

Chapter 6

Work and Kinetic Energy

Conceptual Problems

1 • True or false: (a) If the net or total work done on a particle was not zero, then its speed must have changed. (b) If the net or total work done on a particle was not zero, then its velocity must have changed. (c) If the net or total work done on a particle was not zero, then its direction of motion could not have changed. (d) No work is done by the forces acting on a particle if it remains at rest. (e) A force that is always perpendicular to the velocity of a particle never does work on the particle.

Determine the Concept A force does work on an object when its point of application moves through some distance and there is a component of the force along the line of motion.

(a) True. The total work done on a particle is equal to the change in its kinetic energy (the work-kinetic energy theorem).

(b) True. The total work done on the particle changes the kinetic energy of the particle, which means that its velocity must change.

(c) False. If the total work done on the particle is negative, it could be decreasing the particle's speed and, eventually, reversing its direction of motion.

(d) True. The object could be at rest in one reference frame and moving in another. If we consider only the frame in which the object is at rest, then, because it must undergo a displacement in order for work to be done on it, we would conclude that the statement is true.

(e) True. A force that is always perpendicular to the velocity of a particle changes the direction the particle is moving but does no work on the particle.

2 • You push a heavy box in a straight line along the top of a rough horizontal table. The box starts at rest and ends at rest. Describe the work done on it (including sign) by each force acting on it and the net work done on it.

Determine the Concept The normal and gravitational forces do zero work because they act at right angles to the box's direction of motion. You do positive work on the box and friction does negative work. Overall the net work is zero because its kinetic energy doesn't change (zero to zero).

3 • You are riding on a Ferris wheel that is rotating at constant speed. True or false: During any fraction of a revolution: (a) None of the forces acting on you does work on you. (b) The total work done by all forces acting on you is zero. (c) There is zero net force on you. (d) You are accelerating.

(a) False. Both the force exerted by the surface on which you are sitting and the gravitational force acting on you do work on you.

(b) True. Because you are rotating with a constant speed (ignoring the very brief periods of acceleration at the beginning and end of the ride) and return, at the end of the ride, to the same location from which you started the ride, your kinetic energy has not changed and so the net work done on you is zero.

(c) False. The net force acting on you is the sum of the gravitational force acting on you and the force exerted by the surface on which you are sitting.

(d) True. Because the direction you are moving is continually changing, your velocity is continually changing and, hence, you are experiencing acceleration.

4 • By what factor does the kinetic energy of a particle change if its speed is doubled but its mass is cut in half?

Determine the Concept The kinetic energy of a particle is proportional to the square of its speed. Because $K = \frac{1}{2}mv^2$, replacing v by $2v$ and m by $\frac{1}{2}m$ yields

$$K' = \frac{1}{2}\left(\frac{1}{2}m\right)(2v)^2 = 2\left(\frac{1}{2}mv^2\right) = 2K.$$

Thus doubling the speed of a particle and halving its mass doubles its kinetic energy.

5 • Give an example of a particle that has constant kinetic energy but is accelerating. Can a non-accelerating particle have a changing kinetic energy? If so, give an example.

Determine the Concept A particle moving along a circular path at constant speed has constant kinetic energy but is accelerating (because its velocity is continually changing). No, because if the particle is not accelerating, the net force acting on it must be zero and, consequently, its kinetic energy must be constant.

6 • A particle initially has kinetic energy K . Later it is found to be moving in the opposite direction with three times its initial speed. What is the kinetic energy now? (a) K , (b) $3K$, (c) $23K$, (d) $9K$, (e) $-9K$

Determine the Concept The kinetic energy of a particle is proportional to the square of its speed and is always positive. Because $K = \frac{1}{2}mv^2$, replacing v by $3v$

yields $K' = \frac{1}{2}m(3v)^2 = 9\left(\frac{1}{2}mv^2\right) = 9K$. Hence tripling the speed of a particle increases its kinetic energy by a factor of 9 and **(d)** is correct.

7 • [SSM] How does the work required to stretch a spring 2.0 cm from its unstressed length compare with the work required to stretch it 1.0 cm from its unstressed length?

Determine the Concept The work required to stretch or compress a spring a distance x is given by $W = \frac{1}{2}kx^2$ where k is the spring's stiffness constant. Because $W \propto x^2$, doubling the distance the spring is stretched will require four times as much work.

8 • A spring is first stretched 2.0 cm from its unstressed length. It is then stretched an additional 2.0 cm. How does the work required for the second stretch compare to the work required for the first stretch (give a ratio of second to first)?

Picture the Problem The work required to stretch or compress a spring a distance x is given by $W = \frac{1}{2}kx^2$ where k is the spring's stiffness constant.

Letting x_1 represent the length of the first stretch, express the work (the energy stored in the spring after the stretch) done on the spring during the first stretch:

$$W_1 = \frac{1}{2}kx_1^2$$

The work done in stretching the spring from x_1 to x_2 is given by:

$$\begin{aligned} W_2 &= \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}k(2x_1)^2 - \frac{1}{2}kx_1^2 \\ &= 4\left(\frac{1}{2}kx_1^2\right) - \frac{1}{2}kx_1^2 = 3\left(\frac{1}{2}kx_1^2\right) \end{aligned}$$

Express the ratio of W_2 to W_1 to obtain:

$$\frac{W_2}{W_1} = \frac{3\left(\frac{1}{2}kx_1^2\right)}{\frac{1}{2}kx_1^2} = \boxed{3}$$

9 • The dimension of power is (a) $M \cdot L^2 \cdot T^2$, (b) $M \cdot L^2/T$, (c) $M \cdot L^2/T^2$, (d) $M \cdot L^2/T^3$.

Determine the Concept We can use the definition of power as the scalar product of force and velocity to express the dimension of power.

Power is defined to be the product of force and velocity:

$$\mathbf{P} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Express the dimension of force:

$$\frac{M \cdot L}{T^2}$$

Express the dimension of velocity: $\frac{L}{T}$

Express the dimension of power in terms of those of force and velocity: $\frac{M \cdot L}{T^2} \cdot \frac{L}{T} = \frac{M \cdot L^2}{T^3} \Rightarrow \boxed{(d)}$ is correct.

10 • Show that the SI units of the force constant of a spring can be written as kg/s^2 .

Picture the Problem We can use the relationship $F = kx$ to establish the SI units of k .

Solve $F = kx$ for k to obtain: $k = \frac{F}{x}$

Substitute the units of F/x and simplify to obtain: $\frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}} = \boxed{\text{kg/s}^2}$

11 • True or false: (a) The gravitational force cannot do work on an object, because it is not a contact force. (b) Static friction can never do work on an object. (c) As a negatively charged electron in an atom is pulled from a positively charged nucleus, the electric force on the electron does work that has a positive value. (d) If a particle is moving along a circular path, the total work being done on it is necessarily zero.

(a) False. The definition of work is not limited to displacements caused by contact forces. Consider the work done by the gravitational force on an object in freefall.

(b) False. The force that the earth exerts on your foot as you walk (or run) is a static friction force. Another example is the force that the earth exerts on the tire mounted on a drive wheel of an automobile.

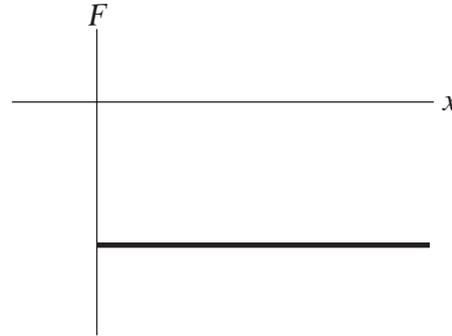
(c) False. The direction of the electric force on the electron during its removal from the atom is opposite that of the displacement of the electron.

(d) False. If the particle is accelerating, there must be a net force acting on it and, hence, work is done on it.

12 •• A hockey puck has an initial velocity in the $+x$ direction on a horizontal sheet of ice. Qualitatively sketch the force-versus-position graph for the (constant) horizontal force that would need to act on the puck to bring it to rest. Assume that the puck is located at $x = 0$ when the force begins to act. Show

that the sign of the area under the curve agrees with the sign of the change in the puck's kinetic energy and interpret this in terms of the work–kinetic–energy theorem.

Determine the Concept The graph of the force F acting on the puck as a function of its position x is shown to the right. Note that, because the force is negative, the area bounded by it and the x axis is negative and, hence, the net work done by the force is negative. In accordance with the work–kinetic energy theorem, the change in kinetic energy is negative and the puck loses all of its initial kinetic energy.



13 •• [SSM] True or false: (a) The scalar product cannot have units. (b) If the scalar product of two nonzero vectors is zero, then they are parallel. (c) If the scalar product of two nonzero vectors is equal to the product of their magnitudes, then the two vectors are parallel. (d) As an object slides up an incline, the sign of the scalar product of the force of gravity on it and its displacement is negative.

(a) False. Work is the scalar product of force and displacement.

(b) False. Because $\vec{A} \cdot \vec{B} = AB \cos \theta$, where θ is the angle between \vec{A} and \vec{B} , if the scalar product of the vectors is zero, then θ must be 90° (or some odd multiple of 90°) and the vectors are perpendicular.

(c) True. Because $\vec{A} \cdot \vec{B} = AB \cos \theta$, where θ is the angle between \vec{A} and \vec{B} , if the scalar product of the vectors is equal to the product of their magnitudes, then θ must be 0° and the vectors are parallel.

(d) True. Because the angle between the gravitational force and the displacement of the object is greater than 90° , its cosine is negative and, hence, the scalar product is negative.

14 •• (a) Must the scalar product of two perpendicular unit vectors always be zero? If not, give an example. (b) An object has a velocity \vec{v} at some instant. Interpret $\sqrt{\vec{v} \cdot \vec{v}}$ physically. (c) A ball rolls off a horizontal table. What is the scalar product between its velocity and its acceleration the instant after it leaves the table? Explain. (d) In Part (c), what is the sign of the scalar product of its velocity and acceleration the instant before it impacts the floor?

Determine the Concept

(a) No. Any unit vectors that are not perpendicular to each other (as are \hat{i} , \hat{j} , and \hat{k}) will have a scalar product different from zero.

(b) $\sqrt{\vec{v} \cdot \vec{v}} = \sqrt{v^2 \cos 0^\circ} = v$ is the object's speed.

(c) Zero, because at the instant the ball leaves the horizontal table, \vec{v} and $\vec{a}(= \vec{g})$ are perpendicular.

(d) Positive, because the final velocity of the ball has a downward component in the direction of the acceleration.

15 •• You lift a package vertically upward a distance L in time Δt . You then lift a second package that has twice the mass of the first package vertically upward the same distance while providing the same power as required for the first package. How much time does lifting the second package take (answer in terms of Δt)?

Picture the Problem Power is the rate at which work is done.

Express the power you exert in lifting the package one meter in Δt seconds:

$$P_1 = \frac{W_1}{\Delta t_1} = \frac{W_1}{\Delta t}$$

Express the power you develop in lifting a package of twice the mass one meter in t seconds:

$$P_2 = \frac{W_2}{\Delta t_2} = \frac{2W_1}{\Delta t_2}$$

Because you exert the same power in lifting both packages:

$$\frac{W_1}{\Delta t} = \frac{2W_1}{\Delta t_2} \Rightarrow \Delta t_2 = \boxed{2\Delta t}$$

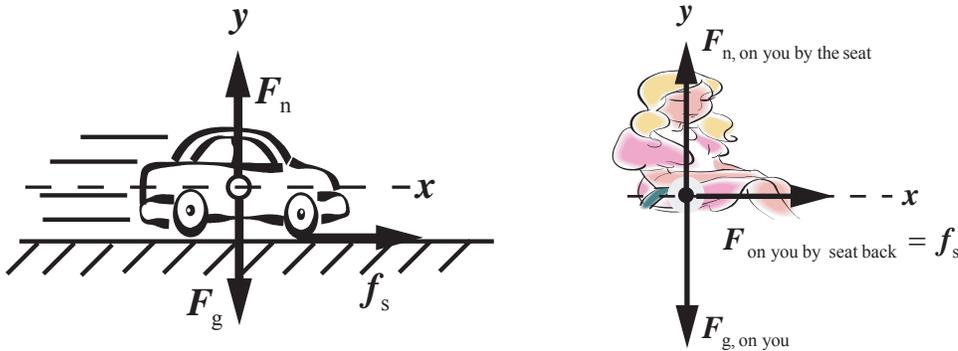
16 •• There are lasers that output more than 1.0 GW of power. A typical large modern electric generation plant typically produces 1.0 GW of electrical power. Does this mean the laser outputs a huge amount of energy? Explain. *Hint: These high-power lasers are pulsed on and off, so they are not outputting power for very long time intervals.*

Determine the Concept No, the power may only last for a short time interval.

17 •• [SSM] You are driving a car that accelerates from rest on a level road without spinning its wheels. Use the center-of-mass work–translational-kinetic-energy relation and free-body diagrams to clearly explain which force (or forces) is (are) directly responsible for the gain in translational kinetic energy of

both you and the car. *Hint: The relation refers to external forces only, so the car's engine is not the answer. Pick your "system" correctly for each case.*

Determine the Concept The car shown in the free-body diagram is accelerating in the positive x direction, as are you (shown to the right). The net external force (neglecting air resistance) acting on the car (and on you) is the static friction force \vec{f}_s exerted by the road and acting on the car tires. The positive work this friction force does is translated into a gain of kinetic energy.



Estimation and Approximation

18 •• (a) Estimate the work done on you by gravity as you take an elevator from the ground floor to the top of the Empire State Building, a building 102 stories high. (b) Estimate the amount of work the normal force of the floor did on you. *Hint: The answer is not zero.* (c) Estimate the average power of the force of gravity.

Picture the Problem According to the work-kinetic energy theorem, the work done on you by gravity and the normal force exerted by the floor equals your change in kinetic energy.

(a) The definition of work is:

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} \quad (1)$$

Assuming a coordinate system in which the upward direction is the $+y$ direction, \vec{F} and $d\vec{s}$ are given by:

$$\vec{F} = -mg \hat{j}$$

and

$$d\vec{s} = dy \hat{j}$$

Substitute for \vec{F} and $d\vec{s}$ in equation (1) to obtain:

$$W_{\text{by gravity}} = \int_0^h -mg \hat{j} \cdot dy \hat{j} = -mg \int_0^h dy$$

where h is the height of the Empire State building.

Evaluating this integral yields:

$$W_{\text{by gravity}} = -mgh$$

Assume that your mass is 70 kg and that the height of the Empire State building is 300 m (102 floors). Then:

$$\begin{aligned} W_{\text{by gravity}} &= -(70 \text{ kg})(9.81 \text{ m/s}^2)(300 \text{ m}) \\ &= -2.060 \times 10^5 \text{ J} \\ &\approx \boxed{-2.1 \times 10^5 \text{ J}} \end{aligned}$$

(b) Apply the work-kinetic energy theorem to obtain:

$$W_{\text{by gravity}} + W_{\text{by floor}} = \Delta K$$

or, because your change in kinetic energy is zero,

$$W_{\text{by gravity}} + W_{\text{by floor}} = 0$$

Solve for $W_{\text{by floor}}$ to obtain:

$$W_{\text{by floor}} = -W_{\text{by gravity}}$$

Substituting for $W_{\text{by gravity}}$ yields:

$$\begin{aligned} W_{\text{by floor}} &\approx -(-2.060 \times 10^5 \text{ J}) \\ &= \boxed{2.1 \times 10^5 \text{ J}} \end{aligned}$$

(c) If it takes 1 min to ride the elevator to the top floor, then the average power of the force of gravity is:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{2.060 \times 10^5 \text{ J}}{60 \text{ s}} = \boxed{3.4 \text{ kW}}$$

Remarks: Some books say the normal force cannot do work. Having solved this problem, what do you think about that statement?

19 •• The nearest stars, apart from the Sun, are light-years away from Earth. If we are to investigate these stars, our space ships will have to travel at an appreciable fraction of the speed of light. (a) You are in charge of estimating the energy required to accelerate a 10,000-kg capsule from rest to 10 percent of the speed of light in one year. What is the minimum amount of energy that is required? Note that at velocities approaching the speed of light, the kinetic energy formula $\frac{1}{2}mv^2$ is not correct. However, it gives a value that is within 1% of the correct value for speeds up to 10% of the speed of light. (b) Compare your estimate to the amount of energy that the United States uses in a year (about 5×10^{20} J). (c) Estimate the minimum average power required of the propulsion system.

Picture the Problem We can find the kinetic energy K of the spacecraft from its definition and compare its energy to the annual consumption in the United States by examining the ratio K/E . Finally, we can use the definition of power to find the average power required of the engines.

(a) The work required by the propulsion system equals the kinetic energy of the spacecraft:

Substitute numerical values and evaluate W :

$$W = \Delta K = K_f = \frac{1}{2} m (0.10c)^2$$

$$W = \frac{1}{2} (10^4 \text{ kg}) [(0.10)(2.998 \times 10^8 \text{ m/s})]^2 \\ = \boxed{4.5 \times 10^{18} \text{ J}}$$

(b) Express this amount of energy as a percentage of the annual consumption in the United States:

$$\frac{K}{E} \approx \frac{4.5 \times 10^{18} \text{ J}}{5 \times 10^{20} \text{ J}} \approx \boxed{1\%}$$

(c) The average power is the rate at which work is done (or energy is delivered):

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{\Delta K}{\Delta t}$$

Substitute numerical values and evaluate $P_{\text{av, min}}$:

$$P_{\text{av, min}} = \frac{4.5 \times 10^{18} \text{ J}}{1 \text{ y} \times \frac{3.156 \times 10^7 \text{ s}}{\text{y}}} \\ = \boxed{1.4 \times 10^{11} \text{ W}}$$

20 •• The mass of the Space Shuttle orbiter is about 8×10^4 kg and the period of its orbit is 90 min. Estimate the kinetic energy of the orbiter and the work done on it by gravity between launch and orbit. (Although the force of gravity decreases with altitude, this affect is small in low-Earth orbit. Use this fact to make the necessary approximation; you do not need to do an integral.) The orbits are about 250 miles above the surface of Earth.

Picture the Problem We can find the orbital speed of the Shuttle from the radius of its orbit and its period and its kinetic energy from $K = \frac{1}{2} mv^2$. We'll ignore the variation in the acceleration due to gravity to estimate the change in the potential energy of the orbiter between its value at the surface of the earth and its orbital value.

Apply the work-kinetic energy theorem to the orbiter to obtain:

$$W_{\text{by gravity}} = \Delta K = K_{\text{orbital}} - K_{\text{launch}} \\ \text{or, because } K_{\text{launch}} = 0, \\ W_{\text{by gravity}} = K_{\text{orbital}} \quad (1)$$

The kinetic energy of the orbiter in orbit is given by:

$$K_{\text{orbital}} = \frac{1}{2} mv^2 \quad (2)$$

Relate the orbital speed of the orbiter to its radius r and period T :

$$v = \frac{2\pi r}{T}$$

Substitute for v in equation (2) and simplify to obtain:

$$K_{\text{orbital}} = \frac{1}{2} m \left(\frac{2\pi r}{T} \right)^2 = \frac{2\pi^2 m r^2}{T^2}$$

Substitute numerical values and evaluate K_{orbital} :

$$K_{\text{orbital}} = \frac{2\pi^2 (8 \times 10^4 \text{ kg}) [(200 \text{ mi} + 3960 \text{ mi})(1.609 \text{ km/mi})]^2}{[(90 \text{ min})(60 \text{ s/min})]^2} \approx \boxed{2 \text{ TJ}}$$

Substitute for K_{orbital} in equation (1)

$$W_{\text{by gravity}} \approx \boxed{2 \text{ TJ}}$$

to obtain:

21 • Ten inches of snow have fallen during the night, and you must shovel out your 50-ft-long driveway (Figure 6-29). Estimate how much work you do on the snow by completing this task. Make a plausible guess of any value(s) needed (the width of the driveway, for example), and state the basis for each guess.

Picture the Problem Let's assume that the width of the driveway is 18 ft. We'll also assume that you lift each shovel full of snow to a height of 1 m, carry it to the edge of the driveway, and drop it. We'll ignore the fact that you must slightly accelerate each shovel full as you pick it up and as you carry it to the edge of the driveway. While the density of snow depends on the extent to which it has been compacted, one liter of freshly fallen snow is approximately equivalent to 100 mL of water.

Express the work you do in lifting the snow a distance h :

$$W = \Delta U = mgh = \rho_{\text{snow}} V_{\text{snow}} gh$$

where ρ is the density of the snow.

Using its definition, express the densities of water and snow:

$$\rho_{\text{snow}} = \frac{m_{\text{snow}}}{V_{\text{snow}}} \quad \text{and} \quad \rho_{\text{water}} = \frac{m_{\text{water}}}{V_{\text{water}}}$$

Divide the first of these equations by the second to obtain:

$$\frac{\rho_{\text{snow}}}{\rho_{\text{water}}} = \frac{V_{\text{water}}}{V_{\text{snow}}} \Rightarrow \rho_{\text{snow}} = \rho_{\text{water}} \frac{V_{\text{water}}}{V_{\text{snow}}}$$

Substitute numerical values and evaluate ρ_{snow} :

$$\rho_{\text{snow}} = (10^3 \text{ kg/m}^3) \frac{100 \text{ mL}}{\text{L}} = 100 \text{ kg/m}^3$$

Calculate the volume of snow covering the driveway:

$$\begin{aligned} V_{\text{snow}} &= (50 \text{ ft})(18 \text{ ft}) \left(\frac{10}{12} \text{ ft} \right) \\ &= 750 \text{ ft}^3 \times \frac{28.32 \text{ L}}{\text{ft}^3} \times \frac{10^{-3} \text{ m}^3}{\text{L}} \\ &= 21.2 \text{ m}^3 \end{aligned}$$

Substitute numerical values in the expression for W to obtain an estimate (a lower bound) for the work you would do on the snow in removing it:

$$W = (100 \text{ kg/m}^3)(21.2 \text{ m}^3)(9.81 \text{ m/s}^2)(1 \text{ m}) = \boxed{21 \text{ kJ}}$$

Remarks: Note that the work done on the snow is zero when it is carried to the end of the driveway because F_{applied} is perpendicular to the displacement.

Work, Kinetic Energy and Applications

22 • A 15-g piece of space junk has a speed of 1.2 km/s. (a) What is its kinetic energy? (b) What is its kinetic energy if its speed is halved? (c) What is its kinetic energy if its speed is doubled?

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the piece of space junk.

(a) The kinetic energy of the piece of space junk is given by: $K = \frac{1}{2}mv^2$

Substitute numerical values and evaluate K : $K = \frac{1}{2}(0.015 \text{ kg})(1.2 \text{ km/s})^2 = \boxed{11 \text{ kJ}}$

(b) Because $K \propto v^2$: $K' = \frac{1}{4}K = \boxed{2.7 \text{ kJ}}$

(c) Because $K \propto v^2$: $K' = 4K = \boxed{43 \text{ kJ}}$

23 • Find the kinetic energy of (a) a 0.145-kg baseball moving with a speed of 45.0 m/s, and (b) a 60.0-kg jogger running at a steady pace of 9.00 min/mi.

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the baseball and the jogger.

(a) Use the definition of K to find the kinetic energy of the baseball: $K_{\text{baseball}} = \frac{1}{2}(0.145 \text{ kg})(45.0 \text{ m/s})^2 = \boxed{147 \text{ J}}$

(b) Convert the jogger's pace of 9.00 min/mi into a speed: $v = \left(\frac{1 \text{ mi}}{9.00 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) = 2.98 \text{ m/s}$

Use the definition of K to find the jogger's kinetic energy:

$$\begin{aligned} K_{\text{jogger}} &= \frac{1}{2}(60.0 \text{ kg})(2.98 \text{ m/s})^2 \\ &= \boxed{266 \text{ J}} \end{aligned}$$

24 • A 6.0-kg box is raised a distance of 3.0 m from rest by a vertical applied force of 80 N. Find (a) the work done on the box by the applied force, (b) the work done on the box by gravity, and (c) the final kinetic energy of the box.

Picture the Problem The work done by the force acting on the box is the scalar product of the force producing the displacement and the displacement of the box. Because the weight of an object is the gravitational force acting on it and this force acts downward, the work done by gravity is negative. We can use the work-kinetic energy theorem to find the final kinetic energy of the box.

(a) Use the definition of work to obtain:

$$W_{\text{on the box}} = \vec{F} \cdot \Delta\vec{y} \quad (1)$$

\vec{F} and $\Delta\vec{y}$ are given by:

$$\begin{aligned} \vec{F} &= (80 \text{ N})\hat{j} \\ \text{and} \\ \Delta\vec{y} &= (3.0 \text{ m})\hat{j} \end{aligned}$$

Substitute for \vec{F} and $\Delta\vec{y}$ in equation (1) and evaluate $W_{\text{on the box}}$:

$$\begin{aligned} W_{\text{on the box}} &= (80 \text{ N})\hat{j} \cdot (3.0 \text{ m})\hat{j} \\ &= \boxed{0.24 \text{ kJ}} \end{aligned}$$

(b) Apply the definition of work again to obtain:

$$W_{\text{by gravity}} = \vec{F}_g \cdot \Delta\vec{y} \quad (2)$$

\vec{F}_g and $\Delta\vec{y}$ are given by:

$$\begin{aligned} \vec{F}_g &= -m\mathbf{g}\hat{j} = -(6.0 \text{ kg})(9.81 \text{ m/s}^2)\hat{j} \\ &= (-58.9 \text{ N})\hat{j} \\ \text{and} \\ \Delta\vec{y} &= (3.0 \text{ m})\hat{j} \end{aligned}$$

Substituting for \vec{F}_g and $\Delta\vec{y}$ and simplifying yields:

$$\begin{aligned} W_{\text{by gravity}} &= (-58.9 \text{ N})\hat{j} \cdot (3.0 \text{ m})\hat{j} \\ &= \boxed{-0.18 \text{ kJ}} \end{aligned}$$

(c) According to the work-kinetic energy theorem:

$$\begin{aligned} \Delta K &= W_{\text{on the box}} + W_{\text{by gravity}} \\ \text{or, because } K_i &= 0, \\ K_f &= W_{\text{on the box}} + W_{\text{by gravity}} \end{aligned}$$

Substitute numerical values and evaluate K_f :

$$K_f = 0.24 \text{ kJ} - 0.18 \text{ kJ} = \boxed{0.06 \text{ kJ}}$$

25 • A constant 80-N force acts on a 5.0 kg box. The box initially is moving at 20 m/s in the direction of the force, and 3.0 s later the box is moving at 68 m/s. Determine both the work done by this force and the average power delivered by the force during the 3.0-s interval.

Picture the Problem The constant force of 80 N is the net force acting on the box and the work it does is equal to the *change* in the kinetic energy of the box. The power of the force is the rate at which it does work on the box.

Using the work-kinetic energy theorem, relate the work done by the constant force to the *change* in the kinetic energy of the box:

$$W = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2)$$

Substitute numerical values and evaluate W :

$$\begin{aligned} W &= \frac{1}{2}(5.0 \text{ kg})[(68 \text{ m/s})^2 - (20 \text{ m/s})^2] \\ &= 10.6 \text{ kJ} \\ &= \boxed{11 \text{ kJ}} \end{aligned}$$

The average power delivered by the force is given by:

$$P = \frac{\Delta W}{\Delta t}$$

Substitute numerical values and evaluate P :

$$P = \frac{10.6 \text{ kJ}}{3.0 \text{ s}} = \boxed{3.5 \text{ kW}}$$

26 •• You run a race with a friend. At first you each have the same kinetic energy, but she is running faster than you are. When you increase your speed by 25 percent, you are running at the same speed she is. If your mass is 85 kg, what is her mass?

Picture the Problem We can use the definition of kinetic energy to find the mass of your friend.

Using the definition of kinetic energy and letting "1" denote your mass and speed and "2" your girlfriend's, express the equality of your kinetic energies:

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 \Rightarrow m_2 = m_1\left(\frac{v_1}{v_2}\right)^2$$

Express the condition on your speed that enables you to run at the same speed as your girlfriend:

$$v_2 = 1.25v_1$$

Substitute for v_2 and simplify to obtain:

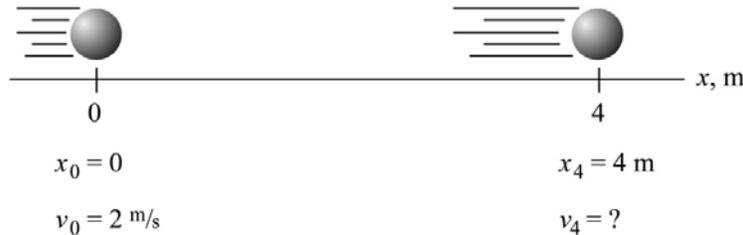
$$m_2 = m_1 \left(\frac{v_1}{1.25v_1} \right)^2 = m_1 \left(\frac{1}{1.25} \right)^2$$

Substitute the numerical value of m_1 and evaluate m_2 :

$$m_2 = (85 \text{ kg}) \left(\frac{1}{1.25} \right)^2 = \boxed{54 \text{ kg}}$$

27 •• [SSM] A 3.0-kg particle moving along the x axis has a velocity of +2.0 m/s as it passes through the origin. It is subjected to a single force, F_x , that varies with position, as shown in Figure 6-30. (a) What is the kinetic energy of the particle as it passes through the origin? (b) How much work is done by the force as the particle moves from $x = 0.0$ m to $x = 4.0$ m? (c) What is the speed of the particle when it is at $x = 4.0$ m?

Picture the Problem The pictorial representation shows the particle as it moves along the positive x axis. The particle's kinetic energy increases because work is done on it. We can calculate the work done on it from the graph of F_x vs. x and relate its kinetic energy when it is at $x = 4.0$ m to its kinetic energy when it was at the origin and the work done on it by using the work-kinetic energy theorem.



(a) Calculate the kinetic energy of the particle when it is at $x = 0$:

$$K_0 = \frac{1}{2}mv^2 = \frac{1}{2}(3.0 \text{ kg})(2.0 \text{ m/s})^2 = \boxed{6.0 \text{ J}}$$

(b) Because the force and displacement are parallel, the work done is the area under the curve. Use the formula for the area of a triangle to calculate the area under the $F(x)$ graph:

$$W_{0 \rightarrow 4} = \frac{1}{2}(\text{base})(\text{altitude}) = \frac{1}{2}(4.0 \text{ m})(6.0 \text{ N}) = \boxed{12 \text{ J}}$$

(c) Express the kinetic energy of the particle at $x = 4.0$ m in terms of its speed and mass:

$$K_{4s} = \frac{1}{2}mv_{4s}^2 \Rightarrow v_{4s} = \sqrt{\frac{2K_{4s}}{m}} \quad (1)$$

Using the work-kinetic energy theorem, relate the work done on the particle to its *change* in kinetic energy:

$$W_{0 \rightarrow 4.0\text{ m}} = K_{4s} - K_0$$

Solve for the particle's kinetic energy at $x = 4.0$ m:

$$K_{4s} = K_0 + W_{0 \rightarrow 4}$$

Substitute numerical values and evaluate K_{4s} :

$$K_{4s} = 6.0\text{ J} + 12\text{ J} = 18\text{ J}$$

Substitute numerical values in equation (1) and evaluate v_4 :

$$v_4 = \sqrt{\frac{2(18\text{ J})}{3.0\text{ kg}}} = \boxed{3.5\text{ m/s}}$$

28 •• A 3.0-kg object moving along the x axis has a velocity of 2.4 m/s as it passes through the origin. It is acted on by a single force, F_x , that varies with x , as shown in Figure 6-31. (a) Find the work done by the force from $x = 0.0$ m to $x = 2.0$ m. (b) What is the kinetic energy of the object at $x = 2.0$ m? (c) What is the speed of the object at $x = 2.0$ m? (d) What is the work done on the object from $x = 0.0$ to $x = 4.0$ m? (e) What is the speed of the object at $x = 4.0$ m?

Picture the Problem The work done on an object can be determined by finding the area bounded by its graph of F_x as a function of x and the x axis. We can find the kinetic energy and the speed of the particle at any point by using the work-kinetic energy theorem.

(a) Express W , the area under the curve, in terms of the area of one square, A_{square} , and the number of squares n :

$$W_{0 \rightarrow 2\text{ m}} = nA_{\text{square}} \quad (1)$$

Determine the work equivalent of one square:

$$A_{\text{square}} = (0.5\text{ N})(0.25\text{ m}) = 0.125\text{ J}$$

Estimate the number of squares under the curve between $x = 0$ and $x = 2.0$ m:

$$n \approx 22$$

Substitute for n and A_{square} in equation (1) and evaluate W :

$$\begin{aligned} W_{0 \rightarrow 2\text{ m}} &= (22)(0.125\text{ J}) = 2.75\text{ J} \\ &= \boxed{2.8\text{ J}} \end{aligned}$$

(b) Relate the kinetic energy of the object at $x = 2.0\text{ m}$, $K_{2\text{ m}}$, to its initial kinetic energy, K_0 , and the work that was done on it between $x = 0$ and $x = 2.0\text{ m}$:

$$K_{2\text{ m}} = K_0 + W_{0 \rightarrow 2\text{ m}}$$

Substitute numerical values and evaluate $K_{2\text{ m}}$:

$$\begin{aligned} K_{2\text{ m}} &= \frac{1}{2}(3.0\text{ kg})(2.4\text{ m/s})^2 + 2.75\text{ J} \\ &= 11.4\text{ J} \\ &= \boxed{11\text{ J}} \end{aligned}$$

(c) The speed of the object at $x = 2.0\text{ m}$ is given by:

$$v_{2\text{ m}} = \sqrt{\frac{2K_{2\text{ m}}}{m}}$$

Substitute numerical values and evaluate $v_{2\text{ m}}$:

$$v = \sqrt{\frac{2(11.4\text{ J})}{3.0\text{ kg}}} = \boxed{2.8\text{ m/s}}$$

(d) Estimate the number of squares under the curve between $x = 0$ and $x = 4.0\text{ m}$:

$$n \approx 26$$

Substitute numerical values and evaluate $W_{0 \rightarrow 4\text{ m}}$:

$$W_{0 \rightarrow 4\text{ m}} = 26(0.125\text{ J}) = 3.25\text{ J} = \boxed{3.3\text{ J}}$$

(e) Relate the kinetic energy of the object at $x = 4.0\text{ m}$, K_4 , to its initial kinetic energy, K_0 , and the work that was done on it between $x = 0$ and $x = 4.0\text{ m}$:

$$K_4 = K_0 + W_{0 \rightarrow 4\text{ m}}$$

Substitute numerical values and evaluate K_4 :

$$\begin{aligned} K_4 &= \frac{1}{2}(3.0\text{ kg})(2.4\text{ m/s})^2 + 3.25\text{ J} \\ &= 11.9\text{ J} \end{aligned}$$

The speed of the object at $x = 4.0\text{ m}$ is given by:

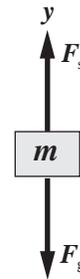
$$v_4 = \sqrt{\frac{2K_4}{m}}$$

Substitute numerical values and evaluate v_4 :

$$v_4 = \sqrt{\frac{2(11.9\text{J})}{3.0\text{kg}}} = \boxed{2.8\text{m/s}}$$

29 •• One end of a light spring (force constant k) is attached to the ceiling, the other end is attached to an object of mass m . The spring initially is vertical and unstressed. You then "ease the object down" to an equilibrium position a distance h below its initial position. Next, you repeat this experiment, but instead of easing the object down, you release it, with the result that it falls a distance H below the initial position before momentarily stopping. (a) Show that $h = mg/k$. (b) Use the work–kinetic-energy theorem to show that $H = 2h$. Try this experiment on your own.

Picture the Problem The free-body diagram shows the object at its equilibrium point. We can use Newton's 2nd law under equilibrium conditions to show that $h = mg/k$ and the work-kinetic energy theorem to prove that $H = 2h$.



(a) Apply $\sum F_y = ma_y$ to the object in its equilibrium position:

$$F_s - F_g = 0$$

or, because $F_s = kh$ and $F_g = mg$,

$$kh - mg = 0 \Rightarrow h = \boxed{\frac{mg}{k}}$$

(b) Apply the work-kinetic energy theorem to the spring-object system to obtain:

$$W_{\text{net,ext}} = \Delta K$$

or, because the object begins and ends at rest,

$$W_{\text{net,ext}} = 0$$

The work done by the net external force is the sum of the work done by gravity and by the spring:

$$W_{\text{by gravity}} + W_{\text{by spring}} = 0$$

Because the force exerted on the object by the spring is upward and the displacement of the object is downward, $W_{\text{by spring}}$ is negative.

$$mgH - \frac{1}{2}kH^2 = 0 \Rightarrow H = \boxed{2h}$$

Substituting for $W_{\text{by spring}}$ and

$W_{\text{by gravity}}$ yields:

30 •• A force F_x acts on a particle that has a mass of 1.5 kg. The force is related to the position x of the particle by the formula $F_x = Cx^3$, where $C = 0.50$ if x is in meters and F_x is in newtons. (a) What are the SI units of C ? (b) Find the work done by this force as the particle moves from $x = 3.0$ m to $x = 1.5$ m. (c) At $x = 3.0$ m, the force points opposite the direction of the particle's velocity (speed is 12.0 m/s). What is its speed at $x = 1.5$ m? Can you tell its direction of motion at $x = 1.5$ m using only the work-kinetic-energy theorem? Explain.

Picture the Problem The work done by this force as it displaces the particle is the area under the curve of F as a function of x . We can integrate F_x to find the work done on the particle and then use the work-kinetic energy theorem to find the particle's speed at $x = 3.0$ m.

(a) Solve the force equation for C to obtain:

$$C = \frac{F_x}{x^3} \text{ where } C \text{ has units of } \boxed{\frac{\text{N}}{\text{m}^3}}.$$

(b) Because F_x varies with position non-linearly, we need to express the work it does as an integral:

$$W_{\text{net, ext}} = \int \vec{F}_x \cdot d\vec{x} = \int -F_x dx$$

where the minus sign is a consequence of the fact that the direction of the force is opposite the displacement of the particle.

Substituting for F_x yields:

$$W_{\text{net, ext}} = \int_{3.0 \text{ m}}^{1.5 \text{ m}} -Cx^3 dx = -C \int_{3.0 \text{ m}}^{1.5 \text{ m}} x^3 dx$$

Evaluate the integral to obtain:

$$\begin{aligned} W_{\text{net, ext}} &= -\left(0.50 \frac{\text{N}}{\text{m}^3}\right) \left[\frac{1}{4}x^4\right]_{3.0 \text{ m}}^{1.5 \text{ m}} = -\frac{1}{4}\left(0.50 \frac{\text{N}}{\text{m}^3}\right) \left[(1.5 \text{ m})^4 - (3.0 \text{ m})^4\right] = 9.492 \text{ J} \\ &= \boxed{9.5 \text{ J}} \end{aligned}$$

(c) Apply the work-kinetic-energy theorem to obtain:

$$W_{\text{net, ext}} = \Delta K = K_{1.5 \text{ m}} - K_{3.0 \text{ m}}$$

Substituting for $K_{1.5 \text{ m}}$ and $K_{3.0 \text{ m}}$ yields:

$$W_{\text{net, ext}} = \frac{1}{2}mv_{1.5 \text{ m}}^2 - \frac{1}{2}mv_{3.0 \text{ m}}^2$$

Solve for $v_{1.5 \text{ m}}$ to obtain:

$$v_{1.5 \text{ m}} = \sqrt{v_{3.0 \text{ m}}^2 + \frac{2W_{\text{net, ext}}}{m}} \quad (1)$$

Substitute numerical values and evaluate $v_{3.0\text{ m}}$:

$$v_{1.5\text{ m}} = \sqrt{(12\text{ m/s})^2 + \frac{2(9.49\text{ J})}{1.5\text{ kg}}} = \boxed{13\text{ m/s}}$$

No. All we can conclude is that the particle gained kinetic energy.

31 •• You have a vacation cabin that has a nearby solar (black) water container used to provide a warm outdoor shower. For a few days last summer, your pump went out and you had to personally haul the water up the 4.0 m from the pond to the tank. Suppose your bucket has a mass of 5.0 kg and holds 15.0 kg of water when it is full. However, the bucket has a hole in it, and as you moved it vertically at a constant speed v , water leaked out at a constant rate. By the time you reached the top, only 5.0 kg of water remained. (a) Write an expression for the mass of the bucket plus water as a function of the height climbed above the pond surface. (b) Find the work done by you on the bucket for each 5.0 kg of water delivered to the tank.

Picture the Problem We can express the mass of the water in your bucket as the difference between its initial mass and the product of the rate at which it loses water and your position during your climb. Because you must do work against gravity in lifting and carrying the bucket, the work you do is the integral of the product of the gravitational field and the mass of the bucket as a function of its position.

(a) Express the mass of the bucket and the water in it as a function of its initial mass, the rate at which it is losing water, and your position, y , during your climb:

$$m(y) = 20.0\text{ kg} - ry$$

Find the rate, $r = \frac{\Delta m}{\Delta y}$, at which your bucket loses water:

$$r = \frac{\Delta m}{\Delta y} = \frac{10.0\text{ kg}}{4.0\text{ m}} = 2.5\text{ kg/m}$$

Substitute for r to obtain:

$$m(y) = \boxed{20.0\text{ kg} - \left(2.5\frac{\text{kg}}{\text{m}}\right)y}$$

(b) Integrate the force you exert on the bucket, $m(y)g$, between the limits of $y = 0$ and $y = 4.0$ m:

$$W = g \int_0^{4.0\text{m}} \left(20.0\text{kg} - \left(2.5 \frac{\text{kg}}{\text{m}} \right) y \right) dy = (9.81\text{m/s}^2) \left[(20.0\text{kg})y - \frac{1}{2} (2.5\text{kg/m})y^2 \right]_0^{4.0\text{m}}$$

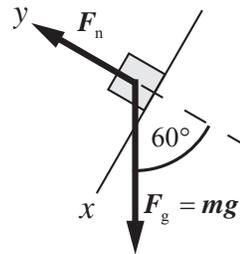
$$= \boxed{0.59\text{kJ}}$$

Remarks: We could also find the work you did on the bucket, at least approximately, by plotting a graph of $m(y)g$ and finding the area under this curve between $y = 0$ and $y = 4.0$ m.

32 •• A 6.0-kg block slides 1.5 m down a frictionless incline that makes an angle of 60° with the horizontal. (a) Draw the free-body diagram of the block, and find the work done by each force when the block slides 1.5 m (measured along the incline). (b) What is the total work done on the block? (c) What is the speed of the block after it has slid 1.5 m, if it starts from rest? (d) What is its speed after 1.5 m, if it starts with an initial speed of 2.0 m/s?

Picture the Problem We can use the definition of work as the scalar product of force and displacement to find the work done by each force acting on the block. When we have determined the work done on the block, we can use the work-kinetic energy theorem to find the speed of the block at any given location and for any initial kinetic energy it may have.

(a) The forces acting on the block are a gravitational force, exerted by the earth, that acts downward, and a normal force, exerted by the incline, that is perpendicular to the incline.



The work done by \vec{F}_g is given by:

$$W_{F_g} = \vec{F}_g \cdot \vec{s} = F_g s \cos \theta$$

where \vec{s} is the displacement of the block and θ is the angle between \vec{F}_g and \vec{s} .

Substituting for F_g yields:

$$W_{F_g} = mgs \cos \theta$$

Substitute numerical values and evaluate W_{F_g} :

$$\begin{aligned} W_{F_g} &= (6.0 \text{ kg})(9.81 \text{ m/s}^2)(2.0 \text{ m})\cos 30^\circ \\ &= \boxed{0.10 \text{ kJ}} \end{aligned}$$

The work done by \vec{F}_n is given by:

$$W_{F_n} = \vec{F}_n \cdot \vec{s} = F_n s \cos \theta$$

where \vec{s} is the displacement of the block and θ is the angle between \vec{F}_n and \vec{s} .

Substitute numerical values and evaluate W_{F_n} :

$$W_{F_n} = F_n (2.0 \text{ m})\cos 90^\circ = \boxed{0}$$

(b) The total work done on the block is the sum of the work done by \vec{F}_n and the work done by \vec{F}_g :

$$W_{\text{tot}} = W_{F_g} + W_{F_n}$$

Substitute numerical values and evaluate W_{tot} :

$$W_{\text{tot}} = 0 + 0.10 \text{ kJ} = \boxed{0.10 \text{ kJ}}$$

(c) Apply the work-kinetic energy theorem to the block to obtain:

$$W_{F_g} = \Delta K = K_f - K_i \quad (1)$$

or, because $v_i = 0$,

$$W_{F_g} = K_f$$

Substituting for W_{F_g} (from Part (b)) and K_f yields:

$$mgs \cos \theta = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{2gs \cos \theta}$$

Substitute numerical values and evaluate v_f for $s = 1.5 \text{ m}$:

$$\begin{aligned} v_f &= \sqrt{2(9.81 \text{ m/s}^2)(1.5 \text{ m})\cos 30^\circ} \\ &= \boxed{5.0 \text{ m/s}} \end{aligned}$$

(d) If $K_i \neq 0$, equation (1) becomes:

$$W_{F_g} = K_f - K_i = \frac{1}{2}mv_f^2 - mv_i^2$$

Solving for v_f and simplifying yields:

$$v_f = \sqrt{2gs \cos \theta + v_i^2}$$

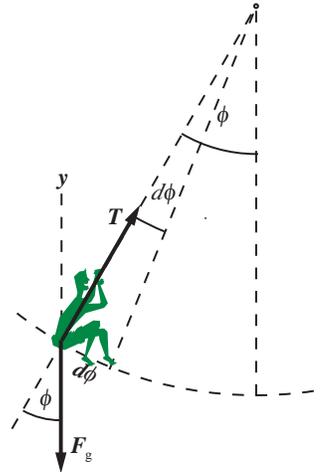
Substitute numerical values and evaluate v_f :

$$v_f = \sqrt{2(9.81 \text{ m/s}^2)(1.5 \text{ m}) \cos 30^\circ + (2.0 \text{ m/s})^2} = \boxed{5.4 \text{ m/s}}$$

33 •• [SSM] You are designing a jungle-vine–swinging sequence for the latest Tarzan movie. To determine his speed at the low point of the swing and to make sure it does not exceed mandatory safety limits, you decide to model the system of Tarzan + vine as a pendulum. Assume your model consists of a particle (Tarzan, mass 100 kg) hanging from a light string (the vine) of length ℓ attached to a support. The angle between the vertical and the string is written as ϕ . (a) Draw a free-body diagram for the object on the end of the string (Tarzan on the vine). (b) An infinitesimal distance along the arc (along which the object travels) is $\ell d\phi$. Write an expression for the total work dW_{total} done on the particle as it traverses that distance for an arbitrary angle ϕ . (c) If the $\ell = 7.0$ m, and if the particle starts from rest at an angle 50° , determine the particle’s kinetic energy and speed at the low point of the swing using the work–kinetic-energy theorem.

Picture the Problem Because Tarzan’s displacement is always perpendicular to the tension force exerted by the rope, this force can do no work on him. The net work done on Tarzan is the work done by the gravitational force acting on him. We can use the definition of work and the work-kinetic energy theorem to find his kinetic energy and speed at the low point of his swing.

(a) The forces acting on Tarzan are a gravitational force \vec{F}_g (his weight) exerted by the earth and the tension force \vec{T} exerted by the vine on which he is swinging.



(b) The work dW_{total} done by the gravitational force \vec{F}_g as Tarzan swings through an arc of length ds is:

$$\begin{aligned} dW_{\text{total}} &= \vec{F}_g \cdot d\vec{s} \\ &= F_g ds \cos(90^\circ - \phi) \\ &= F_g ds \sin \phi \end{aligned}$$

Note that, because $d\phi$ is decreasing as Tarzan swings toward his equilibrium position:

$$ds = -\ell d\phi$$

Substituting for ds and F_g yields:

$$dW_{\text{total}} = \boxed{-mg\ell \sin \phi d\phi}$$

(c) Apply the work-kinetic energy theorem to Tarzan to obtain:

$$W_{\text{total}} = \Delta K = K_f - K_i$$

or, because $K_i = 0$,

$$W_{\text{total}} = K_f$$

Substituting for W_{total} yields:

$$K_f = - \int_{50^\circ}^{0^\circ} mgl \sin \phi d\phi$$

$$= -mgl [-\cos \phi]_{50^\circ}^{0^\circ}$$

Substitute numerical values and evaluate the integral to find K_f :

$$K_f = -(100 \text{ kg})(9.81 \text{ m/s}^2)(7.0 \text{ m})[-\cos \phi]_{50^\circ}^{0^\circ}$$

$$= (100 \text{ kg})(9.81 \text{ m/s}^2)(7.0 \text{ m})(1 - \cos 50^\circ)$$

$$= 2.453 \text{ kJ}$$

$$= \boxed{2.5 \text{ kJ}}$$

Express K_f as a function of v_f :

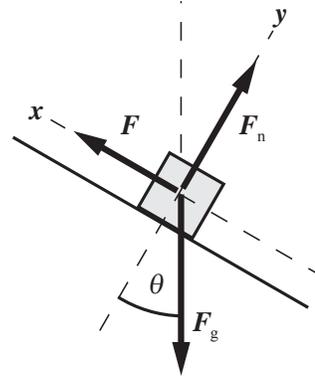
$$K_f = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

Substitute numerical values and evaluate v_f :

$$v_f = \sqrt{\frac{2(2.453 \text{ kJ})}{100 \text{ kg}}} = \boxed{7.0 \text{ m/s}}$$

34 •• *Simple machines* are frequently used for reducing the amount of force that must be supplied to perform a task such as lifting a heavy weight. Such machines include the screw, block-and-tackle systems, and levers, but the simplest of the simple machines is the inclined plane. In Figure 6-32, you are raising a heavy box to the height of the truck bed by pushing it up an inclined plane (a ramp). (a) The *mechanical advantage* MA of the inclined plane is defined as the ratio of the magnitude of the force it would take to lift the block straight up (at constant speed) to the magnitude of the force it would take to push it up the ramp (at constant speed). If the plane is frictionless, show that $MA = 1/\sin \theta = L/H$, where H is the height of the truck bed and L is the length of the ramp. (b) Show that the work you do by moving the block into the truck is the same whether you lift it straight up or push it up the frictionless ramp.

Picture the Problem The free-body diagram, with \vec{F} representing the force required to move the block at constant speed, shows the forces acting on the block. We can apply Newton's 2nd law to the block to relate F to its weight w and then use the definition of the mechanical advantage of an inclined plane. In the second part of the problem we'll use the definition of work.



(a) Express the mechanical advantage MA of the inclined plane:

$$MA = \frac{F_g}{F} \quad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$\begin{aligned} F - F_g \sin \theta &= ma_x \\ \text{or, because } a_x &= 0, \\ F - F_g \sin \theta &= 0 \Rightarrow F = F_g \sin \theta \end{aligned}$$

Substitute for F in equation (1) and simplify to obtain:

$$MA = \frac{F_g}{F_g \sin \theta} = \boxed{\frac{1}{\sin \theta}} \quad (2)$$

Refer to the figure to obtain:

$$\sin \theta = \frac{H}{L}$$

Substitute for $\sin \theta$ in equation (2) and simplify to obtain:

$$MA = \frac{1}{\frac{H}{L}} = \boxed{\frac{L}{H}}$$

(b) The work you do pushing the block up the ramp is given by:

$$W_{\text{ramp}} = FL$$

Because $F = F_g \sin \theta$ and $F_g = mg$:

$$W_{\text{ramp}} = (F_g \sin \theta)L = mgL \sin \theta$$

The work you do lifting the block into the truck is given by:

$$W_{\text{lifting}} = F_g H$$

Because $F_g = mg$ and $H = L \sin \theta$:

$$W_{\text{lifting}} = mgL \sin \theta \Rightarrow \boxed{W_{\text{ramp}} = W_{\text{lifting}}}$$

35 •• Particle a has mass m , is initially located on the positive x axis at $x = x_0$ and is subject to a repulsive force F_x from particle b . The location of particle b is fixed at the origin. The force F_x is inversely proportional to the square of the distance x between the particles. That is, $F_x = A/x^2$, where A is a positive constant. Particle a is released from rest and allowed to move under the influence of the

force. Find an expression for the work done by the force on a as a function of x . Find both the kinetic energy and speed of a as x approaches infinity.

Picture the Problem Because the force varies inversely with distance, we need to use the integral form of the expression for the work done on the particle. We can then apply the work-kinetic energy theorem to the particle to find its kinetic energy and speed as functions of x .

The work done by the force on the particle is given by:

$$W_{x_0 \rightarrow x} = \int_{x_0}^x \vec{F} \cdot d\vec{x} = \int_{x_0}^x F(\cos\theta) dx$$

where θ is the angle between \vec{F} and $d\vec{x}$.

Substituting for F and noting that $\theta = 0$:

$$W_{x_0 \rightarrow x} = \int_{x_0}^x \frac{A}{x^2} (\cos 0^\circ) dx = A \int_{x_0}^x \frac{1}{x^2} dx$$

Evaluate this integral to obtain:

$$W_{x_0 \rightarrow x} = -A \left[\frac{1}{x} \right]_{x_0}^x = \boxed{\frac{A}{x_0} - \frac{A}{x}}$$

Applying the work-kinetic energy theorem to the particle yields:

$$W_{x_0 \rightarrow x} = \Delta K = K_x - K_{x_0}$$

or, because the particle is released from rest at $x = x_0$,

$$W_{x_0 \rightarrow x} = K_x$$

Substitute for $W_{x_0 \rightarrow x}$ to obtain:

$$K_x = \frac{A}{x_0} - \frac{A}{x}$$

As $x \rightarrow \infty$:

$$K_{x \rightarrow \infty} = \boxed{\frac{A}{x_0}}$$

and

$$v_{x \rightarrow \infty} = \sqrt{\frac{2K_{x \rightarrow \infty}}{m}} = \sqrt{\frac{2\left(\frac{A}{x_0}\right)}{m}} = \boxed{\sqrt{\frac{2A}{mx_0}}}$$

36 •• You exert a force of magnitude F on the free end of the rope. If the load moves up a distance h , (a) through what distance does the force move? (b) How much work is done by the rope on the load? (c) How much work do you do on the rope? (d) The mechanical advantage (defined in Problem 34) of this system is the ratio F/F_g , where F_g is the weight of the load. What is this mechanical advantage?

Picture the Problem The object whose weight is \vec{w} is supported by two portions of the rope resulting in what is known as a *mechanical advantage* of 2. The work that is done in each instance is the product of the force doing the work and the displacement of the object on which it does the work.

(a) If w moves through a distance h : F moves a distance $\boxed{2h}$

(b) Assuming that the kinetic energy of the weight does not change, relate the work done on the object to the change in its potential energy to obtain:

$$W = \Delta U = wh \cos \theta = \boxed{wh}$$

(c) Because the force you exert on the rope and its displacement are in the same direction:

$$W = F(2h) \cos \theta = F(2h)$$

Determine the tension in the ropes supporting the object:

$$\sum F_{\text{vertical}} = 2F - w = 0 \Rightarrow F = \frac{1}{2}w$$

Substitute for F :

$$W = F(2h) = \frac{1}{2}w(2h) = \boxed{wh}$$

(d) The mechanical advantage of the inclined plane is the ratio of the weight that is lifted to the force required to lift it, that is:

$$MA = \frac{w}{F} = \frac{w}{\frac{1}{2}w} = \boxed{2}$$

Remarks: Note that the mechanical advantage is also equal to the number of ropes supporting the load.

Scalar (Dot) Products

37 • What is the angle between the vectors \vec{A} and \vec{B} if $\vec{A} \cdot \vec{B} = -AB$?

Picture the Problem Because $\vec{A} \cdot \vec{B} = AB \cos \theta$, we can solve for $\cos \theta$ and use the fact that $\vec{A} \cdot \vec{B} = -AB$ to find θ .

Solve $\vec{A} \cdot \vec{B} = AB \cos \theta$, for θ to obtain:

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{AB} \right)$$

Substitute for $\vec{A} \cdot \vec{B}$ and evaluate θ : $\theta = \cos^{-1}\left(\frac{-AB}{AB}\right) = \cos^{-1}(-1) = \boxed{180^\circ}$

38 • Two vectors \vec{A} and \vec{B} each have magnitudes of 6.0 m and the angle between their directions is 60° . Find $\vec{A} \cdot \vec{B}$.

Picture the Problem We can use the definition of the scalar product to evaluate $\vec{A} \cdot \vec{B}$.

Express the definition of the scalar product $\vec{A} \cdot \vec{B}$: $\vec{A} \cdot \vec{B} = AB \cos \theta$

Substitute numerical values and evaluate $\vec{A} \cdot \vec{B}$: $\vec{A} \cdot \vec{B} = (6.0 \text{ m})(6.0 \text{ m}) \cos 60^\circ = \boxed{18 \text{ m}^2}$

39 • Find $\vec{A} \cdot \vec{B}$ for the following vectors: (a) $\vec{A} = 3\hat{i} - 6\hat{j}$, $\vec{B} = -4\hat{i} + 2\hat{j}$; (b) $\vec{A} = 5\hat{i} + 5\hat{j}$, $\vec{B} = 2\hat{i} - 4\hat{j}$; and (c) $\vec{A} = 6\hat{i} + 4\hat{j}$, $\vec{B} = 4\hat{i} - 6\hat{j}$.

Picture the Problem The scalar product of two-dimensional vectors \vec{A} and \vec{B} is $A_x B_x + A_y B_y$.

(a) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$: $\vec{A} \cdot \vec{B} = (3)(-4) + (-6)(2) = \boxed{-24}$

(b) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$: $\vec{A} \cdot \vec{B} = (5)(2) + (5)(-4) = \boxed{-10}$

(c) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$: $\vec{A} \cdot \vec{B} = (6)(4) + (4)(-6) = \boxed{0}$

40 • Find the angles between the vectors \vec{A} and \vec{B} given: (a) $\vec{A} = 3\hat{i} - 6\hat{j}$, $\vec{B} = -4\hat{i} + 2\hat{j}$; (b) $\vec{A} = 5\hat{i} + 5\hat{j}$, $\vec{B} = 2\hat{i} - 4\hat{j}$; and (c) $\vec{A} = 6\hat{i} + 4\hat{j}$, $\vec{B} = 4\hat{i} - 6\hat{j}$.

Picture the Problem The scalar product of two-dimensional vectors \vec{A} and \vec{B} is $AB \cos \theta = A_x B_x + A_y B_y$. Hence the angle between vectors \vec{A} and \vec{B} is given by

$$\theta = \cos^{-1}\left(\frac{A_x B_x + A_y B_y}{AB}\right)$$

(a) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$:

$$\theta = \cos^{-1} \left(\frac{(3)(-4) + (-6)(2)}{\sqrt{3^2 + (-6)^2} \sqrt{(-4)^2 + (2)^2}} \right) = \boxed{143^\circ}$$

(b) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$:

$$\theta = \cos^{-1} \left(\frac{(5)(2) + (5)(-4)}{\sqrt{5^2 + 5^2} \sqrt{2^2 + (-4)^2}} \right) = \boxed{108^\circ}$$

(c) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$:

$$\theta = \cos^{-1} \left(\frac{(6)(4) + (4)(-6)}{\sqrt{6^2 + 4^2} \sqrt{4^2 + (-6)^2}} \right) = \boxed{90^\circ}$$

41 • A 2.0-kg particle is given a displacement of $\Delta\vec{s} = (3.0\text{ m})\hat{i} + (3.0\text{ m})\hat{j} + (-2.0\text{ m})\hat{k}$. During the displacement, a constant force $\vec{F} = (2.0\text{ N})\hat{i} - (1.0\text{ N})\hat{j} + (1.0\text{ N})\hat{k}$ acts on the particle. (a) Find the work done by \vec{F} for this displacement. (b) Find the component of \vec{F} in the direction of this displacement.

Picture the Problem The work W done by a force \vec{F} during a displacement $\Delta\vec{s}$ for which it is responsible is given by $\vec{F} \cdot \Delta\vec{s}$.

(a) Using the definitions of work and the scalar product, calculate the work done by the given force during the specified displacement:

$$\begin{aligned} W &= \vec{F} \cdot \Delta\vec{s} = [(2.0\text{ N})\hat{i} - (1.0\text{ N})\hat{j} + (1.0\text{ N})\hat{k}] \cdot [(3.0\text{ m})\hat{i} + (3.0\text{ m})\hat{j} - (2.0\text{ m})\hat{k}] \\ &= [(2.0)(3.0) + (-1.0)(3.0) + (1.0)(-2.0)]\text{N} \cdot \text{m} = \boxed{1.0\text{ J}} \end{aligned}$$

(b) Use the definition of work that includes the angle between the force and displacement vectors to express the work done by \vec{F} :

$$W = F\Delta s \cos\theta = (F \cos\theta)\Delta s$$

Solve for the component of \vec{F} in the direction of $\Delta\vec{s}$:

$$F \cos\theta = \frac{W}{\Delta s}$$

Substitute numerical values and evaluate $F \cos \theta$:

$$F \cos \theta = \frac{1.0 \text{ J}}{\sqrt{(3.0 \text{ m})^2 + (3.0 \text{ m})^2 + (-2.0 \text{ m})^2}} = \boxed{0.21 \text{ N}}$$

- 42** •• (a) Find the unit vector that is in the same direction as the $\vec{A} = 2.0\hat{i} - 1.0\hat{j} - 1.0\hat{k}$. (b) Find the component of the vector $\vec{A} = 2.0\hat{i} - 1.0\hat{j} - 1.0\hat{k}$ in the direction of the vector $\vec{B} = 3.0\hat{i} + 4.0\hat{j}$.

Picture the Problem A unit vector that is in the same direction as a given vector and is the given vector divided by its magnitude. The component of a vector that is in the direction of another vector is the scalar product of the former vector and a unit vector that is in the direction of the latter vector.

(a) By definition, the unit vector that is in the same direction as \vec{A} is:

$$\hat{u}_A = \frac{\vec{A}}{A}$$

Substitute for \vec{A} and A and evaluate \hat{u}_A :

$$\begin{aligned} \hat{u}_A &= \frac{2.0\hat{i} - 1.0\hat{j} - 1.0\hat{k}}{\sqrt{(2.0)^2 + (-1.0)^2 + (-1.0)^2}} \\ &= \frac{2.0\hat{i} - 1.0\hat{j} - 1.0\hat{k}}{\sqrt{6}} \\ &= \boxed{0.82\hat{i} - 0.41\hat{j} - 0.41\hat{k}} \end{aligned}$$

(b) The component of \vec{A} in the direction of \vec{B} is given by:

$$A_{\text{direction of } \vec{B}} = \vec{A} \cdot \hat{u}_B \quad (1)$$

The unit vector in the direction of \vec{B} is:

$$\hat{u}_B = \frac{\vec{B}}{B} = \frac{3.0\hat{i} + 4.0\hat{j}}{\sqrt{(3.0)^2 + (4.0)^2}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

Substitute for \vec{B} and \hat{u}_B in equation (1) and evaluate $A_{\text{direction of } \vec{B}}$:

$$\begin{aligned} A_{\text{direction of } \vec{B}} &= (2.0\hat{i} - 1.0\hat{j} - 1.0\hat{k}) \cdot \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} \right) = (2.0) \left(\frac{3}{5} \right) + (-1.0) \left(\frac{4}{5} \right) + (-1.0)(0) \\ &= \boxed{0.40} \end{aligned}$$

- 43** •• [SSM] (a) Given two nonzero vectors \vec{A} and \vec{B} , show that if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then $\vec{A} \perp \vec{B}$. (b) Given a vector $\vec{A} = 4\hat{i} - 3\hat{j}$, find a vector in the

xy plane that is perpendicular to \vec{A} and has a magnitude of 10. Is this the only vector that satisfies the specified requirements? Explain.

Picture the Problem We can use the definitions of the magnitude of a vector and the dot product to show that if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then $\vec{A} \perp \vec{B}$.

$$(a) \text{ Express } |\vec{A} + \vec{B}|^2: \quad |\vec{A} + \vec{B}|^2 = (\vec{A} + \vec{B})^2$$

$$\text{Express } |\vec{A} - \vec{B}|^2: \quad |\vec{A} - \vec{B}|^2 = (\vec{A} - \vec{B})^2$$

$$\text{Equate these expressions to obtain:} \quad (\vec{A} + \vec{B})^2 = (\vec{A} - \vec{B})^2$$

$$\text{Expand both sides of the equation to obtain:} \quad A^2 + 2\vec{A} \cdot \vec{B} + B^2 = A^2 - 2\vec{A} \cdot \vec{B} + B^2$$

$$\text{Simplify to obtain:} \quad 4\vec{A} \cdot \vec{B} = 0 \text{ or } \vec{A} \cdot \vec{B} = 0$$

$$\text{From the definition of the dot product we have:} \quad \vec{A} \cdot \vec{B} = AB \cos \theta$$

where θ is the angle between \vec{A} and \vec{B} .

$$\text{Because neither } \vec{A} \text{ nor } \vec{B} \text{ is the zero vector:} \quad \cos \theta = 0 \Rightarrow \theta = 90^\circ \text{ and } \vec{A} \perp \vec{B}.$$

$$(b) \text{ Let the required vector be } \vec{B}. \text{ The condition that } \vec{A} \text{ and } \vec{B} \text{ be perpendicular is that:} \quad \vec{A} \cdot \vec{B} = 0 \quad (1)$$

$$\text{Express } \vec{B} \text{ as:} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} \quad (2)$$

$$\text{Substituting for } \vec{A} \text{ and } \vec{B} \text{ in equation (1) yields:} \quad (4\hat{i} - 3\hat{j}) \cdot (B_x \hat{i} + B_y \hat{j}) = 0$$

or

$$4B_x - 3B_y = 0 \quad (3)$$

$$\text{Because the magnitude of } \vec{B} \text{ is 10:} \quad B_x^2 + B_y^2 = 100 \quad (4)$$

$$\text{Solving equation (3) for } B_x \text{ and substituting in equation (4) yields:} \quad \left(\frac{3}{4}B_y\right)^2 + B_y^2 = 100$$

$$\text{Solve for } B_y \text{ to obtain:} \quad B_y = \pm 8$$

Substituting for B_y in equation (3) and solving for B_x yields:

$$B_x = \pm 6$$

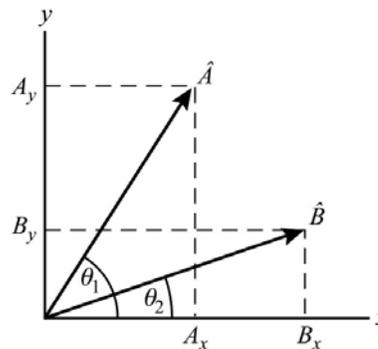
Substitute for B_x and B_y in equation (2) to obtain:

$$\vec{B} = \boxed{\pm 6\hat{i} \pm 8\hat{j}}$$

No. Because of the plus-and-minus signs in our expression for \vec{B} , the required vector is not the only vector that satisfies the specified requirements.

44 •• Unit vectors \hat{A} and \hat{B} are in the xy plane. They make angles of θ_1 and θ_2 , respectively, with the $+x$ axis, respectively. (a) Find the x and y components of the two vectors directly. (Your answer should be in terms of the angles.) (b) By considering the scalar product of \hat{A} and \hat{B} , show that $\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$.

Picture the Problem The diagram shows the unit vectors \hat{A} and \hat{B} arbitrarily located in the 1st quadrant. We can express these vectors in terms of the unit vectors \hat{i} and \hat{j} and their x and y components. We can then form the scalar product of \hat{A} and \hat{B} to show that $\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$.



(a) Express \hat{A} in terms of the unit vectors \hat{i} and \hat{j} :

$$\hat{A} = A_x \hat{i} + A_y \hat{j}$$

where

$$A_x = \boxed{\cos\theta_1} \quad \text{and} \quad A_y = \boxed{\sin\theta_1}$$

Proceed as above to obtain:

$$\hat{B} = B_x \hat{i} + B_y \hat{j}$$

where

$$B_x = \boxed{\cos\theta_2} \quad \text{and} \quad B_y = \boxed{\sin\theta_2}$$

(b) Evaluate $\hat{A} \cdot \hat{B}$:

$$\begin{aligned} \hat{A} \cdot \hat{B} &= (\cos\theta_1 \hat{i} + \sin\theta_1 \hat{j}) \\ &\quad \cdot (\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}) \\ &= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \end{aligned}$$

From the diagram we note that:

$$\hat{A} \cdot \hat{B} = \cos(\theta_1 - \theta_2)$$

Substitute for $\hat{A} \cdot \hat{B}$ to obtain:

$$\cos(\theta_1 - \theta_2) = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2$$

45 •• In Chapter 8, we shall introduce a new vector for a particle, called its *linear momentum*, symbolized by \vec{p} . Mathematically, it is related to the mass m and velocity \vec{v} of the particle by $\vec{p} = m\vec{v}$. (a) Show that the particle's kinetic energy K can be expressed as $K = \frac{\vec{p} \cdot \vec{p}}{2m}$. (b) Compute the linear momentum of a particle of mass 2.5 kg that is moving at a speed of 15 m/s at an angle of 25° clockwise from the $+x$ axis in the xy plane. (c) Compute its kinetic energy using both $K = \frac{1}{2}mv^2$ and $K = \frac{\vec{p} \cdot \vec{p}}{2m}$ and verify that they give the same result.

Picture the Problem The scalar product of two vectors is the product of their magnitudes multiplied by the cosine of the angle between them.

(a) We're to show that:

$$K = \frac{\vec{p} \cdot \vec{p}}{2m} \quad (1)$$

The scalar product of \vec{p} with itself is: $\vec{p} \cdot \vec{p} = p^2$

Because $p = mv$: $\vec{p} \cdot \vec{p} = m^2v^2$

Substitute in equation (1) and simplify to obtain:

$$K = \frac{m^2v^2}{2m} = \boxed{\frac{1}{2}mv^2}$$

(b) The linear momentum of the particle is given by:

$$\vec{p} = m\vec{v}$$

Substitute for m and \vec{v} and simplify to obtain:

$$\begin{aligned} \vec{p} &= (2.5 \text{ kg})[(15 \text{ m/s})\cos 335^\circ \hat{i} + (15 \text{ m/s})\sin 335^\circ \hat{j}] \\ &= \boxed{(34 \text{ kg} \cdot \text{m/s})\hat{i} + (-16 \text{ kg} \cdot \text{m/s})\hat{j}} \end{aligned}$$

(c) Evaluating $K = \frac{1}{2}mv^2$ yields:

$$K = \frac{1}{2}(2.5 \text{ kg})(15 \text{ m/s})^2 = \boxed{0.28 \text{ kJ}}$$

Evaluating $\mathbf{K} = \frac{\vec{\mathbf{p}} \cdot \vec{\mathbf{p}}}{2m}$ yields:

$$\begin{aligned} \mathbf{K} &= \frac{[(34 \text{ m/s})\hat{\mathbf{i}} + (-16 \text{ m/s})\hat{\mathbf{j}}] \cdot [(34 \text{ m/s})\hat{\mathbf{i}} + (-16 \text{ m/s})\hat{\mathbf{j}}]}{2(2.5 \text{ kg})} = \frac{(34 \text{ m/s})^2 + (-16 \text{ m/s})^2}{5.0 \text{ kg}} \\ &= \boxed{0.28 \text{ kJ}} \end{aligned}$$

46 •• (a) Let $\vec{\mathbf{A}}$ be a constant vector in the xy plane with its tail at the origin. Let $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ be a vector in the xy plane that satisfies the relation $\vec{\mathbf{A}} \cdot \vec{\mathbf{r}} = 1$. Show that the points with coordinates (x, y) lie on a straight line. (b) If $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, find the slope and y intercept of the line. (c) If we now let $\vec{\mathbf{A}}$ and $\vec{\mathbf{r}}$ be vectors in three-dimensional space, show that the relation $\vec{\mathbf{A}} \cdot \vec{\mathbf{r}} = 1$ specifies a plane.

Picture the Problem We can form the dot product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{r}}$ and require that $\vec{\mathbf{A}} \cdot \vec{\mathbf{r}} = 1$ to show that the points at the head of all such vectors $\vec{\mathbf{r}}$ lie on a straight line. We can use the equation of this line and the components of $\vec{\mathbf{A}}$ to find the slope and intercept of the line.

(a) Let $\vec{\mathbf{A}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$. Then:

$$\begin{aligned} \vec{\mathbf{A}} \cdot \vec{\mathbf{r}} &= (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}) \cdot (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \\ &= a_x x + a_y y = 1 \end{aligned}$$

Solve for y to obtain:

$$y = \boxed{-\frac{a_x}{a_y}x + \frac{1}{a_y}}$$

which is of the form $y = mx + b$ and hence is the equation of a straight line.

(b) Given that $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$:

$$m = -\frac{a_x}{a_y} = -\frac{2}{-3} = \boxed{\frac{2}{3}}$$

and

$$b = \frac{1}{a_y} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$$

(c) The equation $\vec{\mathbf{A}} \cdot \vec{\mathbf{r}} = 1$ specifies all vectors $\vec{\mathbf{r}}$ whose component r_A (the component of $\vec{\mathbf{r}}$ parallel to $\vec{\mathbf{A}}$) has the constant magnitude A^{-1} .

$$\begin{aligned} 1 &= \vec{\mathbf{A}} \cdot \vec{\mathbf{r}} = A\hat{\mathbf{A}} \cdot \vec{\mathbf{r}} = A(\vec{\mathbf{r}} \cdot \hat{\mathbf{A}}) = Ar_A \\ \text{so} \\ \frac{1}{A} &= r_A \end{aligned}$$

Let \vec{B} represent all vectors \vec{r} such that $\vec{r} = A^{-1}\hat{A} + \vec{B}$. Multiplying both sides by \hat{A} shows that \vec{B} is perpendicular to \hat{A} :

$$\begin{aligned} \vec{r} \cdot \hat{A} &= (A^{-1}\hat{A} + \vec{B}) \cdot \hat{A} \\ &= A^{-1}\hat{A} \cdot \hat{A} + \vec{B} \cdot \hat{A} \\ &= A^{-1} + \vec{B} \cdot \hat{A} \end{aligned}$$

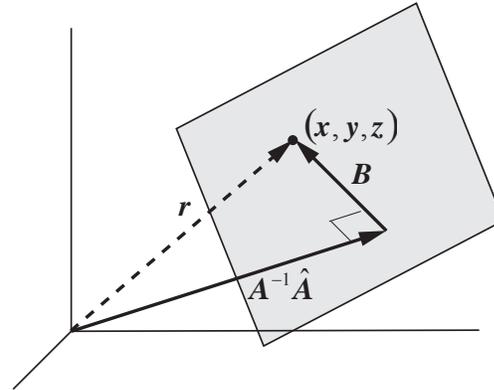
Because $\vec{r} \cdot \hat{A} = r_A = A^{-1}$:

$$A^{-1} = A^{-1} + \vec{B} \cdot \hat{A} \Rightarrow \vec{B} \cdot \hat{A} = 0$$

Because $\vec{B} \cdot \hat{A} = 0$:

$$\vec{B} \perp \hat{A}$$

The relation $\vec{r} = A^{-1}\hat{A} + \vec{B}$ is shown graphically to the right:



From the figure we can see that the tip of \vec{r} is restricted to a plane that is perpendicular to \hat{A} and contains the point a distance A^{-1} from the origin in the direction of \hat{A} . The equation $\vec{A} \cdot \vec{r} = 1$ specifies this plane.

47 **•••** **[SSM]** When a particle moves in a circle that is centered at the origin and the magnitude of its position vector \vec{r} is constant. (a) Differentiate $\vec{r} \cdot \vec{r} = r^2 = \text{constant}$ with respect to time to show that $\vec{v} \cdot \vec{r} = 0$, and therefore $\vec{v} \perp \vec{r}$. (b) Differentiate $\vec{v} \cdot \vec{r} = 0$ with respect to time and show that $\vec{a} \cdot \vec{r} + v^2 = 0$, and therefore $a_r = -v^2 / r$. (c) Differentiate $\vec{v} \cdot \vec{v} = v^2$ with respect to time to show that $\vec{a} \cdot \vec{v} = dv/dt$, and therefore $a_t = dv/dt$.

Picture the Problem The rules for the differentiation of vectors are the same as those for the differentiation of scalars and scalar multiplication is commutative.

(a) Differentiate $\vec{r} \cdot \vec{r} = r^2 = \text{constant}$:

$$\begin{aligned} \frac{d}{dt}(\vec{r} \cdot \vec{r}) &= \vec{r} \cdot \frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt} \cdot \vec{r} = 2\vec{v} \cdot \vec{r} \\ &= \frac{d}{dt}(\text{constant}) = 0 \end{aligned}$$

Because $\vec{v} \cdot \vec{r} = 0$:

$$\boxed{\vec{v} \perp \vec{r}}$$

(b) Differentiate $\vec{v} \cdot \vec{v} = v^2 = \text{constant}$ with respect to time:

$$\begin{aligned} \frac{d}{dt}(\vec{v} \cdot \vec{v}) &= \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v} = 2\vec{a} \cdot \vec{v} \\ &= \frac{d}{dt}(\text{constant}) = 0 \end{aligned}$$

Because $\vec{a} \cdot \vec{v} = 0$:

$$\boxed{\vec{a} \perp \vec{v}}$$

The results of (a) and (b) tell us that \vec{a} is perpendicular to \vec{v} and parallel (or antiparallel) to \vec{r} .

(c) Differentiate $\vec{v} \cdot \vec{r} = 0$ with respect to time:

$$\begin{aligned} \frac{d}{dt}(\vec{v} \cdot \vec{r}) &= \vec{v} \cdot \frac{d\vec{r}}{dt} + \vec{r} \cdot \frac{d\vec{v}}{dt} \\ &= v^2 + \vec{r} \cdot \vec{a} = \frac{d}{dt}(0) = 0 \end{aligned}$$

Because $v^2 + \vec{r} \cdot \vec{a} = 0$:

$$\boxed{\vec{r} \cdot \vec{a} = -v^2} \quad (1)$$

Express a_r in terms of θ , where θ is the angle between \vec{r} and \vec{a} :

$$a_r = a \cos \theta$$

Express $\vec{r} \cdot \vec{a}$:

$$\vec{r} \cdot \vec{a} = ra \cos \theta = ra_r$$

Substitute in equation (1) to obtain:

$$ra_r = -v^2$$

Solving for a_r yields:

$$a_r = \boxed{-\frac{v^2}{r}}$$

Work and Power

48 • Force A does 5.0 J of work in 10 s. Force B does 3.0 J of work in 5.0 s. Which force delivers greater power, A or B? Explain.

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt}$.

Calculate the rate at which force A does work:

$$P_A = \frac{5.0 \text{ J}}{10 \text{ s}} = 0.50 \text{ W}$$

Calculate the rate at which force B does work:

$$P_B = \frac{3.0\text{ J}}{5.0\text{ s}} = 0.60\text{ W}$$

Force B delivers greater power because, although it does only 60% of the work force A does, it does it in 50% of the time required by force A .

49 • A single force of 5.0 N in the $+x$ direction acts on an 8.0-kg object. (a) If the object starts from rest at $x = 0$ at time $t = 0$, write an expression for the power delivered by this force as a function of time. (b) What is the power delivered by this force at time $t = 3.0$ s?

Picture the Problem We can use Newton's 2nd law and the definition of acceleration to express the velocity of this object as a function of time. The power input of the force accelerating the object is defined to be the rate at which it does work; that is, $P = dW/dt = \vec{F} \cdot \vec{v}$.

(a) The power of the force as a function of time is given by:

$$P(t) = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = Fv(t)\cos\theta$$

or, because the force acts in the same direction as the velocity of the object,
 $P(t) = Fv(t)$ (1)

Express the velocity of the object as a function of its acceleration and time:

$$v(t) = at$$

Applying $\sum \vec{F} = m\vec{a}$ to the object yields:

$$a = \frac{F}{m}$$

Substitute for a in the expression for v to obtain:

$$v(t) = \frac{F}{m}t$$

Substituting in equation (1) yields:

$$P(t) = \frac{F^2}{m}t$$

Substitute numerical values and evaluate $P(t)$:

$$\begin{aligned} P(t) &= \frac{(5.0\text{ N})^2}{8.0\text{ kg}}t = (3.125\text{ W/s})t \\ &= \boxed{(3.1\text{ W/s})t} \end{aligned}$$

(b) Evaluate $P(3.0 \text{ s})$:

$$P(3.0 \text{ s}) = (3.125 \text{ W/s})(3.0 \text{ s}) = \boxed{9.4 \text{ W}}$$

- 50** • Find the power delivered by a force \vec{F} acting on a particle that moves with a velocity \vec{v} , where (a) $\vec{F} = (4.0 \text{ N})\hat{i} + (3.0 \text{ N})\hat{k}$ and $\vec{v} = (6.0 \text{ m/s})\hat{i}$; (b) $\vec{F} = (6.0 \text{ N})\hat{i} - (5.0 \text{ N})\hat{j}$ and $\vec{v} = -(5.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{j}$; and (c) $\vec{F} = (3.0 \text{ N})\hat{i} + (6.0 \text{ N})\hat{j}$ and $\vec{v} = (2.0 \text{ m/s})\hat{i} + (3.0 \text{ m/s})\hat{j}$.

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$.

(a) For $\vec{F} = (4.0 \text{ N})\hat{i} + (3.0 \text{ N})\hat{k}$ and $\vec{v} = (6.0 \text{ m/s})\hat{i}$:

$$P = \vec{F} \cdot \vec{v} = [(4.0 \text{ N})\hat{i} + (3.0 \text{ N})\hat{k}] \cdot [(6.0 \text{ m/s})\hat{i}] = \boxed{24 \text{ W}}$$

(b) For $\vec{F} = (6.0 \text{ N})\hat{i} - (5.0 \text{ N})\hat{j}$ and $\vec{v} = -(5.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{j}$:

$$P = \vec{F} \cdot \vec{v} = [(6.0 \text{ N})\hat{i} - (5.0 \text{ N})\hat{j}] \cdot [-(5.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{j}] = \boxed{-50 \text{ W}}$$

(c) For $\vec{F} = (3.0 \text{ N})\hat{i} + (6.0 \text{ N})\hat{j}$ and $\vec{v} = (2.0 \text{ m/s})\hat{i} + (3.0 \text{ m/s})\hat{j}$:

$$P = \vec{F} \cdot \vec{v} = [(3.0 \text{ N})\hat{i} + (6.0 \text{ N})\hat{j}] \cdot [(2.0 \text{ m/s})\hat{i} + (3.0 \text{ m/s})\hat{j}] = \boxed{24 \text{ W}}$$

- 51** • **[SSM]** You are in charge of installing a small food-service elevator (called a *dumbwaiter* in the food industry) in a campus cafeteria. The elevator is connected by a pulley system to a motor, as shown in Figure 6-34. The motor raises and lowers the dumbwaiter. The mass of the dumbwaiter is 35 kg. In operation, it moves at a speed of 0.35 m/s upward, without accelerating (except for a brief initial period, which we can neglect, just after the motor is turned on). Electric motors typically have an efficiency of 78%. If you purchase a motor with an efficiency of 78%, what minimum power rating should the motor have? Assume that the pulleys are frictionless.

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can find P_{in} from the given information that $P_{\text{out}} = 0.78P_{\text{in}}$. We can express P_{out} as the product of the tension in the cable T and the constant speed v of the dumbwaiter. We can apply Newton's 2nd law to the dumbwaiter to express T in terms of its mass m and the gravitational field g .

Express the relationship between the motor's input and output power:

$$P_{\text{out}} = 0.78P_{\text{in}} \Rightarrow P_{\text{in}} = 1.282P_{\text{out}} \quad (1)$$

Express the power required to move the dumbwaiter at a constant speed v :

$$P_{\text{out}} = Tv$$

Apply $\sum F_y = ma_y$ to the dumbwaiter:

$$\begin{aligned} T - mg &= ma_y \\ \text{or, because } a_y &= 0, \\ T &= mg \end{aligned}$$

Substitute for T in equation (1) to obtain:

$$P_{\text{in}} = 1.282Tv = 1.282mgv$$

Substitute numerical values and evaluate P_{in} :

$$\begin{aligned} P_{\text{in}} &= (1.282)(35\text{ kg})(9.81\text{ m/s}^2)(0.35\text{ m/s}) \\ &= \boxed{0.15\text{ kW}} \end{aligned}$$

52 •• A cannon placed at the edge of a cliff of height H fires a cannonball directly upward with an initial speed v_0 . The cannonball rises, falls back down (missing the cannon by a small margin) and lands at the foot of the cliff. Neglecting air resistance, calculate the velocity \vec{v} as a function of time, and show explicitly that the integral of $\vec{F}_{\text{net}} \cdot \vec{v}$ over the time that the cannonball spends in flight is equal to the change in the kinetic energy of the cannonball over the same time.

Picture the Problem Because, in the absence of air resistance, the acceleration of the cannonball is constant, we can use a constant-acceleration equation to relate its velocity to the time it has been in flight. We can apply Newton's 2nd law to the cannonball to find the net force acting on it and then form the dot product of \vec{F} and \vec{v} to express the rate at which the gravitational field does work on the cannonball. Integrating this expression over the time-of-flight T of the ball will yield the desired result.

Express the velocity of the cannonball as a function of time while it is in the air:

$$\vec{v}(t) = 0\hat{i} + (v_0 - gt)\hat{j} = \boxed{(v_0 - gt)\hat{j}}$$

Apply $\sum \vec{F} = m\vec{a}$ to the cannonball to express the force acting on it while it is in the air:

$$\vec{F}_{\text{net}} = -mg\hat{j}$$

Evaluate $\vec{F}_{\text{net}} \cdot \vec{v}$:

$$\begin{aligned} \vec{F}_{\text{net}} \cdot \vec{v} &= -mg\hat{j} \cdot (v_0 - gt)\hat{j} \\ &= -mgv_0 + mg^2t \end{aligned}$$

Relate $\vec{F}_{\text{net}} \cdot \vec{v}$ to the rate at which work is being done on the cannonball:

$$\frac{dW}{dt} = \vec{F}_{\text{net}} \cdot \vec{v} = -mgv_0 + mg^2t$$

Separate the variables and integrate over the time T that the cannonball is in the air:

$$\begin{aligned} W &= \int_0^T (-mgv_0 + mg^2t) dt \\ &= \frac{1}{2}mg^2T^2 - mgv_0T \end{aligned} \quad (1)$$

Using a constant-acceleration equation, relate the time-of-flight T to the initial and impact speeds of the cannonball:

$$v = v_0 - gT \Rightarrow T = \frac{v_0 - v}{g}$$

Substitute for T in equation (1) and simplify to evaluate W :

$$W = \frac{1}{2}mg^2 \frac{v_0^2 - 2vv_0 + v^2}{g^2} - mgv_0 \left(\frac{v_0 - v}{g} \right) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \boxed{\Delta K}$$

53 •• A particle of mass m moves from rest at $t = 0$ under the influence of a single constant force \vec{F} . Show that the power delivered by the force at any time t is $P = F^2t/m$.

Picture the Problem If the particle is acted on by a *single* force, that force is the *net* force acting on the particle and is responsible for its acceleration. The rate at which energy is delivered by the force is $P = \vec{F} \cdot \vec{v}$. We can apply the work-kinetic energy theorem to the particle to show that the total work done by the constant force after a time t is equal to the gain in kinetic energy of the particle.

Express the rate at which this force does work in terms of \vec{F} and \vec{v} :

$$P = \vec{F} \cdot \vec{v} \quad (1)$$

The velocity of the particle, in terms of its acceleration and the time that the force has acted is:

$$\vec{v} = \vec{a}t$$

Using Newton's 2nd law, substitute for \vec{a} :

$$\vec{v} = \frac{\vec{F}}{m}t$$

Substitute for \vec{v} in equation (1) and simplify to obtain:

$$P = \vec{F} \cdot \frac{\vec{F}}{m}t = \frac{\vec{F} \cdot \vec{F}}{m}t = \boxed{\frac{F^2}{m}t}$$

54 •• A 7.5-kg box is being lifted by means of a light rope that is threaded through a single, light, frictionless pulley that is attached to the ceiling. (a) If the box is being lifted at a *constant speed* of 2.0 m/s, what is the power delivered by the person pulling on the rope? (b) If the box is lifted, at *constant acceleration*, from rest on the floor to a height of 1.5 m above the floor in 0.42 s, what average power is delivered by the person pulling on the rope?

Picture the Problem Because, in (a), the box is being lifted at constant speed; the force acting on it is equal to the force due to gravity. We're given the speed and the force and thus calculation of the power is straightforward. In (b) we have a constant acceleration and are given the height to which the box is lifted and the time during which it is lifted. The work done in this case is the force times the displacement and the average power is the ratio of the work done to the time required. We can find the force acting on the box by applying Newton's 2nd law and the acceleration of the box by using a constant-acceleration equation.

(a) The power exerted by the person pulling on the rope is given by:

$$P = \vec{T} \cdot \vec{v} = T v \cos \theta$$

or, because \vec{T} and \vec{v} are in the same direction,

$$P = T v \quad (1)$$

Apply $\sum F_y = ma_y$ to the box to obtain:

$$T - F_g = ma_y$$

or, because $F_g = mg$ and $a_y = 0$,

$$T - mg = 0 \text{ and } T = mg$$

Substituting for T in equation (1) yields:

$$P = mgv$$

Substitute numerical values and evaluate P :

$$P = (7.5 \text{ kg})(9.81 \text{ m/s}^2)(2.0 \text{ m/s})$$

$$= \boxed{0.15 \text{ kW}}$$

(b) The average power exerted by the person pulling the rope is given by:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{F \Delta y}{\Delta t} \quad (2)$$

Use a constant-acceleration equation to relate Δy to the interval of acceleration Δt :

$$\Delta y = v_{0,y} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

or because the box starts from rest,

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 \Rightarrow a_y = \frac{2\Delta y}{(\Delta t)^2}$$

Apply $\sum F_y = ma_y$ to the box to obtain:

$$F = ma_y = m \frac{2\Delta y}{(\Delta t)^2}$$

Substitute for F in equation (2) to obtain:

$$P_{\text{av}} = \frac{m \frac{2\Delta y}{(\Delta t)^2} \Delta y}{\Delta t} = \frac{2m(\Delta y)^2}{(\Delta t)^3}$$

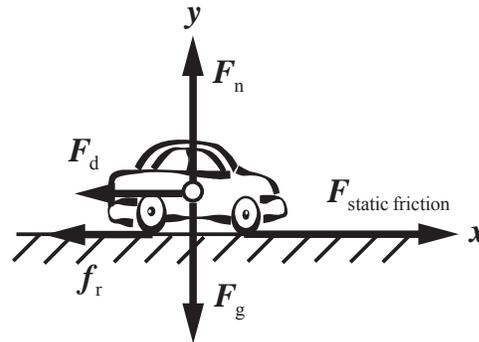
Substitute numerical values and evaluate P_{av} :

$$P_{\text{av}} = \frac{2(7.5 \text{ kg})(1.5 \text{ m})^2}{(0.42 \text{ s})^3} = \boxed{0.46 \text{ kW}}$$

Center of Mass Work and Center of Mass Translational Kinetic Energy

55 **•••** You have been asked to test drive a car and study its actual performance relative to its specifications. This particular car's engine is rated at 164 hp. This value is the *peak* rating, which means that it is capable, at most, of providing energy at the rate of 164 hp to the drive wheels. You determine that the car's mass (including test equipment and driver on board) is 1220 kg. (a) When cruising at a constant 55.0 mi/h, your onboard engine-monitoring computer determines that the engine is producing 13.5 hp. From previous coasting experiments, it has been determined that the coefficient of rolling friction on the car is 0.0150. Assume that the drag force on the car varies as the square of the car's speed. That is, $F_d = Cv^2$. (a) What is the value of the constant, C ? (b) Considering the peak power, what is the maximum speed (to the nearest 1 mi/h) that you would expect the car could attain? (This problem can be done by hand analytically, but it can be done more easily and quickly using a **graphing calculator** or **spreadsheet**.)

Picture the Problem When the car is moving at a constant speed, the static friction force exerted by the road on the tires must be balancing the force of rolling friction exerted by the roadway and the drag force exerted by the air. The power the engine produces is the product of this force and the velocity of the car.



(a) Apply $\sum F_x = ma_x$ to the car as it cruises at constant velocity:

$$F_{\text{static friction}} - F_d - f = 0$$

Solve for $F_{\text{static friction}}$ and substitute for F_d

$$F_{\text{static friction}} = Cv^2 + \mu_r F_g$$

and f to obtain:

$$\text{or, because } F_n = F_g = mg,$$

$$F_{\text{static friction}} = Cv^2 + \mu_r mg$$

Multiplying both sides of this equation by the speed v of the car yields an expression for the power P developed by the engine:

$$F_{\text{static friction}} v = P = (Cv^2 + \mu_r mg)v \quad (1)$$

Solve for C to obtain:

$$C = \frac{P}{v^3} - \frac{\mu_r mg}{v^2}$$

Express $P = 13.5$ hp in watts:

$$P = 13.5 \text{ hp} \times \frac{746 \text{ W}}{\text{hp}} = 10.07 \text{ kW}$$

Express $v = 55$ mi/h in m/s:

$$\begin{aligned} v &= 55 \frac{\text{mi}}{\text{h}} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= 24.58 \text{ m/s} \end{aligned}$$

Substitute numerical values and evaluate C :

$$C = \frac{10.07 \text{ kW}}{(24.58 \text{ m/s})^3} - \frac{(0.0150)(1220 \text{ kg})(9.81 \text{ m/s}^2)}{(24.58 \text{ m/s})^2} = 0.3808 \text{ kg/m} = \boxed{0.381 \text{ kg/m}}$$

(b) Rewriting equation (1) with v as the variable yields the cubic equation:

$$v^3 + \frac{\mu_r mg}{C} v - \frac{P}{C} = 0$$

Express 164 hp in watts:

$$P = 164 \text{ hp} \times \frac{746 \text{ W}}{\text{hp}} = 122.3 \text{ kW}$$

Substitute numerical values in the cubic equation and simplify to obtain:

$$v^3 + \frac{(0.015)(1220 \text{ kg})(9.81 \text{ m/s}^2)}{0.3808 \text{ kg/m}} v - \frac{122.3 \text{ kW}}{0.3808 \text{ kg/m}} = 0$$

or

$$v^3 + (471.4 \text{ m}^2/\text{s}^2)v - 3.213 \times 10^5 \text{ m}^3/\text{s}^3 = 0$$

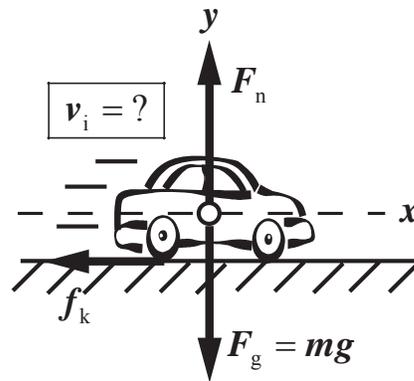
Use your graphing calculator to solve this cubic equation and then use conversion factors from your text to convert your answer to mi/h:

$$v = 66.20 \frac{\text{m}}{\text{s}} \times \frac{0.6818 \frac{\text{mi}}{\text{h}}}{0.3048 \frac{\text{m}}{\text{s}}} = \boxed{148 \text{ mi/h}}$$

56 •• As you drive your car along a country road at night, a deer jumps out of the woods and stands in the middle of the road ahead of you. This occurs just as you are passing from a 55-mi/h zone to a 50-mi/h zone. At the 50-mi/h speed-

limit sign, you slam on the car's brakes, causing them to lock up, and skid to a stop inches from the startled deer. As you breathe a sigh of relief, you hear the sound of a police siren. The policeman proceeds to write you a ticket for driving 56 mi/h in 50-mi/h zone. Because of your preparation in physics, you are able to use the 25-m-long skid marks that your car left behind as evidence that you were not speeding. What evidence do you present? In formulating your answer, you will need to know the coefficient of kinetic friction between automobile tires and dry concrete (see Table 5-1).

Picture the Problem We can apply the work-kinetic energy theorem to the car as it slides to a halt to relate the work done by kinetic friction to its initial speed v_i . Because the sum of the forces acting on the car in the y direction is zero, we can use the definition of the coefficient of kinetic friction to express the kinetic friction force \vec{f}_k in terms of the gravitational force $\vec{F}_g = m\vec{g}$.



Apply the work-kinetic energy theorem to your car to obtain:

$$\begin{aligned} W_{\text{total}} &= \Delta K = K_f - K_i \\ \text{or, because } K_f &= 0, \\ W_{\text{total}} &= -K_i \end{aligned} \quad (1)$$

The total work done on your car is done by the kinetic friction force that brings your car to a stop:

$$\begin{aligned} W_{\text{total}} &= W_{\text{by friction}} = \vec{f}_k \cdot \vec{d} = f_k d \cos \theta \\ \text{or, because } \vec{f}_k \text{ and } \vec{d} &\text{ are in opposite} \\ &\text{directions } (\theta = 180^\circ), \\ W_{\text{total}} &= -f_k d \end{aligned}$$

Substituting for K_i and W_{total} in equation (1) yields:

$$\begin{aligned} -f_k d &= -\frac{1}{2}mv_i^2 \\ \text{or, because } f_k &= \mu_k F_n, \\ \mu_k F_n d &= \frac{1}{2}mv_i^2 \end{aligned} \quad (2)$$

Apply $\sum F_y = ma_y$ to your car to obtain:

$$F_n - F_g = 0 \Rightarrow F_n = F_g = mg$$

Substituting for F_n in equation (2) yields:

$$\mu_k mgd = \frac{1}{2}mv_i^2 \Rightarrow v_i = \sqrt{2\mu_k gd}$$

Referring to Table 5-1 for the coefficient of kinetic friction for rubber on dry concrete, substitute numerical values and evaluate v_i :

$$\begin{aligned} v_i &= \sqrt{2(0.80)(9.81 \text{ m/s}^2)(25 \text{ m})} \\ &= 19.81 \text{ m/s} \end{aligned}$$

Convert v_i to mi/h to obtain:

$$v = 19.81 \frac{\text{m}}{\text{s}} \times \frac{0.6818 \frac{\text{mi}}{\text{h}}}{0.3048 \frac{\text{m}}{\text{s}}} = 44 \text{ mi/h}$$

Thus, at the time you applied your brakes, which occurred as you passed the 50 mi/h sign, you were doing 44 mi/h under the limit. Your explanation should convince the judge to void the ticket.

General Problems

57 • In February 2002, a total of 60.7 billion kW·h of electrical energy was generated by nuclear power plants in the United States. At that time, the population of the United States was about 287 million people. If the average American has a mass of 60 kg, and if 25% of the entire energy output of all nuclear power plants was diverted to supplying energy for a single giant elevator, estimate the height h to which the entire population of the country could be lifted by the elevator. In your calculations, assume that g is constant over the entire height h .

Picture the Problem 25 % of the electrical energy generated is to be diverted to do the work required to change the potential energy of the American people. We can calculate the height to which they can be lifted by equating the change in potential energy to the available energy.

Express the change in potential energy of the population of the United States in this process:

$$\Delta U = Nmgh$$

Letting E represent the total energy generated in February 2002, relate the change in potential to the energy available to operate the elevator:

$$Nmgh = 0.25E \Rightarrow h = \frac{0.25E}{Nmg}$$

Substitute numerical values and evaluate h :

$$h = \frac{(0.25)(60.7 \times 10^9 \text{ kW} \cdot \text{h}) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right)}{(287 \times 10^6)(60 \text{ kg})(9.81 \text{ m/s}^2)}$$

$$= \boxed{3.2 \times 10^5 \text{ m}}$$

58 • One of the most powerful cranes in the world operates in Switzerland. It can slowly raise a 6000-t load to a height of 12.0 m. (Note that 1 t = one tonne is sometimes called a metric ton. It is a unit of mass, not force, and is equal to 1000 kg.) (a) How much work is done by the crane during this lift? (b) If it takes 1.00 min to lift this load to this height at constant velocity, and the crane is 20 percent efficient, find the total (gross) power rating of the crane.

Picture the Problem We can use the definition of the work to find the work done by the crane in lifting this load and the definition of power to find the total power rating of the crane.

(a) The work done by the crane in lifting the load from position 1 to position 2 is given by:

$$W_{\text{by crane}} = \int_1^2 \vec{F} \cdot d\vec{s}$$

where \vec{F} is the force exerted by the crane.

Because \vec{F} and $d\vec{s}$ are in the same direction:

$$W_{\text{by crane}} = \int_1^2 F ds = F(s_2 - s_1) = Fh$$

or, because $F = F_g = mg$,

$$W_{\text{by crane}} = mgh$$

Substitute numerical values and evaluate $W_{\text{by crane}}$:

$$W_{\text{by crane}} = (6.000 \times 10^6 \text{ kg})(9.81 \text{ m/s}^2)(12.0 \text{ m}) = 706.3 \text{ MJ} = \boxed{706 \text{ MJ}}$$

(b) If the efficiency of the crane is η , the total power rating of the crane is given by:

$$P_{\text{total}} = \frac{\Delta W}{\eta(\Delta t)}$$

where ΔW is the work we calculated in part (a).

Substitute numerical values and evaluate P_{total} :

$$P_{\text{total}} = \frac{706.3 \text{ MJ}}{(0.20)(60.0 \text{ s})} = \boxed{59 \text{ MW}}$$

59 • In Austria, there once was a 5.6-km-long ski lift. It took about 60 min for a gondola to travel up its length. If there were 12 gondolas going up, each with a cargo of mass 550 kg, and if there were 12 empty gondolas going down, and the angle of ascent was 30° , estimate the power P the engine needed to deliver in order to operate the ski lift.

Picture the Problem The power P of the engine needed to operate this ski lift is equal to the rate at which it does work on the gondolas. Because as many empty gondolas are descending as are ascending, we do not need to know their mass.

Express the rate at which work is done as the cars are lifted:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{F\Delta s}{\Delta t}$$

where F is the force exerted by the engine on the gondola cars and Δs is the vertical displacement of the cars.

For N gondola cars, each of mass M ,
 $F = F_g = NMg$:

$$P_{\text{av}} = \frac{NMg\Delta s}{\Delta t}$$

Relate Δs to the angle of ascent θ and the length L of the ski lift:

$$\Delta s = L \sin \theta$$

Substituting for Δs yields:

$$P_{\text{av}} = \frac{NMgL \sin \theta}{\Delta t}$$

Substitute numerical values and evaluate P :

$$P = \frac{12(550 \text{ kg})(9.81 \text{ m/s}^2)(5.6 \text{ km}) \sin 30^\circ}{(60 \text{ min})(60 \text{ s/min})} = \boxed{50 \text{ kW}}$$

60 •• To complete your master's degree in physics, your advisor has you design a small linear accelerator capable of emitting protons, each with a kinetic energy of 10.0 keV. (The mass of a single proton is 1.67×10^{-27} kg.) In addition, 1.00×10^9 protons per second must reach the target at the end of the 1.50-m-long accelerator. (a) What the average power must be delivered to the stream of protons? (b) What force (assumed constant) must be applied to each proton? (c) What speed does each proton attain just before it strikes the target, assuming the protons start from rest?

Picture the Problem We can use the definition of average power and the work-kinetic energy theorem to find the average power delivered to the target. A second application of the work-kinetic energy theorem and the use of the definition of

work will yield the force exerted on each proton. Finally, we can use the definition of kinetic energy to find the speed of each proton just before it hits the target.

(a) The average power delivered by the stream of protons is given by:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} \quad (1)$$

Apply the work-kinetic energy theorem to the proton beam to obtain:

$$W_{\text{total}} = \Delta K$$

Substituting ΔK for ΔW in equation (1) yields:

$$P_{\text{av}} = \frac{\Delta K}{\Delta t}$$

Substitute numerical values and evaluate P_{av} :

$$P_{\text{av}} = \frac{10.0 \text{ keV/particle}}{1.00 \times 10^{-9} \text{ s/particle}} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}} = 1.602 \times 10^{-6} \text{ W} = \boxed{1.60 \mu\text{W}}$$

(b) Apply the work-kinetic energy theorem to a proton to obtain:

$$W_{\text{total}} = \Delta K \quad (2)$$

The work done by the accelerating force \vec{F} on each proton is:

$$W_{\text{total}} = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

or, because \vec{F} and \vec{d} are in the same direction,

$$W_{\text{total}} = Fd$$

Substituting for W_{total} in equation (2) yields:

$$Fd = \Delta K \Rightarrow F = \frac{\Delta K}{d}$$

Substitute numerical values and evaluate F :

$$\begin{aligned} F &= \frac{10.0 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}}}{1.50 \text{ m}} \\ &= 1.068 \times 10^{-15} \text{ N} \\ &= \boxed{1.07 \text{ fN}} \end{aligned}$$

(c) The kinetic energy of each proton is related to its speed:

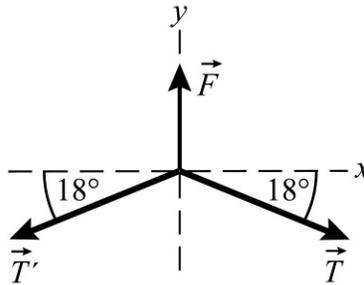
$$K_f = \frac{1}{2} m v_f^2 \Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

Substitute numerical values and evaluate v_f :

$$v_f = \sqrt{\frac{2 \left(10.0 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ J}}{\text{eV}} \right)}{1.67 \times 10^{-27} \text{ kg}}} \\ = \boxed{1.39 \times 10^6 \text{ m/s}}$$

61 ... The four strings pass over the bridge of a violin, as shown in Figure 6-35. The strings make an angle of 72.0° with the normal to the plane of the instrument on either side of the bridge. The resulting total normal force pressing the bridge into the violin is $1.00 \times 10^3 \text{ N}$. The length of the strings from the bridge to the peg to which each is attached is 32.6 cm. (a) Determine the tension in the strings, assuming the tension is the same for each string. (b) One of the strings is plucked out a distance of 4.00 mm, as shown. Make a free-body diagram showing all of the forces acting on the segment of the string in contact with the finger (not shown), and determine the force pulling the segment back to its equilibrium position. Assume the tension in the string remains constant during the pluck. (c) Determine the work done on the string in plucking it out that distance. Remember that the net force pulling the string back to its equilibrium position is changing as the string is being pulled out, but assume that the magnitudes of the tension forces remain constant.

Picture the Problem The free-body diagram shows the forces acting on one of the strings at the bridge. The force whose magnitude is F is one-fourth of the force ($1.00 \times 10^3 \text{ N}$) the bridge exerts on the strings. We can apply the condition for equilibrium in the y direction to find the tension in each string. Repeating this procedure at the site of the plucking will yield the restoring force acting on the string. We can find the work done on the string as it returns to equilibrium from the product of the average force acting on it and its displacement.



(a) Noting that, due to symmetry $T' = T$, apply $\sum F_y = 0$ to the string at the point of contact with the bridge:

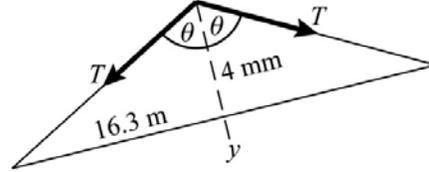
$$F - 2T \sin 18.0^\circ = 0 \Rightarrow T = \frac{F}{2 \sin 18.0^\circ}$$

Substitute numerical values and evaluate T :

$$T = \frac{\frac{1}{4}(1.00 \times 10^3 \text{ N})}{2 \sin 18.0^\circ} = 404.51 \text{ N}$$

$$= \boxed{405 \text{ N}}$$

(b) A free-body diagram showing the forces restoring the string to its equilibrium position just after it has been plucked is shown to the right:



Express the net force acting on the string immediately after it is released:

$$F_{\text{net}} = 2T \cos \theta \quad (1)$$

Use trigonometry to find θ .

$$\theta = \tan^{-1} \left(\frac{16.3 \text{ cm}}{4.00 \text{ mm}} \times \frac{10 \text{ mm}}{\text{cm}} \right) = 88.594^\circ$$

Substitute numerical values for T and θ in equation (1) and evaluate F_{net} :

$$F_{\text{net}} = 2(404.51 \text{ N}) \cos 88.594^\circ$$

$$= 19.851 \text{ N}$$

$$= \boxed{19.9 \text{ N}}$$

(c) Express the work done on the string in displacing it a distance dx' :

$$dW = F dx'$$

If we pull the string out a distance x' , the magnitude of the force pulling it down is approximately:

$$F = (2T) \frac{x'}{L/2} = \frac{4T}{L} x'$$

Substitute to obtain:

$$dW = \frac{4T}{L} x' dx'$$

Integrate dW to obtain:

$$W = \frac{4T}{L} \int_0^x x' dx' = \frac{2T}{L} x^2$$

where x is the final displacement of the string.

Substitute numerical values and evaluate W :

$$W = \frac{2(404.51 \text{ N})}{32.6 \times 10^{-2} \text{ m}} (4.00 \times 10^{-3} \text{ m})^2$$

$$= \boxed{39.7 \text{ mJ}}$$

62 •• The magnitude of the single force acting on a particle of mass m is given by $F = bx^2$, where b is a constant. The particle starts from rest. After it travels a distance L , determine its (a) kinetic energy and (b) speed.

Picture the Problem We can apply the work-kinetic energy theorem and the definition of work to the particle to find its kinetic energy when it has traveled a distance L . We can then use the definition of kinetic energy to find its speed at this location.

(a) Apply the work-kinetic energy theorem to the particle to obtain:

$$W_{\text{total}} = \Delta K = K_f - K_i$$

or, because the particle is initially at rest,

$$W_{\text{total}} = K_f = \frac{1}{2}mv_f^2 \quad (1)$$

The total work done on the particle is also given by:

$$dW_{\text{total}} = \vec{F} \cdot d\vec{x} = Fdx \cos \theta$$

or, because \vec{F} and $d\vec{x}$ are in the same direction,

$$dW_{\text{total}} = K_f = Fdx$$

Substituting for F_x and going from differential to integral form yields:

$$K_f = \int_0^L bx^2 dx = b \int_0^L x^2 dx = \boxed{\frac{1}{3}bL^3}$$

(b) Substitute for K_f in equation (1) to obtain:

$$\frac{1}{3}bL^3 = \frac{1}{2}mv_f^2 \Rightarrow v_f = \boxed{\sqrt{\frac{2bL^3}{3m}}}$$

63 •• [SSM] A single horizontal force in the $+x$ direction acts on a cart of mass m . The cart starts from rest at $x = 0$, and the speed of the cart increases with x as $v = Cx$, where C is a constant. (a) Find the force acting on the cart as a function of x . (b) Find the work done by the force in moving the cart from $x = 0$ to $x = x_1$.

Picture the Problem We can use the definition of work to obtain an expression for the position-dependent force acting on the cart. The work done on the cart can be calculated from its change in kinetic energy.

(a) Express the force acting on the cart in terms of the work done on it:

$$F(x) = \frac{dW}{dx} \quad (1)$$

Apply the work-kinetic energy theorem to the cart to obtain:

$$W = \Delta K = K - K_i$$

or, because the cart starts from rest,

$$W = K$$

Substitute for W in equation (1) to obtain:

$$\begin{aligned} F(x) &= \frac{dK}{dx} = \frac{d}{dx} \left(\frac{1}{2} mv^2 \right) \\ &= \frac{d}{dx} \left[\frac{1}{2} m(Cx)^2 \right] \\ &= \boxed{mC^2 x} \end{aligned}$$

(b) The work done by this force changes the kinetic energy of the cart:

$$\begin{aligned} W &= \Delta K = \frac{1}{2} mv_1^2 - \frac{1}{2} mv_0^2 \\ &= \frac{1}{2} mv_1^2 - 0 = \frac{1}{2} m(Cx_1)^2 \\ &= \boxed{\frac{1}{2} mC^2 x_1^2} \end{aligned}$$

64 •• A force $\vec{F} = (2.0 \text{ N/m}^2)x^2\hat{i}$ is applied to a particle initially at rest in the xy plane. Assume it starts from rest. Find the work done by this force on the particle, and its final speed, as it moves along a path that is (a) in a straight line from point (2.0 m, 2.0 m) to point (2.0 m, 7.0 m), and (b) in a straight line from point (2.0 m, 2.0 m) to point (5.0 m, 6.0 m). (The given force is the only force doing work on the particle.)

Picture the Problem The work done by \vec{F} on the particle depends on whether \vec{F} causes a displacement of the particle in the direction it acts. We can apply the work-kinetic energy theorem to find the final speed of the particle in each case.

(a) The work done on the particle is given by:

$$W = \int \vec{F} \cdot d\vec{\ell}$$

Substitute for \vec{F} and $d\vec{\ell}$ to obtain:

$$W = \int_{2.0 \text{ m}}^{7.0 \text{ m}} (2.0 \text{ N/m}^2)x^2 \hat{i} \cdot dx \hat{j} = \boxed{0}$$

Apply the work-kinetic energy theorem to the particle to obtain:

$$\begin{aligned} W &= \Delta K = K_f - K_i \\ \text{or, because the particle starts from rest} \\ \text{and } W &= 0, \\ K_f &= 0 \Rightarrow v_f = \boxed{0} \end{aligned}$$

(b) Calculate the work done by \vec{F} during the displacement of the particle from $x = 2.0$ m to 5.0 m:

$$\begin{aligned} W &= \int_{2.0\text{m}}^{5.0\text{m}} \left((2.0\text{ N/m}^2)x^2 \right) \hat{i} \cdot dx \hat{i} \\ &= (2.0\text{ N/m}^2) \int_{2.0\text{m}}^{5.0\text{m}} x^2 dx \\ &= (2.0\text{ N/m}^2) \left[\frac{x^3}{3} \right]_{2.0\text{m}}^{5.0\text{m}} \\ &= \boxed{78\text{ J}} \end{aligned}$$

Apply the work-kinetic energy theorem to the particle to obtain:

$$\begin{aligned} W &= \Delta K = K_f - K_i \\ \text{or, because the particle starts from rest,} \\ K_f &= 78\text{ J} \Rightarrow v_f = \frac{2(78\text{ J})}{m} = \boxed{\frac{158\text{ J}}{m}} \end{aligned}$$

where m is the mass of the particle.

65 •• A particle of mass m moves along the x axis. Its position varies with time according to $x = 2t^3 - 4t^2$, where x is in meters and t is in seconds. Find (a) the velocity and acceleration of the particle as functions of t , (b) the power delivered to the particle as a function of t , and (c) the work done by the net force from $t = 0$ to $t = t_1$.

Picture the Problem The velocity and acceleration of the particle can be found by differentiation. The power delivered to the particle can be expressed as the product of its velocity and the net force acting on it, and the work done by the force and can be found from the change in kinetic energy this work causes.

In the following, if t is in seconds and m is in kilograms, then v is in m/s, a is in m/s^2 , P is in W, and W is in J.

(a) The velocity of the particle is given by:

$$v = \frac{dx}{dt} = \frac{d}{dt}(2t^3 - 4t^2) = \boxed{(6t^2 - 8t)}$$

The acceleration of the particle is given by:

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 - 8t) = \boxed{(12t - 8)}$$

(b) Express the rate at which energy is delivered to this particle as it accelerates:

$$P = Fv = mav$$

Substitute for a and v and simplify to obtain:

$$P = m(12t - 8)(6t^2 - 8t) \\ = \boxed{8mt(9t^2 - 18t + 8)}$$

(c) Apply the work-kinetic energy theorem to the particle to obtain:

$$W = \Delta K = K_1 - K_0$$

Substitute for K_1 and K_0 and simplify:

$$W = \frac{1}{2}m[(v(t_1))^2 - (v(0))^2] \\ = \frac{1}{2}m[(6t_1^2 - 8t_1)^2 - 0] \\ = \boxed{2mt_1^2(3t_1 - 4)^2}$$

Remarks: We could also find W by integrating $P(t)$ with respect to time.

66 •• A 3.0-kg particle starts from rest at $x = 0.050$ m and moves along the x axis under the influence of a single force $F_x = 6.0 + 4.0x - 3.0x^2$, where F_x is in newtons and x is in meters. (a) Find the work done by the force as the particle moves from $x = 0.50$ m to $x = 3.0$ m. (b) Find the power delivered to the particle as it passes through the point $x = 3.0$ m.

Picture the Problem We can calculate the work done by the given force from its definition. The power can be determined from $P = \vec{F} \cdot \vec{v}$ and v from the change in kinetic energy of the particle produced by the work done on it.

(a) Express the work done on the particle:

$$W = \int \vec{F}_x \cdot d\vec{s} = \int F_x \hat{i} \cdot ds \hat{i} = \int F_x ds$$

Substitute for F_x and ds and evaluate the definite integral:

$$W = \int_{0.50 \text{ m}}^{3.0 \text{ m}} (6.0 + 4.0x - 3.0x^2) dx \\ = \left[6.0x + \frac{4.0x^2}{2} - \frac{3.0x^3}{3} \right]_{0.50 \text{ m}}^{3.0 \text{ m}} \\ = 5.625 \text{ J} = \boxed{5.6 \text{ J}}$$

(b) Express the power delivered to the particle as it passes through the point at $x = 3.0$ m:

$$P = \vec{F} \cdot \vec{v} = F_{x=3.0 \text{ m}} v_{x=3.0 \text{ m}} \quad (1)$$

Apply the work-kinetic energy theorem to find the work done on the particle as it moves from the point at $x = 0.050$ m to the point at $x = 3.0$ m:

$$W = \Delta K = K_{x=3.0 \text{ m}} - K_{x=0.050 \text{ m}} \\ \text{or, because } K_{x=0.050 \text{ m}} = 0, \\ W = K_{x=3.0 \text{ m}} = \frac{1}{2}mv_{x=3.0 \text{ m}}^2$$

Solve for $v_{x=3.0\text{ m}}$ to obtain:

$$v_{x=3.0\text{ m}} = \sqrt{\frac{2W}{m}}$$

Evaluating W yields:

$$\begin{aligned} W &= \int_{0.050\text{ m}}^{3.0\text{ m}} (6.0 + 4.0x - 3.0x^2) dx \\ &= \left[6.0x + \frac{4.0x^2}{2} - \frac{3.0x^3}{3} \right]_{0.050\text{ m}}^{3.0\text{ m}} \\ &= 8.695\text{ J} \end{aligned}$$

Now we can evaluate $v_{x=3.0\text{ m}}$:

$$v = \sqrt{\frac{2(8.695\text{ J})}{3.0\text{ kg}}} = 2.408\text{ m/s}$$

Evaluate $F_{x=3\text{ m}}$ to obtain:

$$\begin{aligned} F_{x=3\text{ m}} &= 6.0 + 4.0(3.0) - 3.0(3.0)^2 \\ &= -9.0\text{ N} \end{aligned}$$

Substitute for $F_{x=3\text{ m}}$ and $v_{x=3.0\text{ m}}$ in equation (1) and evaluate P :

$$P = (-9.0\text{ N})(2.408\text{ m/s}) = \boxed{-22\text{ W}}$$

67 •• The initial kinetic energy imparted to a 0.0200-kg bullet is 1200 J. (a) Assuming it accelerated down a 1.00-m-long rifle barrel, estimate the average power delivered to it during the firing. (b) Neglecting air resistance, find the range of this projectile when it is fired at an angle such that the range equals the maximum height attained.

Picture the Problem We'll assume that the firing height is negligible and that the bullet lands at the same elevation from which it was fired. We can use the equation $R = (v_0^2/g)\sin 2\theta$ to find the range of the bullet and constant-acceleration equations to find its maximum height. The bullet's initial speed can be determined from its initial kinetic energy.

(a) The average power delivered to the bullet during firing is given by:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} \quad (1)$$

Apply the work-kinetic energy theorem to the bullet to obtain:

$W_{\text{total}} = \Delta K = K_f - K_i$
or, because the initial speed of the bullet is zero,

$$W_{\text{total}} = K_f = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

Because the acceleration of the bullet is (assumed to be) constant, its average speed in the barrel is:

$$v_{\text{av}} = \frac{v_i + v_f}{2} = \frac{1}{2}v_f$$

The time-in-the-barrel is given by:

$$\Delta t = \frac{\ell}{v_{\text{av}}} = \frac{2\ell}{v_f} = \frac{2\ell}{\sqrt{\frac{2K_f}{m}}}$$

Substituting for Δt in equation (1) and simplifying yields:

$$P_{\text{av}} = \frac{K_f}{2\ell} \sqrt{\frac{2K_f}{m}}$$

Substitute numerical values and evaluate P_{av} :

$$\begin{aligned} P_{\text{av}} &= \frac{1200 \text{ J}}{2(1.00 \text{ m})} \sqrt{\frac{2(1200 \text{ J})}{0.0200 \text{ kg}}} \\ &= \boxed{208 \text{ kW}} \end{aligned}$$

(b) Using the "equal-height" range equation, express the range of the bullet as a function of its firing speed and angle of firing:

$$R = \frac{v_0^2}{g} \sin 2\theta$$

Rewrite the range equation using the trigonometric identity $\sin 2\theta = 2\sin\theta \cos\theta$:

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g} \quad (2)$$

Using constant-acceleration equations, express the position coordinates of the projectile along its flight path in terms of the parameter t :

$$\begin{aligned} x &= (v_0 \cos\theta)t \\ \text{and} \\ y &= (v_0 \sin\theta)t - \frac{1}{2}gt^2 \end{aligned}$$

Eliminate the parameter t and make use of the fact that the maximum height occurs when the projectile is at half the range to obtain:

$$h = \frac{(v_0 \sin\theta)^2}{2g}$$

Equate R and h to obtain:

$$\frac{(v_0 \sin\theta)^2}{2g} = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

Simplifying this equation yields:

$$\tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4) = 76.0^\circ$$

Relate the bullet's kinetic energy to its mass and speed and solve for the square of its speed:

$$K = \frac{1}{2}mv_0^2 \Rightarrow v_0^2 = \frac{2K}{m}$$

Substitute for v_0^2 in equation (2) to obtain:

$$R = \frac{2K \sin 2\theta}{mg}$$

Substitute numerical values and evaluate R :

$$R = \frac{2(1200\text{ J})\sin[2(76.0^\circ)]}{(0.0200\text{ kg})(9.81\text{ m/s}^2)} = \boxed{5.74\text{ km}}$$

68 •• The force F_x acting on a 0.500-kg particle is shown as a function of x in Figure 6-36. (a) From the graph, calculate the work done by the force when the particle moves from $x = 0.00$ to the following values of x : -4.00 , -3.00 , -2.00 , -1.00 , $+1.00$, $+2.00$, $+3.00$, and $+4.00$ m. (b) If it starts with a velocity of 2.00 m/s in the $+x$ direction, how far will the particle go in that direction before stopping?

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is positive, so W is negative for $x < 0$.

(a) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square (each square is 1.00 J) to complete the following table:

x , m	-4.00	-3.00	-2.00	-1.00	1.00	2.00	3.00	4.00
W , J	-11.4	-10.2	-7.0	-3.0	1.0	0	-2.0	-3.0

(b) The energy that the particle must lose before it stops is its kinetic energy at $x = 0$:

$$K_{x=0} = \frac{1}{2}mv_{x=0}^2$$

Substitute numerical values and evaluate $K_{x=0}$:

$$K_{x=0} = \frac{1}{2}(0.500\text{ kg})(2.00\text{ m/s})^2 = 1.00\text{ J}$$

The particle is accelerated (recall that $F_x = -dU/dx$) between $x = 0$ and $x = 1.00$ m and gains an additional 1.00 J of kinetic energy. When it arrives at $x = 2.00$ m, however, it will have lost the kinetic energy it gained between $x = 0$ and $x = 1.00$ m and its kinetic energy will again be 1.00 J. Inspection of the graph leads us to conclude that it will have lost its remaining kinetic energy when it reaches $x = \boxed{2.50 \text{ m}}$.

69 •• (a) Repeat Problem 68(a) for the force F_x shown in Figure 6-37. (b) If the object starts at the origin moving to the right with a kinetic energy of 25.0 J, how much kinetic energy does it have at $x = 4.00$ m.

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is negative, so W is positive for $x < 0$. In (b), we can apply the work-kinetic energy theorem to find the kinetic energy at $x = 4.0$ m.

(a) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square (each square is 1.00 J) to complete the following table:

$x, \text{ m}$	-4.00	-3.00	-2.00	-1.00	0	1.00	2.00	3.00	4.00
$W, \text{ J}$	6	4	2	0.5	0	0.5	1.5	2.5	3.0

(b) Apply the work-kinetic energy theorem to the particle to obtain:

$$W_{\text{total}} = \Delta K = K_{x=4\text{ m}} - K_{x=0}$$

Solving for $K_{x=4\text{ m}}$ yields:

$$\begin{aligned} K_{x=4\text{ m}} &= W_{\text{total}} + K_{x=0} \\ &= W_{\text{total}} + 25.0 \text{ J} \end{aligned}$$

From $x = 0$ to $x = 4$ m, the total work done by the force is:

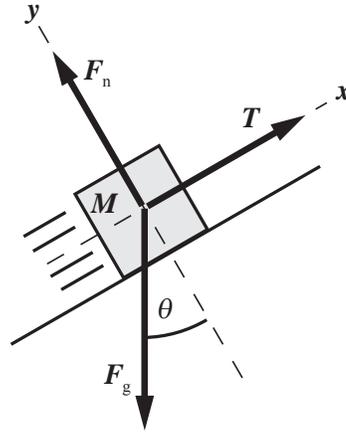
$$W_{\text{total}} = 3 \text{ square} \times \frac{1.00 \text{ J}}{\text{square}} = 3.00 \text{ J}$$

Substitute for W_{total} in the expression for $K_{x=4\text{ m}}$ to obtain:

$$K_{x=4\text{ m}} = 3.00 \text{ J} + 25.0 \text{ J} = \boxed{28.0 \text{ J}}$$

70 •• A box of mass M is at rest at the bottom of a frictionless inclined plane (Figure 6-38). The box is attached to a string that pulls with a constant tension T . (a) Find the work done by the tension T as the box moves through a distance x along the plane. (b) Find the speed of the box as a function of x . (c) Determine the power delivered by the tension in the string as a function of x .

Picture the Problem The forces acting on the block are shown in the free-body diagram. We can use the definition of work to find the work done by \vec{T} in displacing the block a distance x up the incline and the work-kinetic energy theorem to express the speed of the box as a function of x and θ . Finally, we can use the definition of power to determine the power produced by the tension in terms of x and θ .



(a) Use the definition of work to express the work the tension T does moving the box a distance x up the incline:

$$W = \int_0^x \vec{T} \cdot d\vec{x} = \int_0^x T dx \cos \phi$$

or, because the angle between \vec{T} and $d\vec{x}$ is zero,

$$W = T \int_0^x dx$$

Evaluate this integral to obtain:

$$W_{\text{by } T} = \boxed{Tx}$$

(b) Apply the work-kinetic energy theorem to the box to obtain:

$$W_{\text{total}} = \Delta K = K_f - K_i$$

or, because the box is initially at rest,

$$W_{\text{total}} = K_f = \frac{1}{2} M v_f^2 \Rightarrow v_f = \sqrt{\frac{2W_{\text{total}}}{M}}$$

The total work done on the box is the work done by the net force acting on it:

$$W_{\text{total}} = \int_0^x \vec{F}_{\text{net}} \cdot d\vec{x} = \int_0^x F_{\text{net}} dx \cos \phi$$

or, because the angle between \vec{F}_{net}

and $d\vec{x}$ is zero, $W_{\text{total}} = \int_0^x F_{\text{net}} dx$

Referring to the force diagram, we see that the net force acting on the block is:

$$F_{\text{net}} = T - Mg \sin \theta$$

Substitute for F_{net} and evaluate the integral to obtain:

$$\begin{aligned} W_{\text{total}} &= \int_0^x (T - Mg \sin \theta) dx \\ &= (T - Mg \sin \theta)x \end{aligned}$$

Substituting for W_{total} in the expression for v_f and simplifying yields:

$$\begin{aligned} v_f &= \sqrt{\frac{2(T - Mg \sin \theta)x}{M}} \\ &= \sqrt{2\left(\frac{T}{M} - g \sin \theta\right)x} \end{aligned}$$

(c) The power produced by the tension in the string is given by:

$$\begin{aligned} \mathbf{P} &= \vec{T} \cdot \vec{v}_f = T v_f \cos \phi \\ \text{or, because the angle between } \vec{T} \text{ and } \vec{v}_f \text{ is zero, } P &= T v_f \end{aligned}$$

Substitute for v_f to obtain:

$$P = T \sqrt{2\left(\frac{T}{M} - g \sin \theta\right)x}$$

71 ••• [SSM] A force acting on a particle in the xy plane at coordinates (x, y) is given by $\vec{F} = (F_0/r)(y\hat{i} - x\hat{j})$, where F_0 is a positive constant and r is the distance of the particle from the origin. (a) Show that the magnitude of this force is F_0 and that its direction is perpendicular to $\mathbf{r} = x\hat{i} + y\hat{j}$. (b) Find the work done by this force on a particle that moves once around a circle of radius 5.0 m that is centered at the origin.

Picture the Problem (a) We can use the definition of the magnitude of vector to show that the magnitude of \vec{F} is F_0 and the definition of the scalar product to show that its direction is perpendicular to \vec{r} . (b) The work done as the particle moves in a circular path can be found from its definition.

(a) Express the magnitude of \vec{F} :

$$\begin{aligned} |\vec{F}| &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{\left(\frac{F_0}{r}y\right)^2 + \left(-\frac{F_0}{r}x\right)^2} \\ &= \frac{F_0}{r}\sqrt{x^2 + y^2} \end{aligned}$$

Because $r = \sqrt{x^2 + y^2}$:

$$|\vec{F}| = \frac{F_0}{r}\sqrt{x^2 + y^2} = \frac{F_0}{r}r = \boxed{F_0}$$

Form the scalar product of \vec{F} and \vec{r} :

$$\begin{aligned}\vec{F} \cdot \vec{r} &= \left(\frac{F_0}{r}\right)(y\hat{i} - x\hat{j}) \cdot (x\hat{i} + y\hat{j}) \\ &= \left(\frac{F_0}{r}\right)(xy - xy) = 0\end{aligned}$$

Because $\vec{F} \cdot \vec{r} = 0$:

$$\boxed{\vec{F} \perp \vec{r}}$$

(b) The work done in an angular displacement from θ_1 to θ_2 is given by:

$$\begin{aligned}W_{1\text{rev}} &= \int_{\theta_1}^{\theta_2} \vec{F} \cdot d\vec{s} \\ &= \int_{\theta_1}^{\theta_2} \left(\frac{F_0}{r}\right)(y\hat{i} - x\hat{j}) \cdot d\vec{s} \\ &= \frac{F_0}{r} \int_{\theta_1}^{\theta_2} (r \sin \theta \hat{i} - r \cos \theta \hat{j}) \cdot d\vec{s} \\ &= F_0 \int_{\theta_1}^{\theta_2} (\sin \theta \hat{i} - \cos \theta \hat{j}) \cdot d\vec{s}\end{aligned}$$

Express the radial vector \vec{r} in terms of its magnitude r and the angle θ it makes with the positive x axis:

$$\vec{r} = r \cos \theta \hat{i} + r \sin \theta \hat{j}$$

The differential of \vec{r} is tangent to the circle (you can easily convince yourself of this by forming the dot product of \vec{r} and $d\vec{r}$) and so we can use it as $d\vec{s}$ in our expression for the work done by \vec{F} in one revolution:

$$\begin{aligned}d\vec{r} = d\vec{s} &= -r \sin \theta d\theta \hat{i} + r \cos \theta d\theta \hat{j} \\ &= (-r \sin \theta \hat{i} + r \cos \theta \hat{j}) d\theta\end{aligned}$$

Substitute for $d\vec{s}$, simplifying, and integrate to obtain:

$$\begin{aligned}W_{1\text{rev,ccw}} &= F_0 \int_0^{2\pi} (\sin \theta \hat{i} - \cos \theta \hat{j}) \cdot (-r \sin \theta \hat{i} + r \cos \theta \hat{j}) d\theta \\ &= -rF_0 \int_0^{2\pi} (\sin^2 \theta + \cos^2 \theta) d\theta = -rF_0 \int_0^{2\pi} d\theta = -rF_0 \theta \Big|_0^{2\pi} \\ &= -2\pi r F_0\end{aligned}$$

Substituting the numerical value of r yields:

$$\begin{aligned}W_{1\text{rev,ccw}} &= -2\pi(5.0 \text{ m})F_0 \\ &= \boxed{(-10\pi \text{ m})F_0}\end{aligned}$$

If the rotation is clockwise, the integral becomes:

$$W_{\text{rev,cw}} = -rF_0\theta \Big|_{2\pi}^0 = 2\pi rF_0$$

Substituting the numerical value of r yields:

$$W_{\text{rev,cw}} = 2\pi(5.0 \text{ m})F_0 = \boxed{(10\pi \text{ m})F_0}$$

72 **•••** A force acting on a 2.0-kg particle in the xy plane at coordinates (x, y) is given by $\vec{F} = -(b/r^3)(x\hat{i} + y\hat{j})$, where b is a positive constant and r is the distance from the origin. (a) Show that the magnitude of the force is inversely proportional to r^2 , and that its direction is antiparallel (opposite) to the radius vector $\vec{r} = x\hat{i} + y\hat{j}$. (b) If $b = 3.0 \text{ N}\cdot\text{m}^2$, find the work done by this force as the particle moves from $(2.0 \text{ m}, 0.0 \text{ m})$, to $(5.0 \text{ m}, 0.0 \text{ m})$ along a straight-line path. (c) Find the work done by this force on a particle moving once around a circle of radius $r = 7.0 \text{ m}$ that is centered at the origin.

Picture the Problem We can substitute for r and $x\hat{i} + y\hat{j}$ in \vec{F} to show that the magnitude of the force varies as the reciprocal of the square of the distance to the origin, and that its direction is opposite to the radius vector. We can find the work done by this force by evaluating the integral of F with respect to x from an initial position $x = 2.0 \text{ m}$, $y = 0 \text{ m}$ to a final position $x = 5.0 \text{ m}$, $y = 0 \text{ m}$. Finally, we can apply Newton's 2nd law to the particle to relate its speed to its radius, mass, and the constant b .

(a) Substitute for r and $x\hat{i} + y\hat{j}$ in \vec{F} to obtain:

$$\vec{F} = -\left(\frac{b}{(x^2 + y^2)^{3/2}}\right)\sqrt{x^2 + y^2}\hat{r}$$

where \hat{r} is a unit vector pointing from the origin toward the point of application of \vec{F} .

Simplifying yields:

$$\vec{F} = -b\left(\frac{1}{x^2 + y^2}\right)\hat{r} = \boxed{-\frac{b}{r^2}\hat{r}}$$

That is, the magnitude of the force varies inversely with the square of the distance to the origin and its direction (as indicated by the minus sign) is antiparallel (opposite) to the radius vector $\vec{r} = x\hat{i} + y\hat{j}$.

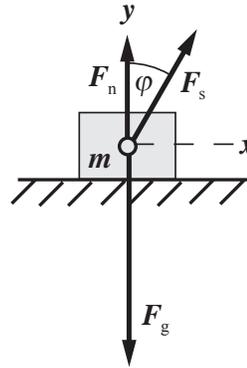
(b) Find the work done by this force by evaluating the integral of F with respect to x from an initial position $x = 2.0 \text{ m}$, $y = 0 \text{ m}$ to a final position $x = 5.0 \text{ m}$, $y = 0 \text{ m}$:

$$\begin{aligned} W &= - \int_{2.0\text{m}}^{5.0\text{m}} \frac{b}{x^2} dx = b \left[\frac{1}{x} \right]_{2.0\text{m}}^{5.0\text{m}} \\ &= 3.0 \text{ N} \cdot \text{m}^2 \left(\frac{1}{5.0 \text{ m}} - \frac{1}{2.0 \text{ m}} \right) \\ &= \boxed{-0.90 \text{ J}} \end{aligned}$$

(c) No work is done as the force is perpendicular to the velocity.

73 ••• A block of mass m on a horizontal frictionless tabletop is attached by a swivel to a spring that is attached to the ceiling (Figure 6-39). The vertical distance between the top of the block and the ceiling is y_0 , and the horizontal position is x . When the block is at $x = 0$, the spring, which has force constant k , is completely unstressed. (a) What is F_x , the x component of the force on the block due to the spring, as a function of x ? (b) Show that F_x is proportional to x^3 for sufficiently small values of $|x|$. (c) If the block is released from rest at $x = x_0$, where $|x_0| \ll y_0$, what is its speed when it reaches $x = 0$?

Picture the Problem We can use trigonometry to express the x -component of \vec{F}_s and the Pythagorean theorem to write it as a function of x . Expanding on of the terms in F_x binomially reveals its cubic dependence on x . In (c), we can apply the work-kinetic energy theorem and use the definition of work to find the speed of the block when it reaches $x = 0$.



(a) F_x , the x -component of \vec{F}_s , is given by:

$$F_x = -F_s \sin \varphi \quad (1)$$

Express F_s when the length of the spring is ℓ :

$$F_s = k(\ell - y_0)$$

Substituting for F_s in equation (1) yields:

$$F_x = -k(\ell - y_0) \sin \varphi \quad (2)$$

Referring to the figure, note that:

$$\sin \varphi = \frac{x}{\ell}$$

Substitute for $\sin\phi$ to obtain:

$$F_x = -k(\ell - y_0)\frac{x}{\ell} = -kx\left(1 - \frac{y_0}{\ell}\right) \quad (3)$$

Refer to the figure again to see that:

$$\ell^2 = x^2 + y_0^2 \Rightarrow \ell = \sqrt{x^2 + y_0^2}$$

Substituting for ℓ in equation (3) yields:

$$F_x = \boxed{-kx\left(1 - \frac{y_0}{\sqrt{x^2 + y_0^2}}\right)} \quad (4)$$

(b) Rewrite the second term in parentheses in equation (4) by dividing numerator and denominator by y_0 to obtain:

$$F_x = -kx\left(1 - \frac{1}{\frac{1}{y_0}\sqrt{x^2 + y_0^2}}\right) = -kx\left(1 - \frac{1}{\sqrt{1 + \frac{x^2}{y_0^2}}}\right) = -kx\left(1 - \left(1 + \frac{x^2}{y_0^2}\right)^{-\frac{1}{2}}\right)$$

Expanding $\left(1 + \frac{x^2}{y_0^2}\right)^{-\frac{1}{2}}$ binomially yields:

$$\left(1 + \frac{x^2}{y_0^2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}\frac{x^2}{y_0^2} + \text{higher order terms}$$

For $x \ll y_0$, we can ignore the higher-order terms and :

$$\left(1 + \frac{x^2}{y_0^2}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{x^2}{y_0^2}$$

Substitute for $\left(1 + \frac{x^2}{y_0^2}\right)^{-\frac{1}{2}}$ in the expression for F_x and simplify to obtain:

$$F_x = -kx\left[1 - \left(1 + \frac{1}{2}\frac{x^2}{y_0^2}\right)\right] = \boxed{-\frac{k}{2y_0^2}x^3}$$

(c) Apply the work-kinetic energy theorem to the block to obtain:

$W_{\text{total}} = \Delta K = K_f - K_i$
or, because the block is released from rest,

$$W_{\text{total}} = K_f = \frac{1}{2}mv_f^2 \quad (5)$$

Integrate F_x to obtain:

$$W_{\text{total}} = -\frac{k}{2y_0^2} \int_L^0 x^3 dx = \frac{kL^4}{8y_0^2}$$

Substitute for W_{total} in equation (5):

$$\frac{kL^4}{8y_0^2} = \frac{1}{2}mv_f^2 \Rightarrow v_f = \boxed{\frac{L^2}{2y_0} \sqrt{\frac{k}{m}}}$$

74 •• Two horses pull a large crate at constant speed across a barn floor by means of two light steel cables. A large box of mass 250 kg sits on the crate (Figure 6-40). As the horses pull, the cables are parallel to the horizontal floor. The coefficient of friction between the crate and the barn floor is 0.25. (a) What is the work done by each horse if the box is moved a distance of 25 m? (b) What is the tension in each cable if the angle between each cable and the direction the crate moves is 15° ?

Picture the Problem We can apply the work-kinetic energy theorem to the crate to relate the work done by the horses and the work done by the friction force. In part (b), we can use the definition of work and our result from Part (a) to find the tension in the cables.

(a) Apply the work-kinetic energy theorem to the crate to obtain:

$$\begin{aligned} W_{\text{total}} &= W_{\text{horses}} + W_{\text{friction}} = \Delta K \\ \text{or, because the horses pull the crate at} \\ \text{constant speed, } \Delta K &= 0, \text{ and} \\ W_{\text{horses}} + W_{\text{friction}} &= 0 \end{aligned} \quad (1)$$

The work done by friction is given by:

$$\begin{aligned} W_{\text{friction}} &= \vec{f}_k \cdot \vec{d} = f_k d \cos \phi \\ \text{where } \phi &\text{ is the angle between } \vec{f}_k \text{ and } \vec{d}. \end{aligned}$$

Because \vec{f}_k and \vec{d} are in opposite directions:

$$W_{\text{friction}} = -f_k d$$

Because the normal force acting on the crate equals the weight of the crate:

$$W_{\text{friction}} = -\mu_k F_n d = -\mu_k mgd$$

Substitute for W_{friction} in equation (1) to obtain:

$$\begin{aligned} W_{\text{horses}} - \mu_k mgd &= 0 \\ \text{and} \\ W_{\text{horses}} &= \mu_k mgd \end{aligned}$$

Because each horse does half the work:

$$W_{\text{each horse}} = \frac{1}{2}W_{\text{horses}} = \frac{1}{2}\mu_k mgd$$

Substitute numerical values and evaluate $W_{\text{each horse}}$:

$$W_{\text{each horse}} = \frac{1}{2}(0.25)(250 \text{ kg})(9.81 \text{ m/s}^2)(25 \text{ m}) = 7.664 \text{ kJ} = \boxed{7.7 \text{ kJ}}$$

(b) Express the work done by each horse as a function of the tension in each cable:

$$W_{\text{each horse}} = \vec{T} \cdot \vec{d} = Td \cos \theta$$

Solving for T yields:

$$T = \frac{W_{\text{each horse}}}{d \cos \theta}$$

Substitute numerical values and evaluate T :

$$T = \frac{7.664 \text{ kJ}}{(25 \text{ m})\cos 15^\circ} = \boxed{0.32 \text{ kN}}$$

