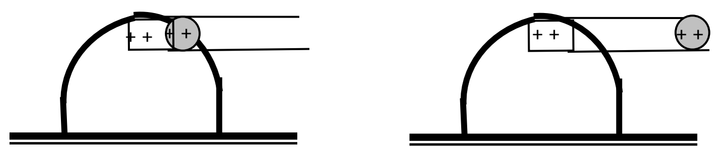
Electric Potential Energy (U_q)

1.1 Derive a Relation

Let us consider two positively charged objects that are held near each other in the muzzle of a cannon (see part (a) of the illustration). Also let us make the cannon point horizontal to the surface. When the “trigger” holding the cannonball is released, the positively charged cannonball flies out the end of the muzzle (part (b)).



(a) (b)

a) Examine a system that contains the positively charged cannonball only. The initial point is when the cannonball is loaded in the muzzle and the final point of the process is when the ball reaches the end the barrel. Using the language of energy, describe in words the process shown, going from the initial state to the final state.

b) Draw an energy bar chart representing the initial and final states.

c) Draw a force diagram for the positively charged cannonball at the initial and final states.

d) Plot a graph of the force exerted by the cannon on the cannonball versus distance between their centers from their initial separation to infinity.

e) Imagine that a person wishes to prevent the cannonball from flying away from the cannon by exerting a force on the ball toward the cannon with the same magnitude that the cannon exerts on the cannonball. If the person pushes the ball just a tiny bit harder, it can be displaced a small distance Δr_1 toward the cannon. Show that work ΔW_1 done by the pushing force during this small displacement is equal to $\Delta W_1 = F_{\text{P on 2}} \Delta r_1 = \frac{k q_1 q_2}{r_i^2} \Delta r_1$.

f) Explain why the amount of work per unit distance needed to push the charged objects closer together increases as the distance between the objects decreases.

1.2 Derive a Relation

For the same process as in 1.1, choose a different system: cannonball – cannon.

a) Using the language of energy, describe in words the process, going from the initial state to the final state.

b) Draw an energy bar chart representing the initial and final states.

c) Use the bar chart and the expression for work that we derived in 1.1 to show that the change in electric potential energy of the system of cannonball-cannon is

$$DU_q = kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right).$$

d) Use the bar chart and the expression for work that we derived in 1.1 to that that the electric potential energy of a system with two charged objects separated by distance r is:

$$U_q = \frac{k\ q_1\ q_2}{r}.$$

e) Plot a graph that shows how the electric potential energy changes as the separation r between two **like-charged** objects is varied.

f) Plot a graph that shows how the electric potential energy changes as the separation r between two **oppositely charged** objects is varied.

Did You Know?

Note that physicists agreed to have the potential energy of interaction of two charged objects to be zero when the distance between them is infinite. This is because being infinitely far from each other, the objects do not interact.

Electric potential energy U_q of two charged particles: The electrical potential energy of two particles with electric charge q_1 and q_2 separated by a distance r is:

$$U_q = \frac{kq_1q_2}{r}$$

Electrical potential energy is measured in units of joules. The relationship above can be used for both positively and negatively charged objects; the signs of the charged particles *must be included*. Notice that electrical potential energy is proportional to $1/r$ and not to $1/r^2$ as in Coulomb’s force law. The reference energy for electric potential energy is zero when the particles are infinitely far apart.

The change in electrical potential energy ΔU_q: When two objects with charges q_1 and q_2 move from an initial separation r_i to a final separation r_f, the change in electric potential energy of the two particle system is:

$$\Delta U_q = U_{qf} - U_{q_i} = kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right).$$

Electric potential energy of multiple charge systems. The total electric potential energy of the system is the sum of the energies of all pairs.

1.3 One very simple and productive (although rarely used in modern physics) model of a hydrogen atom is a positive nucleus (a proton) and a negatively charged electron moving around in a circular orbit. Estimate the electron's speed in this model. The radius of the atom is 0.51 x 10^{-10} m.

1.4 Determine the minimum energy that the proton nucleus-electron system needs to gain for the electron to become free (i.e. to remove the electron very far from its proton nucleus). (Recall, this is a simple model for the hydrogen atom.)

1.5 A 0.50-kg cart with a metal sphere, electrically charged q = + 2.0 x 10^{-5} C, starts at rest with its metal sphere 1.0 m to the right of a fixed sphere with a positive charge Q = + 3.0 x 10^{-4} C. When released, the cart travels toward a fixed sphere on the right with charge $-Q$. The fixed charged objects are separated by 10 m. How fast is the cart moving when its charged metal sphere is 1.0 m from the negatively charged fixed sphere on the right?

1.6 Jeopardy

The equation below describes one or more physical process(es).

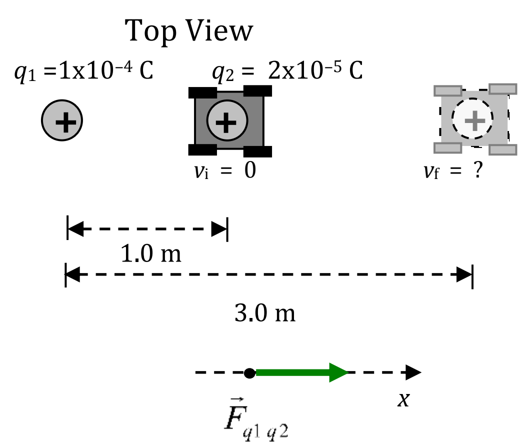
$$\frac{1}{2}(1.67 \text{ x } 10^{-27} \text{ kg})\, v_i^2 + \frac{1}{2}(1.67 \text{ x } 10^{-27} \text{ kg})\, v_i^2 = \frac{(9.0 \text{ x } 10^{9} \text{ N}\bullet\text{m}^2/\text{C}^2)(1.6 \text{ x } 10^{-19} \text{ C})^2}{1.0 \text{ x } 10^{-15} \text{ m}}$$

a) Draw a bar chart that is consistent with the equation.

b) Sketch the initial and final states that the equation might describe.

c) Write a word problem for which the equation could be a solution.

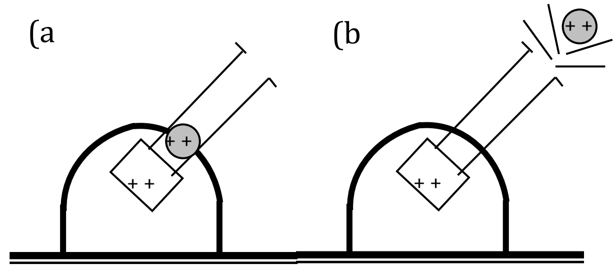
1.7 A 2.0-kg cart with a charge of +2.0 x 10^{-5} C is held at rest 1.0 m to the right of a fixed dome with charge +1.0 x 10^{-4} C. The cart is released. Determine how fast it is moving when it is 3.0 m from the fixed-charged dome.



1.8 Electric Potential Energy in a uniform field. Read pgs 561-562 in your textbook to derive the change in electric potential energy in a uniform field:

$$\Delta U_q = q_t E d$$

Qualitative Two Like-Charged Objects



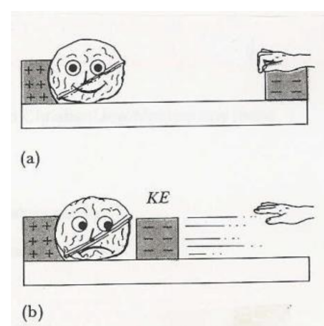
Two positively charged objects are held near each other in the muzzle of a cannon (see part (a) of the illustration). When the "trigger" holding the cannonball is released, the positively charged cannonball flies out the end of the muzzle (part (b)).

a) Identify the system and the objects in your system.
b) Certain types of energy have increased. Describe some type of energy that decreased that you think might compensate for the increase in these other forms of energy.
c) Using the language of energy, describe in words the process, going from the initial state to the final state.
d) Draw an energy bar chart representing the initial and final states.

$K_i + U_{gi} + U_{si} + U_{qi} + W = K_f + U_{gf} + U_{sf} + U_{qf} + \Delta U_{int}$

Qualitative Two Oppositely Charged Objects

Imagine the energy changes of two opposite-sign charged objects used as a nutcracker, as illustrated in the figure to the right.



a) What happens when the negatively charged block shown in (a) is released and moves near the nut, as shown in (b)?
b) What type of energy decreases to make up for the increase in kinetic energy?
c) Using the language of energy, describe in words the process described above, going from the initial state to the final state.
d) Draw an energy bar chart representing the initial and final states.