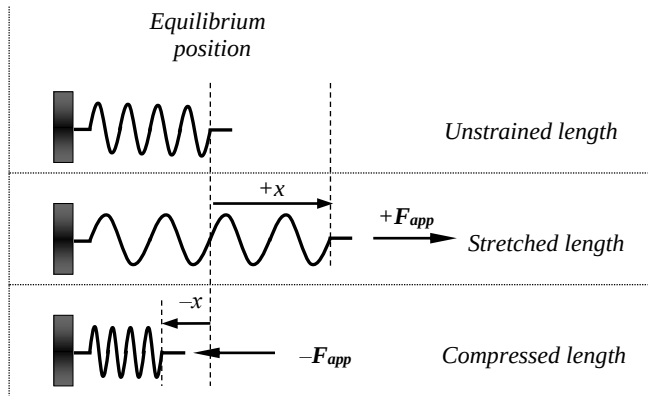
Elastic Potential Energy and Conservation of Mechanical Energy

Level : Physics I **Instructor : Kim**

Hook's Law

Springs are familiar objects that have many applications, ranging from push-button switches on electronic components to mattresses. They are useful because they can be stretched or compressed.

Experiment reveals that for relatively small displacements, the applied force $\boldsymbol{F_{app}}$ required to stretch or compress a spring is directly proportional to the displacement x, that is, $\boldsymbol{F \sim x}$.

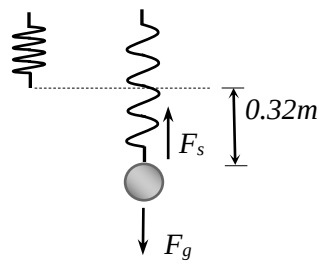


The applied force is also proportional to the type of material of the spring, which is represented as the spring constant $\boldsymbol{k}$. So

$$F_s = kx \text{ : Hooke's Law}$$

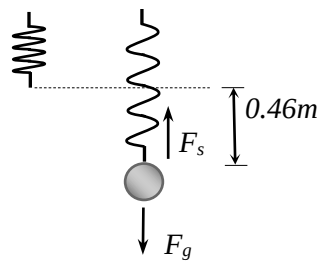
Q1) Finding the spring constant *k*

You hold a spring vertically and hang an object that has a mass of 0.2kg. Then you measure how much the spring is being stretched and find that it has been stretched x=0.32m. Find the spring constant k. Remember that $F_g=mg$ where g=9.8m/s^2.



Q2) You hold a spring and hang an object that has a mass of 0.4kg.

i) If the spring stretches x=0.46m, then find the spring constant k.



ii) If we now know what the spring constant is, we can predict how far the spring will stretch for different masses. Find how much the spring will stretch if we attach the spring with an object of mass 0.6kg.

iii) Find how much the spring will stretch when hung by an object of mass 0.86kg.

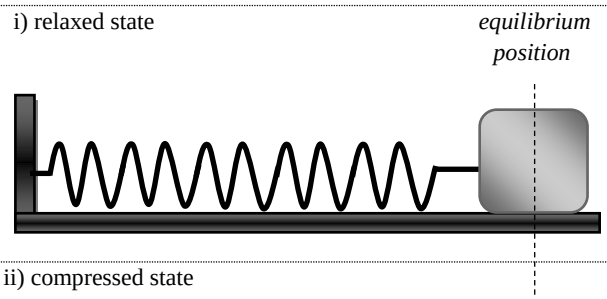
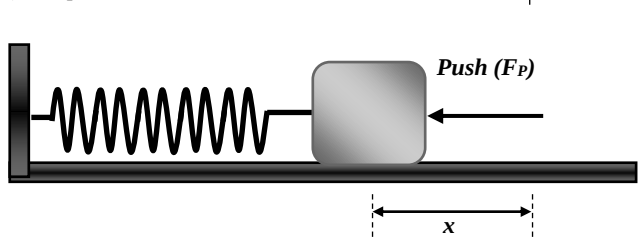
Elastic Potential Energy

When we stretch a rubber band, we are pulling (pulling is a ***force***!) over a certain distance. That is, we do ***Work*** on the object. Doing ***Work*** must C***hange the Energy*** of that object. In this case, we are increasing the Elastic Potential Energy(U_s)

$$U_s = \frac{1}{2}kx^2$$

where k is called the spring constant, and represents the stiffness of the elastic material

In the figures below, a block is attached to a spring on a frictionless surface.

i) relaxed state 	The spring is relaxed. It has no energy in the spring since no work has been done on the spring
ii) compressed state 	Work is done on the spring due to pushing force over a distance 'x'. By pushing against the spring, energy is being stored in the block. Whatever distance the block is pushed, the spring will be compressed the same amount of distance 'x'.

Q3) A block is attached to a spring as shown above. The mass of the block is 3.1kg and the spring constant is k=450(N/m). If the block pushed, compressing the spring 0.04m, then how much elastic potential energy is stored in the block?

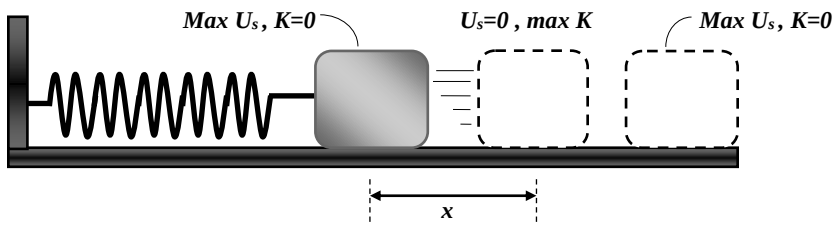
a) 0.12J b) 0.36J c) 0.54J d) 0.71J

Q4) A block is attached to a spring as shown above. The mass of the block is 3.1kg and the spring constant is k=680(N/m). If the block pulled, stretching the spring 0.04m, then how much elastic potential energy is stored in the block?

a) 0.12J b) 0.36J c) 0.54J d) 0.71

Conservation of Mechanical Energy

http://www.wiley.com/college/halliday/0470469080/simulations/fig08_03/fig08_03.html



If the block is compressed and then released, the block will undergo a repeated motion, where there will be continuous transfer between elastic potential energy and kinetic energy.

At the maximum compression or stretch, the block will have maximum elastic potential energy while zero kinetic energy.

When the block is released, the elastic potential energy of the block will decrease while the kinetic energy will increase as it gains more speed.

The instant the block passes the original position(that is, the position where the spring is neither compressed or stretched), the block will have maximum speed, hence, maximum kinetic energy and have zero elastic potential energy.

As the block passes the original position, the kinetic energy will decrease since the block slows down while the elastic potential energy increases, etc, . .

However, the total energy $E_{tot} = U_s + K$, remains the same throughout the motion!!

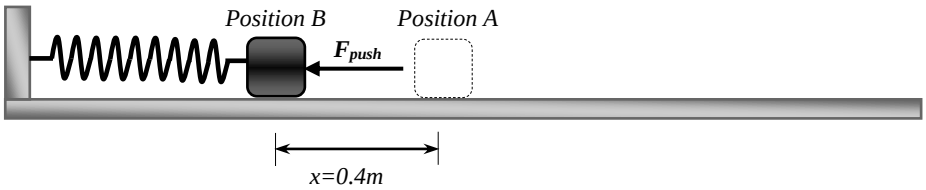
Examples of Application of Elastic Potential Energy(U_s)

- Elastic potential energy can be stored in rubber bands, bungee cords, trampolines, springs, an arrow drawn into a bow, pogo-stick, etc.

$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

Q5) Conservation of Mechanical Energy

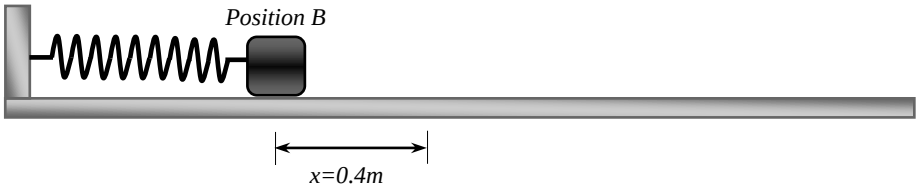
A block of mass 4kg is attached to a spring that has spring constant *k=20N/m*. The block is pushed so that the spring compresses *x*=0.4m from the original position *A*.

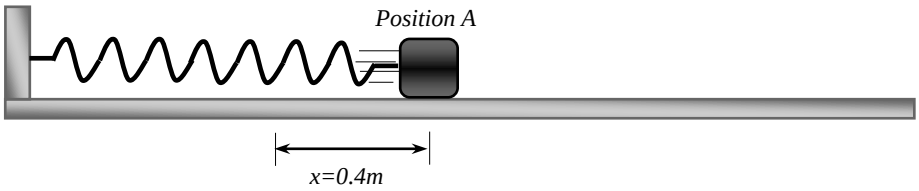


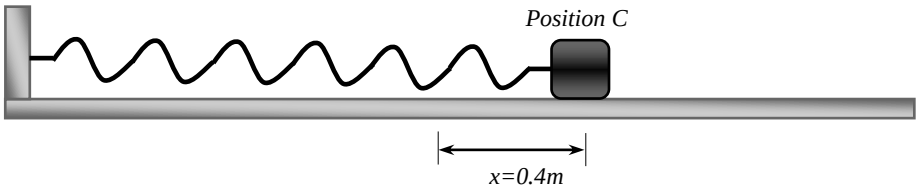
i) How much elastic potential energy is stored in the block when it is at position *B*?

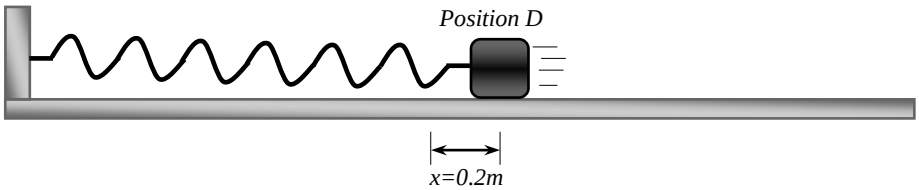
a) 1.2J b) 1.6J c) 2.0J d) 2.4J

Assume the block is attached to the spring. If the block is released, the block will display a back and forth motion as shown below. Answer the questions on next page.









$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

ii) At what position does the block have maximum kinetic energy?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

iii) At what position does the block have maximum elastic potential energy?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

iv) At what position does the block have maximum speed?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

v) At what position does the spring have maximum compression?
a) Position *A* b) Position *B* c) Position *C* d) Position *D*

vi) At what position does the spring have maximum stretch?
a) Position *A* b) Position *B* c) Position *C* d) Position *D*

vii) Find the maximum kinetic energy.
a) 1.2J b) 1.6J c) 2.0J d) 2.4J

viii) Find the total mechanical energy. That is, the sum of the kinetic and elastic potential energy.
a) 1.2J b) 1.6J c) 2.0J d) 2.4J

ix) At what position does the block have zero speed?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

x) At what position does the block have zero kinetic energy?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

xi) At what position does the block have zero elastic potential energy?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

xii) At what position does the block have non-zero kinetic energy <u>and</u> non-zero elastic potential energy?
a) Position *A* b) Position *B* c) Position *B* and *C* d) Position *D*

xiii) What is the total mechanical energy of the block at position *D*?
a) 1.2J b) 1.6J c) 2.0J d) 2.4J

$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

xiv) Find the elastic potential energy at position *D*.

a) 0.4J b) 0.6J c) 0.8J d) 1.0J

xv) What is the kinetic energy at position *D*?

a) 0.4J b) 0.8J c) 1.2J d) 1.6J

xvi) Find the speed at position *D*.

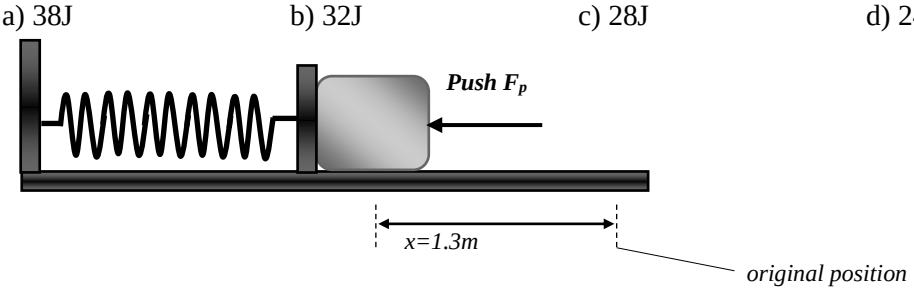
a) 0.12m/s b) 0.34m/s c) 0.56m/s d) 0.77m/s

xvii) According to your answers, is the elastic potential energy of the block at x=0.4m twice as large compared to when it is at x=0.2m? If no, explain why.

Q6) A spring has a spring constant of $k=45N/m$. A block of mass 0.2kg is compressed against a spring of $x=1.3m$ from the equilibrium as seen below.

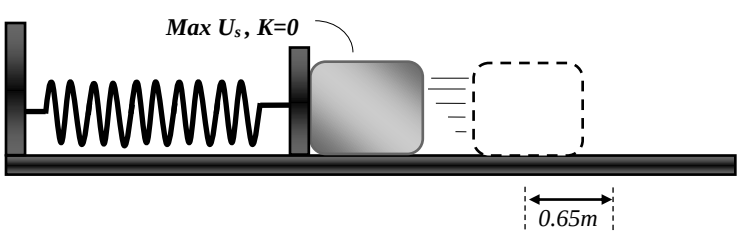
i) Calculate the elastic potential energy stored in the block.

a) 38J b) 32J c) 28J d) 24J



ii) If the block is released, what is the elastic potential energy when the block is 0.65m from the maximum compression?

a) 32.7J b) 21.4J c) 14.2J d) 9.5J



$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

iii) Find the kinetic energy when the block is 0.65m from the maximum compression.

a) 35.5J b) 28.5J c) 17.5J d) 9.5J

iv) Find the speed when the block is 0.65m from the maximum compression.

a) 16.9m/s b) 12.5m/s c) 9.5m/s d) 3.4m/s

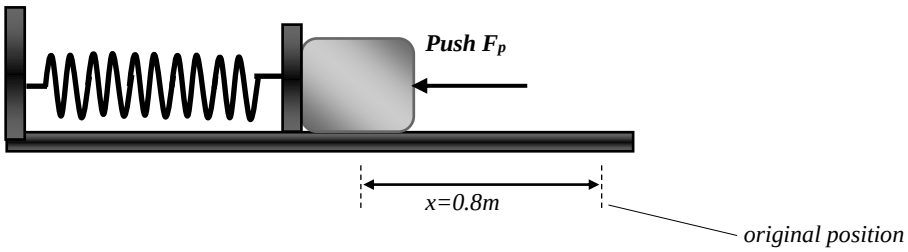
v) What is the maximum kinetic energy of the block?

a) 38J b) 32J c) 28J d) 24J

Q7) A spring has a spring constant of *k=45N/m*. A block of mass 1.4kg is compressed against a spring of *0.8m* from the equilibrium as seen below.

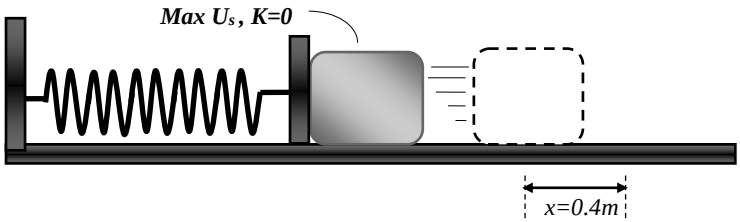
i) Calculate the elastic potential energy stored in the block. Also, what is the total energy?

a) 20.2J b) 14.4J c) 8.5J d) 3.6J



ii) If the block is released, what is the elastic potential energy when the block is 0.4m from the maximum compression?

a) 20.2J b) 14.4J c) 8.5J d) 3.6J



iii) Find the kinetic energy when the block is 0.4m from the maximum compression.

a) 35.5J b) 28.5J c) 17.5J d) 10.8J

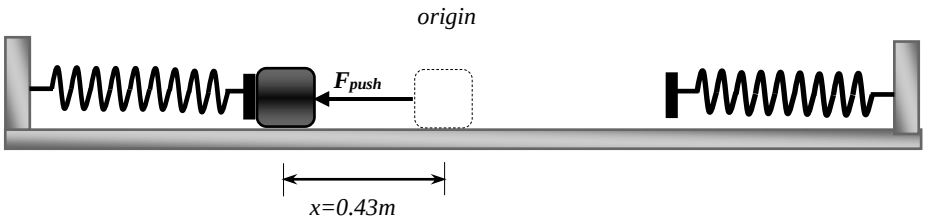
iv) Find the speed when the block is 0.4m from the maximum compression.

a) 16.9m/s b) 12.5m/s c) 9.5m/s d) 3.9m/s

v) What is the maximum kinetic energy of the block?

a) 20.2J b) 14.4J c) 8.5J d) 3.6J

Q8) Two springs are attached to a wall as shown below. The spring constant for the left spring k_1=340N/m and the right spring k_2=440N/m. The block is <u>not</u> attached to the spring and the surface is frictionless. A block of mass 3.2kg is pushed on the left spring and compresses the spring x=0.43m.



i) Find the maximum elastic potential energy of the block.

a) 22.4J b) 31.4J c) 41.8J d) 50.6J

ii) Find the maximum kinetic energy of the block.

a) 22.4J b) 31.4J c) 41.8J d) 50.6J

iii) Find the maximum speed of the block.

a) 4.43m/s b) 7.26m/s c) 9.14m/s d) 12.60m/s

iv) Find the kinetic energy the moment the block hits the second spring.

a) 22.4J b) 31.4J c) 41.8J d) 50.6J

v) Find the maximum elastic potential energy on the block when it is compressed on the right spring.

a) 22.4J b) 31.4J c) 41.8J d) 50.6J

vi) Find the maximum compression of the block on the right spring.

a) 0.92m b) 0.78m c) 0.51m d)0.38m

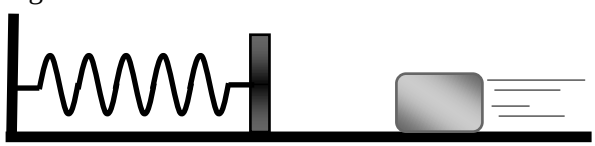
$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

Q9) A 0.01kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124N/m. The block is shoved parallel to the spring axis and is given an initial speed of 8m/s, while the spring is initially unstrained. What is the maximum length the spring is compressed or stretched?

a) 0.134m b) 0.111m c) 0.0967m d) 0.072m

Q10) The horizontal surface on which the block slides is frictionless. The speed of the block before it touches the spring is 6m/s.

i) How fast is the block moving at the instant the spring has been compressed 0.15m? *k*=2000N/m, mass is 2kg.



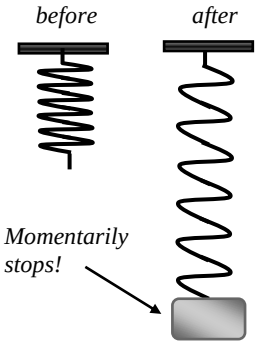
a) 2.34m/s b) 3.67m/s c) 4.89m/s d) 5.22m/s

ii) Find the maximum compression length of the spring

****Vertical Motion** *(U_g changes!)*

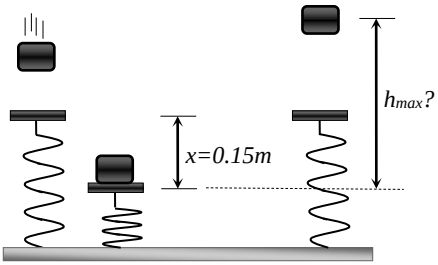
Q11) A spring is hung from the ceiling. A 0.45kg block is then attached to the free end of the spring. If the spring constant *k*=58.8N/m, then how far will the block fall? That is, find the maximum stretched length. *(U_g exists in this system)*

a) 0.42m b) 0.34m c) 0.26m d) 0.15m



$$U_g = mgh, \quad U_s = \frac{1}{2}kx^2, \quad K = \frac{1}{2}mv^2 \quad and \quad E_{tot} = U_{gi} + U_{si} + K_i = U_{gf} + U_{sf} + K_f$$

Q12) A 0.25kg block is dropped straight downward onto a vertical spring. The spring constant of the spring is 58N/m. The block compresses the spring 0.15m before coming to a stop.



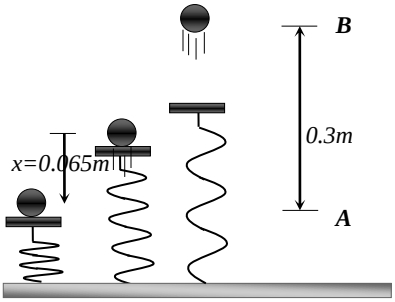
i) What is the speed of the block just before it hits the spring?

a) 1.51m/s b) 1.82m/s c) 2.11m/s d) 2.76m/s

ii) What is the maximum height the block will rise back up?

a) 0.893m b) 0.622m c) 0.452m d) 0.266m

Q13) In preparation for shooting a ball, a vertical spring(*k=675N/m*) is compressed by 0.065m relative to its unstrained length. The ball(*m=0.0585kg*) is at rest against the spring at point *A*. When the spring is released, the ball shoots up and passes point *B*, which is *0.3m* higher than point *A*.



(i) What is the speed of the ball the moment it leaves the spring?

a) 6.9m/s b) 6.55m/s c) 4.21m/s d) 3.5m/s

(ii) How fast is the ball moving at *B*?

a) 6.9m/s b) 6.55m/s c) 4.21m/s d) 3.5m/s

(iii) Find the maximum height the ball will reach from point *A*

a) 0.92m b) 1.54m c) 2.21m d) 2.49m