Objective Analysis Methods in the Mechanics of Sports

by

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Abstract

Sports engineering can be considered as the bridge between the knowledge of sports science and the principles of engineering and has an important role not only in improving the athletic performance, but also in increasing the safety of the athletes. Testing and optimization of sports equipment and athletic performance are essential for supporting athletes in their quest to reach the podium. However, most of the equipment used by world-class athletes is chosen based only on subjective tests and the athletes' feelings. Consequently, one of the aims of this thesis was to combine mechanics and mathematics to develop new objective test methods for sports equipment. Another objective was to investigate the possibility to accurately track and analyse cross-country skiing performance by using a real-time locating system. A long term aim is the contribution to increased knowledge about objective test and analysis methods in sports. The main methodological advancements are the modification of established test methods for sports equipment and the implementation of spline-interpolated measured positioning data to evaluate cross-country skiing performance. The first two papers show that it is possible to design objective yet sport specific test methods for different sports equipment. New test devices and methodologies are proposed for alpine ski helmets and cross-country ski poles. The third paper gives suggestions for improved test setups and theoretical simulations are introduced for glide tests of skis. It is shown, it the fourth paper, that data from a real-time locating system in combination with a spline model offers considerable potential for performance analysis in cross-country sprint skiing. In the last paper, for the first time, propulsive power during a cross-country sprint skiing race is estimated by applying a power balance model to spline-interpolated measured positioning data, enabling in-depth analyses of power output and pacing strategies in cross-country skiing. Even though it has not been a first priority aim in this work, the results from the first two papers have been used by manufacturers to design new helmets with increased safety properties and cross-country ski poles with increased force transfer properties. In summary, the results of this thesis demonstrate the feasibility of using mechanics and mathematics to increase the objectiveness and relevance when analysing sports equipment and athletic performance.

Descriptors: sports equipment, test methods, sports mechanics, biomechanics, performance analysis, tracking, positioning system, pacing, alpine skiing, cross-country skiing, poles, helmets

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Sammanfattning

Sportteknologi kan anses vara länken mellan idrottsfysiologi och ingenjörskonst och är en viktig del vid utvecklandet av ny idrottsutrustning. De senaste decennierna har utvecklingen av ny och bättre idrottsutrustning haft en direkt inverkan på att förbättra idrottsprestationer samt minska skaderiskerna för idrottare. Optimering av idrottsutrustning tillsammans med olika metoder för prestationsanalyser är därför viktigt för elitidrottare i deras strävan att bli bäst i världen. Idag finns det dock få objektiv metoder som på ett idrottsspecifikt sätt testar och analyserar idrottsutrustning och olika prestationsvariabler. En stor del av utrustningen som används av världens bästa idrottare är därför endast testad och utvald baserat på idrottarnas egna tester och subjektiva känslor. Ett syfte med denna avhandling var att kombinera mekanik och matematik för att utveckla objektiva och idrottsspecifika testmetoder för sportutrustning. Ett annat mål har varit att utreda möjligheterna att använda ett lokalpositioneringssystem för att analysera mekaniska prestationsvariabler hos längdskidåkare i fält. Ett mer långsiktigt mål med detta arbete är också att bidra till att höja kunskapsnivån när det gäller objektiva test- och analysmetoder inom idrott. Denna avhandling består av fem olika studier med problemformuleringar utifrån objektiva och idrottsspecifika test- och analysmetoder. De största metodologiska kunskapsvinsterna är modifieringen av etablerade testmetoder tillsammans med integration av konventionella mekanikberäkningar samt implementeringen av splinesinterpolerad positioneringsdata för att analysera längdskidåkning vid sprinttävlingar. De första två studierna visar möjligheterna att konstruera objektiva och idrottsspecifika testmetoder av sportutrustning. Ny testutrustning och testmetodik är föreslagen för alpina skidhjälmar samt för längdskidåkningsstavar. Den tredje studien ger rekommendationer på förbättringar vid glidtester av skidor samt för teoretiska beräkningar av energiförluster och friktionskoefficienter mellan skidbelag och snö. Den fjärde studien visar potentialen i att nyttja ett lokalpositioneringssystem i kombination med splinesinterpolering för att analysera åkprestationer vid längdskidstävlingar. I den femte studien beräknas för första gången den framåtdrivande effekten vid längdskidåkning genom att tillämpa en kraftbalansmodell på splinesinterpolerad positioneringsdata. Detta möjliggör ingående analyser av effektgenerering och farthållningsstrategier vid längdåkning. Trots att det inte varit ett primärt mål har resultaten från de två första studierna använts av hjälm- och stavtillverkare för att utveckla nya och säkrare hjälmar samt längdskidstavar med bättre kraftöverföringsegenskaper. Sammantaget visar resultaten i denna avhandling att objektiva och idrottsspecifika testmetoder, tillsammans med teoretiska beräkningar kan bidra till att höja kvaliteten och funktionaliteten av idrottsutrustning, möjliggöra noggrannare och mer relevanta analyser av idrottsliga prestationer samt i ett längre perspektiv, även höja idrottarnas prestationer.

Nyckelord: sportutrustning, testmetoder, idrottsmekanik, biomekanik, prestationsanalys, positioneringssystem, farthållningsstrategi, alpin skidåkning, längdskidåkning, stavar, hjälmar

Preface

This thesis studies the possibilities to increase the objectivity in the testing procedures of sports equipment and athletic performance based on experimental approaches accompanied by mechanics simulations. The first part gives a concise background, description of methods, results and conclusions. The second part of the thesis consists of the following five papers:

Paper 1.
Swarén, M., Holmberg, H.-C., Eriksson, A.
"Repeated low impacts in alpine ski helmets"
Published in Sports Technology, 2012, 6(1): p. 43-52

Paper 2.
Swarén, M., Therell, M., Eriksson, A., Holmberg, H.-C.
"Testing methods for objective evaluation of cross-country ski poles"
Published in Sports Engineering, 2013, 16(4): p. 255-264

Paper 3.
Swarén, M., Karlöf L., Holmberg, H.-C., Eriksson, A.
"Validation of test setup to evaluate glide performance in skis"
Published in Sports Technology, 2014, 7(1-2): p. 89-97

Paper 4.
Swarén, M., Stöggl, T., Supej, M., Holmberg, H.-C., Eriksson, A.
"Usage and validation of a real-time locating system to monitor position and velocity during cross-country skiing"
Published in International Journal of Performance Analysis in Sports, 2016, 16(2): p. 769-785

Paper 5.
Swarén, M., Eriksson, A.
"Power and pacing calculations based on real-time locating data from a cross-country sprint skiing race"
Submitted

Division of work between authors

The research program was originated by Prof. Anders Eriksson (AE) who also was the main supervisor for all five papers. Papers 1-3 were initiated and guided by Prof. Hans-Christer Holmberg (HCH). Dr. Lars Karlöf (LK) initiated and acted as advisor for the project which resulted in Paper 3. For all five papers, Mikael Swarén (MS) has continuously reported and discussed the different projects and their progress with AE with input from HCH, LK and other co-authors.

Paper 1

The design of the study, the development of the test rig, all data collection and analysis were done by MS. The paper was written by MS with input from AE and HCH.

Paper 2

MS was responsible for the experimental setup and the design of the test rig. The building of the test rig and data collection was performed by MS with support from Mikael Therell (MT). All data analyses were done by MS. The paper was written by MS in dialogue with AE and HCH.

Paper 3

The experimental setup and data collection were performed by MS, LK and AE. Data analysis was performed by MS. The paper was written my MS with input from LK, HCH and AE.

Paper 4

The design and setup of the study were done by MS. The installation of the real-time locating system was done by MS. The data collection was performed by MS with assistance from the co-authors. All data analysis was done by MS. The paper was written by MS in dialogue with the co-authors.

Paper 5

The data which were used in the paper were collected during the project which resulted in Paper 4 and were collected by MS with assistance from the co-authors. All data preparation was done by MS. The paper was written by MS with input from AE

Work not included in the thesis

The work in the five appended papers have also been reported in other forms. The author has also been involved and contributed in several other scientific projects.

Papers

Born, D.-P., Stöggl, T., **Swarén, M.**, Björklund, G. (2016). Running in hilly terrain: NIRS is more accurate to monitor intensity than heart rate. *International Journal of Sports Physiology and Performance, Epub ahead of print.*

Sperlich, B., Born, D.-P., **Swarén, M.**, Kilian, Y., Geesmann, B., Kohl-Bareis, M., & Holmberg, H.-C. (2013). Is leg compression beneficial for alpine skiers? *BMC Sports Science, Medicine and Rehabilitation, 5*(1), 1-12.

Tesch, P. A., Pozzo, M., Ainegren, M., **Swarén, M.,** & Linnehan, R. M. (2013). Cardiovascular Responses to Rowing on a Novel Ergometer Designed for Both Resistance and Aerobic Training in Space. *Aviation, Space, and Environmental Medicine, 84*(5), 516-521.

Hurst, H. T., **Swarén, M.**, Hébert-Losier, K., Ericsson, F., Sinclair, J., Atkins, S., & Holmberg, H.-C. (2013). GPS-Based Evaluation of Activity Profiles in Elite Downhill Mountain Biking and the Influence of Course Type. *Journal of Science and Cycling, 2*(1), 25.

Hurst, H. T., **Swaren, M.**, Hébert-Losier, K., Ericsson, F., Sinclair, J., Atkins, S., & Holmberg, H.-C. (2012). Influence of course type on upper body muscle activity in elite cross-country and downhill mountain bikers during off road downhill cycling. *Journal of Science and Cycling, 1*(2), 2-9.

Book Chapters

Swarén, M., Holmberg, H., & Eriksson, A. (2014). Repetitive low energy impacts on Alpine ski helmets. *Science and Skiing VI*.

Swarén, M., Danvind, J., & Holmberg, H. (2012). Acceleration of the head during alpine skiing. *Science and Skiing V, 5*.

Swarén, M., Therell, M., Eriksson, A., Holmberg, H.-C., (2012). How to test cross-country ski poles?. *Science and Nordic Skiing II*

Swarén, M., Supej, M., Eriksson, A., & Holmberg, H.-C. (2012). Treadmill simulation of Olympic cross-country ski tracks. *Science and Nordic Skiing II*

Conference Proceedings

Swarén, M., Larsson, O., Björklund, G., (2016). *Using telecasting to identify turn times in alpine skiing.* Paper presented at the 2016 International Society of Performance Analysis of Sport 6th International Workshop, Carlow, Ireland.

Swarén, M., Soehnlein, Q., Holmberg, M., Stöggl, T., & Björklund, G. (2015). *Using 3D motion capture to analyze ice-hockey shooting technique on ice.* Paper presented at the 20th Annual Congress of the European College of Sport Science, Malmö, Sweden.

Swarén, M., Born, D., Stöggl, T., & Björklund, G. (2015). *Biomechanical 3D field measurements of trail runners.* Paper presented at the 20th Annual Congress of the European College of Sport Science, Malmö, Sweden.

Born, D.P., Stöggl, T., **Swarén, M.**, Sperlich, B., Björklund, G. (2015). *Is Heart Rate a Valid Measure To Monitor Exercise Intensity.* Paper presented at the 20th Annual Congress of the European College of Sport Science, Malmö, Sweden.

Swarén, M., Björklund, G., (2014). *Equipment and textiles for optimizing sports performance.* Paper presented at Chalmers Annual Matereials Science Inititive Seminar and Molecular Frontiers Symposium, 2014, Gothenburg, Sweden.

Swarén, M., Supej, M., Eriksson, A., & Holmberg, H.-C. (2012). *Treadmill simulation of Olympic cross-country ski tracks.* Paper presented at the Second International Congress on Science and Nordic Skiing. Vuokatti, Finland.

Swarén, M., Therell, M., Eriksson, A., Holmberg, H.-C., (2012). *How to test cross-country ski poles?* Paper presented at the Second International Congress on Science and Nordic Skiing. Vuokatti, Finland.

Hurst, H., **Swarén, M.**, Hébert-Losier, K., Ericsson, F., & Holmberg, H.-C. (2012). *Anaerobic Power and Cadence Characteristics of Elite Cross-Country and Downhill Mountain Bikers.* Paper presented at the 17th annual Congress of the Eurpean College of Sport Science.

Contents

Part I

Overview and Summary

Chapter 1. Introduction

Sports science is defined as the analysis in terms of motion, physiology, biomechanics and psychology of an athlete, whereas sports engineering is the technical application of mathematics and physics to assist athletes to enhance their performance [1]. Over the past decades, the advances in sports engineering have been essential in the evolution of sports and athletic performances by improving sports equipment. The development of new materials and equipment have assisted athletes in improving their performances while decreasing the risk of injuries.

Biomechanics is frequently used to study human movements and has been defined as the study of the movement of living things, using the science of mechanics [2, 3]. In sports engineering, biomechanical studies are required to enhance the knowledge regarding movements, performance and injury mechanisms in order to improve sports equipment and athletic performance. Elite sports push the human body to its maximal capacity and athletes must commit to total dedication and training discipline to reach the podium in the Olympics or the World Championships. For example, world-class cross-country skiers train between 700-1000 hours per year and international elite rowers around 1100 hours per year [4-7]. However, it is not only the number of training hours that is important to become an elite athlete; food as well as rest are both highly significant to performance. Overall, elite sport is today a full time commitment where everything has to be planned, carried out, analysed and optimized. Hence, every detail is important and this applies not only to the athletes, but also the equipment.

Objective test and analysis methods are needed to support athletes in their quest to perform at their peak capacity. Sports engineering is especially important in sports like skiing, pole-vaulting, speed-skating, golf or cycling where the equipment plays an essential role, affecting the athlete's performance. Hence, objective test methods of sports equipment are needed to provide reliable and repeatable data to manufacturers when designing new products. These tests of sports equipment need to be robustly designed and test the equipment in ways as similar to reality as possible. This thesis deals with these aspects with emphasis on the methodology rather than individual results.

1.1. Objective analysis methods in sports

Competitive alpine- and cross-country skiing are today well-developed athletic activities. Athletes are well-trained and the equipment is well developed with respect to both materials and geometric forms. Both alpine- and cross-country skiing require specific equipment and even though thousands of hours are spent on optimizing sports equipment for elite athletes, the selection process is still in many cases only based on subjective feelings. Margins between athletes are often measured in 10 milliseconds, which calls for further development of the athletic techniques, the equipment and increased tactical knowledge.

1.2. Background

The combination of sports engineering and biomechanics create a broad field which is used for various human motion analyses to increase the understanding of different movements, injuries, performances as well as product development and optimization. For a given athlete and exercise, equipment can play an important role for optimizing the performance by, e.g., reducing the rate of fatigue or by minimizing energy losses. Today, objective test methods for sports equipment are predominantly used for evaluating the equipment's effects on the athlete's performance or safety. Examples of this can be found in most sports for, e.g., clothing, helmets, bikes, shoes, vaulting poles, ice-hockey sticks or golf clubs [8-33]. Sperlich and Holmberg [34] investigated the physiological effects of different racing suits in cross-country skiing and found that a newly developed suit with better ventilation capacity, compared to an older one, decreased the athletes' core temperatures, lowered the oxygen uptake, minute ventilation and heart rate during an incremental intensity test on roller skis. In addition, engineering and mechanics play a central role in sports performance and also from a biomechanical point of view, as equipment often affects the athlete's movements, technique and performance. For example, Worobets et al. [29] investigated the influence of shaft stiffness on potential energy and puck speed in ice-hockey. They found that the stiffness influences puck speed during wrist shots and that more flexible sticks could store the most energy, resulting in higher puck speed. However, the way an ice-hockey player loads the stick has as much influence on puck speed as the design and properties of the stick. Another example of how mechanics can influence sports performance is the development of the klapskate in speed skating [35, 36]. The klapskate has a hinge mechanism underneath the ball of the foot instead of the rigid connection in conventional skates. The hinge mechanism allows the foot to plantarflex during the end push off, while the whole blade still glides on the ice [36, 37]. This enhances the effectiveness of the plantarflexion which increases the work per stroke, mean power output and gross efficiency compared to conventional skates [36, 38].

Biomechanical research plays a critical role in the development of sports equipment as a perfectly engineered mechanical piece of sports equipment can still obstruct an athlete due to poor athlete-equipment interaction [39]. The human factor, however, is always present when studying the athlete-equipment interaction, which decreases the repeatability as every test will be different. For example, in alpine and cross-country skiing, athletes spend hundreds of hours each season to find the best performing equipment, based on unrepeatable tests and subjective feelings. Many scientific studies have also an element of uncertainty due to the human factor when investigating equipment properties and how these affect performance [18, 40-54]. Objective and repeatable studies are therefore needed to investigate how the equipment might influence the performance, based on mechanics, biomechanics, mathematics, physiology and tactics.

1.3. Sports applications

Skiing is an example where the equipment plays a significant role for performance and the target groups of athletes, coaches and manufacturers are all depending on performance analysis and objective test methods to succeed in their occupations. Hence, from a scientific point of view, all groups strive for the best possible performance, but within their individual branches of occupation.

All studies in this thesis each have their own applications. In the helmet study, Paper 1, the mechanical methodology was chosen based on actual impact observations from alpine ski racing, where the helmets break due to the multiple low energy impacts, caused by the skiers hitting the gates. Most studies regarding head impact and brain injuries are focused on impacts caused by situations which are obvious elements in the particular sports, such as tackles in ice hockey or crashes in alpine skiing. However, to the author’s knowledge, research regarding the capacity of alpine helmets to withstand repeated low energy impacts in alpine skiing is non-existent. A testing method for repeated violence is also of interest when designing helmets in other sports, e.g., snowboarding, biking and skateboarding.

Paper 2 deals with objective test methods for cross-country ski poles as the currently used method when testing poles are based upon the perception of the shaft during skiing, which is a very subjective method for choosing one of the most important pieces of equipment in cross-country skiing. An objective and sport specific test method is developed to compare and categorize cross-country ski poles. This is perhaps most important for manufacturers who want to compare pole models and/or control the production quality. Still, an objective test value for comparison purposes can be beneficial to customers as well as elite athletes who want to get an objective value on certain pole characteristics. It is thus conceivable to study the effects from differently characterized poles by analytic methods.

Paper 3 uses the law of energy transformation to determine the friction between ski and snow when analyzing the glide of different cross-country skis. The field of application is highly relevant in all ski-sports but the testing methods have not developed far beyond a general comparison of feelings. A test setup which enables accurate and continuous measurements of the test track and skiing speeds will be beneficial for skiers, coaches and their aids when analyzing the glide of different skis and also in future biomechanical studies where the friction coefficient needs to be considered.

Paper 4 uses advanced positioning technology to validate the possibilities to use tracking devices in cross-country skiing. Different tracking solutions are already being used for performance analysis in sports like rugby, basketball and orienteering. Except for orienteering, which uses GPS based system, most sports with implemented tracking solutions are performed in arenas or on open fields, whereas cross-country skiing is performed along a narrow track between trees and other obstacles. The implementation of a local positioning system in cross-country skiing allows coaches, skiers and sport scientists to perform more advanced performance analyses. It can also be used to enhance the audience's experience by visualizing more race data.

A possible continuation of the measurements in Paper 4 is also given in Paper 5, which deals with the question formulation of how different skiers economize their energy in order to reach maximal performance. This is a frequent topic in endurance sports and numerous papers have been written about pacing strategies in e.g. cycling, running, rowing and cross-country skiing [55-62]. However, no previous study has used collected real time positioning data from a cross-country skiing sprint race to post-analyse different pacing strategies between the qualifying time-trial and the finals. Together with numerical fatigue models [63], numerical pacing simulations can be used to improve pacing strategies. In addition to cross-country skiing, the method enables in-depth performance analyses and pacing simulations in numerous sports, e.g., rowing, canoeing, running, speed skating and harness racing.

1.3.1. Helmets

Alpine skiing involves a risk of injury and head injuries are common among alpine skiers [64-67]. Numerous studies report a reduced risk of head injuries with helmet use, which makes the alpine ski helmet an essential piece of safety equipment [10, 64, 68, 69]. Helmets, including alpine ski helmets, typically consist of a shock absorbing core, most commonly made of Expanded polystyrene (EPS) or Expanded polypropylene (EPP) with an outer shell of Polycarbonate (PC) or/and Acrylonitrile-butadiene-styrene (ABS) [70]. The International ski federation (FIS) rules say that it is mandatory to use an approved ski helmet in all competition events. An FIS approved ski helmet must have a smooth surface, have shell and padding that cover the head and ears, with an exception for the slalom event where soft ear padding is permitted [71]. Accepted helmets shall have been CE marked and be in line with appropriate standards [71]. In addition, in giant slalom, super G and downhill, approved helmets must fulfill further requirements and have a maximum deceleration (equal to or) lower than 230 g during a specified impact test.. The speed events helmet must also pass an additional specific test with even higher demands [72].

The increased impact velocity for the speed event helmets is enhancing the similarity between test and an actual crash situation in skiing. However, the test procedure only includes a straight falling helmet with a linear impact, which seldom occurs in a crash. Gennarelli [73] reported that the most frequent brain injuries from motor vehicle accidents are due to rotational acceleration. New helmet designs and test methods to minimize rotational acceleration are being developed, especially in activities where the head hits a surface, like in alpine skiing, which enhances the tangential forces [74-76]. This development reduces the gap between the test setup and the real crash situation.

However, alpine ski helmets are not only exposed to one major impact during their lifespan, but also to minor low energy impacts from different types of collisions caused by normal handling and smaller tumbles. In alpine ski racing, where the skiers often aim to ski as short a line as possible, the helmets are exposed to repetitive impacts from hitting the gates. In giant slalom the helmet is particularly exposed to violence, due to the trajectory of the skier relative to the gate. Observations made by ski-coaches, race-skiers and manufacturers report co-occurrence breakage of race helmets due to repetitive impacts from the gates. The front and temple regions are the most exposed areas and also where the shell and core fracture. Thus, helmets need to be able to withstand the violence caused by hitting gates without losing their protective capacities.

1.3.2. Cross-country ski poles

Cross-country ski poles consist of a handle with a wrist strap, a shaft and a ski-basket with a metallic tip. The ski-basket generates support in the snow while the metallic tip creates grip on icy and hard ski-tracks or on roads when roller skiing. The shafts were originally constructed of solid wood, later bamboo, which remained the standard shaft material until the 1970s when shafts made of aluminium or fibreglass arrived [77]. Modern racing poles are constructed of carbon fibres in an epoxy resin, a design which generates high shaft strength with minimum weight, but also rather a high level of flexibility.

Cross-country skiers normally use pole lengths of 83 – 85 % of body height in classical skiing and 90 % of body height in the skating technique. Nilsson and co-workers [48] investigated the influence of pole length on ground reaction forces and propulsive impulse and found that longer poles significantly increase propulsive force and impulse, due to a longer thrust phase. In addition, Hansen & Losnegard [49] reported 0.11 s shorter sprint time with 7.5 cm longer poles than the self-selected pole length during an 80 m all-out double poling trial on snow. Hence, longer poles could be beneficial in certain situations where the highest possible propulsive pole force and maximal skiing velocity are essential.

Longer poles combined with higher peak forces and the more explosive poling techniques, described by numerous authors [78-81], generate higher demands on the poles regarding shaft strength, rigidity, durability and power transfer. Stöggl & Karlöf [47] found that the bending properties of cross-country ski poles affects skiers' abilities to ski at high velocities and that higher performance skiers should use poles with more homogeneous bending behavior. It is therefore surprising that in contrast to the amount of time and resources invested in ski testing, cross-country skiers evaluate and select poles based upon the perception of the shaft during skiing.

1.3.3. Glide testing for skis

It can be hypothesized that in all skiing events (with a minor exception being the need for grip in classical style cross-country skiing) the lowest possible friction between ski and snow is desirable. Ski bases are normally made of ultra-high molecular polyethylene (UHMWPE) which gives a surface that easily can be manipulated and optimized for maximal glide capacity. Lots of time and resources are invested in ski optimization through the use of different base structures and waxes. Today's waxing chemicals are highly sophisticated, as are the micro-machining techniques for the UHMWPE-bases of the skis.

Numerous studies [82-88] have investigated different parameters that directly or indirectly affect the ski-snow friction in order to increase the understanding of the basis for optimizing glide in cross-country skis. With the obvious objective to choose a surface with the lowest possible friction coefficient, the testing and verification methods are, however, not developed far beyond a general comparison of feelings. Today's comparison of the best possible surface properties for the expected race conditions are based on a direct comparison between different skis, waxes and base structures in a glide-track close to the arena. This method is practical to use out in the field but is sensitive to variation of the initial velocity, air resistance and the body masses of the skiers. In addition, photocells are sometimes used to measure the time between two selected points, to give more objective and comparable data. This setup only provides information regarding time used to glide a certain distance. This distance is often about 100 m, where the test-skiers normally starts from standing still and hence includes an acceleration phase as well as a gliding phase with higher velocity. Therefore, the test only gives a rough mean glide velocity value which could differ significantly compared to a racing situation.

Breitschädel et al. [89] used inertial measurements units (IMU), attached to cross-country skis, to measure the acceleration during glide tests to attain continuous and accurate glide characteristics. The results could be used to roughly differ between skis ($\Delta\mu \sim 0.01$), where $\Delta\mu$ is the difference in friction coefficient, but did not have the desired accuracy to distinguish between the best skis ($\Delta\mu \sim 0.001$), due to drift and noise in the IMU units [89]. Still, such a system could, if sufficiently accurate, together with mathematical modeling of the friction coefficient give a more objective analysis of how the friction coefficient differs for various skis, grinds and waxes vary with skiing velocity. This insight is beneficial, when preparing skis for optimal performance as a reduction of the kinetic ski-snow friction of 0.001 corresponds to a reduction of the run time of approximately 1 s/km [89].

1.3.4. Positioning tracking of cross-country skiers

Accurate tracking of athletes and the analyses of their movements are essential parts of performance analysis in many sports. Positioning data are normally collected either by Global Positioning Systems (GPS), radio frequency systems (Local Position Measurement, LPM) or video based systems. Most GPS-solutions can be operated without any time-consuming hardware and infrastructure installations, whereas both video based systems and LPM-systems need installations of either cameras or locators. Also, a GPS is not limited to a certain measuring volume or arena and can hence be used to collect positioning data in sports where the athletes cover large distances or navigate on their own. In orienteering for instance, the athletes are often equipped with an individual GPS which allows coaches and spectators to track and follow all contestants in real time. The collected positioning data also enable structured race and performance analysis.

Cross-country sprint skiing is an event with race times of approximate three minutes and an average track length of 1350 m [90]. Good tactics and positioning skills are essential abilities for a cross-country sprint skier. However, besides the official timing from start to finish, no other objective analysis tools e.g. split times, stationary cameras or tracking devices are normally provided by the race organizers. Multiple split times, positioned around the course at different key location, would give the coaches, skiers and spectators valuable information regarding where a skier lost or gained time compared to the contestants. Like orienteering, cross-country sprint skiing contains a large element of tactics, which increases the importance of accurate, continuous and objective performance analyses. Andersson et al. [91] did employ a Real-Time Kinematic Global Navigation Satellite System (RTK GNSS) to analyze and evaluate skiing techniques and their relationship to performance. Unfortunately, due to its costliness and bulkiness, such a system is not suited for use during races or everyday training [91, 92]. Hence, a small, light and accurate positioning system would be beneficial in terms of performance analysis in cross-country sprint skiing and could also increase the spectator experience with new race visualization solutions.

1.3.5. Using positioning data to calculate power in cross-country skiing

Pacing strategy in locomotive sports may be defined as the athlete's conscious variation of speed along the course [93]. For improving the performance, the pacing strategy aims to optimize the power output distribution along the course to minimize the finishing time. A power balance model in regard to power production and power dissipation is often used when simulating pacing strategies and predicting performance in endurance sports like cycling, rowing, running and cross-country skiing [57-62, 94-97].

Carlson et al. [94], used the power balance model to simulate cross-country skiing and further development of the skiing motion equations was completed by Sundström et al. [55] and [95], when optimizing pacing strategies in a synthetic cross-country skiing race. Moxnes et al. [98] compared simulated with experimental results where the power balance model was used to simulate cross-country skiing performance, defining the skier's locomotive power as a function of skiing speed and Eriksson et al. [63] developed a numerical model to simulate the fatigue effects on power production during whole-body exercises. However, none of the previous studies have been based on pre-collected skiing data where known skiing velocities along a measured track are used to calculate the propulsive power output used by the athlete in different parts of a race. Applying a power balance model to spline-interpolated measured positioning data enables a novel approach to estimate propulsive power in cross-country skiing and other endurance sports. The suggested method enables in-depth analyses of power output and pacing strategies and can be used in both training and racing situations as well as to improve the algorithms used in computer simulations.

1.4. Aim and scope

The overall aim of this work was to develop applicable and repetitive methods to objectively, yet sport specifically, collect and analyse data regarding equipment testing and performance analysis to support athletes, coaches and manufacturers to increase their athletic performances. Also, the purpose of combining mechanics and mathematics was to develop new and comprehensive methods to enable in-depth analysis of sports equipment as well as of athletic performance. A further aim was to simulate and compare the obtained test results with theoretical calculations to identify possibilities to use a standard mechanics approach for applicable sport specific testing.

Paper 1, Helmet testing

The main objective was to develop an objective test method for alpine ski helmets which could be used to evaluate the effects of repetitive impacts that arise when race skiers hit the gates. The test method should simulate similar impact patterns as in the field, and allow the user to test and evaluate different helmet designs to increase the level of safety in alpine ski helmets. The methodology and relevance in this paper were primarily related to slalom and giant slalom helmets, as the impact velocity of the gate in the test setup corresponded most closely to those events.

Paper 2, Objective test method for cross-country ski poles

The objective was to develop a quantitative and reproducible methodology to establish mechanical properties of slender tapered shafts, made of carbon fibre reinforced polymers (CFRP). The methodological development in this paper was primarily related to cross-country ski poles with associated loading cases. Also, an objective was to compare and validate the measured results with theoretical methods which are commonly used in mechanics. The work was aimed to result in theoretically founded indices of cross-country ski poles which could be used for comparison purposes.

Paper 3, Evaluation of test setup for glide evaluation of skis

The aim of this study was to investigate various test setups for analyzing the glide properties of different cross-country skis and to analyze the friction coefficients for different ski pairs by using theoretical calculations. Also to further increase the accuracy of the methodologies that are commonly used for glide evaluation in cross-country skiing. A qualitative measure for the dependence of friction on velocity was a particular aim. In addition, the work included measurements of the test slope which were described as a mathematical function allowing usage of the law of energy transformation to calculate the theoretical friction coefficients during the acceleration phase of the glide tests.

Paper 4, Using a real-time tracking system to track cross-country skiers

The aim of this investigation was to explore the possibility of using a real-time locating system to track and evaluate cross-country skiers during training and competition. A further aim was to validate the collected positioning data to those from a real-time kinematic global navigation satellite system of acknowledged accuracy.

Paper 5, Using positioning data to calculate propulsive power during cross-country skiing

The purpose was to investigate the possibility to use collected positioning data from a cross-country skiing sprint race, to calculate the skiers' propulsive power and pacing strategies. To further increase understanding of the strategies used by cross-country sprint skiers, the differences in pacing and power production between the individual time-trial and the final were calculated.

1.5. Outline of the thesis

The methods used in the five different papers are described in Chapter 2. Since the methods used are different in each paper, they are presented with reference to their respective applications. Chapter 3 presents some main results which also are discussed. In Chapter 4, the main conclusions from the papers are found. The five papers, which this thesis is based on, are presented in the second part of the thesis.

Chapter 2. Methods

All five papers approach relevant problem formulations from both applied and objective perspectives. These question formulations have been identified by input from athletes, coaches, manufacturers as well as from previous scientific papers. The different methods in the five papers are designed with respect to the sporting community and have been user driven as the aim of this work is to develop and investigate new applicable methods to analyse sports equipment and performance.

The method description will attend each paper separately as each paper has its own applied problem setting. The full descriptions of the methods of the five studies are given in the appended papers in Part II of this thesis.

Paper 1

The design of test device

A special test device was developed, made from aluminium profiles to which an electrical motor with pertained gearbox was attached, Figure 1. A driveshaft transferred the rotational movement from the gearbox to two asymmetrically mounted rods where a shortened slalom pole was attached to the second rod via a short driveshaft, Figure 1. The asymmetrically mounted rods underwent a sling-like motion which increased the rotational velocity of the slalom pole in the sector where it impacted the helmets. The test helmets were off-the-shelf competition helmets with the same design except from the core material used. Six helmets had a core of EPS, and the other six helmets had a core of EPP. The helmets where divided into two groups, EPS and EPP (n = 6 + 6), depending on the used core material. Beside the disparity in core material, both designs consisted of an inner liner of pneumatic honeycomb pads made of polyurethane and an outer shell of PC/ABS. The helmets were firmly fitted, using adhesive tape, to a headform which was monitored with a triaxial accelerometer, mounted at the headform's center of gravity. The headform was positioned so the rotating pole impacted the helmet to the left of the helmet's centerline with an impact velocity of approximately 13 m/s.

Figure 1. To the left, the test device and the used test setup where the rotating pole impacted the helmet on the left side of the centerline. To the right, a close-up on the drive chain with the motor in bottom right corner and with the attached gearbox which drives the two drive shafts that are connected with asymmetrical mounted rods. (Reproduced from [99].)

Study design

Acceleration data were collected at 10 kHz, for each axis, for 10 consecutive impacts around impact numbers 0, 200, 400, 600, 800 and 1000. The resulting acceleration acting on the headform's center of gravity was calculated from the collected acceleration data. Mean resulting acceleration, mean peak resulting acceleration and time to peak acceleration were calculated for all ten impacts in every test series. A Head Injury Criterion (HIC) was calculated using:

$$HIC = \left[(t_2 - t_1)^{-1} \int_{t_1}^{t_2} a(t)dt\right]^{2.5} (t_2 - t_1) \quad (1)$$

where a is the resultant head acceleration, t_1 and t_2 are any two points in time, with a fixed interval, but chosen to give the maximum HIC during the impact [100]. The HIC value was calculated for each impact using a 15 milliseconds time interval, unfavorably placed.

One-way analysis of variance (ANOVA) for repeated measures was used to analyze HIC and acceleration differences between the two helmet designs for each sample series, as well as for the two groups, using group mean values. The probability level accepted for statistical significance was set to $P<0.05$.

Paper 2

The design of the test device

A specially designed test device was used for measuring the bending behavior of seven different cross-country ski poles that where commonly used in the cross-country ski World Cup 2011/2012. The length for each pole was measured from the bottom of the ski-basket to the entry of the strap in the handle. Three poles for each length (145 cm, 155 cm, 165 cm and 170 cm) and model were tested.

The test device was designed to bend the poles in a similar way as during skiing, by applying the load via the handle strap. The load was applied by a sled, connected to the handle strap and sliding on two horizontally placed aluminium beams. The sled applied the load via a nut on the threaded rod which was operated by a handheld drilling machine at a constant speed. The pitch of the threaded rod resulted in a linear velocity of 1.2 m/s. Also, the sled could be connected to a wire that was led through a system of pulleys, where a weight was dropped, creating a rapid eccentric axial impact load on the pole shaft**.**

The ski-basket at the end of the shaft was mounted to a load cell. Only brand-specific ski-baskets with a centrally placed tip were used. Shaft bending was measured using a laser distance meter mounted on a sled attached to two horizontal aluminium beams above the pole. The sled with the laser distance meter could be moved along the entire length of the test device and a linear encoder was attached to the laser unit to position the laser along the pole. Two guide pins were positioned on each side of the pole shaft to direct the bending direction of the shaft upwards, directly underneath the laser distance meter, Fig. 2. All data collection was performed at 100 Hz.

Study design

Three different tests were performed for each pole length and model. First, a maximal loading test was performed by increasing the load until no more force could be transferred by the shaft and only buckling-like bending occurred. The load was controlled via the nut on the threaded rod and the drilling machine. Second, a deflection test was performed, where each shaft was pre-loaded with 10 N as a baseline. The load was thereafter increased to 100 N and then with an increment of 100 N until the next level could not be reached due to breakage. The deflection of the pole was measured at each load level. Third, an impact test was performed, where a 10 kg weight was dropped from a height of 25 cm. Each pole was tested three times. The dropped weight landed on an oil damper which lengthened the impact and reduced the force. The same measuring setup was used as that for the load test with the exception of the laser distance meter which was placed in the middle of the poles. The mean and standard deviation of the impact forces were calculated for each pole model and length.

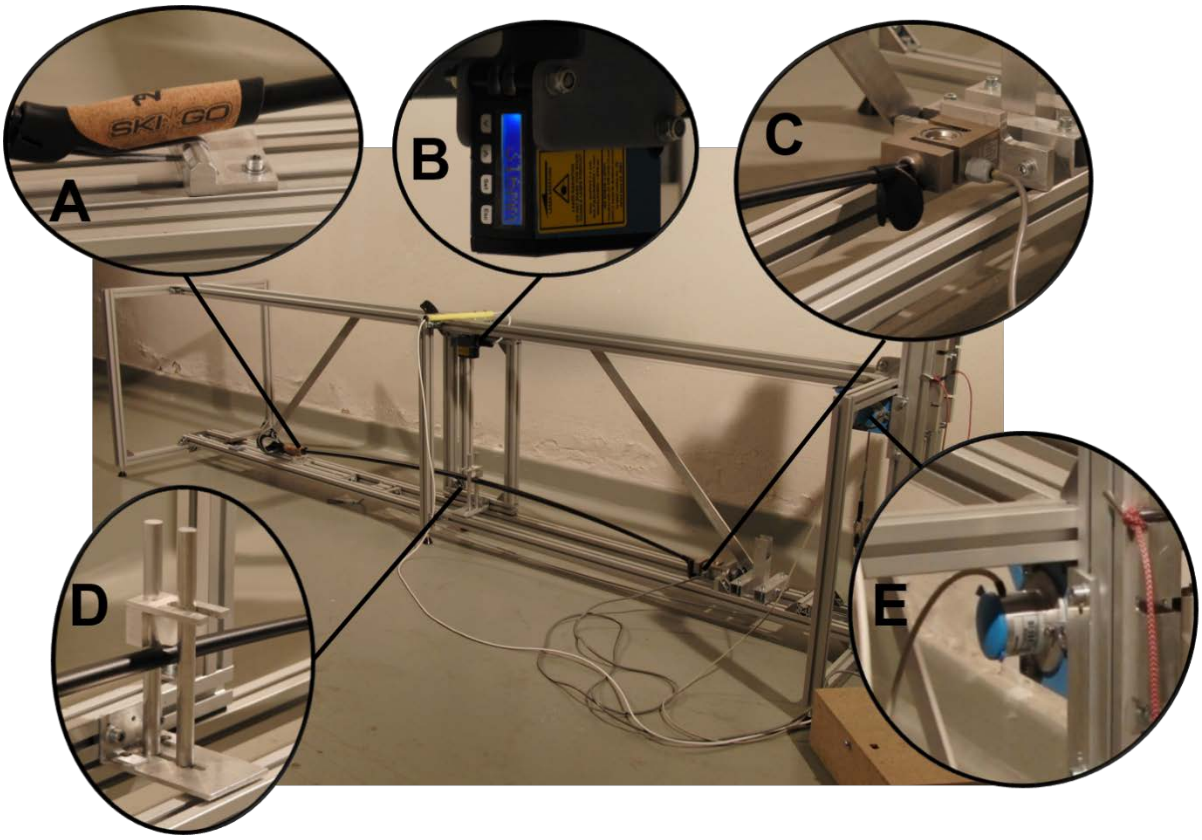

Figure 2. Close-up on the test setup. A) The handle strap attached to the adjustable sled for applying the load. B) The laser distance meter to measure shaft bending. C) The ski-basket and the load cell. D) The movable sled along the pole for directing and measuring the shaft bending. E) The linear encoder to measure the laser distance meter along the shaft. (Reproduced from [101]).

Data processing

As a theoretical reference, a mean *B*-value (flexural rigidity) was calculated for each pole. The *B*-value represents the effective bending stiffness of a beam section. It is possible to calculate the B-value by regarding the attached pole to be pinned at both ends by inserting the Euler buckling expression for a free-free case, with known length *L*, force *F* and force eccentricity *e*.

$$B = \frac{0.0642FeL^2}{\delta_{\max}} + \frac{FL^2}{\pi^2} \tag{2}$$

where $\delta_{\max}$ is the maximum measured deflection for the shaft along its length for a given axial load *F*.

High deflections typically occurred for *F*>200 N which decreases the accuracy for calculations based on buckling instability. *F*=200 N was the load where minimum deflection occurred, but still with deflection differences between the 145 cm and 170 cm shafts and was hence the load used in the flexural rigidity calculations corresponding deflections values δ were used, measured in the deflection test, and *e* = 0.017 m.

The Southwell plot [102] was used to validate the maximal loading test with an established method to calculate the critical buckling load. The mean deflection value *w* for every load for each shaft model and length at various loads *F* were plotted as the value of *w*/F on the vertical axis versus *w* on the horizontal axis. The inverse slope of the Southwell plot gives the theoretical buckling failure load (P_C) for this eccentrically compressed shaft as the load when deflections go towards infinity.

To verify the calculated buckling loads from the Southwell plots and the measured maximal static force, one FEM analysis was performed for one shaft with a tapered shape at the tip and one shaft with constant outer and inner diameters.

The shockwave propagation velocity for a general shaft model was used for investigating if the impact test could be analyzed in a quasi-static manner or if the impact could cause a dynamic buckling scenario. Calculating the shockwave propagation velocity was done by:

$$c_p = \sqrt{(E\rho^{-1})} \tag{3}$$

where *E* is the estimated *E*-modulus [103] for the carbon fibre reinforced polymer used in the shaft and *ρ* is the density of the shaft. The shockwave propagation time to move along the length *L* of a pole was calculated by:

$$t = L/c_p \tag{4}$$

Paper 3

The design of the test setup

Pairs of skate skis were glide tested inside a ski-tunnel (Torsby, Sweden). Temperature and humidity in air and snow inside the tunnel were constant. The test track consisted of a slope with a vertical drop of about 7%. The slope was followed by a flatter glide-section with a vertical drop of about 1%.

Two different pairs of skis were prepared by a professional ski-waxer, working for the Swedish national cross-country team. One pair of skis was waxed for maximal glide and high performance (HP). The other pair of skis, was deliberately less well waxed (LP). All glide tests were performed by two professional test-skiers (TS1 and TS2) from the Swedish national cross-country team.

Speed-traps, consisting of two photocells, were used to measure the skiers' instantaneous velocities at three positions along the track. A calibrated Doppler radar connected to a computer was positioned at the second speed-trap and aligned along the tracks, providing distance data based on the measured velocity.

Also, each ski was equipped with one inertia measurement unit (IMU) consisting of a 3D-accelerometer, a magnetometer and a gyroscope. Stationary magnets were positioned next to the ski-track and in the middle of each speed-trap to generate a spike in the magnetometer data when passing each speed-trap. Each test run was performed to a total stand-still in order to get a zero-acceleration point for re-calibration of any drift in the IMUs.

Study design

Each test skiers performed 7 glide test for the HP and LP skis respectively. The skiing velocity *v* at each speed-trap was calculated by:

$$v = s/t \tag{5}$$

where *s* and *t* are the distance and the measured time between the photocells in each speed-trap, respectively. Based on the velocity mean values, the four most representative trials, one for each test-skier and ski-pair based on mean values, were selected for detailed analysis between speed-traps one and two. Distance from the Doppler radar data was calculated and compared with the measured distance between speed-trap one and speed-trap two.

Data processing
For evaluation of friction, the normal force was considered to be equal to the gravity force *mg*, and the measured distance along the track equal to its horizontal projection, disregarding the small slope.

Potential energy W_p along the track was calculated from the vertical elevation at each point based on the spline model of the test slope. The kinetic energy was calculated from a 9:th degree polynomial, fitted to the raw-velocity data provided by the Doppler radar. The friction coefficient between the first two speed-traps could then be described by:

$$\mu_1 = \frac{P-P_D}{mgv} \tag{6}$$

where *P* is the total power loss, $P_D = 0.5\rho v^2 C_d A v$ is the power from air drag with ρ the air density, *v* the velocity against air, C_d the drag coefficient and *A* the frontal area.

As described by Leino et al [104], an average friction coefficient μ_2 on flat ground was calculated as a reference value, neglecting the small effect of the slope α:

$$\mu_2 = \left(m(v_0^2 - v_1^2) - A'\rho\left(\frac{v_0^2+v_1^2}{2}\right)^2\right)/2mgs \tag{7}$$

where *s* is the distance between the two last speed-traps, and v_0 and v_1 represent the skiing velocities at the second and third speed-traps respectively. The differences between μ_1 and μ_2 were calculated for each of the four tests at the same velocities as used when calculating μ_2.

Paper 4

The design of the test setup
The study contained two parts, where the first part was a validation study of a real-time locating system (RTLS) used for tracking cross-country skiers. The second part was the collection of positioning data for 70 participants in a cross-country sprint skiing race.

The RTLS (Quuppa Oy, Espoo, Finland) consisted of 20 locators, positioned around a 1.4 km long cross-country skiing sprint course. Each locator was mounted to a rigid object with a height of 3.5 – 11.5 m above the track and was connected to a POE (Power Over Ethernet) switch. Each locator position was precisely measured by using a total station connected to a Global Navigation Satellite System (GNSS) receiver. Each locator comprised a directional antenna array which was calibrated in order to retrieve their orientation in space. The reference points for the calibration of the whole RTLS system were 26 fixed positions around the sprint course, all of which had location coordinates measured by the GNSS receiver. Positioning data were collected at 50 Hz from the tags attached to the athletes. The track had a 29 m altitude difference between the lowest to the highest point on the course which caused problems for the RTLS to accurately measure the vertical coordinates along the *z*-axis. Therefore, the coordinates of the middle of the course were measured approximately every 0.5 m around the entire track, using a Real-Time Kinematic Global Navigation Satellite System (RTK GNSS) with a 99.99% position survey reliability [91, 92]. The track profile obtained from the RTK GNSS data was then used as a lookup topography table by the RTLS to estimate the *z*-coordinates by using the RLTS measured *x*- and *y*-coordinates.

Part one, validation study
The validation was performed by a test skier, equipped with a backpack containing the rover for the RTK GNSS, as described by Andersson et al. [91]. Three RTLS tags were attached to the RTK GNSS antenna. The skier performed three laps with different intensities around the track (low, medium and high intensity), while simultaneously recording positioning data with both the RTLS and RTK GNSS. The RTK GNSS collected data at a 20 Hz sampling rate and the RTLS at 50 Hz.

A Kalman filter was applied to the RTLS data and the data from the three different validation runs were synchronized with the RTK GNSS data by detecting the start movement and the coordinates for the finishing line. All RTLS data were decimated from 50 Hz to 1 Hz and 0.5 Hz and the typical error (TE) between the three RTLS tags was calculated.

The d-GNSS measured the actual movement of the skier's upper body while skiing which included the flexion/extension of the upper body. Hence, the RTK GNSS overestimates the skied distance and skiing velocity as the travelled distance of the upper body is longer than the skied distance.

Spline models, based on the 1 Hz and 0.5 Hz positioning data, were constructed. Instantaneous skiing velocities and distances were numerically calculated by enumerating the spline functions into segments of 1 ms. Knots for the spline interpolation were taken at even, specified intervals not considering the estimated quality of these measurements. For comparing purposes, both spline models and numerical analysis were applied for the 1 Hz and 0.5 Hz RTLS data sets.

Part two, data collection from a cross-country skiing sprint race

Continuous RTLS data were collected for the 30 best FIS-ranked female skiers and the 40 best FIS-ranked male skiers during the Scandinavian Cup Sprint Cross-Country Race in Falun, Sweden, 2015. A synchronization marker was added manually for each start in order to determine the starting time for each skier. RTLS data were collected for the time-trials and for the skiers who qualified to the following finals. As the RTLS data from the finals contained poor quality data, only five time-trials with continuous positioning data, combined with finish times corresponding to the official results where chosen for further investigations.

Paper 5

The design of the study

The collected positioning data from Paper 4 were also used in Paper 5. For analysis purposes, only two skiers (one male and one female) who qualified to "big" finals (positions 1-6) were chosen for further power calculations. Spline models for coordinates as functions of time were constructed and the instantaneous skiing velocities and distances were numerically calculated by enumerating the spline functions into segments of 1 ms.

In the calculation model, the track was described in a 2D vertical plane where *s* is a horizontal coordinate, defined as the accumulated distance in the horizontal plane (*x,y*), and *h* is the height above an arbitrary zero level. The resulting skiing speed *v* was calculated from the RTLS positioning data.

The propulsive power was defined as: [105]

$$P_{\text{prop}} = mva + \mu mg\cos(\alpha)v + mg\sin(\alpha)\,v + 0.5\rho C_d A v^3 \quad (8)$$

where *a* is the acceleration of the skier along the simulated plane track profile, *μ* the friction coefficient between snow and skis, *m* the mass of the skier, *g* the gravitational constant and *α* the instantaneous slope angle. The last term defines the air drag where ρ is the air density, C_d is the drag coefficient and *A* is the projected frontal area of the skier. The drag area $A' = C_d A$ varies depending on the skier's size, clothing and skiing position [98, 104, 106]. As only propulsive power while skiing in an upright position was considered in the current study, A' was set to 0.45 m^2 [106]. Only forward generating P_{prop} was considered in this work. For skiing velocities $v \geq 10$ m/s, the skiers were assumed to use the deep-tuck position to minimize air drag, while gliding, and the propulsive power produced by skiers was hence set to zero [98]. Wind speed was not considered in the simulations as the weather during the race was calm.

From the enumerated instantaneous values of power, two different averages were calculated, active propulsive power $\bar{P}_{\text{prop}}^{\text{active}}$, which only included the sections where the skiers actively created propulsion, and one full race propulsive power $\bar{P}_{\text{prop}}^{\text{race}}$, which in the average included the sections where the propulsive power was assumed as zero.

Chapter 3. Results and discussion

3.1. Objective test methods

Papers 1 and 2 both resulted in new objective test methods which allow alpine ski helmets and cross-country ski poles to be tested and evaluated in an objective, yet sports specific, context.

Paper 3 combined standard glide test procedures and theoretical calculations which resulted in a basic method to evaluate detailed ski glide parameters as functions of glide velocity. The experiments showed possible measurements techniques for position, velocity and acceleration data, but also pointed out the need for detailed and accurate topography measurements.

Paper 4 employed a real-time locating system to track cross-country skiers during a sprint race. Skiing trajectories and velocities could be analysed by interpolating splines where the decimated positioning data were used as knots.

Paper 5 showed the possibilities to estimate propulsive power variation during cross-country sprint skiing by applying a power balance model to spline-interpolated measured positioning data. The suggested method enables in-depth analyses of power output and pacing strategies and can be used in both training and racing situations.

3.2. Paper 1

The main result in this study was the development of a new methodology for testing alpine ski helmets in a realistic way in regard to the identified issue with repetitive impacts, when hitting the gates in alpine ski racing. The use of a 10 kHz sampling frequency for the 3D accelerometer, mounted at the centre of mass of the headform, provided reliable translational acceleration data of the headform when exposed to impacts from the rotating pole. The foremost finding was the identification of the differences between expanded polystyrene EPS and expanded polypropylene EPP as core materials, where EPS showed a higher level of breakage, higher peak accelerations and a shorter time to peak acceleration compared to EPP. Hence, EPP is a better suited core material in alpine ski helmets when these are exposed to repetitive low energy impacts. In addition, the findings suggest that helmets' capacity to withstand and absorb repetitive impacts is an important factor to consider when designing alpine ski helmets.

The impact velocity was restricted to 13.3 m/s, which is similar to reported velocities in slalom but lower than measured velocities in events like giant slalom, super G and downhill [107-110]. Another limitation in the study is how the rotating pole hits the helmets instead of vice versa, as in real skiing. The rotating slalom gate has less mass than a fast moving alpine ski racer, resulting in a smaller impact impulse in the test setup compared to the impulse that arises when skiing. However, the impracticality of building a test setup with a fast moving monitored headform, hitting a fixed slalom gate with the right movement, is considerable.

The results showed that repetitive violence, although if it is of low energy, is a significant factor for the deterioration of alpine ski helmets and should therefore be considered when designing and testing the helmets. However, the study only investigated how the repetitive violence affects the protective materials of the helmets, and designed an objective test method for testing helmets with regard to repetitive violence. The energy absorption capacities of the helmets after withstanding 1000 impacts were not investigated here. This would require extensive test equipment which was not available at the time of the study, but should be included in future studies. However, it can be hypothesized, based on visible damage to the tested helmets, that the energy absorption capacity of a helmet after being exposed to 1000 repetitive low- to medium level impacts will decrease, especially if the impact area is the same as the area which previously has been exposed to the repetitive impacts.

3.3. Paper 2

The newly developed test device allowed a more skiing specific way of loading and evaluating cross-country ski poles. In the maximal load tests, all poles were deemed to be moving towards a buckling-type failure, with increasing transversal deflection, but without primary material failure. The main data finding show how each shaft reached a plateau, here named Maximal Force Transfer (MFT) where no more force could be transferred by the shaft. The MFT value was individual for each pole but clear differences in MFT were observed between the different models and pole lengths. In general, MFT decreased with increased pole length, coupled to an increasing shaft deflection with increased pole length and applied load. The measurements also gave a possibility to evaluate the bending flexibility, and indirectly the energy storing capacity of the poles. Both these aspects contributes to the “feel” of the poles in skiing.

Peak forces from the impact test were on average 24% higher than the MFT values. The results showed up to 25% reduction of force transfer caused by the poles. The average time to peak pole force was about 18.0 ms for all tests, which is longer than the mean shockwave propagation time between the ends. This suggests that a quasi-static view on the loading situation is appropriate, and that the dynamic effects are limited, even in an impact situation.

On average, the MFT values were somewhat lower than the calculated critical loads based on the Southwell plots. Critical load from the performed FEM simulation was close to the critical load result for the shaft with constant diameter. It is likely that the materials in the tested shafts are non-linear when exposed to high loads and deflections, due to the decreasing cross-sectional area of the shafts and the eccentrically applied load. The issue with a tapering shape of the shaft is mentioned by Huntley et al. [111], who compared static and dynamic behavior of carbon fibre composite golf club. The difference between measured and theoretical values shows the difficulty in using theoretical calculations to estimate the flexural rigidity in carbon fiber reinforced polymer shafts

3.4. Paper 3

The inertia measurement units IMUs attached to the skis provided no usable data due to high drift and variations in the units. The Doppler radar provided accurate velocity readings when compared to the speed-traps. The mean maximal velocity difference between speed-trap two and the Doppler radar was 0.6% for all tests. The distance and time data provided by the Doppler radar overestimated somewhat the distance and time between speed-traps one and two

The low performance LP skis were in total 5.6% slower than the high performance HP skis along the whole test track. The HP skis had higher acceleration between speed traps one and two compared to

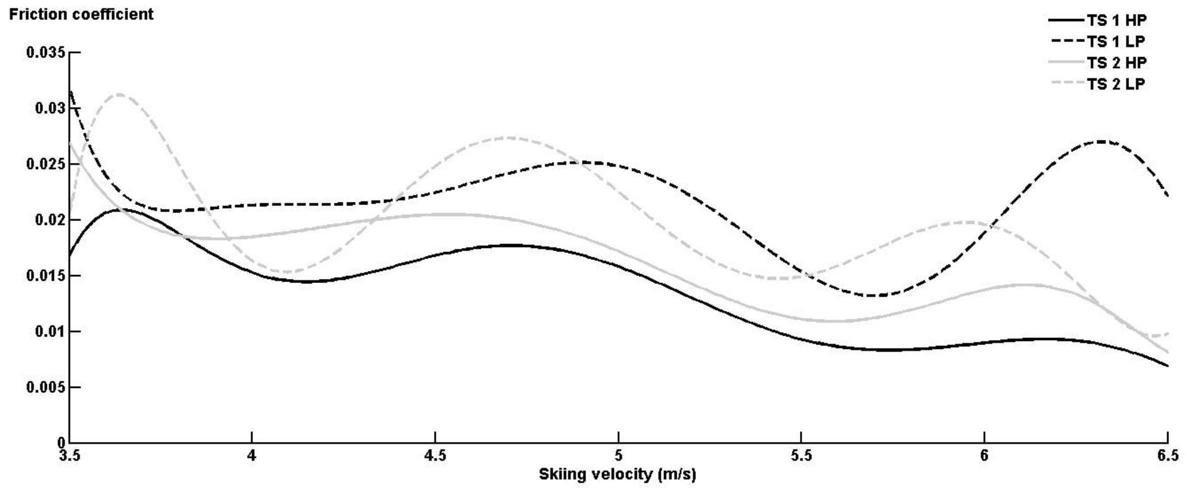

Figure 3. Comparison of the friction coefficient between the four selected trials during the acceleration phase. (Reproduced from [112].)

the LP skis with a velocity increase of 96.1% compared to 95.2% for the LP skis. The decrease in velocity between speed traps two and three was 25.7% for the HP skis compared to 31.9% for the LP skis. An explanation of the higher velocity loss with the LP skis in the more flat final part would have needed more detailed measurements but point to the need for velocity dependent glide parameters.

An underestimation of the energy loss W_p occurred due to too few data points for the spline model describing the track. The energy loss became too small at certain points, resulting in calculated negative values for power *P*. To avoid this, the spline model used to describe the track was therefore modified to a straight line.

The calculated friction coefficients during the acceleration phase all showed similar sinusoidal tendencies, Figure 3. The mean friction coefficients for HP and LP during the acceleration phase were 0.014 and 0.020, respectively, and the mean velocity was 5.23 m/s and 5.07 m/s. In gliding with lower velocity, the HP skis tended to give a friction coefficient of 0.002 lower than the LP skis.

The few measurement points of the test hill resulted in a poor spline model which affected the energy calculations. For future studies, the precision of the test track estimations need to be increased to achieve higher accuracy of the friction coefficient calculations. A differential global navigation satellite system (dGPS), as used by several authors [91, 109, 113, 114], can be used to achieve higher accuracy when measuring the test hill. Future evaluations of the friction coefficient, based on energy balance considerations, must also include more accurate estimations of the test skiers' drag area to include the correct energy dissipation due to air resistance in relevant human postures and at different velocities.

3.5. Paper 4

The data from the three real-time locating system RTLS tags attached to the real-time kinematics global navigation satellite system RTK GNSS antenna could not be used to calculate the centre of the antenna, since for all three test trials, the mean distance between the tags in the *x, y* and *z* axis was 0.09 ± 1.65, 0.17 ± 1.15 and 0.00 ± 0.15 m, respectively. As a consequence of the differences in the positioning accuracy between the tags, the maximal time differences between two tags for reaching the coordinates for the finish line during the three validation runs were 0.24 s, 3.16 s and 1.62 s for low, medium and high intensity, respectively. The mean typical error TE between the three tags for the three different intensities were 1.36 m, 1.36 m and 0.20 m for the *x*-, *y*- and *z*-axes respectively at 50

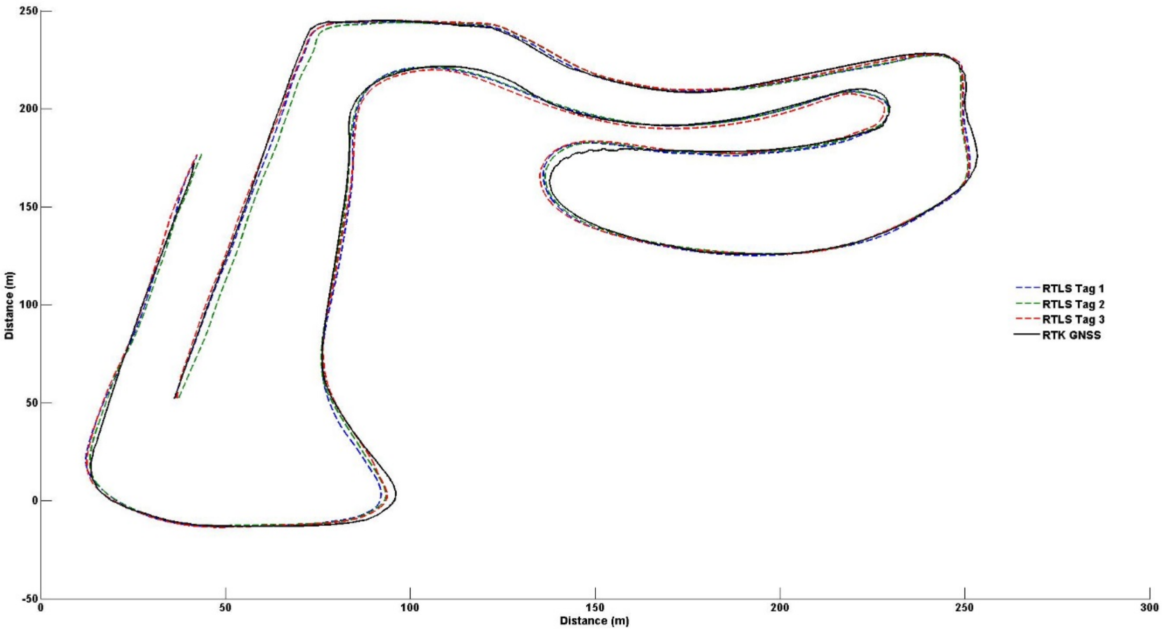

Figure 4. The in *xy*-positioning from the three RTLS tags (real time-locating system), and the RTK GNSS (real-time kinematics global navigation satellite system) while skiing around the track. (Reproduced from [115].)

Hz sampling frequency. The corresponding TE values for 0.5 Hz sampling frequency were 0.53 m, 0.31 m and 0.8 m.

The *xy*-positioning data from the three RTLS tags and the RTK GNSS during high intensity skiing are presented in Figure 4. The graphical comparison between the RTLS tags and the RTK GNSS show only small differences, which suggests that the RTLS solution can be useful for comparing the routes taken by skiers along a course.

No data from the finals were analyzed due to too many artifacts and uncertain accuracy. This was possibly due to an increased number of spectators. Hence, the method of using splines was only applied to the RTLS data from the five chosen time-trials where the spline model based on reduced sampling frequency proved applicable. Based on the five analyzed time trials, the 0.5 Hz spline model underestimated the total skied distance by an average of 6 m, compared to the 1 Hz spline model, suggesting that 0.5 Hz positioning data is sufficient to use for spline interpolation for analyzing skiing velocities, skied trajectories and distances.

In the current study, this method is shown to work well. However, decimation does not take the accuracy of the data points into account and obvious outliers may therefore be included. One possible solution to identify outliers is Chauvenet's criterion, which defines an acceptable scatter around a mean value from a given sample size [116]. The criterion combined with a condition which defines an acceptable variance, can be useful for detecting outliers and used together with a stepping function

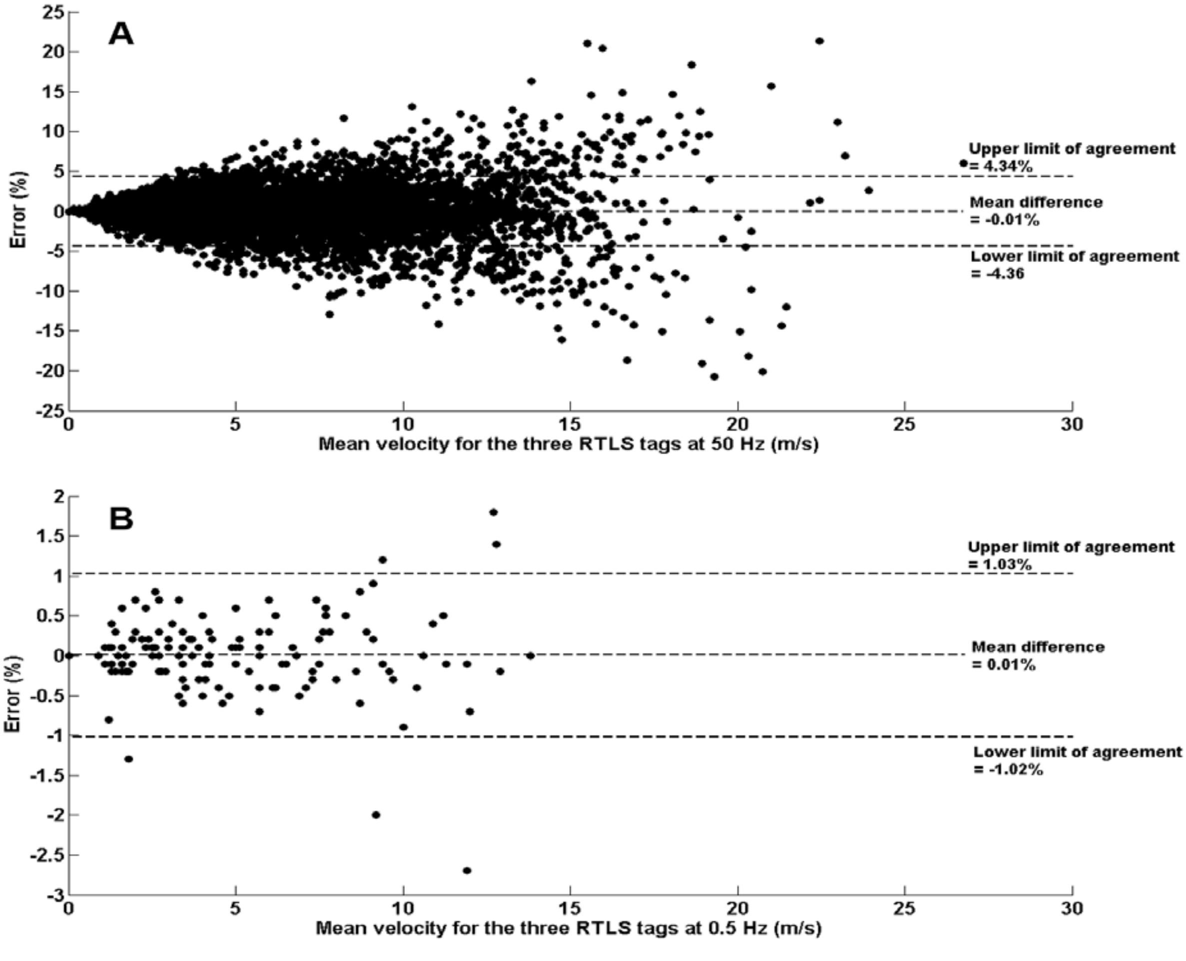

Figure 5 A-B. Bland-Altman plots for the reliability of the velocity data of the three RTLS tags with sampling at (A) 50 Hz and (B) 0.5 Hz. Notice the different scales on the *y*-axes between A and B. (Reproduced from [115].)

when seeking knots for the spline function. It can be assumed that such a procedure could have somewhat improved the quality in the interpolated data.

A decrease in the reliability of the velocity data with increased skiing velocity can be noted, Figure 5A. During high intensity skiing, the reliability of the RTLS velocity data was higher for the 0.5 Hz than for the 50 Hz sampled data (Figure 5B vs. 5A). These results are in line with findings by Frencken et al. [117] and Stevens et al. [118], who reported velocity and intensity dependent accuracy and TE for local position measurement systems LPM in soccer. In addition, Ogris et al. [119] states that LPM is suitable for estimating average velocities, but is less reliable for high intensity movements and when measuring instantaneous velocities. This gives support to our findings.

3.6 Paper 5

The mean skiing speed was 0.4 m/s higher in the time trials compared to the finals for both skiers. The average full race propulsive power $\bar{P}_{\mathrm{prop}}^{\mathrm{race}}$ was 5 % and 4 % lower in the final compared to the time trial

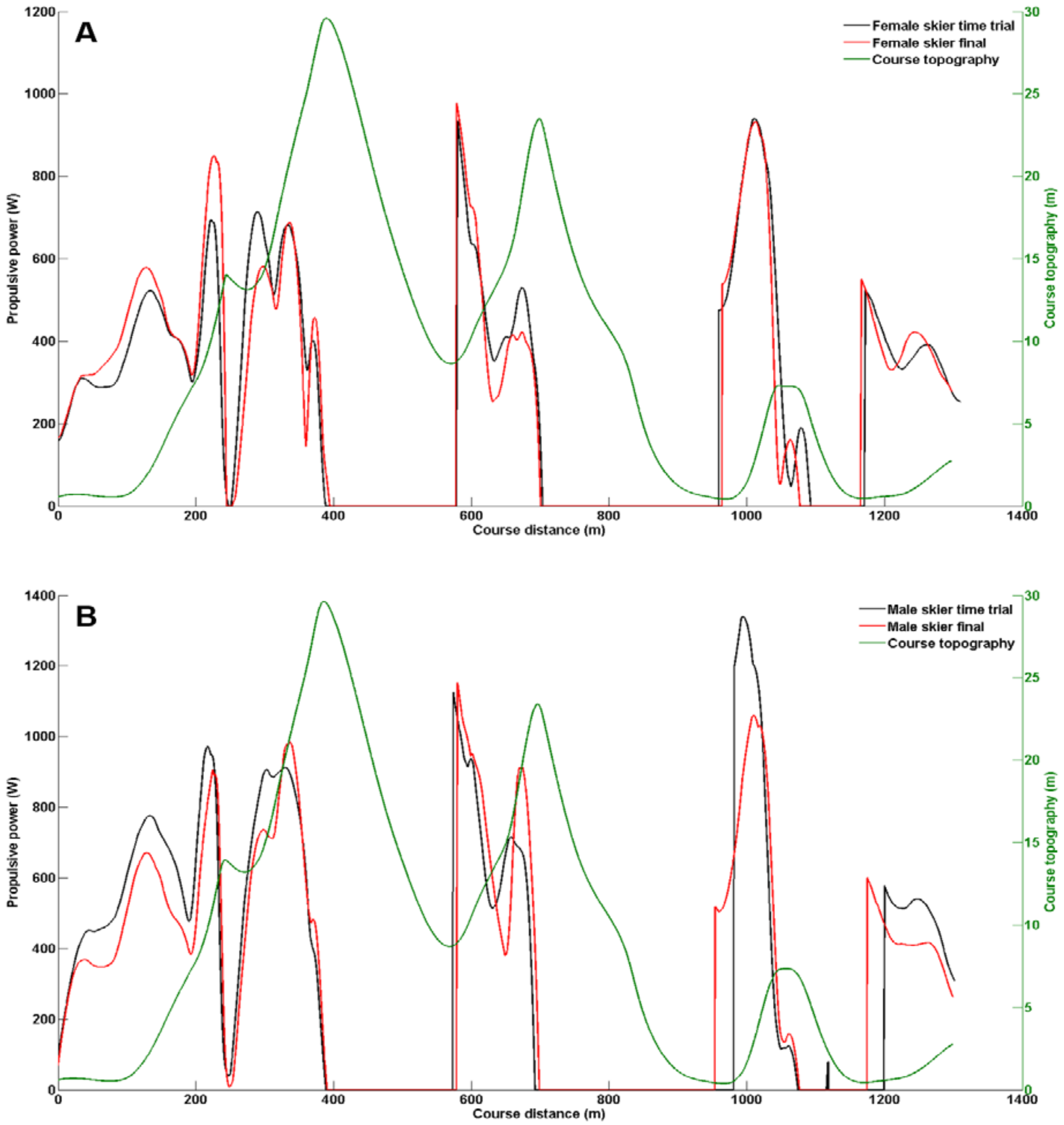

Figure 6. A-B. Propulsive power along the course for the female (A) and the male (B) for the time trial and the final.

(296 W vs 311 W and 386 W vs 400 W) for the female and male skier respectively. The active propulsive power in the time trial and final were calculated to be $\bar{P}_{\mathrm{prop}}^{\mathrm{active}}$ = 414 W and $\bar{P}_{\mathrm{prop}}^{\mathrm{active}}$, = 404 W for the female and $\bar{P}_{\mathrm{prop}}^{\mathrm{active}}$ = 577 and $\bar{P}_{\mathrm{prop}}^{\mathrm{active}}$ = 530 W for the male. The P_{prop} distribution along the course for both skiers and situations is presented in Figure 6, and compared to the height profile of the track.

The mean P_{prop} values in the current study are similar to those presented by Sundström et al. [55, 95], who reported a mean P_{prop} of 346.8 and 376 W, when simulating the optimal pacing strategy for skiers with body masses of 75 kg and 82 kg, respectively, along cross-country skiing sprint course. Anderson et al. [120] also presented similar values when simulating a cross-country skiing sprint race on a treadmill. Even if the course in the present study is different to the simulated courses the values are similar, suggesting that the method used in the current study provides relevant results regarding P_{prop}. However, as shown in the present study, the average active propulsive power is higher compared to the full race propulsive power (33% and 44% in the time trials, and 36% and 37% in the finals for the female and male, respectively). Hence, the concept of average propulsive power must be clearly defined when used to compare different skiing performances and computer simulations.

The results show differences in power output strategies between genders and heats. Both the female and the male skier used higher average propulsive power and skiing speeds during the time-trials than in the finals. In the final, compared to the time trial, the female skier increased the average propulsive power during the first 20 s by 11%, whereas the male skier had a 17% decrease in average propulsive power. In addition, the female had a higher average propulsive power during the spirt on the final stretch while the male produced less power in the final compared to the time trial. However, the benefits of slip streaming was not considered in the calculations and the friction coefficient, the drag area, friction coefficient and the added mass of the equipment were estimated based on previous studies. Furthermore, the suggested cutoff speed for being in the deep-tuck position could vary between skiers and heats and might also be affected by the level of physical fatigue. The accuracy of the propulsive power estimate can hence be increased. To further improve the presented method, future studies are needed, where respiratory values are collected simultaneously with the positioning data and with time-synchronized video footage.

In summary, the paper presents a novel approach to estimate propulsive power during cross-country sprint skiing by applying a power balance model to spline-interpolated measured positioning data. The results show that cross-country sprint skiers during some race phases produce higher propulsive power than what is shown in previously published studies [55, 95, 98, 120]. The evaluations show that the average power over a full race is based on a combination of relatively high propulsive power during some parts of the race, and some phases where gliding is used a recuperation. Hence, the suggested method enables in-depth analyses of power output and pacing strategies, which makes it suitable for both training and racing situations.

Chapter 4. Conclusions

This thesis presents an attempt to introduce more strict mechanics and mathematics into the analysis of sports and sports equipment. New methods for objective and sports specific analysis methods have been developed. The resulting settings generate more relevant results which are directly applicable for enhancing sports performance. Previous analysis methods have only used conventional methods or theoretical simulations, whereas the papers in this thesis are presenting new sport specific methods for testing sports equipment and analysing athletic performance.

A central part of the work has been to identify question formulations for important variables which affect the overall performance. Sports engineers and performance analysts should always strive to analyse equipment and performance in the most realistic setting possible. The results from these studies show the possibilities in designing tests devises, data collection and analyses which are sports specific oriented to increase the relevance of the results. Also, the findings show how theoretical calculations can be implemented to increase the objectivity and the understanding of the results when testing sports equipment and analysing athletic performance. In a widening perspective, the presented results and insights can, although here mainly focussing on skiing, also be applied to other sports, such as kayaking, canoeing, cycling, rowing and sailing where sports engineering and performance analysis are of vast importance.

The results from the five different papers relate to different fields of sports engineering, sports mechanics and performance analysis. They have approached measuring and analysis issues in sports in an objective, yet sports specific way.

Paper 1 presents a method to test helmets in order to increase the protective capacities of alpine ski race helmets. The developed test device allows easy adjustment of the impact velocity and easy replacement of the impact tool. The findings suggest that helmets constructed with expanded polypropylene EPP cores are more suitable for absorbing multiple low impacts caused by alpine gates. The capacity to withstand and absorb repetitive impacts is an important factor to consider when designing alpine ski helmets. In addition, the presented method can be used for repetitive impact tests for other helmets than alpine ski helmets, for instance ice-hockey helmets, climbing helmets or boxing headgear. A next step in development could be to investigate how repetitive low energy impacts affect the energy absorbing capacity of helmets during standardized drop tests. Another important step could be to develop a test method where objective and repetitive drop tests can be performed on snow. This would increase the authenticity of such a test and also increase the insights regarding the mechanisms which can cause concussion and brain injury.

Paper 2 presents a new objective, yet sport specific, test device for testing cross-country ski poles. The suggested Maximal Force Transfer MFT value and deflection measurements offer accurate and reliable data for establishing the flexural rigidity in carbon fiber reinforced polymer cross-country ski poles. Hence, MFT could be a fair parameter to use as a comparison standard between different shaft models and brands. The results also show extra complexities introduced when using established mechanics calculations for calculating, e.g., the buckling failure load for struts as poles, when these are constructed from highly optimized material combinations, have a tapered shape and are eccentrically loaded. The next step could be to investigate how the MFT value affects skiing performance and also how it correlates to the skiers' subjective feelings. The energy storage in the poles, can also be further investigated. Similar mechanical testing characterization is believed to be highly relevant for, e.g., pole vaulting poles, and for sticks used in several sports.

Paper 3 examines different methods to analyze the glide of different cross-country skis and to determine the friction coefficients for skis by employing energy balance calculations along a glide test track. The findings reveal that cross-country skis must be tested and evaluated at different velocities in order to fully describe their gliding characteristics and friction coefficients. The Doppler radar data proved to be both accurate and consistent. However, theoretical friction calculations, based on the transformation of potential energy to kinetic energy, are very sensitive to the accuracy of the topography measurements of the track. Future studies should therefore focus on accurate and very detailed topography measurements in order to accurately evaluate momentary track height and inclination, to allow more accurate evaluations of momentary energy balances and thereby the friction between skis and snow.

Paper 4 investigates the possibilities to employ a real-time locating system RTLS to follow cross-country skiers on a track. This a tool for deducing mathematical expression for further analysis of performance. When validated against a real-time kinematics global satellite system RTK GNSS of acknowledged accuracy, the RTLS provides precise estimates of the mean skiing velocity but is lacking in accuracy for determination of momentary velocities, even if spline interpolation can improve this. The collected continuous positioning data enables detailed analyses of trajectories and skiing speeds in cross-country ski racing. The next step should be to evaluate different methods to detect and exclude outliers in the positioning in order to improve the accuracy of the spline interpolation. Future projects should also implement the presented method in other sports, such as rowing, canoeing, running and alpine skiing where the collected data can be used for evaluating racing tactics and pacing strategies. Accurate position and velocity measurements also have many other interesting applications in sports itself, but also for, e.g. media coverage of different sport events.

Paper 5 presents a novel approach to evaluate pacing strategies and race tactics in cross-country sprint skiing. Based on collected real-time positioning data, the power-balance model provides mean propulsive power in line with previous research [55, 95, 120]. However, the presented method enables post-race analysis regarding propulsive power distribution during a cross-country sprint skiing race, whereas previous studies regarding pacing have been theoretical computer simulations [55, 94, 95, 98, 105]. The post-race analysis provides the actual pacing strategies, employed by the skiers during the race. This information allows in-depth analysis regarding pacing, tactics and the physical demands in cross-country ski racing. The data can also be used to further develop individually adapted physiological fatigue models to simulate human power output under regimes of variable work rates. A next step could be to, based on individual status parameter, seek the mathematical optimal pacing strategies on a specific track.

In general, it can be concluded from the performed studies that common engineering and mathematical tools can be used to advantage in analysing sports and sports equipment. Variables used when analysing sports equipment and in sports performance analysis need to be valid and reliable and collected in an objective manner. Objectivity exists where automated systems measure variables without human judgement involved [121]. In spite of the rather simplified situations in *Papers 1-3*, the presented approach of objectively analyse the performance of sports equipment, allows for this kind of objectively evaluation of sports equipment. However, the increased use of sports engineering for enhancing athletic performance can also cause controversies and be considered to challenge athletic skills. On the other hand, sports engineering and the development of new equipment is important to enhance safety and prevent injuries. Still, the key aspect regarding development and optimization of sports equipment should always be to follow the current regulations, as any sort of doping contradicts the very spirit of sport. It is believed that the implantation of objective measurement techniques can, and will, have an impact on these regulations.

The presented method to analyse skiing performance in *Papers 4-5*, offers accurate positioning tracking of larger volumes and can easily be implemented in other sports, such as rowing, speed skating and alpine skiing. This would make studies more efficient and representative as more detailed data can be gathered and analysed without any subjective human influence. However, tracking systems need to be set up carefully as misplacement of a locator can lead to inaccurate positioning data. Smoothing algorithms can remove noise in the collected data but objective and automatic methods to detect and remove outliers need to be developed and applied for different sports. Hence, sport specific knowledge and close collaboration between athletes, coaches, technicians, biomechanical experts and engineers are needed to successfully develop and implement new objective analyses methods in the mechanics of sports.

This thesis presents new tools and knowledge regarding objective analyses in sports which is of great importance as small adjustments to a piece of equipment and/or changes in racing tactics can help optimize athletic performance and increase safety, for elite and recreational athletes.

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