1 An overview of the kinetic energy cycle in the atmosphere

Atmospheric fluid motions may be divided into two broad classes, both of which owe their existence to the uneven distribution of diabatic heating in the atmosphere:

1. Motions driven either directly or indirectly by horizontal heating gradients in a stably stratified atmosphere account for more than 98% of the atmospheric kinetic energy. Nearly all this kinetic energy is associated with the synoptic- and planetary-scale horizontal wind field, which has a globally averaged root mean square velocity of about 12-15 m s-1.

2. Motions driven by convective instability account for the remainder of the atmospheric kinetic energy. Convection is continually breaking out within discrete regions of the atmosphere as a consequence of the vertical gradient of diabatic heating.

The resulting motions have space scales ranging from about 30 km in the largest thunderstorms down to less than 1 mm in microscale motions within the surface layer. Despite their small contribution to the atmospheric kinetic energy, convectively driven motions play an important role in the upward transport of latent and sensible heat.

The term general circulation is used by some meteorologists to denote the totality of atmospheric fluid motions, while others in the field use the term in a more restrictive sense, to denote motions described under (a).

Defined either way, the general circulation can be viewed in the context of a kinetic energy cycle in which atmospheric fluid motions continually draw on the reservoir of potential energy inherent in the spatial distribution of atmospheric mass in order to maintain themselves against frictional dissipation, which is continually transforming the kinetic energy of fluid motions into the internal energy of random molecular motions. In the presence of this continual drain, the potential energy of the mass field is maintained by the spatial gradients of diabatic heating which, in a statistical sense, are always acting to lift the center of gravity of the atmosphere.

From a dynamical point of view, large-scale horizontal motions owe their existence to the pressure gradient force which drives a slow horizontal flow across the isobars from higher toward lower pressure. In the presence of the earth's rotation this flow across the isobars induces a circulation parallel to the isobars whose speed tends toward a state of geostrophic balance[1] with the horizontal pressure gradient (and thermal wind balance with the temperature gradient). From an energetic point of view, the same cross-isobar flow, together with its attendant vertical motions, is responsible for the conversion from potential to kinetic energy.

---

[1] Gradient wind balance when the flow is strongly curved.

In lower latitudes, most of the atmospheric kinetic energy is contained in quasi-steady, thermally driven circulations which are directly related to the geographical distribution of sources and sinks of heat; as a result, the observed weather over most of the tropics varies relatively little from day to day (apart from diurnal variations) at a fixed location, although it may vary greatly from one location to another. These thermally driven circulations include the seasonally varying monsoons, which are the atmospheric response to land-sea heating contrasts, and a large-scale meridional overturning over the mid-Atlantic and Pacific Oceans which gives rise to the intertropical convergence zone, a narrow east-west band of heavy cloudiness and precipitation.

In middle and high latitudes much of the kinetic energy is associated with moving disturbances called baroclinic waves, which develop spontaneously within zones of strong horizontal temperature gradients. Most of the significant day-to-day weather changes at these latitudes can be attributed to the passage of these systems with their attendant mesoscale frontal zones.

Planetary- and synoptic-scale atmospheric disturbances are subject to frictional dissipation, which causes them to gradually lose their kinetic energy. The energy is not transferred directly from the large-scale motions into random molecular motions. Through processes such as shear instability, and convection, and mechanically driven turbulence in the presence of vertical wind shear, it is siphoned off by small-scale fluid motions which interact among themselves to transfer energy to smaller and smaller scales and ultimately down to the random molecular motions.

Roughly half the frictional dissipation takes place within the lowest kilometer of the atmosphere, as a result of turbulent motions generated mechanically by flow over irregularities in the underlying surface. The other half takes place higher in the atmosphere in discrete patches where small-scale disturbances are generated as a result of convection or shear instability of the vertical wind profile.

In a gross sense the atmosphere may be viewed as a vast but inefficient heat engine to which heat is added at a high temperature and removed at a somewhat lower temperature. The mechanical output of the heat engine is the supply of kinetic energy required to maintain the general circulation against frictional dissipation. Since the atmosphere receives heat at a higher effective temperature than it radiates it away to space, it may be viewed as working to increase the entropy of the universe.

The physical processes responsible for the generation and maintenance of large-scale atmospheric motions can be demonstrated by means of two simple laboratory analogs.

1.1   A "before and after" analog

Figure 1.1a shows a tank filled with equal volumes of two homogeneous immiscible liquids of differing densities, placed side by side and separated by a movable partition. The shaded liquid on the right is more dense than the unshaded liquid. In (b) the partition has been removed and fluid motion has developed. We

will not be concerned about the details of the motion, but only the final state after friction has brought the liquids to rest. In the new equilibrium configuration shown in (c) the heavy fluid occupies the bottom half of the tank.

Now let us consider this sequence of events from the point of view of the energetics. In the initial configuration, the center of gravity of the fluid, denoted by the dot, is exactly halfway between the top and bottom. After the partition is removed, the center of gravity drops as the denser liquid slides under the lighter one. Through the sinking of denser liquid and the rising of lighter liquid [indicated in (b)] potential energy is converted into the kinetic energy of fluid motions. Frictional dissipation eventually converts all the fluid motions to random molecular motions so that, in the final state (c), the only evidence of the conversion that took place is the drop in the center of gravity of the system and a very slight increase in the temperatures (or internal energy) of the liquids. The energy cycle is summarized in Fig. 1.2. Note that only a small fraction of the potential energy inherent in the initial state is available for conversion to kinetic energy, since no matter what kind of motions develop the center of gravity cannot possibly drop below the level shown in (c).

-

Questions:

1.1 In the experiment described above the depth of the tank is 1 m and two fluids have specific gravities of 1.1 and 0.9, and the same specific heat of 4 x 103 J kg -1 deg-1. What is the maximum possible root mean square (rms) velocity of fluid motion that can be realized in the experiment. How much does the temperature increase as a result of frictional dissipation? (Assume that the rms velocity and the temperature rise are the same for the two fluids.)

1.2 What kinds of processes contribute to the irreversible mixing of denser and lighter fluid in the atmosphere?

1.3 By how much would the center of mass of the atmosphere have to drop to release enough potential energy to account the observed kinetic energy of the general circulation? (For the kinetic energy, assume that the root-mean squared velocity associated with fluid motions is 15 m s^{-1}.)

1.2 A "steady-state" analog

In the real atmosphere, potential energy is constantly being replenished by diabatic heating so that there is a continuous flow of energy through the cycle indicated in Fig. 1.2. The following "steady-state" laboratory analog will help to illustrate this situation. Figure1.3 shows a tank full of liquid, which has internal heat sources along the bottom and left wall and matching heat sinks along the top and right wall. The liquid expands with increasing temperature.

The gradient of heating drives a slow, clockwise circulation cell, as represented in the figure. As a parcel of fluid is carried around the tank in this cell it grows warmer as it passes along the bottom of the tank and as it ascends along the left wall. Then it grows cooler as it moves across the top of the tank and down along the right wall to complete the circuit. In order to be consistent with these temperature changes, the isotherms must slope from lower left to upper

right, as indicated by the dashed lines in the figure. Since temperature increases with height, the fluid in the tank is stably stratified.

In applying the above arguments to the atmosphere, the compressibility of air must be taken into account by considering changes in potential temperature (rather than temperature) as an air parcel moves around the circulation cell. The isotherms in the figure are thus indicative of isentropes in the atmosphere, where an increase of potential temperature with height is the criterion for stable stratification.

At any given level in the tank, lighter fluid is rising along the left-hand side of the tank and an equal volume of heavier fluid is sinking along the right-hand side. This vertical exchange of equal volumes of fluid with different densities produces a net downward flux of mass which should tend to lower the center of gravity of the fluid, just as in the previous analog, converting potential energy to kinetic energy. In this steady-state model, the conversion proceeds at exactly the same rate as the kinetic energy of the fluid motions is being destroyed by frictional dissipation. The lowering of the center of gravity of the fluid is opposed by diabatic heating which is always warming and expanding the fluid near the bottom of the tank, and cooling and compressing the fluid near the top. In order to accommodate this expansion and compression, a very small mean upward motion is required at intermediate levels. This mean upward mass flux exactly cancels the downward mass flux due to the circulation cell so that the center of gravity of the fluid remains at a constant level.

In the laboratory analog, the heat source and sink adjacent to the side walls of the tank represent the horizontal gradients of diabatic heating in the earth's atmosphere. The heat source near the bottom of the tank represents the combined effects of the absorption of solar radiation, the exchange of infrared radiation with the earth's surface, and, most important, the input of sensible heat associated with convectively driven motions in the mixed layer and the release of latent heat in clouds. Collectively, these processes result in a strong input of energy into the lower troposphere. The atmospheric heat sources and sinks overlap in the vertical, but the 'center of mass' of the sinks is slightly higher in the troposphere than that of the sources.

An idealized example of such a circulation cell in the atmosphere is shown in Fig. 1.4. The rising branch corresponds to the regions of deep convection over the tropical monsoons and the intertropical convergence zone, where air parcels may ascend from the troposphere to the upper troposphere in a matter of an hour. Vast quantities of latent heat are released as the water vapor in these updrafts condenses into liquid water and ice. The air detrained from the anvils of these clouds subsides slowly, over a period of a week or so, cooling by the emission of infrared radiation, which nearly always exceeds the heating due to the absorption of solar and infrared radiation. Hence, these subsiding air parcels cross the isentropes toward lower values of potential temperature. When they reach the ~1.5 km level are entrained into the planetary boundary layer (PBL), where the air is being heated and moistened by the fluxes through the underlyng surface. Hence, air parcels cross the isentropes toward higher values of potential temperature as they flow toward the tropical rain belts.in the lower

branch of the circulation cell. Shallow convection has the effect of vertically mixing the heat and moisture added at the lower surface through the depth of the PBL. A discrete jump in moist static energy is observed at the top of the PBL.

The importance of atmospheric water vapor in these large-scale thermally driven circulations is worth emphasizing. Latent heat added to the atmosphere in the lower branch of the cell is converted into sensible heat when the water vapor condenses in the rising branch. This additional source of heating serves to enhance the horizontal heating contrasts that would have existed in the absence of the cell. As a result, thermally driven circulations tend to be much stronger than they would be in an atmosphere without water vapor.

The maintenance of large-scale thermally driven circulations requires both horizontal and vertical gradients of diabatic heating. In the absence of horizontal heating contrasts, the heat source / sink at the bottom and top of the tank destroy the stable stratification and initiate convectively driven motions on a scale much smaller than the dimensions of the tank. In these convection cells, bubbles of warm light fluid rise, and are replaced by cooler, denser fluid from above. In the equilibrium situation the downward mass flux in the convection cells is just enough to maintain the center of gravity of the fluid at a constant level.

-

Questions:

1.4 Consider how the slopes of the isotherms (or isentropes) in Fig. 1.3 depends upon the relative magnitudes of the horizontal and vertical heating gradients

1.5 How would the circulation cell in Fig. 1.3 be affected if the vertical heating gradients were turned off? How could a steady state ultimately be achieved? Hint: as an analogue, consider a room with a radiator on one side and a cold window on the other side.

1.3 Kinetic energy generation in a hydrostatic fluid

In applying the two above examples to the atmosphere, one important distinction needs to be kept in mind. In experiments of laboratory size proportions, the horizontal and vertical components of the motion are likely to be of the same order of magnitude so that an appreciable fraction of the kinetic energy associated with the fluid motions resides in the vertical component. This energy is converted from potential energy when imbalances between the forces in the vertical equation of motion propel buoyant parcels of fluid upward and denser parcels downward. In effect, gravity does work on the fluid, just as it does work on a falling object. In a hydrostatic fluid there is no vertical equation of motion and no kinetic energy associated with the vertical motion component. Kinetic energy is realized when the horizontal component of the pressure gradient force does work on parcels of fluid as they move across the isobars from higher toward lower pressure on level surfaces. The relation between this cross-isobar flow and the release of potential energy will become fully apparent when we consider the

governing equations. However, one can get some intuitive feel for it from Fig. 1.5, which shows an the slope of the pressure surfaces that would develop in association with the steady state circulation considered in the previous figure. In agreement with the hypsometric equation, the vertical spacing between pressure surfaces (i.e., the thickness) is higher on the warm side of the tank than on the cold side. It follows that the horizontal flow in the clockwise circulation cell must be primarily directed down the horizontal pressure gradient, as required for the generation of kinetic energy. Hence, the rising of warmer fluid and sinking of colder fluid implies cross-isobar horizontal flow toward lower pressure and vice versa.

Question:

1.6 For a homogeneous (constant density) fluid in a tank with a flat bottom, vertical walls, and no top:

(a) show that the average potential energy per unit area is given by $\frac{1}{2}\overline{\rho g z^2}$, where z is the height of the free surface of the fluid and the overbar denotes an average over the area of the tank.

(b)

Show that the average potential energy per unit area that is available for conversion into

kinetic energy is given by $\frac{1}{2}\overline{\rho g z'^2}$, where $z' = z - \overline{z}$. {Hint: Begin by substituting in

the expression for potential energy per unit area. Note that $\overline{z z'} = \overline{z} \overline{z'} = 0$.}

(c)

Show that the average rate of conversion from potential to kinetic energy is given by

$-\rho g w z$, where w is the vertical velocity of the free surface. Give a physical interpretation

of this result in terms of flow across the isobars.

2   Lorenz's concept of available potential energy

The foundation for quantitative studies of the atmospheric kinetic energy cycle is the formalism introduced by Lorenz (1955), which was inspired by much earlier work by Margules (1903) on the energetics of storms. Central to this formalism is the concept of available potential energy. In Lorenz's words:

> The strengths of the cyclones, anticyclones, and other systems which form the weather pattern are often measured in terms of the kinetic energy that they possess. Intensifying and weakening systems are then regarded as those which are gaining or losing kinetic energy. When such gains or losses occur, the source or sink of kinetic energy is a matter of importance. Under adiabatic motion, the total energy of the whole atmosphere would remain constant. The only sources

or sinks for the kinetic energy of the whole atmosphere would then be potential energy and internal energy.

In general the motion of the atmosphere is not adiabatic. The only nonadiabatic process which directly alters kinetic energy is friction, which ordinarily generates internal energy while it destroys kinetic energy.... The remaining nonadiabatic processes, including the release of latent energy, alter only the internal energy directly. Hence the only sources for the kinetic energy of the whole atmosphere are atmospheric potential energy and internal energy, while the environment may also act as a sink.

It is easily shown (cf. Haurwitz, 1941) that the potential and internal energies within a column extending to the top of the atmosphere bear a constant ratio to each other, to the extent that hydrostatic equilibrium prevails. Hence, net gains of kinetic energy occur in general at the expense of both potential and internal energy, in this same ratio. It is therefore convenient to treat potential and internal energy as if they were a single form of energy. The sum of the potential and internal energy has been called total potential energy by Margules (1903).

Evidently the total potential energy is not a good measure of the amount of energy available for conversion into kinetic energy under adiabatic flow. Some simple cases will serve to illustrate this point. Consider first an atmosphere whose density stratification is everywhere horizontal. In this case, although total potential energy is plentiful, none at all is available for conversion into kinetic energy. Next suppose that a horizontally stratified atmosphere becomes heated in a restricted region. This heating adds total potential energy to the system, and also disturbs the stratification, thus creating horizontal pressure forces which may convert total potential energy into kinetic energy. But next suppose that a horizontally stratified atmosphere becomes cooled rather than heated. The cooling removes total potential energy into kinetic energy. Evidently removal of energy can be as effective as addition of energy in making more energy available.

We therefore desire a quantity which measures the energy available for conversion into kinetic energy under adiabatic flow. A quantity of this sort was discussed by Margules (1903) in his famous paper concerning the energy of storms. Margules considered a closed system possessing a certain distribution of mass. Under adiabatic flow, the mass may become redistributed, with an accompanying change in total potential energy, and an equal and opposite change in kinetic energy. If the stratification becomes horizontal and statically stable, the total potential energy reaches its minimum possible value, and the gain of kinetic energy thus reaches its maximum. This maximum gain of kinetic energy equals the maximum amount of total potential energy available for conversion into kinetic energy under any adiabatic redistribution of mass, and as such may be called the available potential energy.

Available potential energy in this sense can be defined only for a fixed mass of atmosphere which becomes redistributed within a fixed region. The storms

with which Margules was primarily concerned do not consist of fixed masses within fixed regions, nor do any other systems having the approximate size of storms. It is perhaps for this reason that available potential energy has not become a more familiar quantity. It is in considering the general circulation that we deal with a fixed mass within a fixed region – the whole atmosphere. It is thus possible to define the available potential energy of the whole atmosphere as the difference between the total potential energy of the whole atmosphere and the total potential energy which would exist if the mass were redistributed under conservation of potential temperature to yield a horizontal stable stratification.

The available potential energy so defined possesses these important properties:

(1) The sum of the available potential energy and the kinetic energy is conserved under adiabatic flow.

(2) The available potential energy is completely determined by the distribution of mass.

(3) The available potential energy is zero if the stratification is horizontal and statically stable.

It seems fairly obvious that the available potential energy so defined is the only quantity possessing these properties, although a rigorous proof would be somewhat involved. Moreover, it possesses the further property:

(4) The available potential energy is positive if the stratification is not both horizontal and statically stable.

It follows from property (1) that available potential energy is the only source for kinetic energy. On the other hand, it is not the only sink. When friction destroys kinetic energy it creates internal energy, but in doing so it increases the minimum total potential energy as well as the existing total potential energy. Thus the loss of kinetic energy exceeds the gain of available potential energy.

There is no assurance in any individual case that all the available potential energy will be converted into kinetic energy. For example, if the flow is purely zonal, and the mass and momentum distributions are in dynamically stable equilibrium, no kinetic energy at all can be realized. It might seem desirable to redefine available potential energy, so that, in particular, it will be zero in the above example. But the available potential energy so defined would depend upon both the mass and momentum distributions. If it is desired to define available potential energy as a quantity determined by the mass distribution, the definition already introduced must be retained.

2.1 A simplified derivation

Consider an atmosphere at rest with a globally averaged temperature profile $T(p)$ or $T(z)$, but with undulations in the potential temperature surfaces (or isentropes) as sketched in Fig. 1.6 so that locally

$$T(x, y, z) = T(z) + T"(x, y, z) \qquad (1)$$

If the isentropes are allowed to flatten out by means of an adiabatic readjustment of mass, potential energy will be released as cold, heavy air sinks and warm, light

air rises in the process, so that the center of gravity of the atmosphere drops by a small amount. Alternatively, we may view this release of potential energy as work done on the atmosphere by gravity. This so-called available potential energy of the atmosphere, divided by the area of the atmosphere $(4\pi R_E^2)$, will be denoted by the symbol $\overline{A}$. We will now derive an expression for $\overline{A}$.

The downward directed force that exists on any air parcel, of unit mass, by virtue of the undulations in the isentropes is given by

$$\frac{\rho"}{\overline{\rho}}g = \frac{-T"}{\overline{T}}g = \frac{g(\Gamma_d - \Gamma)}{\overline{T}}z" \tag{2}$$

where $z"$ is the vertical displacement of the air parcel from the mean level of its isentrope. (For a more thorough derivation, see Wallace and Hobbs 2nd ed., Exercise 3.11, p. 88). The work done by gravity (or the potential energy released) in allowing this parcel to move down (or up) to the mean level of its isentrope is given by

$$W = \int_{z"}^{0} F dz = \frac{g(\Gamma_d - \Gamma)}{\overline{T}} \int_{z"}^{0} z" dz = \frac{g(\Gamma_d - \Gamma)}{\overline{T}} \frac{z"^2}{2} \tag{3}$$

where we have assumed that the undulations are small enough so that T and $(\Gamma_d - \Gamma)$ can be taken outside the integral sign. Since $z" = T"/(\Gamma_d - \Gamma)$, we can write

$$W = \frac{g}{2\overline{T}} \frac{T"^2}{(\Gamma_d - \Gamma)} \tag{4}$$

To obtain an expression for available potential energy, we simply integrate W over the entire mass of the atmosphere and divide it by the area of the earth's surface. These operations are equivalent to integrating (1.4) over pressure divided by g, and averaging it over the area of the earth: hence

$$\overline{A} = \frac{1}{2} \int_0^{p_0} \frac{1}{\overline{\overline{T}}} \frac{\overline{T"}^2}{(\Gamma_d - \Gamma)} dp \tag{5}$$

where p_0 is the globally averaged value of pressure at the earth's surface, and $\overline{T}$ and $\overline{\Gamma}$, the environmental temperature and lapse rate now refer to globally averaged values at pressure level p.

In the above derivation we have made three kinds of approximations:

(1) We have done some linearizing in getting to equations. (1.2) and (1.3). This approximation is based on the assumption that the isentropes in the earth's atmosphere are relatively flat.

(2) In the final step we have treated the lapse-rate $\overline{\Gamma}$ as if it were a function of pressure only. Strictly speaking, we should have written

$$\overline{A} = \frac{1}{2} \int_0^{p_0} \frac{1}{\overline{\overline{T}}} \overline{\frac{T"^2}{(\Gamma_d - \Gamma)}} dp \tag{6}$$

In effect, the approximation in (1.5) amounts to a further linearization. Since $\overline{T"}^2$ is everywhere positive and $(\Gamma_d - \Gamma)$ is always positive and does not vary

widely in the horizontal (except within a thin layer near the tropopause) this approximation should be acceptable, provided that one doesn't choose to worry too much about the PBL, where the expression locally "blows up"in neutrally stratified regions.

(3) By terminating our vertical integration at the pressure surface p_0 rather than at a potential temperature surface, we have, for the sake of convenience, chosen to ignore the fact that isentropes intersect the earth's surface both because of topography and, more importantly, because of the substantial horizontal temperature gradients at the earth's surface. This complication is dealt with more explicitly in Lorenz's derivation.

(4) Available potential energy, as defined in (1.5) would vanish if all the isentropes coincided with surfaces of constant pressure. If the pressure surfaces were not flat, such an atmosphere could still contain a reservoir of energy that could be converted into kinetic energy through horizontal motions across the isobars. Available potential energy of the kind defined by Lorenz could exist by virtue of the horizontal density gradients in the fluid associated with "internal waves". Not included in Lorenz's formulation is the potential energy associated with "external waves" which involve undulations in the free surface.

Following Lorenz, if we assume that T" is the earth's atmosphere is on the order of 15K, T is about 250 K and $\overline{\Gamma} \sim \frac{2}{3}\Gamma_d \sim 6.5$ K km^{-1}, then $\overline{A}$ is given by

$$\frac{1}{2} \times \frac{15^2 \text{ K}^2}{250 \text{ K} \times 3.5 \text{ K km}^{-1}} \times 10^3 \text{ m km}^{-1} \times 10^5 \text{ N m}^{-2} = 10^7 \text{ K m}^{\text{-2}}$$

It is of interest to compare this figure with the average kinetic energy per unit vertical column of the atmosphere based on an r.m.s. velocity of 15 m s^{-1}:

$$\frac{1}{2} \times 15 \text{ m}^2\text{s}^{-2} \times \frac{10^5 \text{ N m}^{-2}}{9.8 \text{ m s}^{-2}}$$

Hence the existing reservoir of available potential energy is about an order of magnitude larger than typical values of the global kinetic energy.

It is important to note that A is by nature a globally integrated quantity that can't be separated neatly into the contributions that come from various geographical regions (e.g., tropics vs. extratropics) as kinetic energy can be. For example, suppose that the temperature on a certain pressure level is a uniform value T_1 in the tropical half of the atmosphere and another uniform value T2 in the extratropics. Then the variance of temperature about the global mean $(T_1 + T_2)/2$ is $(T_1 - T_2)^2/4$. However the variance of the temperature within these two uniform regions about their respective means T_1 and T_2 is identically equal to zero. In other words, the variance of the whole is greater than the sum of the variances of the parts. It follows that A has a unique and unambiguous definition only if it is based on a global domain (or a hemispheric domain, where one makes the assumption of equatorial symmetry).

It is possible to partition A into the contributions from various layers or from various harmonic components. For example, one may speak of the available potential energy of zonal wavenumber 1 in the 100-10 mb layer. We will

also have occasion to speak of the zonal symmetry and "eddy" components of the available potential energy. In certain situations, one may also speak of the available potential energy associated with some particular dynamical entity which is confined to a limited region of the atmosphere (e.g., African waves, the quasi-biennial oscillation, a hurricane).

Because of the constraint of thermal wind equilibrium, the only way that all the available potential energy inherent in the mass distribution can be realized is to generate disturbances that are purely barotropic. Since atmospheric disturbances always have some degree of baroclinity, it follows that A never really approaches the theoretical limit of zero; that is to say, not all the available potential energy is really available.

-

Questions:

1.7 Show that the sum of the potential plus internal energy of a unit column of theatmosphere is given by

$$P + I = \frac{c_p}{g} \int_0^{p_0} T\,dp \tag{7}$$

Hint: To derive this expression, you may wish to make use of the identity

$$\int_0^\infty p\,dz = \int_0^{p_0} z\,dp$$

which is valid if sea level is assumed to coincide with a constant pressure surface.

1.8 Are the assumptions that underlie the definition of available potential energy more valid in the troposphere or in the stratosphere?

1.9 How does the ratio of $\overline{K}/\overline{A}$ differ between troposphere and stratosphere? How does it vary with the horizontal and vertical scales of atmospheric disturbances?

1.10 Can you think of any large-scale atmospheric phenomenon for which $K \gg A$?

1.11 Suppose that the observed rate of kinetic energy dissipation in the earth's atmosphere is 10^5 J m^2 d^{-1}. On the basis of the estimates of $\overline{K}$ and A given in this section, estimate the characteristic decay time of atmospheric motions if sources of K were turned off. (Assume an exponential decay with a constant ratio of K/A.)

1.12 Define the available potential energy of a stratified Boussinesq fluid enclosed within a container with a flat bottom, vertical walls, and no top. Be sure to take the free surface into account. Assume that fluid parcels conserve density.

1.13 Derive an expression for the true available potential energy of a purely barotropic
atmosphere.

3   The Budget of Available Potential Energy

It is evident from (1.5) that available potential energy can be changed by any process that is capable of changing:

(a) the variance of temperature on pressure surfaces;

(b) the mean static stability of the atmosphere;

(c) the mean temperature of the atmosphere.

We regard (c) as unimportant for the purposes of this discussion. The observed distribution of diabatic heating in the earth's atmosphere tends to increase the variance of temperature on pressure surfaces, since in the long term mean diabatic heating is acting to increase the equator to pole temperature gradient, particularly in the winter hemisphere. Diabatic heating also tends to destabilize the atmosphere by heating it from below (by sensible heat flux from the earth's surface and latent heat release) and cooling at upper tropospheric levels (by infrared emission to space). Hence the observed distribution of diabatic heating functions as a source of available potential energy.

In a similar manner, atmospheric motions draw on the reservoir of available potential energy in two different ways when warm (light) air rises and cold (dense) air sinks:

(a) horizontal temperature contrasts, which determine the variance of temperature on pressure surfaces, tend to be reduced as the rising warm air undergoes adiabatic cooling as it expands while the sinking cold air undergoes adiabatic warming as it is compressed.

(b) the upward transport of heat by these same vertical motions increases the static stability.

Hence, if we are interested in doing a really comprehensive diagnosis of the budget of available potential energy, we have to consider changes in variance $T^{''2}$ and in static stability $(\Gamma - \Gamma_d)$. There have been a number of such studies by Dutton and Johnson (1967) and collaborators over the years, based on isentropic analysis techniques.

In most general circulation studies a further simplification is employed: the static stability is treated as if it doesn't vary with time; processes which contribute to time variations of static stability are assumed to be in a state of balance, so that there is no need to be concerned about them. With this simplification, the task of keeping track of the budget of available potential energy is essentially reduced to the task of accounting for changes in the variance of temperature on pressure surfaces. When we consider the general circulation from a global perspective this simplification is justifiable because globally averaged static stability does change very little with time.

The nature of this simplification can perhaps be made a little clearer by the use of an analogue from the realm of economics. Let us think of available potential energy as the analogue of the wealth in the federal treasury and kinetic energy as the analogue of the material goods that could be purchased with that wealth. The wealth in the treasury is equal to the amount of currency in the treasury (the analogue of $\overline{T^{''2}}$) divided by the amount of currency required to buy material goods, or the "cost of living index" (the analogue of static

stability). If the federal treasury were to rapidly accumulate or dispose of large amounts of currency the value of that currency might be affected and with it, the national wealth. For example, a wild spending spree would not only decrease the amount of currency remaining in the federal coffers, but it would also deflate the value of that currency by inflating the "cost of living index". [Likewise, a large uncompensated conversion from $A \rightarrow K$ would reduce $\overline{T''^2}$ and it would also deflate $(\Gamma - \Gamma_d\ )$.] So long as the value of the currency is stable, it is sufficient for budgeting purposes to keep track only of the amount of currency held in reserve. Fortunately, the static stability of the earth's atmosphere does not appear to be subject to "inflation", or large short-term fluctuations, and thus, we are justified in regarding $\overline{T''^2}$ as if it alone is a true measure of $\overline{A}$.

In recognition of this simplification let us rewrite the expression for available potential energy in the form

$$\overline{A} = \frac{1}{2}\frac{R}{g}\int_0^{p_0}\frac{\overline{T''^2}}{s}d\ln p \tag{8}$$

where

$$s(p) \equiv \frac{\overline{RT}}{pg}(\overline{\Gamma} - \Gamma_d) \tag{9}$$

-

Questions:

1.14 Contrast large scale motions in a stably stratified atmosphere versus convection with respect to the way in which they draw upon the reservoir of available potential energy, as defined in (1.7).

1.15 Is the assumption made in this section more easily justified for the troposphere or for the stratosphere? Is it more easily justified for a global study or for the study of an individual storm?

4   Generation and Release of Available Potential Energy

Consider the First Law of Thermodynamics written in the form

$$\frac{dT}{dt} = -\mathbf{V}\cdot\nabla T + s\omega + Q \tag{10}$$

where $s = s(p)$ only. If we average (1.9) over the entire earth's surface on a given pressure level, we obtain

$$\frac{d\overline{T}}{dt} = Q \tag{11}$$

Horizontal advection can move the isotherms around on a pressure surface, but it can't act to systematically raise or lower the mean temperature of the layer. Furthermore, it is clear that $\blacksquare = 0$ on any pressure level, since there are no

13

appreciable sources or sinks of atmospheric mass. (Here we ignore the small, systematic upward mass flux of water vapor.) Subtracting (1.10) from (1.9) we obtain

$$T^{''} \frac{dT}{dt} = -T^{''} \mathbf{V} \cdot \nabla T + s\omega T^{''} + Q^{''} T^{''} \tag{12}$$

Finally, we average over the surface of the earth to obtain

$$\frac{1}{2} \frac{d}{dt} \overline{T'^2} = s\overline{\omega T} + \overline{Q^{''} T^{''}} \tag{13}$$

where we have thrown out the horizontal advection term because horizontal advection can only distort the shapes of the isotherms; it can't change the area enclosed by any isotherm, and therefore it can in no way change the variance or any of the higher moments of temperature. We have also made use of the fact that $\omega T^{''} = \omega T$, since the continuity of mass requires that $\omega = 0$ on pressure surfaces. The first term on the right hand side is associated with the conversion from available potential energy to kinetic energy. On average in the earth's atmosphere warm air rises and cold air sinks, so there is a negative correlation between ω and T on pressure surfaces, and since $\omega = 0$, this effect contributes to a decrease in the variance of temperature; that is, a release of available potential energy. The reservoir of available potential energy is maintained by the diabatic heating term $Q^{''} T^{''}$, which involves the spatial correlation between diabatic heating and temperature. At tropospheric levels Q and T tend to be positively correlated since on average, the tropical atmosphere receives more heat than it radiates to space and the polar atmosphere radiates more away than it receives. [For a further discussion, see Wallace and Hobbs 2nd ed.,§4.6. 10.1.1]. There is an additional positive correlation between Q and T, even at a given latitude, because of the tendency for precipitation and latent heat release to occur preferentially in rising warm air masses; the most spectacular example is the case of hurricanes,but from a global perspective the positive correlation between Q and T due to latent heat release in monsoons is far more significant.

There are limited regions in the atmosphere in which Q and T are negatively correlated so that diabatic heating is acting to destroy the existing temperature gradients. For example desert regions undergo strong radiative cooling in comparison to their surroundings, as reflected in the net radiation at the top of the atmosphere. The equatorial tropopause is far below its radiative equilibrium temperature, while in middle latitudes temperatures at these levels are above radiative equilibrium. Hence in the lower stratosphere, diabatic heating functions as a sink of available potential energy. The same is true of the mesopause level, where the summer pole is cold and the winter pole is warm. From a thermodynamic point of view, these localized regions of the atmosphere function as refrigerators rather than heat engines, with temperature gradients being maintained by an influx of mechanical energy (work) supplied by other regions of the atmosphere.

Combining (1.7) and (1.11) we can express the conversion from available potential energy to kinetic energy in the form

$$\frac{d\overline{A}}{dt} = -C + G \tag{14}$$

where

$$C = -\frac{R}{g} \int_0^{p_0} \overline{\omega T} d\ln p \tag{15}$$

We have introduced a minus sign so that the conversion $A \to K$, which corresponds to the usual situation in the Earth's atmosphere, will correspond to a positive value of C. Substituting for T from the equation of state $p\alpha = RT$, we can write

$$C = -\frac{1}{g} \int_0^{p_0} \overline{\omega \alpha} dp \tag{16}$$

which can be interpreted as the rising of lighter air and the sinking of denser air which results in a net downward flux of mass and a lowering of the center of gravity of the atmosphere. If we substitute from the hydrostatic equation for dp, the conversion can be written

$$C = - \int_0^{\infty} \overline{\omega} dz \tag{17}$$

which tells us that if $A \to K$ then in an average over the volume of the atmosphere air parcels are undergoing a decrease in pressure.[2] The same is true of the working fluid in any heat engine,.as implied by the clockwise sense of the loop in a plot of pressure versus volume.

Returning to (1.14) we can substitute for α from the hydrostatic equation and integrate by parts so that the conversion takes the form

$$C = \frac{1}{g} \int_0^{p_0} \overline{\omega \frac{\partial \Phi}{\partial p}} dp = \frac{1}{g} \int_0^{p_0} \overline{\omega \Phi} dp - -\frac{1}{g} \int_0^{p_0} \overline{\Phi \frac{\partial \omega}{\partial p}} dp \tag{18}$$

The term $\omega \Phi / g$ represents the rate at which work is being done by (on) the part of the atmosphere below the pressure level in question on (by) the part of the atmosphere above that level by vertical motions. It can be viewed as the mechanical stirring of one part of the atmosphere by another part. Note that $\omega \Phi / g$ has units of Force $\times$ Velocity / Area or, alternatively Work / (Area $\times$ Time). If the air parcels on a pressure surface are being pushed up in high pressure regions and down in lows, then $\omega \Phi < 0$ and the layer of air below the pressure surface is doing work on the layer of air above it. The term g^{-1} $\partial/\partial p(\omega \Phi)$ represents the energy exported upward and downward from level p through work done by vertical motions. We will discuss this term further in the next section.

_____

[2] Note that this expression involves the global average of ω on constant height, rather than constant pressure surfaces. On constant height surfaces, $\partial p/\partial t$ must vanish in the global average, but not dp/dt.

Now it is clear that $\omega\Phi = 0$ at the top of the atmosphere, since $\omega \to 0$ as $p \to 0$, and the atmosphere cannot be doing mechanical work on empty space. The spatial correlation between ω and Φ does not vanish at the earth's surface or on the 1000 hPa level. However, it is true that at the earth's surface tends to be about an order of magnitude smaller than at mid-tropospheric levels. Were it not for the existence of large mountain ranges like the Himalayas and Rockies the work done on the atmosphere by the earth's surface or vice versa would play only a minor role in the atmospheric kinetic energy budget. For the present, let us treat the surface of the earth as if it were flat, so that the lower boundary corresponds to a surface of constant geopotential Φ, and let us also assume that the 1000 hPa pressure surface is also rather flat and/or ω is small there so that $\omega\Phi$ is vanishingly small at the bottom boundary. With this assumption

$$\frac{1}{g}\int_0^{p_0} \overline{\omega\Phi}dp = \left[\overline{\omega\Phi}\right]_{p-p_0} - \left[\overline{\omega\Phi}\right]_{p-0} = 0 \tag{19}$$

so that

$$C = -\frac{1}{g}\int_0^{p_0} \overline{\Phi\frac{\partial\omega}{\partial p}}dp \tag{20}$$

At this point we begin to see the conversion C in the context of atmospheric dynamics (as opposed to thermodynamics). The term $\partial\omega/\partial p$ may be viewed as the vertical stretching of "vortex lines" associated with the absolute vorticity of the horizontal flow. Following air parcels in large-scale motions, the primary source/sink of vorticity is the stretching term

$$\frac{d\eta}{dt} = f\frac{\partial\omega}{\partial p}$$

where η is absolute vorticity. Since $\partial\omega/\partial p = 0$, we can go through an analogous set of operations to that which we performed in (1.9)-(1.12) to show that

$$\frac{1}{2}\frac{\partial}{\partial t}\overline{\eta''^2} = f\overline{\eta\frac{\partial\omega}{\partial p}}$$

that is to say, the variance of absolute vorticity on a pressure surface will be increased if vertical stretching is occurring in regions of high vorticity (i.e., "lows") and vertical squashing is occurring in regions of low vorticity (in highs). In other words, vertical stretching accelerates the circulation around lows and vertical squashing accelerates the circulation around highs, so that if $\Phi\partial\omega/\partial p < 0$, the geostrophic flow in the atmosphere will be speeded up. Next we substitute from the continuity equation to obtain

$$C = \frac{1}{g}\int_0^{p_0} \overline{\Phi\nabla\cdot\mathbf{V}}dp \tag{21}$$

Energy is being converted from A to K if Φ and $\nabla\cdot\mathbf{V}$ are positively correlated on pressure surfaces so that air diverges out of highs and converges into lows (as

16

is observed, for example, in the planetary boundary layer). It is impossible to draw a realistic picture of air diverging out of highs and converging into lows in the horizontal without having air flowing across the isobars from higher toward lower pressure. We can express C explicitly in terms of the cross isobar flow by integrating (1.19) by parts to obtain

$$C = \frac{1}{g} \int_0^{p_0} \overline{\nabla \cdot \mathbf{V}\mathbf{\Phi}} dp - \frac{1}{g} \int_0^{p_0} \overline{\mathbf{V} \cdot \nabla \mathbf{\Phi}} dp \qquad (22)$$

The term $\mathbf{V}\mathbf{\Phi}$ is the horizontal counterpart of the $\omega\Phi$ term discussed above. It too is a work term involving the horizontal flux of geopotential or, in (x, y, z) coordinates, it involves Pressure $\times$ Velocity or (Force/Area) $\times$ Velocity. It represents the horizontal flux of energy across a vertical "wall", such as a latitude circle, by mechanical stirring. Now since the atmosphere has no horizontal boundaries there is no external region for it do work on. Formally, from the divergence theorem

$$\int\int \nabla \cdot \mathbf{V}\mathbf{\Phi} dS = \oint V_n \Phi ds$$

where dS is an element of surface area and the horizontal integration is carried out over the entire surface of the earth, s is an element of arc length around the perimeter of S, and V_n represents the component of the horizontal velocity directed outward through this perimeter. For an integration over the entire earth's surface there is no perimeter and so the integral vanishes. Hence, (1.20) reduces to

$$C = -\frac{1}{g} \int_0^{p_0} \overline{\mathbf{V} \cdot \nabla \mathbf{\Phi}} dp \qquad (23)$$

and the conversion can be expressed in terms of the cross-isobar flow from higher toward lower pressure.

It is possible to derive (1.21) directly from the horizontal equation of motion, expressed in pressure coordinates:

$$\frac{d\mathbf{V}}{dt} = -\nabla\mathbf{\Phi} - f\mathbf{k} \times \mathbf{V} + \mathbf{F} \qquad (24)$$

where $\mathbf{F}$ is the frictional force per unit mass. Taking the dot product with $\mathbf{V}$, averaging over the area of the earth's surface and integrating over the vertical coordinate p/g, we obtain

$$\frac{\partial}{\partial t} \frac{1}{2g} \int_0^{p_0} \mathbf{V} \cdot \mathbf{V} dp = -\frac{1}{g} \int_0^{p_0} \overline{\mathbf{V} \cdot \nabla \mathbf{\Phi}} dp + \frac{1}{g} \int_0^{p_0} \overline{\mathbf{F} \cdot \mathbf{V}} dp \qquad (25)$$

where D, the frictional dissipation term is always positive as defined here, since and $\mathbf{V}$ tend to be in the opposite direction. Equation (1.12) can also be expressed in symbolic form

$$\frac{\partial A}{\partial t} = -C + G \qquad (26)$$

17

If the existing reservoirs of A and K are maintained over long periods of time, then C, G and D must all be equal to one another. We can express this equality in terms of the following "flow chart" of the kinetic energy cycle, which is really just an elaboration of Fig. 1.2.

Thermally direct circulations are those in which $A \rightarrow K$, so that warm, light air is rising, cold, dense air is sinking, and air is flowing across the isobars toward lower pressure; and thermally indirect circulations are those in which the opposite conditions prevail. In the average,motions in the atmosphere tend to be thermally direct, but there are local exceptions which correspond to those phenomena or regions of the atmosphere which behave as refrigerators. The work required to run these refrigerators is supplied by the $-\frac{\partial}{\partial p}\omega\Phi$ and or $-\nabla \cdot \nabla\Phi$ terms, which will be discussed in more detail in the next section.

Since G, C and D are equal in the long-term mean, it is possible to get a sense of how fast kinetic energy is generated and dissipated in the atmosphere by evaluating C in any one of its various forms. A particularly convenient form is (1.21), for which the absolute value of the integrand can be expressed in the form $f\mathbf{V}\mathbf{k} \times \mathbf{V}$, making use of the geostrophic equation.

Since and are of comparable magnitude, the absolute upper limit of this expression is

$$fV^2\frac{p_0}{g} \sim 10^{-4}\text{s}^{-1} \times \left(15 \text{ m s}^{-1}\right)^2 \times \frac{10^5 \text{ Pa}}{9.8 \text{ m s}^{-2}} \sim 200 \text{ W m}^{-2}$$

and this limit would be approached only if and were systematically oriented at right angles to one another. Since the cross-isobar flow tends to be smaller than the geostrophic wind by a factor of the Rossby Number (almost an order of magnitude), it follows that C cannot be more than 20-30 W m^{-2} and that value would be approached only if the cross-isobar flow were systematically toward lower (or higher) pressure. Within the planetary boundary layer, the cross-isobar flow tends to be systematically directed toward lower pressure, but in the free atmosphere there is a good deal of compensation between up- and down-gradient flow. Hence, C is not likely to be more than a few W m^{-2}. Estimates by Oort (1964) and many others since then place it in the range of 2-3 W m^{-2}, which amounts to ~1% of the.solar radiation incident on the top of the atmosphere reduced by the fraction reflected directly back to space. Hence, the thermal efficiency of the atmospheric heat engine is on the order of 1%. This low figure indicates that the effective temperatures of the the atmospheric heat source and heat sink cannot differ by more than a few degrees.

-

Questions:

1.16 If C as defined in (1.14) tends to be positive in the earth's atmosphere, why doesn't the center of gravity drop?

1.17 Why is it that $\omega = 0$ on pressure surfaces and not $\omega\alpha = 0$? Hint: Remember the

definition of ω and think of a pressure surface as a material surface.

1.18 If C as defined in (1.15) tends to be positive in the earth's atmosphere, what keeps the pressure of air parcels from continually dropping?

1.19 How would the treatment in this section have to be modified in order to make it applicable to convective scale motions?

1.20 Write an expression for C in terms of velocity $\mathbf{V}$ and the ageostrophic velocity $\mathbf{V}_a$ a in component form.

5   The Local Kinetic Energy Cycle

It should be emphasized that the equality between the release of available potential energy, as manifested in the $-\omega T$ and $-\omega\alpha$ terms, and the generation of kinetic energy, as manifested in the $-\ -\mathbf{V}\cdot\nabla\Phi$ term holds only for integrals over the entire mass of the atmosphere. Locally there may be large imbalances between these terms which are compensated by imports or exports of energy through the $-\frac{\partial}{\partial p}\omega\Phi$ or $-\boldsymbol{\nabla}\cdot\nabla\boldsymbol{\Phi}$ terms. For example, most of the release of potential energy in the earth's atmosphere takes place in the middle troposphere where the vertical motions are largest, whereas most of the kinetic energy generation occurs in the planetary boundary layer and in the vicinity of the jetstream level, where the strongest frictional dissipation is taking place. The middle troposphere is continually doing work on the upper and lower troposphere, through the $\omega\Phi$ term to complete the balance. Let us consider this situation in more detail.

Combining (1.13), (1.14), (1.16), (1.18), (1.19), and (1.20) it is (more easily than it sounds) shown that

$$-\overline{\omega\alpha} = \frac{\partial}{\partial p}\overline{\omega\Phi} - \overline{\mathbf{V}\cdot\nabla\boldsymbol{\Phi}} \tag{27}$$

Hence, all the available potential energy released at a particular level can be accounted for either in terms of export, through the work term, or generation of kinetic energy at that same level. Fig. 1.7 shows idealized vertical profiles of the three terms in (1.26) in the earth's atmosphere. We know that $-\overline{\omega\Phi} < 0$ at the top of the planetary boundary layer for the following reasons:

(1) high pressure regions at the earth's surface tend to be clear and dry and low cloudy with precipitation

(2) cross-isobar flow toward lower pressure associated with frictional veering within the boundary layer implies a downward mass flux in the highs and an upward mass flux in the lows at the top of that layer.

In contrast to conditions at the earth's surface, at upper troposphere levels, regions of ascent and enhanced precipitation tend to correspond to upper air ridges which is consistent with a reversal in the sign of $\overline{\omega\Phi}$ between upper and lower troposphere. The shape of the $\overline{-\mathbf{V}\cdot\nabla\boldsymbol{\Phi}}$ profile is indicated both by direct measurements of Kung (1966) and many others, and by our knowledge of the vertical distribution of kinetic energy dissipation in turbulence in the boundary layer and in the free atmosphere. The $-\overline{\omega\alpha}$ and profiles are also well established on the basis of observations. There are some minor differences between tropics and mid-latitudes, but the shapes of all the profiles are qualitatively similar to those shown in Fig. 1.8.

It is also evident from Fig. 1.8 that the troposphere does work on the stratosphere through the $\omega\Phi$ term. As noted above, the lower stratosphere is one of those passive regions of the earth's atmosphere, whose motions are driven by an influx of energy from elsewhere. This influx of energy is mainly accomplished through the "mechanical stirring" brought about by the term. There also exist situations in which the mechanical stirring is provided by the term. For example, some of the transient disturbances in the tropics are driven by an influx of energy from higher latitudes as manifested in the observed negative correlation between v and Φ in the subtropical upper troposphere (Mak, 1969). Evidently, some of the available potential energy released in middle latitudes is being used to generate kinetic energy in the tropics. A full treatment of the kinetic energy cycle for localized regions of the atmosphere requires consideration of additional terms involving boundary fluxes of kinetic energy and available potential energy. In this discussion we have considered only the leading terms. A comprehensive formulation has been developed by Muench (1964) for studying the kinetic energy cycle in the stratosphere.

-

Questions

1.21 Consider the hypothetical situation in which the flow in the free atmosphere above the PBL is purely barotropic so that all the work done on the PBL by the $-\mathbf{V} \cdot \nabla \mathbf{\Phi}$ term is at the expense of the kinetic energy of the free atmosphere. Complete the sketch of the pressure perturbations and the cross-isobar flow in an idealized vertical cross-section (i.e., fill in the cross-isobar flow (the horizontal arrows) in the free atmosphere, assuming that the vertical motion and the cross-isobar flow satisfy continuity in the plane of the section and the vertical integrated cross-isobat flow is equal to zero.

Make an analogous sketch for the contrasting situation in which the kinetic energy generated in the PBL is all at the expense of the available potential energy of the free atmosphere? [Hint: imagine a two-layer free atmosphere consisting of a baroclinic lower layer and an upper layer with no horizontal pressure gradients. Assume that all the cross-isobar flow required to satisfy continuity is concentrated in the upper layer.]

1.22 Describe the latitudinal profile of $\boxed{v^*\Phi^*}$ in relation to the "storm track" or monsoon disturbances centered at latitude ϕ_0.

1.23 What other boundary term besides the work term $\omega\Phi$ enters into the kinetic energy budget for the stratosphere? (Hint: start with (1.22) with the time rate of change term expressed as a local derivative and repeat the operations that led to (1.23), except perform the vertical integration only down to a pressure level which corresponds to the base of the stratosphere.

6 Partitioning of the kinetic energy cycle

The Lorenz (1955) paper is notable not only for its useful definition of available potential energy, but also for its subdivision of the kinetic energy cycle in terms of zonally averaged and eddy components as illustrated in Fig. 1.9. Here the

zonal kinetic energy, K_z is the kinetic energy of the zonal wind component, $[u]^2$ /2; and eddy kinetic energy, K_E is $(u^{*2} + v^{*2})/2$. The kinetic energy associated with mean meridional motions $[v]^2$ /2 is neglected for reasons that will become more clear in subsequent chapters.

In order to partition A into zonal and eddy parts, A_z and A_E , we expand the temperature into global mean, the zonally averaged departure from the global mean and the eddy part $T = \overline{T} + [T]" + T^*$, from which it is easily shown that

$$T^{"2} = [T]^{"2} + T^{*2} \tag{28}$$

The various quadratic terms associated with C (i.e., ωT, $\omega \alpha$, $-\mathbf{V} \cdot \boldsymbol{\nabla}\Phi$, etc.) can be partitioned into C_Z and C_E in a similar manner, where the zonal component

$$C_Z = -\frac{1}{g}\int_0^{p_0} \overline{[\omega][\alpha]}dp = -\frac{1}{g}\int_0^{p_0} \overline{[v]\frac{\overline{\partial[\Phi]}}{\partial y}}dp \tag{29}$$

is associated entirely with mean meridional motions because the zonally averaged zonal flow $[u]$ is, by definition, nondivergent and parallel to the zonally averaged geopotential height contours. Hence the mean meridional motions play a crucial role in the kinetic cycle even though they account for only a minute fraction of the kinetic energy reservoir. These simple two-dimensional circulations provide a convenient illustration of the energy conversion process: The Hadley circulation is obviously thermally direct, with $C > 0$. It is characterized by the rising of warm, light air in equatorial latitudes and the sinking of colder, denser air in subtropical latitudes ($-\omega\alpha > 0$); low level equatorward flow out of the subtropical high pressure belt into a belt of low pressure along the equator ($-\mathbf{V} \cdot \boldsymbol{\nabla}\Phi > 0$); and upper tropospheric flow out of the equatorial belt and into subtropical latitudes in the presence of a westerly flow in which $[\Phi]$ decreases with latitude (again $-\mathbf{V} \cdot \boldsymbol{\nabla}\Phi > 0$). The middle latitude "Ferrel cell" which extends between approximately $30°$ and $60°$ latitude is, in many respects, a mirror image of the Hadley cell. Relatively warm, light air sinks in the subtropics while colder, denser air rises at higher latitudes. The cross-isobar flow is poleward, toward lower pressure at low levels, and equatorward, toward higher pressure at high levels. However, since $[u]$ increases with height at these latitudes, (i.e., $-\partial[\Phi]/\partial y$ is larger aloft) it is clear that the up-gradient flow in the upper troposphere must dominate in the average over the cell. Hence the Ferrel cell is thermally indirect with $C < 0$. Observations indicate that the conversions in the Ferrel and Hadley cells nearly cancel one another, so that in the global average the numerical value of C_Z is, in effect, a small difference between two large numbers, each of which has some uncertainty associated with it. Hence, even the sign of C_Z is uncertain in a global average.

Returning to Fig. 1.9 we note that C_E represents the eddy part of the conversion $-\omega^*\alpha^*$ or $-\mathbf{V}^* \cdot \boldsymbol{\nabla}\Phi^* > 0$, where it is readily verified that

$$\overline{\omega\alpha} = \overline{[\omega][\alpha]} + \overline{\omega^*\alpha^*} \tag{30}$$

and

$$\overline{\mathbf{V} \cdot \boldsymbol{\nabla}\Phi} = \overline{[v]\frac{\partial[\Phi]}{\partial y}} + \mathbf{V} \cdot \boldsymbol{\nabla}\Phi + u^*\frac{\partial\Phi^*}{\partial x} + v^*\frac{\partial\Phi^*}{\partial y} \tag{31}$$

The eddy conversion is associated with phenomena such as monsoons and baroclinic waves which are characterized by strong deviations from zonal symmetry with large and systematically interrelated fluctuations in T^*, α^*, ω^*, u^*, v^*, and Φ^*. We will discuss these phenomena further in Chapter 5.

The conversions C_A and C_K involve the advection of temperature and momentum, respectively. For example, if horizontal temperature advection is tending to distort the isotherms from a zonally symmetric configuration into a more perturbed one, with larger values of T^*, then $A_Z \rightarrow A_E$. In a similar manner, if the horizontal advection of zonal momentum is tending to smooth out a zonally averaged jet, then $K_Z \rightarrow K_E$. We will consider these processes in further detail in Chapter 5.

The generation term $Q"T"$ in (1.12) can be partitioned in a manner entirely analogous to (1.29) and dissipation can be classified as D_Z or D_E . It is evident that G constitutes a major input of energy into the kinetic energy cycle because of the large equator to pole heating gradient in the troposphere. Latent heat release in the troposphere makes strong positive contributions to both G_Z and G_E while Newtonian cooling at stratospheric levels represents a sink of eddy available potential energy ($G < 0$). It is possible to imagine an idealized steady state atmosphere, not drastically unlike our own, in which all the input of energy into the kinetic energy cycle is through G_Z and energy flows from the A_Z reservoir to the other reservoirs and is destroyed mainly by frictional dissipation D_Z and D_E .

In Lorenz's formulation, the zonal mean and eddy fields are spatially orthogonal so that spatial variance and covariance quantities such as u, v, T, ωT, $Q"T"$,$-\mathbf{V} \cdot \boldsymbol{\nabla}\Phi$, etc. can be partitioned as in (1.27), (1.29) and (1.30). A similar partitioning can be carried out among any set of spatially orthogonal fields. For example, the eddy energies and conversion processes in Fig. 1.9 can be expanded in terms of zonal wavenumber components so that A is partitioned into A_1 , A_2 , A_3 ...A_k, where k refers to zonal wavenumber, and similarly for G, C, and D, as illustrated in Fig. 1.10.

A and K are exchanged among the various harmonic components by means of advective processes similar to those described above. The nonlinear wave-wave interactions as well as the interactions between each wave component can be expressed in terms of formulae involving the coefficients of the various harmonics (Saltzman, 1970) and different components in the wavenumber-frequency domain (Kao, 1968; Kao and Chi, 1978). With these formulations it is possible to gain some insight into the contributions of various dynamical entities to G and C. For example, the processes associated with the monsoon circulations are represented, for the most part, by $k = 1$-3, whereas baroclinic waves fall largely within the range $k = 6$-10. Because of the large number of possible interactions between the various wave components, the job of keeping track of the kinetic energy cycle depicted in Fig. 1.10 is rather complex and tedious. A considerable reduction in complexity can be realized if the waves are grouped into a few categories such as planetary waves ($k = 1$-3), long waves ($k = 4$-5), synoptic scale waves ($k = 6$-15) etc.

Relationships analogous to (1.27), (1.29) and (1.30) can also be derived by

components of the general circulation which are orthogonal in the time domain. The simplest subdivision of this type would be a partitioning between time mean components, A_M, K_M, C_M, etc. and time varying or transient components A_T, K_T, C_T, etc. Such a breakdown would allow one to distinguish between geographically fixed phenomena such as the monsoons and the wintertime standing waves in the Northern Hemisphere, versus propagating disturbances such as baroclinic waves and transient planetary waves. Temporal subdivision can be extended in a manner analogous to Fig. 1.10 by means of power spectrum analysis in the time domain. Mixed space/time subdivisions are possible and spherical harmonics or other spatially orthogonal functions can be used in place of zonal harmonics. It is also possible to subdivide motion systems in terms of vertical structure (e.g., barotropic vs. baroclinic systems). Hence there are numerous possible variants of Lorenz's formulation. In practice, it seems to have worked out that the number of valuable new insights derived from such studies is inversely proportional to the number of subdivisions. The most successful studies have tended to be those in which dynamical considerations have motivated the choice of subdivisions.

-

Questions:

1.24 Prove that $A_M + A_T = A$, $C_M + C_T = C$, etc., where M refers to time mean and T to
transient.

7 References

Haurwitz, B. 1941: *Dynamic Meteorology*, McGraw-Hill, New York, 365pp.

Dutton, J.A. and D.R. Johnson, 1967: The theory of available potential energy and a variational approach to atmospheric energetics. *Advances in Geophysics,* **12**.

Kao, S.K., 1968: Governing equations and spectra for atmospheric motion and transports in wavenumber frequency space. *J. Atmos. Sci.,* **25**, 32-38.

Kao, S.K.and C.N. Chi, 1978: Mechanism for the growth and decay of long- and synoptic-scale waves in the mid-troposphere. *J. Atmos. Sci.,* **35**, 1375-1387.

Kung, E.C.,1966: Large-scale balance of kinetic energy in the atmosphere. *Mon. Wea. Rev.,* **94**, 627-640.

Lorenz, E.N., 1955: Available potential energy and the maintenance of the general circulation. *Tellus,* **7**, 157-167.

Mak, M., 1968: Laterally driven stochastic motions in the tropics. *J. Atmos. Sci.,* **26**, 41-64..

Margules, M., 1903: Über die Energie der Stürme. Jahrb. Zentralanst. Meteor., Vienna, 126,

English translation: Abbe, C., 1910: The Mechanics of the Earth's Atmosphere. 3rd. Coll.. Washington, Smithsonian Inst., 533-595.

Muench, H.S.,1964: *Stratospheric Energy Processes and Associated Long-wave Structure in Winter.* PhD thesis, University of Washington, 129pp.

Oort, A.H., 1964: On estimates of the atmospheric energy cycle. *Mon. Wea. Rev.,* , 483-493.

Saltzman, B., 1970: Large-scale atmospheric energetics in the wavenumber domain. *Rev. Geophys. Space Phys.,* **8**, 289-302.

Wiin Nielsen, A., 1962: On the transformation of kinetic energy between the vertical shear flow and the vertical mean flow in the atmosphere. *Mon. Wea. Rev.,* **90**, 311-323.