Math 150 Lecture Notes
Real Numbers

The **natural** or **counting numbers** are 1, 2, 3, 4,…

The **whole numbers** consist of the natural numbers and 0:
0, 1, 2, 3, 4,…

The **integers** consist of the natural numbers together with the negatives and 0:
…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …

The **rational numbers** are numbers that can be written as an integer divided by an integer (or a ratio of integers). Examples: ½ -¼ 0.19 4.27 31

The **irrational numbers** are numbers that cannot be written as an integer divided by an integer. Examples: $\sqrt{3}$ π $\sqrt[3]{5}$ e

Properties of Real Numbers

Commutative Property
for Addition: $a + b = b + a$
for Multiplication: $ab = ba$

Associative Property
for Addition: $(a + b) + c = a + (b + c)$
for Multiplication: $(ab)c = a(bc)$

Distributive Property
$a(b + c) = ab + ac$ or $(b + c)a = ab + ac$

Additive Identity
$a + 0 = 0 + a = a$

Subtraction is the inverse operation for addition ("undoes" addition) and is the same as adding the negative of the number to be subtracted: $a - b = a + (-b)$

Properties of Negatives
1. $(-1)a = -a$
2. $-(-a) = a$
3. $(-a)b = a(-b) = -(ab)$
4. $(-a)(-b) = ab$
5. $-(a + b) = -a - b$
6. $-(a - b) = b - a$

Multiplicative Identity

$a \cdot 1 = 1 \cdot a = a$

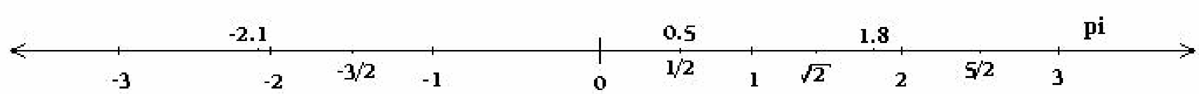
Division is the inverse operation for multiplication ("undoes" multiplication) and is the same as multiplying by the reciprocal: $a \div b = a \cdot \frac{1}{b}$

Properties of Fractions

1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
5. $\frac{ac}{bc} = \frac{a}{b}$
6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$

<u>The Real Line</u>

The real numbers can be represented by points on a line as shown below. The real numbers are **ordered**. Geometrically, if $a < b$, then a lies to the left of b on the number line.



<u>Sets and Intervals</u>

A **set** is a collection of object, called **elements** of the set.

Notation: $a \in S$ means "a is an element of set S."

$\notin$ means "is not an element of"

$A = \{x \mid x \text{ is an integer and } 0 < x < 5\}$ is read "A is the set of all x such that x is an integer between 0 and 5".

$\cap$ - intersection $\cup$ - union $\subseteq$ - subset $\subset$ - proper subset $\not\subset$ - not a subset

Interval Notation:

$(a, b) = \{x \mid a < x < b\}$ $[a, b] = \{x \mid a \le x \le b\}$ ∞ - infinity

Absolute Value and Distance

The **absolute value** of a number is the distance from the number to 0 on the number line.

$$|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$$

Properties of Absolute Value

1. $|a| \geq 0$
2. $|a| = |-a|$
3. $|ab| = |a|\,|b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$d(a, b) = |b - a|$

Is $|a - b| = |b - a|$? Why or why not?