,h}*} \{LMP - c(x)\}\, dx \tag{1}$$

Here, $\pi_{g,h}$ is the generator opportunity cost for an hour h, $x_{g,h}^{*}$ is the economic dispatch setpoint without ancillary

service provision, $x_{g,h}^{AS}$ is the dispatch setpoint including ancillary service provision, LMP is the forecasted locational marginal price at the generator node, and $c(x)$ is the generator offer curve into the wholesale energy market.

Ancillary service opportunity costs exist for HVAC systems as they do for generators, in that they account for deviations from an economically optimal operational strategy with respect to energy consumption or production in order to provide ancillary services. There are, however, two important differences.

1) Opportunity costs for generators result from decreased energy profits due to potential energy generation capacity used instead to provide ancillary services. For HVAC systems, opportunity costs result from increased energy costs due to deviations from an economic baseline operating schedule to provide ancillary services.

2) Opportunity costs for generators are linked to an alternative dispatch setpoint within the hour of consideration (with ramping exceptions noted in the PJM and ISO New England markets [26, 29]). Meanwhile, for HVAC systems, opportunity costs are linked to a required alternative operating schedule over the planning horizon due to load shifting. This also requires the inclusion of HVAC opportunity costs in performance payments in addition to capacity, as dispatch instructions may impact future operation.

To the authors' knowledge, there is no practical method to account for HVAC opportunity costs through time, as there is for generators, as defined in this section. Therefore, this paper presents a method to calculate opportunity costs for HVAC systems using MPC techniques. Such costs could then be either included in market-clearing mechanisms to more accurately determine a marginal price at high penetrations of HVAC ancillary service provision, or they can be used by the building operator to determine fair, or required, remuneration for ancillary services provided.

Overall, this method enables buildings and electric operators to account for the cost of meeting ancillary service requirements as well as anticipate the capabilities of HVAC systems providing the ancillary services.

III. HVAC Opportunity Cost Quantification

A. Framework

The framework used to quantify opportunity costs is summarized in Fig. 2. Here, we consider that an HVAC system can provide two types of ancillary services: reserves and regulation. Reserves refers to the sudden curtailment of energy over a timescale of minutes to an hour according to system operator dispatch instructions in response to a grid contingency or sudden loss of supply energy. Regulation refers to adjustments in energy consumption up and/or down over a timescale of seconds according to a system operator dispatch signal to help maintain grid frequency.

Each ancillary service is split into two components: capacity and performance. Note the difference between the two is the same as that which Order 755 defines for regulation [11]. Capacity represents the capacity the HVAC system has to provide an ancillary service. Performance represents the work the HVAC system does to provide the ancillary service in response to grid operator dispatch instructions. For example, applying the terminology to reserve provision, capacity represents the capability to provide reserves, while performance represents the actuality of delivering zero, some, or all of that capacity in the case of a contingency.

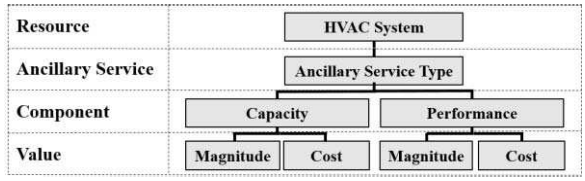

Fig. 2. Relationship of terms used for quantification of ancillary service opportunity costs for HVAC systems.

Finally, each ancillary service component has two associated values: magnitude and cost. The magnitude represents the amount of potential or actual energy modification for ancillary service capacity or performance provision respectively. The cost represents the financial penalty of providing the corresponding component. We are interested in quantifying the opportunity portion of this cost; such other factors as maintenance and equipment efficiency costs are outside the scope of this paper, though have been recognized in another work [31].

Before explicitly quantifying opportunity costs, we must first define the economic baseline schedule and discuss the effects of ancillary service provision.

B. The Economic Baseline and Sources of Opportunity Costs

Consider a general formulation of the HVAC MPC energy cost minimization problem described by Equations 2-6 below. This problem will be referred to as Problem 1.

$$\boldsymbol{x}^*, \boldsymbol{T}^*, \boldsymbol{q}^* = \text{argmin} \sum_{i=1}^{n} P_i x_i \quad (2)$$

$$s.t.$$

$$T_i = f(q_{1:i}, B_{1:i}) \quad \forall i \quad (3)$$

$$x_i = g(q_i, S_i) \quad \forall i \quad (4)$$

$$T_i^{ll} \leq T_i \leq T_i^{ul} \quad \forall i \quad (5)$$

$$q_i^{ll} \leq q_i \leq q_i^{ul} \quad \forall i \quad (6)$$

Here, $\boldsymbol{x}$, $\boldsymbol{T}$, and $\boldsymbol{q}$ are vectors of electric energy, zone air temperature, and cooling thermal energy respectively for each time interval, i, within the planning horizon of length n. Equation 2 expresses the objective function, which quantifies the cost of energy over the planning horizon, subject to electricity prices $\boldsymbol{P}$. At the retail level, prices may result from day-ahead locational marginal prices (LMP) [32] or utility rate structures. Equation 3 represents the functional dependence of room air temperature on the cooling power schedule and the set of all other variables that contribute to room air temperature physics, B, including environmental loads, internal loads, and building construction. Equation 4 represents the functional dependence of electrical power on cooling power and the set of all other variables that contribute to HVAC system efficiency, S, including environmental conditions, operating conditions,

and system design. Equation 5 implements upper, $\boldsymbol{T}^{ul}$, and lower, $\boldsymbol{T}^{ll}$, temperature limits in order to maintain thermal comfort. These limits may vary over the planning horizon with occupancy schedule. Lastly, Equation 6 implements upper, $\boldsymbol{q}^{ul}$, and lower, $\boldsymbol{q}^{ll}$, cooling power limits that may be derived from capacity constraints or minimum IAQ requirements.

Solving Problem 1 with forecasts of weather, internal loads, and electricity prices gives an economic baseline cooling schedule, $\boldsymbol{q}^*$, and associated baseline room air temperature, $\boldsymbol{T}^*$, and energy consumption, $\boldsymbol{x}^*$, trajectories, that minimize energy costs and obey system limits. A common, though hypothetical, solution to the problem is depicted in Fig. 3 and consists of an early morning period of cooling towards the lower comfort limit followed by a period during the day in which room air temperature rises towards the upper comfort limit. Once it is reached, the temperature is kept at the upper limit for the remainder of the day. This strategy takes advantage of shifting cooling to the morning hours when cooling efficiencies tend to be higher and electricity prices tend to be lower. Note, however, that both system design and exogenous conditions will dictate the extent to which this strategy can be utilized [34].

Opportunity costs arise from adjusting the economic baseline cooling scheduled for an hour of interest in order to provide magnitudes of capacity or performance ancillary service. As depicted in Fig. 3, a modification to the baseline cooling at one hour impacts the cooling required during future hours, creating an alternative cooling schedule. Decreased cooling at one hour will cause a rise in room air temperature, which requires increased cooling at future hours in order to maintain the room air temperature below an upper limit. Higher ambient air temperatures and electricity prices at these future hours will make this alternative cooling schedule more costly. Meanwhile, increased cooling at one hour will cause a drop in room air temperature, which will lessen the requirement of cooling at future hours. However, the reduction in cooling cost for future hours will not offset the increased cost of overcooling at the hour of interest. Therefore, this alternative cooling schedule will also be more costly.

C. Quantifying Opportunity Costs

The process of quantifying opportunity costs as a function of capacity and performance magnitudes is outlined in Fig. 4 and detailed in the rest of this section.

1) Capacity: First, the smart HVAC system must solve Problem 1 in order to obtain an economic baseline for the planning horizon (Fig. 4.i). Once Problem 1 is solved, a smart HVAC system may estimate its capacity magnitude for an ancillary service at a given hour, h, using Equation 7 (Fig 4.ii).

$$x_h^C = x_h^* + x_h^a - x_h^{ll} \tag{7}$$

The capacity magnitude available for an ancillary service at a given hour, x_h^C, is equal to the baseline energy consumption at that hour, x_h^*, plus additional energy consumption to provide extra capacity, x_h^a, less a lower system limit of energy consumption, x_h^{ll}. The sign convention here is natural for

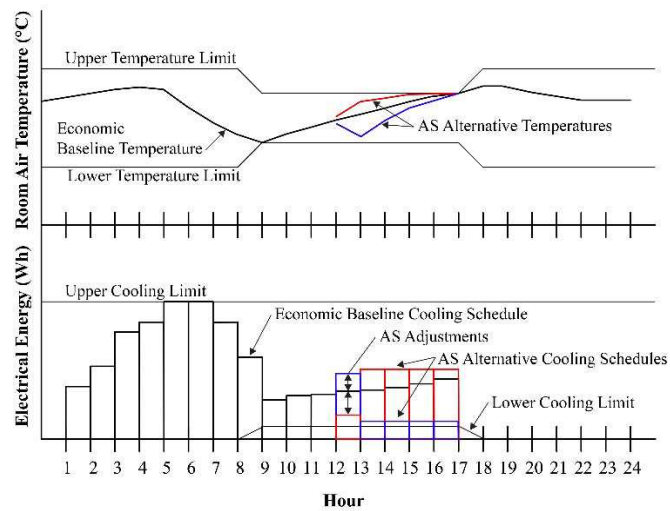

Fig. 3. Hypothetical baseline (black) and alternative cooling schedules with corresponding zone air temperatures for decreased (red) and increased (blue) consumption modification at hour ending 13.

ancillary services that require reduction in energy consumption (such as reserves or regulation up), where higher energy consumption and lower system limit give the opportunity for higher capacity. Ancillary services that require an increase in energy consumption (such as regulation down), capacity may be calculated similarly: by replacing the lower limit of power consumption with an upper limit and multiplying the right side of the equation by (-1).

While the baseline energy consumption, x_h^*, is given from the solution of Problem 1, the values of additional energy consumption, x_h^a, and the lower system limit, x_h^{ll}, are to be chosen or calculated. The lower system limit should be calculated as a function of operating conditions, HVAC system capacity, thermal comfort constraints, and indoor air quality constraints. A method of calculating this limit will be included in a future article. Assuming, however, that a lower limit value is calculated, quantifying additional energy consumption is the only term over which the building operator has control to determine how much ancillary service capacity is provided.

The impact of the choice of this additional energy consumption value is important to consider, as a non-zero value will impact the cooling schedule for future hours, as described in Section III. B. A positive value, indicating greater energy consumption than the economic baseline, will have the impact of the increased cooling case, while a negative value, indicating less energy consumption, will have the impact of the decreased cooling case. The value of additional energy consumption may be varied to determine opportunity cost as a function of capacity magnitude for market bidding.

Once a capacity magnitude is determined, the effect of providing this capacity on the economic baseline can be found by solving the optimization problem described by Equations 8-11, which are a modification to Problem 1 and are referred to as Problem 2 (Fig. 4.iii). Note that constraint Equations 3-5 still apply.

$$\overline{\boldsymbol{x}}_h^C, \overline{\boldsymbol{T}}_h^C, \overline{\boldsymbol{q}}_h^C = \text{argmin} \sum_{i=1}^{n} P_i x_i$$

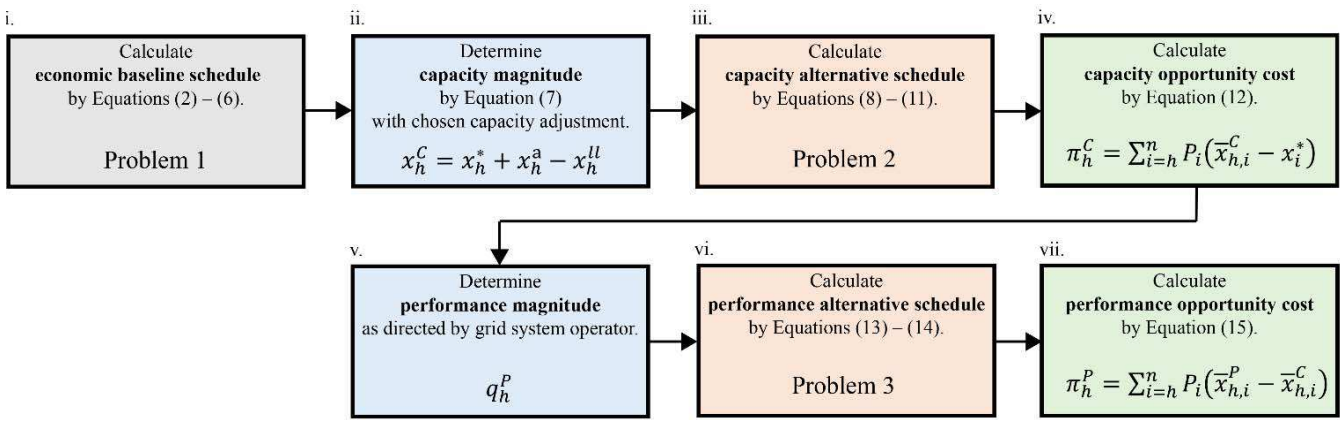

Fig. 4. Flowchart for quantifying opportunity costs as a function of capacity and performance magnitudes.

$$s.t. \tag{8}$$

$$q_i = q_i^* \quad \forall i < h \tag{9}$$

$$q_i = q_i^* + q_h^a \quad \forall i = h \tag{10}$$

$$q_i^{ll} \leq q_i \leq q_i^{ul} \quad \forall i > h \tag{11}$$

Here, $\overline{\boldsymbol{x}}_h^C$, $\overline{\boldsymbol{T}}_h^C$, and $\overline{\boldsymbol{q}}_h^C$ represent capacity alternative schedules of electric energy, zone air temperature, and cooling energy, respectively. The objective function in Equation 8 has not changed from Problem 1. Equation 9 implements the assumption that until the hour of interest, the HVAC system will operate according to the economic baseline. Equation 10 implements a change in cooling power at the hour of interest, q_h^a, in order to modify the ancillary service capacity magnitude by x_h^a. Note the relationship between the two is defined by Equation (4). Lastly, Equation 11 implements upper and lower cooling power limits for the remainder of the planning horizon.

Upon solving Problem 2, the opportunity cost associated with a capacity of ancillary service at an hour of interest, π_h^C, can be calculated by Equation 12 (Fig. 4.iv) The opportunity cost is equal to the energy cost difference between the economic baseline and capacity alternative cooling schedules over the planning horizon.

$$\pi_h^C = \sum\nolimits_{i=h}^n P_i \left(\overline{x}_{h,i}^C - x_i^*\right) \tag{12}$$

Note that both Equations 1 and 12 consist of valuing the difference between the economic and ancillary service alternative operating strategies with the forecasted price of energy. Meanwhile, the order of this difference in Equation 12 is opposite of that in Equation 1 and Equation 12 sums the valued difference over multiple hours. These two points are consistent with the two key differences between HVAC and generator opportunity costs that were discussed in Section II.B.

Capacity opportunity cost curves can be calculated for a number of capacity magnitudes at an hour of interest by varying the value of x_h^a and resolving Problem 2 appropriately. We expect capacity opportunity costs to be zero if x_h^a is zero, and positive if x_h^a is non-zero. This is by definition of the economic baseline optimal solution. In practice, cost curves could be calculated on an hourly basis with the most recent performance information and exogenous variable forecasts in order to be submitted in advance of the next hour's real-time market clearing. Upon market-clearing, the capacity alternative schedule that results from the assigned magnitude of capacity for the following hour should be used as the economic baseline schedule for future capacity calculations.

2) Performance: Once ancillary service capacity has been assigned to the HVAC system, performance opportunity costs may be calculated based on forecasted or actual performance ancillary service provided over an hour of interest. Here, we are interested in the effect providing a magnitude of performance ancillary service has on changing the total amount of cooling delivered over the hour. The difference is relative to the amount of cooling required to provide the assigned magnitude of capacity, the cost of which has already been calculated according to the previous section. The difference can be non-zero due to a call for reserve energy, an asymmetric regulation signal, or symmetric regulation signal that is not cooling energy-neutral (Fig. 4.v). A non-zero difference will again impact the cooling schedule for future hours, as described in Section III.B.

In order to determine the alternative cooling schedule associated with a magnitude of ancillary service performance, the optimization problem described by Equations 13-14 must be solved. This problem is a modification to Problem 2 and referred to as Problem 3 (Fig. 4.vi). Note that constraint Equations 3-5, 9, and 11 still apply.

$$\overline{\boldsymbol{x}}^P, \overline{\boldsymbol{T}}^P, \overline{\boldsymbol{q}}^P = \text{argmin} \sum\nolimits_{i=1}^n P_i x_i \tag{13}$$

$$s.t.$$

$$q_i = \overline{q}_{h,i}^C - q_h^P \quad \forall i = h \tag{14}$$

Here, $\overline{\boldsymbol{x}}_h^P$, $\overline{\boldsymbol{T}}_h^P$, and $\overline{\boldsymbol{q}}_h^P$ represent performance alternative schedules of electric energy, zone air temperature, and cooling energy respectively. The objective function in Equation 13 has not changed from Problem 1. Equation 14 implements a change in cooling energy at the hour of interest, q_h^P, in order to provide an ancillary service performance magnitude as directed by an electric grid system operator. Once again, the notation is positive for ancillary services that require a decrease in energy consumption.

Upon solving Problem 3, the opportunity cost associated

with a magnitude of ancillary service performance at an hour of interest, π_h^P, can be calculated by Equation 15 (Fig. 4.vii). The opportunity cost is equal to the energy cost difference between the performance alternative and capacity alternative cooling schedules over the planning horizon.

$$\pi_h^P = \sum_{i=h}^{n} P_i \left(\overline{x}_{h,i}^P - \overline{x}_{h,i}^C\right) \tag{15}$$

Similar to the case of capacity, a performance opportunity cost curve corresponding to various amounts of performance magnitudes may be calculated ex-ante and bid into an ancillary service performance market, though these do not yet exist in practice. This could allow aggregators or electric grid operators to optimize the dispatch of ancillary service performance. Alternatively, the opportunity cost may be calculated ex-post, in order to serve as a corrective payment. While we expect capacity opportunity costs to be strictly positive for non-zero x_h^a, performance opportunity costs may be positive or negative for non-zero x_h^P. It can be positive if the performance dispatch instructions move the cooling schedule away from the economic baseline and negative if the dispatch instructions move the cooling schedule towards the economic baseline. Lastly, similar to the schedule after the assignment of capacity magnitude, in practice the performance alternative schedule should be used as the economic baseline schedule for updates to the availability and costs of ancillary service capacity in future hours.

D. Discussion

It is important to explore some implications of the work presented in the previous section with semi-quantitative examples. Consider a first example where, in a particular hour, the HVAC system overcools compared to the economic baseline to provide more reserve capacity, but is not called to deliver this capacity as energy. In this case, $q_h^a > 0$ and $q_h^P = 0$. Here, the HVAC system should be remunerated the full capacity opportunity cost, as the performance opportunity cost is zero. Consider a second example where, in a particular hour, the HVAC system does not overcool to provide more capacity, but is called to deliver available reserve capacity as energy. In this case, $q_h^a = 0$ and $q_h^P > 0$. Here, the HVAC system should be remunerated the full performance opportunity cost, which is positive in this case, while the capacity opportunity cost is zero. Lastly, consider a third example where an HVAC system, in a particular hour, overcools to provide more reserve capacity and is called to deliver some or all of this extra reserve capacity in the form of energy. In this case, $q_h^P \leq q_h^a$ for $q_h^P, q_h^a > 0$. Here, providing some of this extra reserve capacity back to the electric grid moves the building state back towards the original economic baseline. Therefore, the performance opportunity cost is negative and reduces the total remuneration expected from the capacity opportunity cost alone.

Given the exploration above, it is convenient to define a total opportunity cost for a particular hour as the sum of that hour's capacity and performance opportunity costs; described by Equation 16

$$\pi_h^T = \pi_h^C + \pi_h^P \tag{16}$$

The relative magnitudes of capacity and performance opportunity costs are likely to vary with HVAC system design, operating conditions, and energy pricing.

Finally, there are two types of uncertainties associated with opportunity cost accounting. First, there are uncertainties around the realized dispatch of ancillary service. One example of this is whether reserve capacity is called to be provided as energy in the case of a contingency, or not. Note that the method presented here is in a predictor-corrector format. That is, capacity opportunity costs are corrected for by performance opportunity costs, which represent the impact of realized dispatch. Therefore, the use of total opportunity costs account for the impact of realized ancillary service dispatch. An important conclusion here is that an ideal market-clearing and dispatch mechanism could optimally consider the hedging of potential performance payments with capacity payments. This would require probabilistic modeling of realized dispatch situations, leading to expected performance, and therefore total, opportunity costs rather than deterministic.

The second type of uncertainty lies in the forecasting of HVAC load conditions and energy pricing for the three optimization problems used in the method. In general, this method does not specifically account for this uncertainty. An economic baseline and ancillary service availability should be updated if realized operating conditions are significantly different than forecasted and made available to market operators. Resources are typically allowed to update ancillary service availability on an hourly basis in U.S. RTOs. Often for consumers, energy prices are set based on a pre-determined retail rate structure or can be set daily based on day-ahead prices, in order to incentivize load shifting [32]. Energy prices in these majority cases are not uncertain for the calculation of opportunity costs.

IV. Numeric Examples

To examine the magnitudes and implications of HVAC opportunity costs, we describe the HVAC system model and calculate of capacity opportunity cost curves for three hours during an example day. We then calculate performance opportunity cost curves associated with an assigned value of capacity at one hour during the example day.

A. Model Description

The model describes a single-zone test chamber on MIT's campus meant to represent an office. Cooling is supplied by an air-to-air heat pump. A linear equation, known as an iCRTF [33], is used to describe the room temperature dynamics in response to ambient air temperature, T_i^{amb}, adjacent room temperature, T_i^{adj}, interior convective heating loads, q_i^{con}, interior radiative heating loads, q_i^{rad}, and delivered cooling, q_i. The linear equation coefficient vectors $\boldsymbol{a} \in R^3, \boldsymbol{b} \in R^4, \boldsymbol{c} \in R^4, \boldsymbol{e} \in R^4, \boldsymbol{f} \in R^4$, correspond to a model of order $N = 3$. Use of these iCRTF models for MPC was validated experimentally in [34]. A third-order polynomial function, g, is used to model the coefficient of performance of the heat pump as a function of evaporator air inlet temperature, which is the same as room air temperature T_i, condenser air inlet temperature, which is the same as ambient air temperature T_i^{amb}, and

TABLE I

NUMERIC EXAMPLE MODELING CONDITIONS AND ASSUMPTIONS

Variable	Assumption
n	= 24 hours
T_i^{adj}	= 22.5 °C $\forall i$
q_i^{con}	= 18 W/m^2 if $i \in$ [6,18], = 0 W/ m^2 otherwise.
q_i^{rad}	= 18 W/m^2 if $i \in$ [6,18], = 0 W/ m^2 otherwise.
T_i^{ll}	= 20 °C if $i \in$ [6,18], = 15 °C otherwise.
T_i^{ul}	= 25 °C if $i \in$ [6,18], = 30 °C otherwise.
q_i^{ll}	= 0 Wh $\forall i$
q_i^{ul}	= 1500 Wh $\forall i$
P_i	Fig. 5(a)
T_i^{amb}	Fig. 5(b)

part-load ratio, q_i/q_i^{ul}. Detailed development, validation, and former use of this model can be found in [13,35]. These models are applied to the generalized Problems 1-3 in order to demonstrate our numeric examples. Specifically, the function f is defined by the iCRTF equation and the function g is defined by the third-order polynomial heat pump performance function. The models applied to Problem 1 are shown by Equations 17-21.

$$\boldsymbol{x}^*, \boldsymbol{T}^*, \boldsymbol{q}^* = \text{argmin} \sum_{i=1}^{n} P_i x_i \tag{17}$$

$$s.t.$$

$$T_i = \sum_{k=1}^{N} a_k T_{i-k} + \sum_{k=0}^{N} b_k T_{i-k}^{adj} + \sum_{k=0}^{N} c_k T_{i-k}^{amb} + \sum_{k=0}^{N} e_k \left(q_{i-k}^{con} - q_{i-k} \right) + \sum_{k=0}^{N} f_k q_{i-k}^{rad} \quad \forall i \tag{18}$$

$$x_i = \frac{q_i}{g\left(\frac{q_i}{q_i^{ul}}, T_i, T_i^{amb}\right)} \forall i \tag{19}$$

$$T_i^{ll} \le T_i \le T_i^{ul} \quad \forall i \tag{20}$$

$$q_i^{ll} \le q_i \le q_i^{ul} \forall i \tag{21}$$

Notice that Equation 17 is identical to Equation 2 and Equations 18-21 are explicit representations of Equations 3-6 respectively.

For this example, the modeling conditions and assumptions are displayed in Table 1 and Fig. 5. For simplicity, humidity and fresh air delivery control are not considered. The optimizations for this example are implemented in MATLAB with an interior-point algorithm [36]. Finding the economic baseline solution takes approximately 1.6 seconds on a desktop computer with a 2.5 GHz CPU and 8 GB RAM.

B. Capacity

We explore capacity opportunity cost curves for several increments of reserve or regulation down capacity magnitude provided at hours 5, 10, and 15. For each hour, we assume the lower limit of electrical cooling energy, x_h^{ll}, to be zero and vary the value of q_h^a from -200 Wh to 200 Wh by

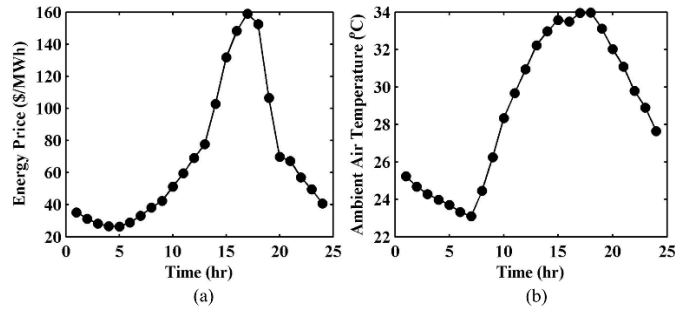

Fig. 5. Day-ahead LMP (a) and ambient temperature (b) in Willow Grove, PA for July 15th 2013 [37-38].

TABLE II

DETAILED RESULTS FOR CAPACITY OPPORTUNITY COST CALCULATIONS

h	x_h^* (Wh)	q_h^+ (Wh)	x_h^a (Wh)	x_h^C (Wh)	$\sum_{i=1}^{n} P_i \overline{x_i^C}$ (¢)	π_h^C (¢)
5	145	-200	-49	96	8.646	0.021
		-100	-26	118	8.631	0.005
		0	0	145	8.626	0.000
		100	31	176	8.635	0.009
		200	67	212	8.662	0.036
10	76	-200	-33	43	8.648	0.022
		-100	-17	59	8.632	0.006
		0	0	76	8.626	0.000
		100	19	95	8.633	0.007
		200	42	118	8.660	0.033
15	80	-200	NS	NS	NS	NS
		-100	NS	NS	NS	NS
		0	0	80	8.626	0.000
		100	21	100	8.742	0.119
		200	45	124	8.902	0.279

increments of 100 Wh. As was mentioned in Section III.C, a systematic approach to calculating the actual lower limit of electrical cooling energy required to maintain acceptable occupant service is under development for a future article. Using the models described in Section IV.A, Problem 1 is solved once and Problem 2 is solved for each hour and each value of q_h^a.

Fig. 6 presents the economic baseline electric energy consumption and room air temperature for the day. The capacity alternatives for hour 10 and q_{10}^a values of 200 Wh and -200 Wh are also shown beginning at hour 10. The economic baseline strategy is to precool the building in the early morning hours when energy is cheap and allow the temperature to rise to an upper limit during the day. This results in an economic baseline energy cost for the day of 8.626 ?. The effects of cooling power adjustments at hour 10 are noticeable in both Fig. 6(a) and Fig. 6(b). If more power is used at hour 10, the temperature decreases and future hours require less cooling. If less power is used, the temperature increases and future hours require more cooling.

Fig. 7 presents capacity opportunity cost curves for each hour as a function of capacity magnitude, while Table 2 gives more detailed values. Notice that the capacity opportunity costs are zero when x_h^a is zero and positive when x_h^a is non-zero, as hypothesized in Section III.C.1). The exception to

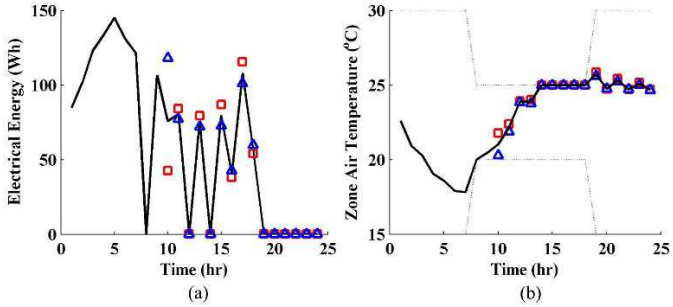

Fig. 6. Hourly electrical energy (a) and zone air temperature (b) for the economic baseline (line), $q_{10}^{a} = -200$ Wh (red square), and $q_{10}^{a} = 200$ Wh (blue triangle).

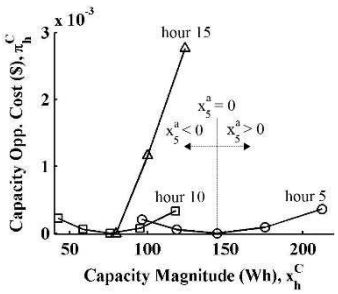

Fig. 7. Capacity opportunity cost as a function of capacity magnitude for hour 5 (circle), hour 10 (square), and hour 15 (triangle).

this is for the cases in hour 15 where cooling was adjusted downward, and no feasible capacity alternative solution could be found. In these cases, the economic baseline temperature is already at the upper temperature limit. Providing 100 or 200 Wh less cooling during this hour causes the temperature to rise above this limit and makes the solutions infeasible according to Equation 19. This indicates downward adjustments in cooling of these magnitudes are not available.

Also, the intersection of the cost curves with the zero-axis at capacity magnitudes greater than zero indicates that this amount of capacity magnitude may be provided without opportunity cost. Note that it is equal to the difference between the economic baseline energy consumption and the lower limit of energy consumption for the hour.

Lastly, the rate at which capacity cost changes per unit change of capacity magnitude increases with hour, as seen by the relative slopes of the curves in Fig. 7 for each hour. This indicates that the marginal cost of capacity is greater at later hours in the day, when energy prices are high, room air temperatures have nearly reached the upper limit, and there is less time to recover from alternative operation. Approximating these marginal cost values from the data in Table 2 gives a range of \$1.85/MWh at hour 5 to \$66.66/MWh at hour 15. The magnitude of these marginal costs agree with the specific costs of load shifting to and from a period of one hour found in [24], which were between 0 and 80 €/MWh for the heating of an office building.

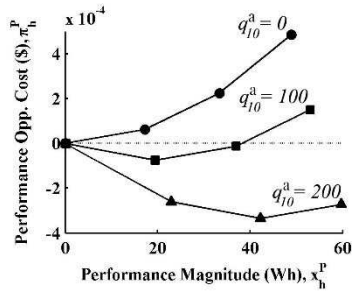

Fig. 8. Performance opportunity cost as a function of performance magnitude for hour 10 and $q_{10}^{a} = 0$ Wh (circle), $q_{10}^{a} = 100$ Wh (square), and $q_{10}^{a} = 200$ Wh (triangle).

C. Performance

In this example, we look to explore performance opportunity cost curves for several increments of reserve or regulation down performance magnitude using particular capacity alternatives. We consider hour 10 and the capacity alternative cases of q_{10}^{a} equal to 0, 100, and 200 Wh. For each case, q_{10}^{P} is varied from 0 to 300 Wh by increments of 100 Wh. Problem 3 is solved for each case using the models described in Section IV.A. and results obtained in Section IV.B.

Fig. 8 presents performance opportunity costs as a function of performance magnitude for each value of capacity magnitude. Notice that the costs are not strictly positive, agreeing with the hypothesis in Section III. C. 2). If more cooling is delivered than the economic baseline to provide extra capacity magnitude, use of the extra capacity for performance moves the building operation back towards the economic baseline and the performance opportunity cost is negative. However, providing performance magnitude in excess of extra capacity leads to a building state that is still worse than that of the capacity alternative, causing the slope of performance opportunity cost to be positive. Note that if the state of the building is still better than the full capacity state, the performance opportunity cost will remain negative. The slope changes from negative to positive when the performance magnitude equals the additional capacity magnitude. The magnitude changes from negative to positive when the building state after performance is worse than the building state after capacity.

Finally, Fig. 9 presents the total opportunity cost as a function of performance magnitude for each value of capacity magnitude. Note here the balance between capacity and performance. The case in which no additional capacity was provided experiences increasing total costs with increasing performance magnitude. The cases in which additional capacity was provided experience a period of decreasing total costs with increasing performance magnitude, followed by increasing total costs. Meanwhile, all total cost curves are non-negative and intercept the zero-axis when the performance magnitude equals the extra capacity magnitude. The nature of ancillary services makes it difficult to predict exactly the performance magnitude that will be required from an HVAC resource during a given hour. However, these results indicate that an HVAC system may hedge its total opportunity

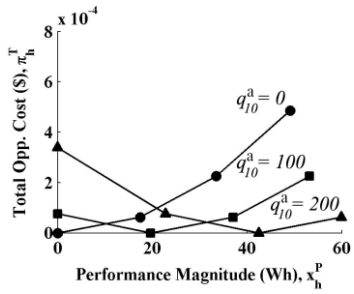

Fig. 9. Total opportunity costs as a function of performance magnitude for hour 10 and $q^a_{10} = 0$ (circle), $q^a_{10} = 100$ (square), and $q^a_{10} = 200$ (triangle).

costs of providing an amount of performance magnitude by adjusting its capacity magnitude appropriately. At the same time, adjusting capacity too much and not having to provide as much performance can be costly. Note that distinguishing performance magnitude and cost from capacity magnitude and cost is an important difference between the HVAC flexibility costing method proposed here and that proposed in [24], which enables bidding into ancillary service markets and accounting for uncertain operation requirements.

V. Conclusion

In response to growing interest in the use of commercial building HVAC systems to provide electric grid ancillary services, the primary contributions of this paper are the development of a definition for ancillary service opportunity costs for HVAC systems and a method to sequentially account for them in time. This method enables buildings and electric grid operators to account for cost of meeting ancillary service requirements with HVAC systems as well as anticipate the capabilities of HVAC systems providing the ancillary services.

There are three main areas of future work from this paper:

1) Continued development and validation of HVAC MPC and optimization methods for multi-zone commercial buildings. Specifically needed are better methods to predict system limits of ancillary service capacity. These limits are not only set by thermal comfort and indoor air quality constraints for each occupied zone, but also equipment operating limits under various exogenous and part-load conditions.

2) Exploration of opportunity cost calculation for various building forms, HVAC system types, and electricity pricing environments. The impact of thermal storage availability on marginal opportunity cost prices is particularly interesting.

3) Development of strategies for how these opportunity costs may best be integrated with electricity market clearing mechanisms and analysis of the effect on market-clearing prices for large amounts of HVAC-provided ancillary services. Due to the amount of state information required and large number of buildings, it could be most practical for buildings or building aggregators to calculate opportunity costs, instead of an RTO/ISO.

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