Chapter 2

Real Numbers

Chapter Overview and Pacing

LESSON OBJECTIVES

	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
2-1 Rational Numbers on the Number Line *(pp. 68–72)* • Graph rational numbers on a number line. • Find absolute values of rational numbers.	1	optional	0.5	optional
2-2 Adding and Subtracting Rational Numbers *(pp. 73–78)* • Add integers and rational numbers. • Subtract integers and rational numbers.	2	optional	1	optional
2-3 Multiplying Rational Numbers *(pp. 79–83)* • Multiply integers. • Multiply rational numbers.	1	optional	0.5	optional
2-4 Dividing Rational Numbers *(pp. 84–87)* • Divide integers. • Divide rational numbers.	1	optional	0.5	optional
2-5 Statistics: Displaying and Analyzing Data *(pp. 88–94)* • Interpret and create line plots and stem-and-leaf plots. • Analyze data using mean, median, and mode.	1	optional	1	optional
2-6 Probability: Simple Probability and Odds *(pp. 96–102)* • Find the probability of a simple event. • Find the odds of a simple event. ***Follow-Up:*** Use tables to investigate probability and Pascal's Triangle.	2 (with 2-6 Follow-Up)	optional	1 (with 2.6 Follow-Up)	optional
2-7 Square Roots and Real Numbers *(pp. 103–109)* • Find square roots. • Classify and order real numbers.	1	optional	0.5	optional
Study Guide and **Practice Test** *(pp. 110–115)* **Standardized Test Practice** *(pp. 116–117)*	1	3	0.5	1
Chapter Assessment	1	1	0.5	0
TOTAL	**11**	**4**	**6**	**1**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter Resource Manager

Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Prerequisite Skills Workbook	Applications*	Parent and Student Study Guide Workbook	5-Minute Check Transparencies	Interactive Chalkboard	AlgePASS: Tutorial Plus (lessons)	Materials
CHAPTER 2 RESOURCE MASTERS											
75–76	77–78	79	80		1–4, 15–16, 19–20, 45–46, 55–56, 63–66, 75–76	SM 33–36	11	2-1	2-1		
81–82	83–84	85	86	131	15–16, 19–24, 39–40, 55–60, 65–66, 75–76	GCS 26	12	2-2	2-2		
87–88	89–90	91	92		15–16, 19–20, 25–28, 39–40, 47–50, 65–66, 75–76	SC 3	13	2-3	2-3		
93–94	95–96	97	98	131, 133	15–16, 19–20, 29–32, 39–40, 47–48, 51–54, 63–66, 75–76	SC 4	14	2-4	2-4	3	
99–100	101–102	103	104		15–16, 61–62, 75–76		15	2-5	2-5		grid paper
105–106	107–108	109	110	132	17–18, 37–38, 67–70, 99–100		16	2-6	2-6		
111–112	113–114	115	116	132	75–76	GCS 25	17	2-7	2-7		grid paper
				117–130, 134–136			18				

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

ELL Study Guide and Intervention, Skills Practice, Practice, and Parent and Student Study Guide Workbooks are also available in Spanish.

Chapter 2

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

In previous courses, students learned to perform the operations of adding, subtracting, multiplying, and dividing with whole numbers. They also found square roots of whole numbers. In Chapter 1, students simplified expressions using the order of operations, the Distributive, Commutative, and Associative Properties.

This Chapter

This chapter explores basic operations with rational numbers. The number line is used as a model to develop rules for addition and subtraction of real numbers. Rules for multiplying and dividing rational numbers are also explored, as well as finding square roots. Students also create and use the statistical tools of line plots and stem-and-leaf plots to solve problems involving the measures of central tendency. Probability and odds of simple events are also explored.

Future Connections

The rules for adding, subtracting, multiplying, dividing, and finding the square root of rational numbers are essential to simplifying and solving equations correctly. Creating line plots and stem-and-leaf plots will help students better understand data in the future.

2-1 Rational Numbers on the Number Line

Natural numbers, whole numbers, and integers can be shown on a number line. Rational numbers, numbers that can be expressed in the form of $\frac{a}{b}$ where b does not equal 0, can also be displayed on number lines. Positive numbers are to the right of 0, and negative numbers are to the left of 0. Number lines can help students with the concept of absolute value. When a number is graphed on a number line, students can see the distance that number is from 0. When evaluating expressions containing an absolute value, treat the absolute value bars as a grouping symbol.

2-2 Adding and Subtracting Rational Numbers

Number lines can be used to add rational numbers. Start at the first number in an expression, and then move right when adding positive numbers or left when adding negative numbers. Students soon understand that if the signs of the numbers are the same, they are to add the absolute values and the sum has the same sign as the addends. If the signs are different, subtract the absolute values and the sum has the sign of the number with the greater absolute value.

The additive inverse of a rational number is its opposite. When you add a number and its additive inverse, the sum is 0. To subtract rational numbers, rewrite the expression to add the inverse of the second number. Then use a number line or the rules for addition.

2-3 Multiplying Rational Numbers

Multiplication is a representation of repeated addition of the same number. This concept can be used to arrive at the rules for multiplying rational numbers. Tables of repeated addition of the same number can be used to verify the "rules" for multiplying a positive number by either a negative or a positive number. The product of two negative numbers can be explained as the opposite of the result of a positive times a negative.

2-4 Dividing Rational Numbers

Multiplication and division are inverse operations. Therefore, they share the same rules. If the two numbers have the same sign, the quotient is positive. If the signs are different, the quotient is negative.

A fraction bar represents division. It is also considered a grouping symbol. Be sure to simplify the numerator and denominator separately before dividing the numerator by the denominator. One method for dividing is to multiply by the reciprocal of the second number.

2-5 Statistics: Displaying and Analyzing Data

Line plots are used to compare data. They resemble a number line with Xs written in columns above their corresponding numbers on the number line. The number line must contain all of the data and use a scale with equal intervals. The Xs represent the frequency of a number in a specific set of data.

Stem-and-leaf plots are another representation of frequency, but of numbers in a category, not the numbers themselves. The greatest common place value is used for the stems, and the numbers in the next greatest place value are written as the leaves.

Measures of central tendency are numbers that help analyze data. Mean, median, and mode are the most common. Carefully choose which measure of central tendency best describes a set of data. An outlier can affect the mean and not affect the median or mode. A number with a high frequency of repetition can cause the mode to be a poor representation of the data. If most numbers in a set of data are relatively close in value with a few extreme outliers, the median can be too low or too high.

2-6 Probability: Simple Probability and Odds

Probability describes the likelihood of an event happening. The list of all possible outcomes of an event is referred to as the sample space. The probability of an event is a comparison, in ratio form, of the number of favorable outcomes to the total number of possible outcomes for the event. The ratio can be expressed in fraction, decimal, or percent form. The value of the ratio is from 0 to 1. If the probability of an event is 0, then it will never occur. If the probability of an event is 1, then it will always occur.

The odds of an event is a comparison of the number of ways an event can occur to the number of ways it cannot occur.

2-7 Square Roots and Real Numbers

Finding a square root is the inverse of squaring a number. When a number is squared, it is multiplied by itself. The square root is the factor that was multiplied by itself to get the original number. All positive numbers have a positive square root and a negative square root that are additive inverses of each other. When there is no sign in front of a radical sign, find the positive or principal square root only. If a negative sign precedes the radical sign, find the negative square root. A $\pm$ sign before the radical sign indicates finding both square roots. There are no real numbers that are square roots of negative numbers.

All the numbers students have studied up to now are real numbers. All real numbers are either rational or irrational. Rational numbers can be written as a ratio and can be expressed as terminating or repeating decimals. Within the set of rational numbers are integers, whole numbers, and natural numbers. Irrational numbers cannot be expressed as terminating or repeating decimals. They go on forever with no repeating pattern. Pi is an irrational number.

www.algebra1.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 1 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra1.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Adding Integers (Lesson 2)

Chapter 2

DAILY INTERVENTION and Assessment

INTERVENTION

ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 67, 72, 78, 83, 87, 94, 101 Practice Quiz 1, p. 83 Practice Quiz 2, p. 101	5-Minute Check Transparencies *Prerequisite Skills Workbook,* pp. 1–4, 15–28, 31–32, 37–40, 45–54, 57–70, 75–76 Quizzes, *CRM* pp. 131–132 Mid-Chapter Test, *CRM* p. 133 Study Guide and Intervention, *CRM* pp. 75–76, 81–82, 87–88, 93–94, 99–100, 105–106, 111–112	AlgePASS: Tutorial Plus www.algebra1.com/self_check_quiz www.algebra1.com/extra_examples
Mixed Review	pp. 72, 78, 83, 87, 94, 101, 109	Cumulative Review, *CRM* p. 134	
Error Analysis	Find the Error, pp. 76, 98 Common Misconceptions, p. 104	Find the Error, *TWE* pp. 76, 99 Unlocking Misconceptions, *TWE* pp. 70, 91, 97	
Standardized Test Practice	pp. 72, 78, 83, 87, 94, 101, 106, 109, 115, 116–117	*TWE* pp. 116–117 Standardized Test Practice, *CRM* pp. 135–136	Standardized Test Practice CD-ROM www.algebra1.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 72, 78, 82, 87, 94, 100, 109 Open Ended, pp. 70, 76, 81, 86, 91, 98, 107 Standardized Test, p. 117	Modeling: *TWE* pp. 72, 109 Speaking: *TWE* pp. 78, 94 Writing: *TWE* pp. 83, 86, 101 Open-Ended Assessment, *CRM* p. 129	
Chapter Assessment	Study Guide, pp. 110–114 Practice Test, p. 115	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 117–122 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 123–128 Vocabulary Test/Review, *CRM* p. 130	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra1.com/vocabulary_review www.algebra1.com/chapter_test

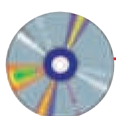
Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*

The Princeton Review's *Cracking the ACT*

ALEKS



TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

AlgePASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 1 Lesson	AlgePASS Lesson
2-4	**3** *Working with Signed Numbers*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Parent and Student Study Guide

Parents and students may work together to reinforce the concepts and skills of this chapter. (Workbook, pp. 11–18 or log on to www.algebra1.com/parent_student)

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra1.com/extra_examples
  www.algebra1.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra1.com/vocabulary_review
  www.algebra1.com/chapter_test
  www.algebra1.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 1 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 67
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 70, 76, 81, 86, 91, 98, 107)
- Reading Mathematics, p. 95
- Writing in Math questions in every lesson, pp. 72, 78, 82, 87, 94, 100, 109
- Reading Study Tip, pp. 69, 96, 97, 103
- WebQuest, p. 100

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 67, 110
- Study Notebook suggestions, pp. 70, 76, 81, 85, 91, 95, 99, 102, 107
- Modeling activities, pp. 72, 109
- Speaking activities, pp. 78, 94
- Writing activities, pp. 83, 86, 101
- Differentiated Instruction, (Verbal/Linguistic), p. 72
- ELL Resources, pp. 66, 72, 71, 77, 82, 87, 93, 95, 100, 108, 110

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 2 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 2 Resource Masters*, pp. 79, 85, 91, 97, 103, 109, 115)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*
- *Hot Words/Hot Topics* Sections 1.5, 2.1–2.6, 3.1, 3.2, 3.4, 4.2–4.6, 6.3

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 2

Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
2-1	1, 6, 8, 9, 10	
2-2	1, 6, 8, 9, 10	
2-3	1, 6, 8, 9, 10	
2-4	1, 6, 8, 9, 10	
2-5	1, 5, 6, 8, 9, 10	
2-6	1, 5, 6, 7, 8, 9, 10	
2-6 Follow-Up	1, 5, 6, 7, 8, 9, 10	
2-7	1, 6, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 2

Real Numbers

What You'll Learn

- **Lesson 2-1** Classify and graph rational numbers.
- **Lessons 2-2 through 2-4** Add, subtract, multiply, and divide rational numbers.
- **Lesson 2-5** Display and interpret statistical data on line graphs and stem-and-leaf plots.
- **Lesson 2-6** Determine simple probability and odds.
- **Lesson 2-7** Find square roots and compare real numbers.

Key Vocabulary

- rational number (p. 68)
- absolute value (p. 69)
- probability (p. 96)
- square root (p.103)
- real number (p. 104)

Why It's Important

The ability to work with real numbers lays the foundation for further study in mathematics and allows you to solve a variety of real-world problems. For example, temperatures in the United States vary greatly from cold arctic regions to warm tropical regions. You can use real numbers and absolute value to compare these temperature extremes. *You will use absolute value and real numbers to compare temperatures in Lessons 2-1 and 2-2.*

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 2 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 2 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

For Lessons 2-1 through 2-5 — Operations with Decimals and Fractions

Perform the indicated operation. *(For review, see pages 798 and 799.)*

1. $2.2 + 0.16$ **2.36** **2.** $13.4 - 4.5$ **8.9** **3.** $6.4 \cdot 8.8$ **56.32** **4.** $76.5 \div 4.25$ **18**

5. $\frac{1}{4} + \frac{2}{3}$ $\mathbf{\frac{11}{12}}$ **6.** $\frac{1}{2} - \frac{1}{3}$ $\mathbf{\frac{1}{6}}$ **7.** $\frac{5}{4} \cdot \frac{3}{10}$ $\mathbf{\frac{3}{8}}$ **8.** $\frac{4}{9} \div \frac{1}{3}$ $\mathbf{\frac{4}{3}}$ **or** $\mathbf{1\frac{1}{3}}$

For Lessons 2-1 through 2-5 — Evaluate Expressions

Evaluate each expression if $a = 2$, $b = \frac{1}{4}$, $x = 7$, and $y = 0.3$. *(For review, see Lesson 1-2.)*

9. $3a - 2$ **4** **10.** $2x + 5$ **19** **11.** $8(y + 2.4)$ **21.6** **12.** $4(b + 2)$ **9**

13. $a - \frac{1}{2}$ $\mathbf{1\frac{1}{2}}$ **14.** $b + 3$ $\mathbf{3\frac{1}{4}}$ **15.** xy **2.1** **16.** $y(a \div b)$ **2.4**

For Lesson 2-5 — Find Mean, Median, and Mode

Find the mean, median, and mode for each set of data. *(For review, see pages 818 and 819.)*

17. 2, 4, 7, 9, 12, 15 $\mathbf{8\frac{1}{6}}$**; 8; none** **18.** 23, 23, 23, 12, 12, 14 $\mathbf{17\frac{5}{6}}$**; 13; 23** **19.** 7, 19, 2, 7, 4, 9 **8; 7; 7**

For Lesson 2-7 — Square Numbers

Simplify. *(For review, see Lesson 1-1.)*

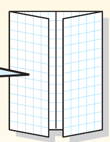
20. 11^2 **121** **21.** 0.9^2 **0.81** **22.** $\left(\frac{2}{3}\right)^2$ $\mathbf{\frac{4}{9}}$ **23.** $\left(\frac{4}{5}\right)^2$ $\mathbf{\frac{16}{25}}$

FOLDABLES™ Study Organizer

Make this Foldable to collect examples and notes about operations with real numbers. Begin with a sheet of grid paper.

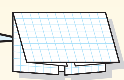
Step 1 Fold

Fold the short sides to meet in the middle.



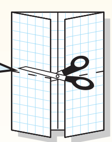
Step 2 Fold Again

Fold the top to the bottom.



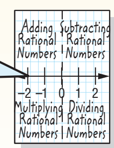
Step 3 Cut

Open. Cut along second fold to make four tabs.



Step 4 Label

Add a number line and label the tabs as shown.



Reading and Writing As you read and study the chapter, use the number line to help you solve problems. Write examples and notes under each tab.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 2. Page references are included for additional student help.

Additional review is provided in the *Prerequisite Skills Workbook*, pp. 1–4, 15–32, 37–40, 45–70, 75–76, 99–100.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
2-2	Adding and Subtracting Fractions, p. 72
2-3	Multiplying Fractions, p. 78
2-4	Dividing Fractions, p. 83
2-5	Mean, Median, and Mode, p. 87
2-6	Simplifying Fractions, p. 94
2-7	Evaluating Expressions, p. 101

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Main Ideas and Note Taking Use this Foldable to promote student writing. Note taking is a skill that is based upon listening or reading for main ideas and then recording those ideas for future reference. Under the tabs of the Foldable, have students take notes about what they need to know to add, subtract, multiply, and divide rational numbers. Encourage students to apply these concepts by writing original addition, subtraction, multiplication, and division sentences using real numbers.

2-1 Lesson Notes

1 Focus

5-Minute Check Transparency 2-1 Use as a quiz or a review of Chapter 1.

Mathematical Background notes are available for this lesson on p. 66C.

Building on Prior Knowledge

In previous course material, students learned about fractions, decimals, and negative numbers. In this lesson, they should recognize that rational numbers are a larger set of numbers containing types of numbers that students already know.

How can you use a number line to show data?

Ask students:

- What does a positive change in river level mean? **The level of the river rose.**
- What does a negative change in river level mean? **The level of the river dropped.**
- Do you think any of these rivers are flooding? How do you know? **If any of the rivers were flooding, the river level would likely be greatly increasing.**

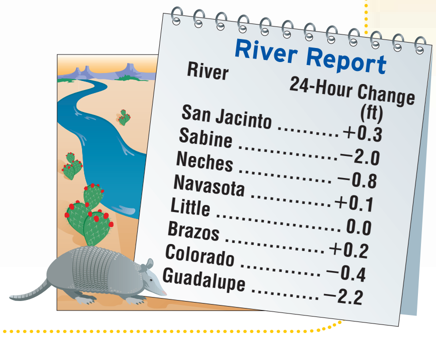
2-1 Rational Numbers on the Number Line

What You'll Learn

- Graph rational numbers on a number line.
- Find absolute values of rational numbers.

How can you use a number line to show data?

A river's level rises and falls depending on rainfall and other conditions. The table shows the percent of change in river depths for various rivers in Texas over a 24-hour period. You can use a number line to graph these values and compare the changes in each river.

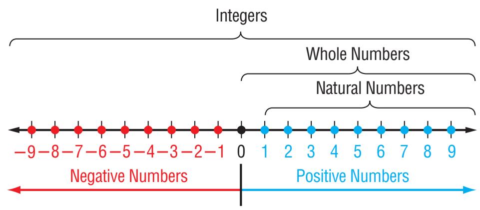


Vocabulary

- natural number
- whole number
- integers
- positive number
- negative number
- rational number
- infinity
- graph
- coordinate
- absolute value

GRAPH RATIONAL NUMBERS

A number line can be used to show the sets of **natural numbers**, **whole numbers**, and **integers**. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.



Study Tip

Number Line
Although only a portion of the number line is shown, the arrowheads indicate that the line and the set continue to **infinity**, which means that they never end.

Another set of numbers you can display on a number line is the set of rational numbers. A **rational number** is any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Some examples of rational numbers are shown below.

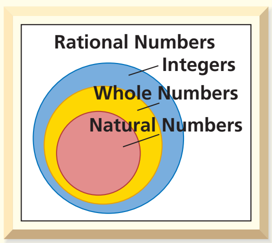
$$\frac{1}{2} \qquad \frac{-2}{3} \qquad \frac{17}{5} \qquad \frac{15}{-3} \qquad \frac{-14}{-11} \qquad \frac{3}{1}$$

A rational number can also be expressed as a decimal that terminates, or as a decimal that repeats indefinitely.

$$0.5 \qquad -0.\overline{3} \qquad 3.4 \qquad 2.6767\ldots \qquad -5 \qquad 1.\overline{27} \qquad -1.23568994141\ldots$$

Concept Summary — *Rational Numbers*

Natural Numbers	{1, 2, 3, …}
Whole Numbers	{0, 1, 2, 3, …}
Integers	{…, −2, −1, 0, 1, 2, …}
Rational Numbers	numbers expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$



Later in this chapter, you will be introduced to numbers that are not rational.

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters
- Study Guide and Intervention, pp. 75–76
- Skills Practice, p. 77
- Practice, p. 78
- Reading to Learn Mathematics, p. 79
- Enrichment, p. 80

Parent and Student Study Guide Workbook, p. 11
Prerequisite Skills Workbook, pp. 1–4, 15–16, 19–20, 45–46, 55–56, 63–66, 75–76
Science and Mathematics Lab Manual, pp. 33–36

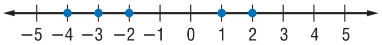
Transparencies
5-Minute Check Transparency 2-1
Answer Key Transparencies

Technology
Interactive Chalkboard

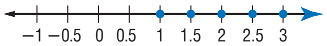
To **graph** a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the **coordinate** of that point.

Example 1 Identify Coordinates on a Number Line

Name the coordinates of the points graphed on each number line.

a. 

The dots indicate each point on the graph.
The coordinates are $\{-4, -3, -2, 1, 2\}$.

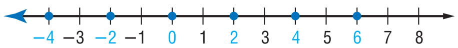
b. 

The bold arrow on the right means that the graph continues indefinitely in that direction. The coordinates are $\{1, 1.5, 2, 2.5, 3, \ldots\}$.

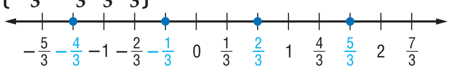
Example 2 Graph Numbers on a Number Line

Graph each set of numbers.

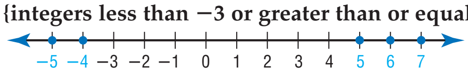
a. $\{\ldots, -4, -2, 0, 2, 4, 6\}$



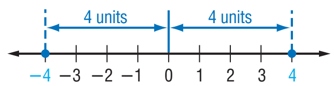
b. $\left\{-\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{5}{3}\right\}$



c. {integers less than −3 or greater than or equal to 5}



ABSOLUTE VALUE On a number line, 4 is four units from zero in the positive direction, and −4 is four units from zero in the negative direction. This number line illustrates the meaning of **absolute value**.



Key Concept *Absolute Value*

- **Words** The absolute value of any number n is its distance from zero on a number line and is written as $|n|$.
- **Examples** $|-4| = 4$ $|4| = 4$

Since distance cannot be less than zero, absolute values are always greater than or equal to zero.

Study Tip

Reading Math
$|-7| = 7$ is read *the absolute value of negative 7 equals 7.*

Example 3 Absolute Value of Rational Numbers

Find each absolute value.

a. $|-7|$

−7 is seven units from zero in the negative direction.

$|-7| = 7$

www.algebra1.com/extra_examples

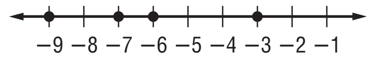
2 Teach

GRAPH RATIONAL NUMBERS

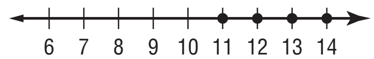
In-Class Examples Power Point®

Teaching Tip The real numbers also include the set of irrational numbers, which is introduced in Lesson 2-7.

1 Name the coordinates of the points graphed on each number line.

a. 

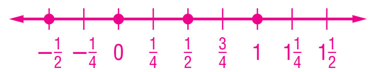
$\{-9, -7, -6, -3\}$

b. 

$\{11, 12, 13, 14, \ldots\}$

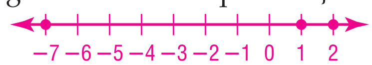
2 Graph each set of numbers.

a. $\left\{-\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$



b. $\{-1.5, 0, 1.5, \ldots\}$

c. {integers less than −6, or greater than or equal to 1}



ABSOLUTE VALUE

In-Class Example Power Point®

3 Find each absolute value.

a. $\left|-\frac{5}{8}\right|$ **$\frac{5}{8}$**

b. $|0.25|$ **0.25**

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

In-Class Example Power Point®

4 Evaluate $|y - 8| + 5$ if $y = 12$. **9**

Teaching Tip After presenting Example 4, give students the expression $|2 - x| - 7$ to evaluate for $x = 5$. **−4** Point out that while absolute values will always be positive, expressions involving absolute value may be negative.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- include the descriptions of the sets of natural numbers, whole numbers, integers, and rational numbers.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Graph Rational Numbers:** 18–33, 42, 58
- **Absolute Value:** 34–41, 43–57, 59

Odd/Even Assignments
Exercises 18–41 and 45–56 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 19–41 odd, 42–44, 45–57 odd, 60–77

Average: 19–41 odd, 42–44, 45–57 odd, 60–77

Advanced: 18–40 even, 46–56 even, 57–69 (optional: 70–77)

b. $\left|\frac{7}{9}\right|$

$\frac{7}{9}$ is seven-ninths unit from zero in the positive direction.

$\left|\frac{7}{9}\right| = \frac{7}{9}$

You can also evaluate expressions involving absolute value. The absolute value bars serve as grouping symbols.

Example 4 Expressions with Absolute Value

Evaluate $15 - |x + 4|$ if $x = 8$.

$15 - \lvert x + 4\rvert = 15 - \lvert 8 + 4\rvert$	Replace x with 8.
$= 15 - \lvert 12\rvert$	$8 + 4 = 12$
$= 15 - 12$	$\lvert 12\rvert = 12$
$= 3$	Simplify.

Check for Understanding

Concept Check

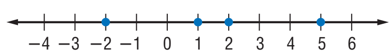
1–3. See pp. 117A–117B.

1. **Tell** whether the statement is *sometimes*, *always*, or *never* true. *An integer is a rational number.*
2. **Explain** the meaning of absolute value.
3. **OPEN ENDED** Give an example where absolute values are used in a real-life situation.

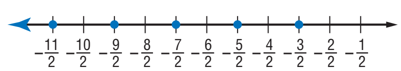
Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
4, 5	1
6–9	2
10–13	3
14–16	4

Name the coordinates of the points graphed on each number line.

4. 

$\{-2, 1, 2, 5\}$

5. 

$\left\{\ldots, -\frac{11}{2}, -\frac{9}{2}, -\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}\right\}$

Graph each set of numbers. 6–9. See pp. 117A–117B.

6. $\{-4, -2, 1, 5, 7\}$
7. $\{-2.8, -1.5, 0.2, 3.4\}$
8. $\left\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{5}{3}\right\}$
9. {integers less than or equal to −4}

Find each absolute value.

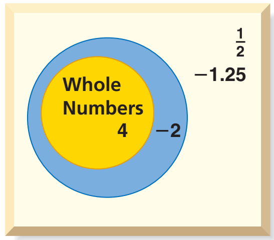
10. $|-2|$ **2**
11. $|18|$ **18**
12. $|2.5|$ **2.5**
13. $\left|-\frac{5}{6}\right|$ $\frac{5}{6}$

Evaluate each expression if $x = 18$, $y = 4$, and $z = -0.76$.

14. $57 - |x + 34|$ **5**
15. $19 + |21 - y|$ **36**
16. $|z| - 0.26$ **0.50**

Application

17. **NUMBER THEORY** Copy the Venn diagram at the right. Label the remaining sets of numbers. Then place the numbers $-3, -13, 0, 53, \frac{2}{3}, -\frac{1}{5}$, 0.33, 40, 2.98, −49.98, and $-\frac{5}{2}$ in the most specific categories. See pp. 117A–117B.



DAILY INTERVENTION

Unlocking Misconceptions

Absolute Value Many students associate absolute value with opposites. While it is true that the absolute value of n is the opposite of n if n is negative, this definition fails if n is positive or zero. Therefore, it is important for students to think of the absolute value of a number as the distance between that number and zero on a number line.

Practice and Apply

Homework Help

For Exercises	See Examples
18–23	1
24–33	2
34–41	3
42–44, 58, 59	2, 3
45–56	4

Extra Practice

See page 823.

Name the coordinates of the points graphed on each number line.

18. −5 −4 −3 −2 −1 0 1 2 3 4 5 **{−4, −2, 0, 2, 4}**

19. −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 **{−7, −6, −5, −3, −2}**

20. −4 −3 −2 −1 0 1 2 3 4 5 6 **{2, 3, 4, 5, 6, ...}**

21. 0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 **{..., 0, 0.2, 0.4, 0.6, 0.8}**

22. $-2\ -\frac{5}{3}\ -\frac{4}{3}\ -1\ -\frac{2}{3}\ -\frac{1}{3}\ 0\ \frac{1}{3}\ \frac{2}{3}\ 1\ \frac{4}{3}$ $\left\{-2, -\frac{5}{3}, -1, \frac{2}{3}, 1\right\}$

23. $0\ \frac{1}{5}\ \frac{2}{5}\ \frac{3}{5}\ \frac{4}{5}\ 1\ \frac{6}{5}\ \frac{7}{5}\ \frac{8}{5}\ \frac{9}{5}\ 2$ $\left\{\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{8}{5}, 2\right\}$

Graph each set of numbers. **24–33. See pp. 117A–117B.**

24. {−4, −2, −1, 1, 3}
25. {0, 2, 5, 6, 9}
26. {−5, −4, −3, −2, ...}
27. {..., −2, 0, 2, 4, 6}
28. {−8.4, −7.2, −6.0, −4.8}
29. {−2.4, −1.6, −0.8, 3.2, ...}
30. $\left\{..., -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, ...\right\}$
31. $\left\{-3\frac{2}{5}, -2\frac{1}{5}, -1\frac{4}{5}, -\frac{4}{5}, 1\right\}$
32. {integers less than −7 or greater than −1}
33. {integers greater than −5 and less than 9}

Find each absolute value.

34. $|-38|$ **38**
35. $|10|$ **10**
36. $|97|$ **97**
37. $|-61|$ **61**
38. $|3.9|$ **3.9**
39. $|-6.8|$ **6.8**
40. $\left|-\frac{23}{56}\right|$ $\frac{23}{56}$
41. $\left|\frac{35}{80}\right|$ $\frac{35}{80}$

POPULATION For Exercises 42–44, refer to the table below. **42. See pp. 117A–117B.**

Population of Various Counties, 1990–1999

County	Percent Change	County	Percent Change
Kings, NY	−1.4	Wayne, MI	−0.2
Los Angeles, CA	5.3	Philadelphia, PA	−10.6
Cuyahoga, OH	−2.9	Suffolk, NY	4.7
Santa Clara, CA	10.0	Alameda, CA	8.5
Cook, IL	1.7	New York, NY	4.3

Source: *The World Almanac*

42. Use a number line to order the percent of change from least to greatest.
43. Which population had the greatest percent increase or decrease? Explain.
44. Which population had the least percent increase or decrease? Explain.

Evaluate each expression if $a = 6$, $b = \frac{2}{3}$, $c = \frac{5}{4}$, $x = 12$, $y = 3.2$, and $z = -5$.

45. $48 + |x - 5|$ **55**
46. $25 + |17 + x|$ **54**
47. $|17 - a| + 23$ **34**
48. $|43 - 4a| + 51$ **70**
49. $|z| + 13 - 4$ **14**
50. $28 - 13 + |z|$ **20**
51. $6.5 - |8.4 - y|$ **1.3**
52. $7.4 + |y - 2.6|$ **8**
53. $\frac{1}{6} + \left|b - \frac{7}{12}\right|$ $\frac{1}{4}$
54. $\left(b + \frac{1}{2}\right) - \left|-\frac{5}{6}\right|$ $\frac{1}{3}$
55. $|c - 1| + \frac{2}{5}$ $\frac{13}{20}$
56. $|-c| + \left(2 + \frac{1}{2}\right)$
57. CRITICAL THINKING Find all values for x if $|x| = -|x|$. **0**

Career Choices

Demographer

A demographer analyses the size, nature, and movement of human populations. Many demographers specialize in one area such as health, housing, education, agriculture, or economics.

Online Research For information about a career as a demographer, visit www.algebra1.com/careers

43. Philadelphia, PA; Sample answer: It had the greatest absolute value.
44. Wayne, MI; Sample answer: It had the least absolute value.
56. $\frac{15}{4}$ **or** $3\frac{3}{4}$

www.algebra1.com/self_check_quiz

Study Guide and Intervention, p. 75 (shown) and p. 76

Graph Rational Numbers The figure at the right is part of a number line. A number line can be used to show the sets of **natural numbers, whole numbers,** and **integers. Positive numbers,** are located to the right of 0, and **negative numbers** are located to the left of 0.

Another set of numbers that you can display on a number line is the set of **rational numbers.** A rational number can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Some examples of rational numbers are $\frac{1}{4}, \frac{-3}{5}, \frac{-7}{-8}$, and $\frac{12}{-3}$.

Example 1 Name the coordinates of the points graphed on each number line.

a. The dots indicate each point on the graph. The coordinates are {−3, −1, 1, 3, 5}

b. The bold arrow to the right means the graph continues indefinitely in that direction. The coordinates are {2, 2.5, 3, 3.5, 4, 4.5, 5, ...}.

Example 2 Graph each set of numbers.

a. {..., −3, −2, −1, 0, 1, 2}

b. $\left\{-\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}\right\}$

Exercises

Name the coordinates of the points graphed on each number line.

1. {−2, 0, 2, 4, 6}
2. {1, 3, 5, 7, ...}
3. $\left\{-\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$
4. $\left\{-3, -\frac{1}{2}, \frac{1}{3}, 2, 3\right\}$

Graph each set of numbers.

5. {−3, −1, 1, 3}
6. {−5, −2, 1, 2}
7. {integers less than 0}
8. {..., −2, −1, 0, 1}
9. $\left\{-2\frac{1}{2}, -1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$
10. {..., −4, −2, 0, 2, ...}

Skills Practice, p. 77 and Practice, p. 78 (shown)

Name the coordinates of the points graphed on each number line.

1. $\left\{..., -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}\right\}$
2. {2.4, 2.8, 3.2, 3.6, 4, ...}

Graph each set of numbers.

3. $\left\{..., -\frac{7}{5}, -\frac{6}{5}, -1, -\frac{4}{5}, -\frac{3}{5}\right\}$
4. {integers less than −4 or greater than 2}

Find each absolute value.

5. $|-11|$ 11
6. $|100|$ 100
7. $|-0.35|$ 0.35
8. $\left|-\frac{28}{53}\right|$ $\frac{28}{53}$

Evaluate each expression if $a = 4$, $b = \frac{3}{5}$, $c = \frac{3}{2}$, $x = 14$, $y = 2.4$, and $z = -3$.

9. $41 - 16 - |z|$ 22
10. $|3a + 20| - 15$ 17
11. $|2x + 4| - 6$ 26
12. $2.5 - |3.8 - y|$ 1.1
13. $\left(b - \frac{1}{5}\right) + \left|-\frac{3}{10}\right|$ $\frac{7}{10}$
14. $\frac{2}{15} + \left|b - \frac{2}{5}\right|$ $\frac{1}{3}$
15. $|c - 1| - \frac{1}{3}$ $\frac{1}{6}$
16. $|-c| - \frac{3}{4}$ $\frac{3}{4}$

ASTRONOMY For Exercises 17–19, use the following information.

The *absolute magnitude* of a star is how bright the star would appear from a standard distance of 10 parsecs, or 32.6 light years. The lower the number, the greater the magnitude, or brightness, of the star. The table gives the magnitudes of several stars.

Star	Magnitude
Altair	2.3
Betelgeuse	−7.2
Castor	0.5
Deneb	−4.7
Pollux	0.7
Regulus	−0.3
Rigel	−8.1
Sirius	1.4

Source: www.astro.wisc.edu

17. Use a number line to order the magnitudes from least to greatest.
18. Which of the stars are the brightest and the least bright? brightest: Rigel; least bright: Altair
19. Write the absolute value of the magnitude of each star. 2.3, 7.2, 0.5, 4.7, 0.7, 0.3, 8.1, 1.4

20. **CLIMATE** The table shows the mean wind speeds in miles per hour at Daytona Beach, Florida.

Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
9.0	9.7	10.1	9.7	9.0	7.9	7.4	7.1	8.3	9.1	8.7	8.5

Source: National Climatic Data Center

Graph the wind speeds on a number line. Which month has the greatest mean wind speed? March

Reading to Learn Mathematics, p. 79 ELL

Pre-Activity How can you use a number line to show data?

Read the introduction to Lesson 2-1 at the top of page 68 in your textbook.

In the table, what does the number +0.2 tell you? The level of the Brazos River increased by 0.2 foot in 24 hours.

Reading the Lesson

1. Refer to the number line on page 68 in your textbook. Write *true* or *false* for each of the following statements.
   a. All whole numbers are integers. true
   b. All natural numbers are integers. true
   c. All whole numbers are natural numbers. false
   d. All natural numbers are whole numbers. true
   e. All whole numbers are positive numbers. false

2. Use the words *denominator*, *fraction*, and *numerator* to complete the following sentence.
You know that a number is a rational number if it can be written as a fraction that has a numerator and denominator that are integers, where the denominator is not equal to zero.

3. Explain why $\frac{-3}{5}$, $0.\overline{6}$, and 15 are rational numbers.
Each number is in or can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. $0.\overline{6}$ can be written as $\frac{2}{3}$, and 15 can be written as $\frac{15}{1}$.

Helping You Remember

4. Connecting a mathematical concept to something in your everyday life is one way of remembering. Describe a situation or setting in your life that reminds you of absolute value.
Sample answer: The distance from each goal line to the 50-yard line is 50 yards.

Enrichment, p. 80

Intersection and Union

The intersection of two sets is the set of elements that are in both sets. The intersection of sets A and B is written A ∩ B. The union of two sets is the set of elements in either A, B, or both. The union is written A ∪ B.

In the drawings below, suppose A is the set of points inside the circle and B is the set of points inside the square. Then, the shaded areas show the intersection in the first drawing and the union in the second drawing.

Intersection A ∩ B   Union A ∪ B

Write A ∩ B and A ∪ B for each of the following.

1. A = {p, q, r, s, t} A ∩ B = {q, r, s} A ∪ B = ...

4 Assess

Open-Ended Assessment

Modeling Use masking tape to create a large number line on the floor in front of the classroom. Write sets of numbers on the board and have students stand on the line to "graph" the points. Also write absolute value statements on the board and have students step the "distance" equivalent to the absolute value on the number line.

Getting Ready for Lesson 2-2

PREREQUISITE SKILL Students will learn about adding and subtracting rational numbers in Lesson 2-2. They will apply the rules of adding and subtracting integers to computations with fractions. Use Exercises 70–77 to determine your students' familiarity with the addition and subtraction of fractions.

Answers

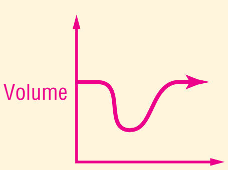
58.

59. Bismark, ND 11; Caribou, ME 5; Chicago, IL 4; Fairbanks, AK 9; International Falls, MN 13; Kansas City, MO 7; Sacramento, CA 34; Shreveport, LA 33

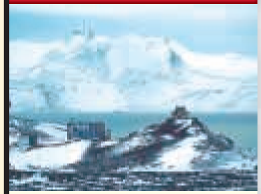
60. Sample answer: You can plot the data on a number line to visualize its relationship. Answers should include the following.

- Determine the least and greatest values of the data, and use those as the endpoints of the line.
- Find the absolute value of each number.

66.



More About. . .



Weather

The lowest temperature ever recorded in the world was −129°F at the Soviet Antarctica station of Vostok.

Source: *The World Almanac*

WEATHER For Exercises 58 and 59, use the table at the right and the information at the left.

Same Day Low Temperatures for Certain U.S. Cities

City	Low Temperature (°F)
Bismarck, ND	−11
Caribou, ME	−5
Chicago, IL	−4
Fairbanks, AK	−9
International Falls, MN	−13
Kansas City, MO	7
Sacramento, CA	34
Shreveport, LA	33

Source: *The World Almanac*

58. Draw a number line and graph the set of numbers that represents the low temperatures for these cities. **58–59. See margin.**

59. Write the absolute value of the low temperature for each city.

60. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you use a number line to show data?

Include the following in your answer:
- an explanation of how to choose the range for a number line, and
- an explanation of how to tell which river had the greatest increase or decrease.

Standardized Test Practice

61. Which number is a natural number? **D**

Ⓐ −2.5 Ⓑ $5 - |5|$ Ⓒ $-|3 + 5|$ Ⓓ $|-8| - 2$

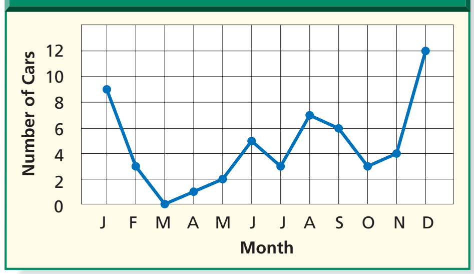
62. Which sentence is *not* true? **C**

Ⓐ All natural numbers are whole numbers.
Ⓑ Natural numbers are positive numbers.
Ⓒ Every whole number is a natural number.
Ⓓ Zero is neither positive nor negative.

Maintain Your Skills

Mixed Review

SALES For Exercises 63–65, refer to the graph. *(Lesson 1-9)*



63. In which month did Mr. Michaels have the greatest sales? **December**

64. Between which two consecutive months did the greatest change in sales occur? **Nov. and Dec.**

65. In which months were sales equal? **Feb., July, Oct.**

66. **ENTERTAINMENT** Juanita has the volume on her stereo turned up. When her telephone rings, she turns the volume down. After she gets off the phone, she returns the volume to its previous level. Sketch a reasonable graph to show the volume of Juanita's stereo during this time. *(Lesson 1-8)* **See margin.**

Simplify each expression. *(Lesson 1-6)*

67. $8x + 2y + x$ $\mathbf{9x + 2y}$ **68.** $7(5a + 3b) - 4a$ $\mathbf{31a + 21b}$ **69.** $4[1 + 4(5x + 2y)]$ $\mathbf{4 + 80x + 32y}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each sum or difference.
*(To review **addition and subtraction of fractions**, see pages 798 and 799.)*

70. $\frac{3}{8} + \frac{1}{8}$ $\mathbf{\frac{1}{2}}$ **71.** $\frac{7}{12} - \frac{3}{12}$ $\mathbf{\frac{1}{3}}$ **72.** $\frac{7}{10} + \frac{1}{5}$ $\mathbf{\frac{9}{10}}$ **73.** $\frac{3}{8} + \frac{2}{3}$ **73.** $\mathbf{\frac{25}{24}}$ **or** $\mathbf{1\frac{1}{24}}$

74. $\frac{5}{6} + \frac{1}{2}$ $\mathbf{\frac{4}{3}}$ **or** $\mathbf{1\frac{1}{3}}$ **75.** $\frac{3}{4} - \frac{1}{3}$ $\mathbf{\frac{5}{12}}$ **76.** $\frac{9}{15} - \frac{1}{2}$ $\mathbf{\frac{1}{10}}$ **77.** $\frac{7}{9} - \frac{7}{18}$ $\mathbf{\frac{7}{18}}$

DAILY INTERVENTION

Differentiated Instruction ELL

Verbal/Linguistic Have students look up the word *absolute* in a dictionary and find meanings that relate to the mathematical meaning. Also have them read the definitions of terms beginning with *absolute*, such as *absolute ceiling, absolute humidity*, or *absolute pitch*. Have students read aloud the definitions they found and invite students to define in their own words the mathematical meaning of *absolute value* based on any insights they have gained from the dictionary definitions.

2-2 Adding and Subtracting Rational Numbers

What You'll Learn

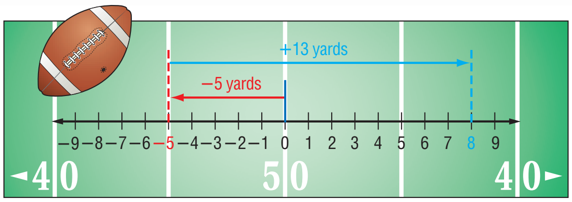
- Add integers and rational numbers.
- Subtract integers and rational numbers.

Vocabulary

- opposites
- additive inverses

How can a number line be used to show a football team's progress?

In one series of plays during Super Bowl XXXV, the New York Giants received a five-yard penalty before completing a 13-yard pass.



The number line shows the yards gained during this series of plays. The total yards gained was 8 yards.

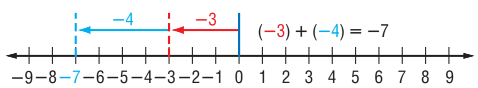
ADD RATIONAL NUMBERS The number line above illustrates how to add integers on a number line. You can use a number line to add any rational numbers.

Example 1 Use a Number Line to Add Rational Numbers

Use a number line to find each sum.

a. $-3 + (-4)$

Step 1 Draw an arrow from 0 to -3.

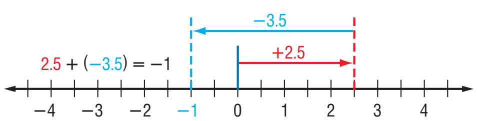


Step 2 Then draw a second arrow 4 units to the left to represent adding -4.

Step 3 The second arrow ends at the sum -7. So, $-3 + (-4) = -7$.

b. $2.5 + (-3.5)$

Step 1 Draw an arrow from 0 to 2.5.



Step 2 Then draw a second arrow 3.5 units to the left.

Step 3 The second arrow ends at the sum -1. So, $2.5 + (-3.5) = -1$.

2-2 Lesson Notes

1 Focus

5-Minute Check Transparency 2-2 Use as a quiz or a review of Lesson 2-1.

Mathematical Background notes are available for this lesson on p. 66C.

Building on Prior Knowledge

Apply what students know already from previous courses by demonstrating adding and subtracting positive integers on a number line. In this lesson, students will extend this understanding to add and subtract all rational numbers.

How can a number line be used to show a football team's progress?

Ask students:

- How is a football field similar to a number line? **Football fields have yard markers that tell the distance from the goal line, which are similar to the tic marks on a number line that tell the distance to zero.**
- How are the yard markers on a football field different from the marks on a number line? **The yard markers on a football field increase in number until the 50-yard line, and then they decrease until they reach the opposite goal.**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 81–82
- Skills Practice, p. 83
- Practice, p. 84
- Reading to Learn Mathematics, p. 85
- Enrichment, p. 86
- Assessment, p. 131

Graphing Calculator and Spreadsheet Masters, p. 26

Parent and Student Study Guide Workbook, p. 12

Prerequisite Skills Workbook, pp. 15–16, 19–24, 39–40, 55–60, 65–66, 75–76

Transparencies

5-Minute Check Transparency 2-2

Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

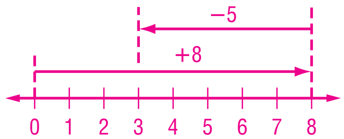
ADD RATIONAL NUMBERS

In-Class Examples Power Point®

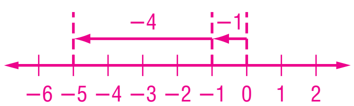
Teaching Tip Tell students that when using a number line to add rational numbers, they should always start at zero. If the first number in the expression is positive, then move to the right. If it is negative, move to the left. Then, if the second number is positive, move to the right and if it is negative, move to the left. The sum is indicated by wherever they end up on the number line.

1 Use a number line to find each sum.

a. $8 + (-5)$ **3**



b. $-1 + (-4)$ **−5**



2 Find each sum.

a. $6 + (-14)$ **−8**

b. $-\frac{3}{7} + \left(-\frac{2}{7}\right)$ $-\frac{5}{7}$

You can use absolute value to add rational numbers.

Same Signs		Different Signs	
+ +	− −	+ −	− +
$3 + 5 = 8$	$-3 + (-5) = -8$	$3 + (-5) = -2$	$-3 + 5 = 2$
3 and 5 are positive, so the sum is positive.	−3 and −5 are negative, so the sum is negative.	Since −5 has the greater absolute value, the sum is negative.	Since 5 has the greater absolute value, the sum is positive.

The examples above suggest the following rules for adding rational numbers.

Key Concept *Addition of Rational Numbers*

- To add rational numbers with the *same sign*, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different signs*, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.

Example 2 *Add Rational Numbers*

Find each sum.

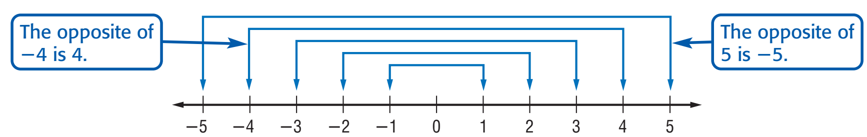
a. $\mathbf{-11 + (-7)}$

$$\begin{aligned} -11 + (-7) &= -(|-11| + |-7|) && \text{Both numbers are negative, so the sum is negative.} \\ &= -(11 + 7) \\ &= -18 \end{aligned}$$

b. $\frac{7}{16} + \left(-\frac{3}{8}\right)$

$$\begin{aligned} \frac{7}{16} + \left(-\frac{3}{8}\right) &= \frac{7}{16} + \left(-\frac{6}{16}\right) && \text{The LCD is 16. Replace } -\tfrac{3}{8} \text{ with } -\tfrac{6}{16}. \\ &= +\left(\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right|\right) && \text{Subtract the absolute values.} \\ &= +\left(\frac{7}{16} - \frac{6}{16}\right) && \text{Since the number with the greater absolute value is } \tfrac{7}{16}\text{, the sum is positive.} \\ &= \frac{1}{16} \end{aligned}$$

SUBTRACT RATIONAL NUMBERS Every positive rational number can be paired with a negative rational number. These pairs are called **opposites**.



Study Tip

Additive Inverse

Since 0 + 0 = 0, zero is its own additive inverse.

A number and its opposite are **additive inverses** of each other. When you add two opposites, the sum is always 0.

Key Concept — *Additive Inverse Property*

- **Words** The sum of a number and its additive inverse is 0.
- **Symbols** For every number a, $a + (-a) = 0$.
- **Examples** $2 + (-2) = 0$ $\quad -4.25 + 4.25 = 0$ $\quad \frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Additive inverses can be used when you subtract rational numbers.

Subtraction	**Addition**
$9 - 4 = 5$	$9 + (-4) = 5$

additive inverses

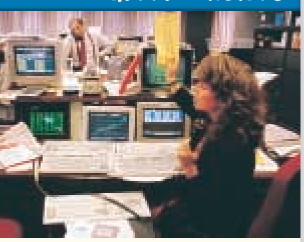
same result

This example suggests that subtracting a number is equivalent to adding its inverse.

Key Concept — *Subtraction of Rational Numbers*

- **Words** To subtract a rational number, add its additive inverse.
- **Symbols** For any numbers a and b, $a - b = a + (-b)$.
- **Examples** $8 - 15 = 8 + (-15)$ or -7
  $-7.6 - 12.3 = -7.6 + (-12.3)$ or -19.9

Career Choices



Stockbroker

Stockbrokers perform various duties, including buying or selling stocks, bonds, mutual funds, or other financial products for an investor.

Online Research For information about a career as a stockbroker, visit: www.algebra1.com/careers

Example 3 *Subtract Rational Numbers to Solve a Problem*

STOCKS **During a five-day period, a telecommunications company's stock price went from \$17.82 to \$15.36 per share. Find the change in the price of the stock.**

Explore The stock price began at \$17.82 and ended at \$15.36. You need to determine the change in price for the week.

Plan Subtract to find the change in price.

$$\underbrace{15.36}_{\text{ending price}} \quad \underbrace{-}_{\text{minus}} \quad \underbrace{17.82}_{\text{beginning price}}$$

Solve

$$\begin{aligned} 15.36 - 17.82 &= 15.36 + (-17.82) && \text{To subtract 17.82, add its inverse.} \\ &= -(|-17.82| - |15.36|) && \text{Subtract the absolute values.} \\ &= -(17.82 - 15.36) && \text{The absolute value of } -17.82 \text{ is greater, so the result is negative.} \\ &= -2.46 \end{aligned}$$

The price of the stock changed by $-\$2.46$.

Examine The problem asks for the change in a stock's price from the beginning of a week to the end. Since the change was negative, the price dropped. This makes sense since the ending price is less than the beginning price.

www.algebra1.com/extra_examples

SUBTRACT RATIONAL NUMBERS

In-Class Example Power Point®

Teaching Tip Explain that a positive change in price indicates that the price of the stock rose. A negative change in price indicates that the price of the stock fell.

3 **STOCKS** In the past year, a publishing company's stock went from \$52.08 per share to \$70.87 per share. Find the change in the price of the stock. **The price of the stock changed by \$18.79.**

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Students may find the rules for adding and subtracting rational numbers confusing, especially when determining which sign the sum or difference will have. Have students check their answers by adding or subtracting with a number line until they are comfortable with addition or subtraction without the number line.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- include worked-out examples of adding and subtracting rational numbers.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Remind students to review the rules for adding and subtracting rational numbers when solving this problem. Remember, when adding rational numbers with different signs, you must *subtract* the lesser absolute value from the greater absolute value. Did both Gabriella and Nick follow this rule?

About the Exercises...

Organization by Objective
- **Add Rational Numbers:** 17–38
- **Subtract Rational Numbers:** 39–56

Odd/Even Assignments

Exercises 17–56 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–33 odd, 37–53 odd, 57–59, 63–82

Average: 17–55 odd, 57–59, 63–82

Advanced: 18–56 even, 60–76 (optional: 77–82)

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a subtraction expression using rational numbers that has a difference of $-\frac{2}{5}$. **Sample answer: $\frac{1}{5} - \frac{3}{5}$**
2. **Describe** how to subtract real numbers.
3. **FIND THE ERROR** Gabriella and Nick are subtracting fractions.

2. Sample answer: To subtract a real number, add its opposite.

3. Gabriella; subtracting $-\frac{6}{9}$ is the same as adding $\frac{6}{9}$.

Gabriella

$$\left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) = \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right)$$
$$= \left(-\frac{4}{9}\right) + \left(\frac{6}{9}\right)$$
$$= \left(\frac{6}{9} - \frac{4}{9}\right)$$
$$= \frac{2}{9}$$

Nick

$$\left(-\frac{4}{9}\right) - \left(-\frac{2}{3}\right) = \left(-\frac{4}{9}\right) - \left(-\frac{6}{9}\right)$$
$$= \left(-\frac{4}{9}\right) + \left(-\frac{6}{9}\right)$$
$$= -\left(\frac{6}{9} + \frac{4}{9}\right)$$
$$= -\frac{10}{9}$$

Who is correct? Explain your reasoning.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
4–9	1, 2
10–16	3

Find each sum.

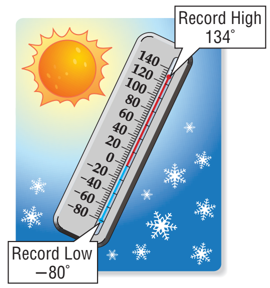
4. $-15 + (-12)$ **−27**
5. $-24 + (-45)$ **−69**
6. $38.7 + (-52.6)$ **−13.9**
7. $-4.62 + (-12.81)$ **−17.43**
8. $\frac{4}{7} + \left(-\frac{1}{2}\right)$ **$\frac{1}{14}$**
9. $-\frac{5}{12} + \frac{8}{15}$ **$\frac{7}{60}$**

Find each difference.

10. $18 - 23$ **−5**
11. $12.7 - (-18.4)$ **31.1**
12. $(-3.86) - 1.75$ **−5.61**
13. $-32.25 - (-42.5)$ **10.25**
14. $-\frac{2}{9} - \frac{3}{10}$ **$-\frac{47}{90}$**
15. $\left(-\frac{7}{10}\right) - \left(-\frac{11}{12}\right)$ **$\frac{13}{60}$**

Application

16. **WEATHER** The highest recorded temperature in the United States was in Death Valley, California, while the lowest temperature was recorded at Prospect Creek, Alaska. What is the difference between these two temperatures? **214°**



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
17–38	1, 2
39–62	3

Extra Practice

See page 823.

34. $-\frac{21}{20}$ or $-1\frac{1}{20}$

Find each sum.

17. $-8 + 13$ **5**
18. $-11 + 19$ **8**
19. $41 + (-63)$ **−22**
20. $80 + (-102)$ **−22**
21. $-77 + (-46)$ **−123**
22. $-92 + (-64)$ **−156**
23. $-1.6 + (-3.8)$ **−5.4**
24. $-32.4 + (-4.5)$ **−36.9**
25. $-38.9 + 24.2$ **−14.7**
26. $-7.007 + 4.8$ **−2.207**
27. $43.2 + (-57.9)$ **−14.7**
28. $38.7 + (-61.1)$ **−22.4**
29. $\frac{6}{7} + \frac{2}{3}$ **$\frac{32}{21}$ or $1\frac{11}{21}$**
30. $\frac{3}{18} + \frac{6}{17}$ **$\frac{53}{102}$**
31. $-\frac{4}{11} + \frac{3}{5}$ **$\frac{13}{55}$**
32. $-\frac{2}{5} + \frac{17}{20}$ **$\frac{9}{20}$**
33. $-\frac{4}{15} + \left(-\frac{9}{16}\right)$ **$-\frac{199}{240}$**
34. $-\frac{16}{40} + \left(-\frac{13}{20}\right)$

★ 35. Find the sum of $4\frac{1}{8}$ and $-1\frac{1}{2}$. **$2\frac{5}{8}$**

★ 36. Find the sum of $1\frac{17}{50}$ and $-3\frac{17}{25}$. **$-2\frac{17}{50}$**

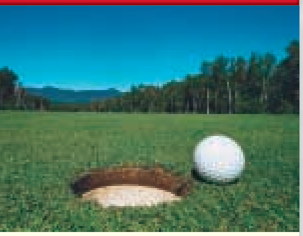
37. **GAMES** Sarah was playing a computer trivia game. Her scores for round one were +100, +200, +500, −300, +400, and −500. What was her total score at the end of round one? **400 points**

38. **FOOTBALL** The Northland Vikings' offense began a drive from their 20-yard line. They gained 6 yards on the first down, lost 8 yards on the second down, then gained 3 yards on third down. What yard line were they on at fourth down? **21-yard line**

Find each difference.

39. $-19 - 8$ **−27**
40. $16 - (-23)$ **39**
41. $9 - (-24)$ **33**
42. $12 - 34$ **−22**
43. $22 - 41$ **−19**
44. $-9 - (-33)$ **24**
45. $-58 - (-42)$ **−16**
46. $79.3 - (-14.1)$ **93.4**
47. $1.34 - (-0.458)$ **1.798**
48. $-9.16 - 10.17$ **−19.33**
49. $67.1 - (-38.2)$ **105.3**
50. $72.5 - (-81.3)$ **153.8**
51. $-\frac{1}{6} - \frac{2}{3}$ $-\frac{5}{6}$
52. $\frac{1}{2} - \frac{4}{5}$ $-\frac{3}{10}$
53. $-\frac{7}{8} - \left(-\frac{3}{16}\right)$ $-\frac{11}{16}$
54. $-\frac{1}{12} - \left(-\frac{3}{4}\right)$ $\frac{2}{3}$
★ 55. $2\frac{1}{4} - 6\frac{1}{3}$ $-\frac{49}{12}$ or $-4\frac{1}{12}$
★ 56. $5\frac{3}{10} - 1\frac{31}{50}$ $\frac{184}{50}$ or $3\frac{17}{25}$

More About. . .



Golf

In the United States, there are more than 16,000 golf courses played by 26 million people each year.

Source: Encarta Online

GOLF For Exercises 57–59, use the following information.

In golf, scores are based on *par*. Par 72 means that a golfer should hit the ball 72 times to complete 18 holes of golf. A score of 67, or 5 under par, is written as −5. A score of 3 over par is written as +3. At the Masters Tournament (par 72) in April, 2001, Tiger Woods shot 70, 66, 68, and 68 during four rounds of golf.

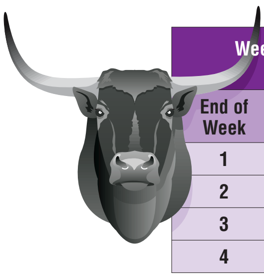
57. Use integers to write his score for each round as over or under par.
58. Add the integers to find his overall score. **−16**
59. Was his score under or over par? Would you want to have his score? Explain.

57. −2, −6, −4, −4

59. Under; yes, it is better than par 72.

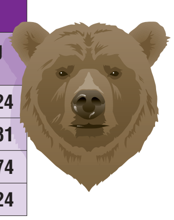
Online Research **Data Update** Find the most recent winner of the Masters Tournament. What integer represents the winner's score for each round as over or under par? What integer represents the winner's overall score? Visit www.algebra1.com/data_update to learn more.

STOCKS For Exercises 60–62, refer to the table that shows the weekly closing values of the stock market for an eight-week period.



Weekly Dow Jones Industrial Average (April – May 2000)

End of Week	Closing Value	End of Week	Closing Value
1	9791.09	5	10,951.24
2	10,126.94	6	10,821.31
3	10,579.85	7	11,301.74
4	10,810.05	8	11,257.24



Source: *The Wall Street Journal*

60. Find the change in value from week 1 to week 8. **1466.15**
61. Which week had the greatest change from the previous week? **week 7**
62. Which week had the least change from the previous week? **week 8**

63. **CRITICAL THINKING** Tell whether the equation $x + |x| = 0$ is *always*, *sometimes*, or *never* true. Explain.

63. Sometimes; if x is a negative number, then its absolute value is positive and the two values are additive inverses.

www.algebra1.com/self_check_quiz

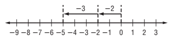
Study Guide and Intervention, p. 81 (shown) and p. 82

Add Rational Numbers

Adding Rational Numbers, Same Sign	Add the numbers. If both are positive, the sum is positive; if both are negative, the sum is negative.
Adding Rational Numbers, Different Signs	Subtract the number with the lesser absolute value from the number with the greater absolute value. The sign of the sum is the same as the sign of the number with the greater absolute value.

Example 1 **Use a number line to find the sum −2 + (−3).**

Step 1 Draw an arrow from 0 to −2.
Step 2 From the tip of the first arrow, draw a second arrow 3 units to the left to represent adding −3.
Step 3 The second arrow ends at the sum −5. So −2 + (−3) = −5.



Example 2 **Find each sum.**

a. $-8 + 5$

$$-8 + 5 = -(|-8| - |5|) = -(8 - 5) = -3$$

b. $\frac{3}{4} + \left(-\frac{1}{2}\right)$

$$\frac{3}{4} + \left(-\frac{1}{2}\right) = \frac{3}{4} + \left(-\frac{2}{4}\right) = +\left(\left|\frac{3}{4}\right| - \left|-\frac{2}{4}\right|\right) = +\left(\frac{3}{4} - \frac{2}{4}\right) = \frac{1}{4}$$

Exercises

Find each sum.

1. $12 + 24$ **36**
2. $-6 + 14$ **8**
3. $-12 + (-15)$ **−27**
4. $-21.5 + 34.2$ **12.7**
5. $8.2 + (-3.5)$ **4.7**
6. $23.5 + (-15.2)$ **8.3**
7. $90 + (-105)$ **−15**
8. $108 + (-62)$ **46**
9. $-84 + (-90)$ **−174**
10. $\frac{5}{7} + \frac{1}{3}$ $\frac{22}{21}$ or $1\frac{1}{21}$
11. $\frac{3}{14} + \frac{6}{17}$ $\frac{135}{238}$
12. $-\frac{4}{9} + \frac{3}{5}$ $\frac{7}{45}$
13. $-\frac{2}{3} + \left(-\frac{1}{4}\right)$ $-\frac{11}{12}$
14. $-\frac{1}{5} + \frac{7}{11}$ $\frac{24}{55}$
15. $-\frac{18}{40} + \left(-\frac{10}{20}\right)$ $-\frac{19}{20}$
16. $-\frac{3}{5} + \left(-\frac{5}{6}\right)$ $-\frac{43}{30}$ or $-1\frac{13}{30}$
17. $-1.6 + (-1.8)$ **−3.4**
18. $-0.008 + (-0.25)$ **−0.258**

Skills Practice, p. 83 and Practice, p. 84 (shown)

Find each sum.

1. $-82 + 14$ **−68**
2. $-33 + 47$ **14**
3. $-17 + (-39)$ **−56**
4. $8 + (-11)$ **−3**
5. $-1.7 + 3.2$ **1.5**
6. $-13.3 + (-0.9)$ **−14.2**
7. $-51.8 + 29.7$ **−22.1**
8. $7.34 + (-9.06)$ **−1.72**
9. $\frac{5}{9} + \frac{5}{6}$ $\frac{25}{18}$ or $1\frac{7}{18}$
10. $-\frac{3}{5} + \frac{2}{3}$ $\frac{1}{15}$
11. $-\frac{3}{4} + \left(-\frac{3}{5}\right)$ $-\frac{27}{20}$ or $-1\frac{7}{20}$
12. $\frac{3}{8} + \left(-\frac{2}{3}\right)$ $-\frac{7}{24}$

Find each difference.

13. $65 - 93$ **−28**
14. $-42 - (-17)$ **−25**
15. $13 - (-19)$ **32**
16. $-8 - 43$ **−51**
17. $82.8 - (-12.4)$ **95.2**
18. $1.27 - 2.34$ **−1.07**
19. $-9.26 - 12.05$ **−21.31**
20. $-18.1 - (-4.7)$ **−13.4**
21. $-\frac{1}{5} - \frac{2}{3}$ $-\frac{13}{15}$
22. $\frac{4}{3} - \frac{5}{6}$ $\frac{1}{2}$
23. $-\frac{5}{2} - \left(-\frac{3}{7}\right)$ $-\frac{29}{14}$ or $-2\frac{1}{14}$
24. $\frac{1}{8} - \left(-\frac{5}{6}\right)$ $\frac{23}{24}$

FINANCE For Exercises 25–27, use the following information.

The table shows activity in Ben's checking account. The balance before the activity was $200.00. Deposits are added to an account and checks are subtracted.

Number	Date	Transaction	Amount	Balance
	5/2	deposit	52.50	252.50
101	5/10	check to Castle Music	25.50	?
102	6/1	check to Comp U Save	235.40	?

25. What is the account balance after writing check number 101? **$227.00**
26. What is the account balance after writing check number 102? **−$8.40**
27. Realizing that he has just written a check for more than is in the account, Ben immediately deposits $425. What will this make his new account balance? **$416.60**
28. **CHEMISTRY** The melting points of krypton, radon, and sulfur in degrees Celsius are −156.6, −61.8, and 112.8, respectively. What is the difference in melting points between radon and krypton and between sulfur and krypton? **94.8°C and 269.4°C**

Reading to Learn Mathematics, p. 85 ELL

Pre-Activity **How can a number line be used to show a football team's progress?**

Read the introduction to Lesson 2-2 at the top of page 73 in your textbook.

Use *positive* or *negative* to complete the following sentences.

The five-yard penalty is shown by the **negative** number −5.

The 13-yard pass is shown by the **positive** number 13.

Reading the Lesson

1. To add two rational numbers, you can use a number line. Each number will be represented by an arrow.
   a. Where on the number line does the arrow for the first number begin? **at 0**
   b. Arrows for negative numbers will point to the **left** (left/right). Arrows for positive numbers will point to the **right** (left/right).
2. Two students added the same pair of rational numbers. Both students got the correct sum. One student used a number line. The other student used absolute value. Then they compared their work.
   a. How do the arrows show which number has the greater absolute value? **The number with the greater absolute value matches the longer arrow.**
   b. If the longer arrow points to the left, then the sum is **negative** (positive/negative). If the longer arrow points to the right, then the sum is **positive** (positive/negative).
3. If two numbers are additive inverses, what must be true about their absolute values? **The absolute values of the two numbers are equal.**
4. Write each subtraction problem as an addition problem.
   a. $12 - 4$ **12 + (−4)**
   b. $-15 - 7$ **−15 + (−7)**
   c. $0 - 9$ **0 + (−9)**
   d. $-20 - 34$ **−20 + (−34)**

Helping You Remember

5. Explain why knowing the rules for adding rational numbers can help you to subtract rational numbers.
   Sample answer: Since subtraction is the same as adding the opposite, you can change every subtraction problem to an addition problem. Then you can use the rules for adding rational numbers to get the final answer.

Enrichment, p. 86

Rounding Fractions

Rounding fractions is more difficult than rounding whole numbers or decimals. For example, think about how you would round $\frac{4}{9}$ inches to the nearest quarter-inch. Through estimation, you might realize that $\frac{4}{9}$ is less than $\frac{1}{2}$. But, is it closer to $\frac{1}{2}$ or to $\frac{1}{4}$?

Here are two ways to round fractions. Example 1 uses only the fractions; Example 2 uses decimals.

Example 1

Subtract the fraction twice. Use the two nearest quarters.

$\frac{1}{2} - \frac{4}{9} = \frac{1}{18}$ $\quad \frac{4}{9} - \frac{1}{4} = \frac{7}{36}$

Compare the differences.

Example 2

Change the fraction and the two nearest quarters to decimals.

$\frac{4}{9} = 0.4\overline{4}$, $\frac{1}{2} = 0.5$, $\frac{1}{4} = 0.25$

Find the decimal halfway between the two nearest quarters.

4 Assess

Open-Ended Assessment

Speaking Have a student volunteer write an expression involving the addition or subtraction of rational numbers on the chalkboard or overhead projector. Have other students explain the steps involved in performing the operation. Ask the volunteer to record the steps as they are explained.

Getting Ready for Lesson 2-3

PREREQUISITE SKILL Students will learn about multiplying rational numbers in Lesson 2-3. This includes applying special rules to products involving fractions. Use Exercises 77–82 to determine your students' familiarity with the multiplication of fractions.

Assessment Options

Quiz (Lessons 2-1 and 2-2) is available on p. 131 of the *Chapter 2 Resource Masters*.

Answer

64. Sample answer: If a team gains yards, move right on the number line. If a team loses yards, move left on the number line. Answers should include the following.

- **Move right or left, depending on whether the Giants gained or lost yards on each play. Where you end will tell you how many yards the Giants lost or gained.**
- **Instead of using a number line, you can use the rules for adding and subtracting real numbers.**

64. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How can a number line be used to show a football team's progress?

Include the following in your answer:
- an explanation of how you could use a number line to determine the yards gained or lost by the Giants on their next three plays, and
- a description of how to determine the total yards gained or lost without using a number line.

Standardized Test Practice

65. What is the value of n in $-57 - n = -144$? **D**

(A) -201 (B) 201 (C) -87 (D) 87

66. Which expression is equivalent to $5 - (-8)$? **B**

(A) $(-5) + 8$ (B) $8 + 5$ (C) $8 - 5$ (D) $5 - 8$

Maintain Your Skills

Mixed Review

Evaluate each expression if $x = 4.8$, $y = -7.4$, and $z = 10$. *(Lesson 2-1)*

67. $12.2 + |8 - x|$ **15.4**

68. $|y| + 9.4 - 3$ **13.8**

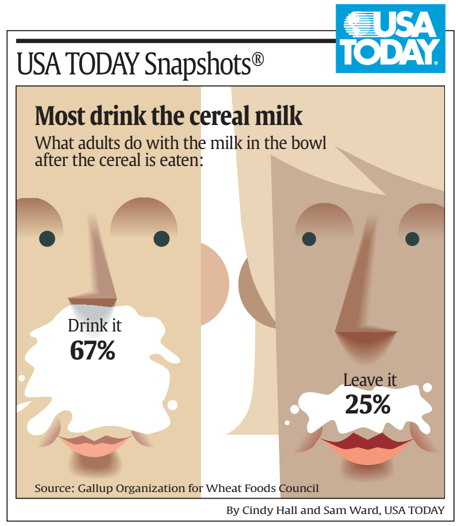
69. $24.2 - |18.3 - z|$ **15.9**

For Exercises 70 and 71, refer to the graph. *(Lesson 1-9)*

70. If you wanted to make a circle graph of the data, what additional category would you have to include so that the circle graph would not be misleading?

70. Sample answer: a category labeled "other" representing 8%

71. Construct a circle graph that displays the data accurately. **See pp. 117A–117B.**



Find the solution sets for each inequality if the replacement sets are $A = \{2, 3, 4, 5, 6\}$, $B = \{0.3, 0.4, 0.5, 0.6, 0.7\}$, and $C = \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}\right\}$. *(Lesson 1-3)*

72. $b + 1.3 \geq 1.8$ **{0.5, 0.6, 0.7}**

73. $3a - 5 > 7$ **{5, 6}**

74. $c + \frac{1}{2} < 2\frac{1}{4}$ $\mathbf{\left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}\right\}}$

Write an algebraic expression for each verbal phrase. *(Lesson 1-1)*

75. eight less than the square of q $\mathbf{q^2 - 8}$

76. 37 less than 2 times a number k $\mathbf{2k - 37}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**
*(To review **multiplication of fractions**, see pages 800 and 801.)*

77. $\frac{1}{2} \cdot \frac{2}{3}$ $\mathbf{\frac{1}{3}}$

78. $\frac{1}{4} \cdot \frac{2}{5}$ $\mathbf{\frac{1}{10}}$

79. $\frac{3}{4} \cdot \frac{5}{6}$ $\mathbf{\frac{5}{8}}$

80. $4 \cdot \frac{3}{5}$ $\mathbf{2\frac{2}{5}}$

81. $8 \cdot \frac{5}{8}$ **5**

82. $\frac{7}{9} \cdot 12$ $\mathbf{9\frac{1}{3}}$

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

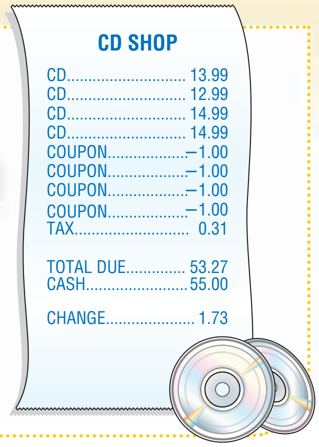
2-3 Multiplying Rational Numbers

What You'll Learn

- Multiply integers.
- Multiply rational numbers.

How do consumers use multiplication of rational numbers?

Stores often offer coupons to encourage people to shop in their stores. The receipt shows a purchase of four CDs along with four coupons for $1.00 off each CD. How could you determine the amount saved by using the coupons?



MULTIPLY INTEGERS One way to find the savings from the coupons is to use repeated addition.

$$-\$1.00 + (-\$1.00) + (-\$1.00) + (-\$1.00) = -\$4.00$$

An easier way to find the savings would be to multiply $-\$1.00$ by 4.

$$4(-\$1.00) = -\$4.00$$

Suppose the coupons were expired and had to be removed from the total. You can represent this by multiplying $-\$1.00$ by -4.

$$(-4)(-\$1.00) = \$4.00$$

In other words, $4.00 would be added back to the total.

These examples suggest the following rules for multiplying integers.

Study Tip

Multiplying Integers
When multiplying integers, if there are an even number of negative integers, the product is positive. If there are an odd number of negative integers, the product is negative.

Key Concept *Multiplication of Integers*

- **Words** The product of two numbers having the *same sign* is positive. The product of two numbers having *different signs* is negative.
- **Examples** $(-12)(-7) = 84$ same signs → positive product
  $15(-8) = -120$ different signs → negative product

Example 1 *Multiply Integers*

Find each product.

a. $\mathbf{4(-5)}$

$4(-5) = -20$ different signs → negative product

b. $\mathbf{(-12)(-14)}$

$(-12)(-14) = 168$ same signs → positive product

2-3 Lesson Notes

1 Focus

5-Minute Check Transparency 2-3 Use as a quiz or a review of Lesson 2-2.

Mathematical Background notes are available for this lesson on p. 66C.

How do consumers use multiplication of rational numbers?

Ask students:

- What is the total value of the coupons used in this transaction? **$4**
- What are two ways that you could find the total value of the coupons? **You can add the amounts of the coupons, or since the amount is the same for all four coupons, you can multiply the amount of one coupon by 4.**
- Why might multiplying the amount of one coupon by 4 be easier than adding all the values? **Multiplying involves fewer numbers.**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 87–88
- Skills Practice, p. 89
- Practice, p. 90
- Reading to Learn Mathematics, p. 91
- Enrichment, p. 92

Parent and Student Study Guide Workbook, p. 13

Prerequisite Skills Workbook, pp. 15–16, 19–20, 25–28, 39–40, 47–50, 65–66, 75–76

School-to-Career Masters, p. 3

Transparencies

5-Minute Check Transparency 2-3
Real-World Transparency 2
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

MULTIPLY INTEGERS

In-Class Examples (PowerPoint®)

Reading Tip Remind students to look carefully at the signs of both numbers before deciding on the sign of the product.

1 Find each product.

a. $(-8)(-6)$ **48**

b. $(10)(-11)$ **−110**

Teaching Tip The same multiplication rules apply whether or not variables are involved in the multiplication. For example, $(-2)(x) = -2x$ because the product of numbers with different signs is negative.

2 Simplify the expression $13x + (-6)(4x)$. $\mathbf{-11x}$

MULTIPLY RATIONAL NUMBERS

In-Class Examples (PowerPoint®)

3 Find $\left(-\frac{2}{3}\right)\left(-\frac{3}{4}\right)$. $\frac{1}{2}$

4 **STOCKS** The value of a company's stock dropped by \$1.25 per share. By what amount did the total value of the company's stock change if the company has issued 500,000 shares of stock? **−\$625,000**

5 Evaluate $\left(-\frac{3}{7}\right)x^3$ if $x = \left(-\frac{1}{2}\right)$. $\frac{3}{56}$

You can simplify expressions by applying the rules of multiplication.

Example 2 Simplify Expressions

Simplify the expression $4(-3y) - 15y$.

$4(-3y) - 15y = 4(-3)y - 15y$	Associative Property (×)
$= -12y - 15y$	Substitution
$= (-12 - 15)y$	Distributive Property
$= -27y$	Simplify.

MULTIPLY RATIONAL NUMBERS

Multiplying rational numbers is similar to multiplying integers.

Example 3 Multiply Rational Numbers

Find $\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right)$.

$\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right) = -\frac{9}{32}$ different signs → negative product

Example 4 Multiply Rational Numbers to Solve a Problem

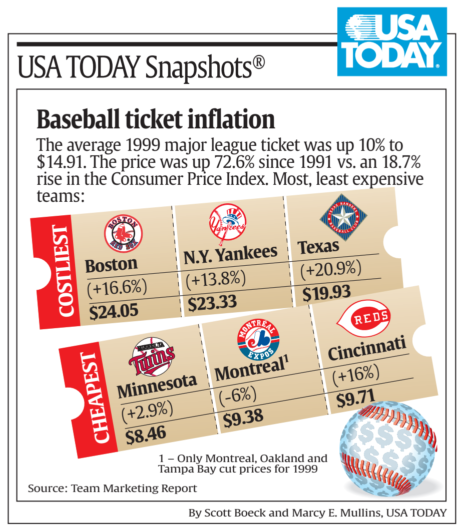
BASEBALL **Fenway Park, home of the Boston Red Sox, is the oldest ball park in professional baseball. It has a seating capacity of about 34,000. Determine the approximate total ticket sales for a sold-out game.**

To find the approximate total ticket sales, multiply the number of tickets sold by the average price.

$34{,}000 \cdot 24.05 = 817{,}770$

same signs → positive product

The total ticket sales for a sold-out game are about \$817,770.



You can evaluate expressions that contain rational numbers.

Example 5 Evaluate Expressions

Evaluate $n^2\left(-\frac{5}{8}\right)$ if $n = -\frac{2}{5}$.

$n^2\left(-\frac{5}{8}\right) = \left(-\frac{2}{5}\right)^2\left(-\frac{5}{8}\right)$	Substitution
$= \left(\frac{4}{25}\right)\left(-\frac{5}{8}\right)$	$\left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)$ or $\frac{4}{25}$
$= -\frac{20}{200}$ or $-\frac{1}{10}$	different signs → negative product

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In Lesson 1-4, you learned about the Multiplicative Identity Property, which states that any number multiplied by 1 is equal to the number. Another important property is the Multiplicative Property of −1.

Key Concept *Multiplicative Property of −1*

- **Words** The product of any number and −1 is its additive inverse.
- **Symbols** For any number a, $-1(a) = a(-1) = -a$.
- **Examples** $(-1)(4) = (4)(-1) = -4$ $\quad$ $(-1)(-2.3) = (-2.3)(-1) = 2.3$

Check for Understanding

Concept Check

1. **List** the conditions under which the product ab is negative. Give examples to support your answer. **1–3. See margin**
2. **OPEN ENDED** Describe a real-life situation in which you would multiply a positive rational number by a negative rational number. Write a corresponding multiplication expression.
3. **Explain** why the product of two negative numbers is positive.

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–9	1, 3
10, 11	2
12–14	5
15	4

Find each product.

4. $(-6)(3)$ **−18**
5. $(5)(-8)$ **−40**
6. $(4.5)(2.3)$ **10.35**
7. $(-8.7)(-10.4)$ **90.48**
8. $\left(\frac{5}{3}\right)\left(-\frac{2}{7}\right)$ $\mathbf{-\frac{10}{21}}$
9. $\left(-\frac{4}{9}\right)\left(\frac{7}{15}\right)$ $\mathbf{-\frac{28}{135}}$

Simplify each expression.

10. $5s(-6t)$ $\mathbf{-30st}$
11. $6x(-7y) + (-15xy)$ $\mathbf{-57xy}$

Evaluate each expression if $m = -\frac{2}{3}$, $n = \frac{1}{2}$, and $p = -3\frac{3}{4}$.

12. $6m$ **−4**
13. np $\mathbf{-\frac{15}{8}}$ **or** $\mathbf{-1\frac{7}{8}}$
14. $n^2(m + 2)$ $\mathbf{\frac{1}{3}}$

Application

15. **NATURE** The average worker honeybee makes about $\frac{1}{12}$ teaspoon of honey in its lifetime. How much honey do 675 honeybees make? $\mathbf{56\frac{1}{4}}$ **t**

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
16–33	1, 3
34–39	2
40, 41	4
42–49	5
50–54	4

Extra Practice

See page 823.

Find each product.

16. $5(18)$ **90**
17. $8(22)$ **176**
18. $-12(15)$ **−180**
19. $-24(8)$ **−192**
20. $-47(-29)$ **1363**
21. $-81(-48)$ **3888**
22. $\left(\frac{4}{5}\right)\left(\frac{3}{8}\right)$ $\mathbf{\frac{3}{10}}$
23. $\left(\frac{5}{12}\right)\left(\frac{4}{9}\right)$ $\mathbf{\frac{5}{27}}$
24. $\left(-\frac{3}{5}\right)\left(\frac{5}{6}\right)$ $\mathbf{-\frac{1}{2}}$
25. $\left(-\frac{2}{5}\right)\left(\frac{6}{7}\right)$ $\mathbf{-\frac{12}{35}}$
26. $\left(-3\frac{1}{5}\right)\left(-7\frac{1}{2}\right)$ **24**
27. $\left(-1\frac{4}{5}\right)\left(-2\frac{1}{2}\right)$ $\mathbf{4\frac{1}{2}}$
28. $7.2(0.2)$ **1.44**
29. $6.5(0.13)$ **0.845**
30. $(-5.8)(2.3)$ **−13.34**
31. $(-0.075)(6.4)$ **−0.48**
★ 32. $\frac{3}{5}(-5)(-2)$ **6**
★ 33. $\frac{2}{11}(-11)(-4)$ **8**

Simplify each expression.

34. $6(-2x) - 14x$ $\mathbf{-26x}$
35. $5(-4n) - 25n$ $\mathbf{-45n}$
36. $5(2x - x)$ $\mathbf{5x}$
37. $-7(3d + d)$ $\mathbf{-28d}$
38. $-2a(-3c) + (-6y)(6r)$ $\mathbf{6ac + (-36ry)}$
39. $7m(-3n) + 3s(-4t)$ $\mathbf{-21mn + (-12st)}$

www.algebra1.com/extra_examples

3 Practice/Apply

Study Notebook

Have students—

- record the rules for multiplying integers, and the Multiplicative Property of −1.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective

- **Multiply Integers:** 16–21, 34–39
- **Multiply Rational Numbers:** 22–33, 41–51

Odd/Even Assignments

Exercises 16–39 and 42–49 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–31 odd, 35–39 odd, 40, 41, 43–47 odd, 51, 55–76

Average: 17–39 odd, 40, 41, 43–51 odd, 55–76

Advanced: 16–38 even, 42–52 even, 53–68 (optional: 69–76)

All: Practice Quiz 1 (1–10)

Answers

1. Sample answer: ab will be negative if either a or b is negative. Let $a = -2$ and $b = 3$: $-2(3) = -6$. Let $a = 2$ and $b = -3$: $2(-3) = -6$.
2. Sample answer: Calculating a \$2.00 monthly banking fee for the entire year: $12 \times (-\$2) = -\$24$.
3. Since multiplication is repeated addition, multiplying a negative number by another negative number is the same as adding repeatedly in the opposite, or positive direction.

DAILY INTERVENTION

Differentiated Instruction

Logical Students may need help understanding why the product of two numbers with the same sign is positive, especially when both are negative. Start by explaining that $3(-4)$ means 3 groups of -4 or $-4 + (-4) + (-4)$, which is -12. Write $3(-4) = -12$ on the board. Now explain that $-3(-4)$ means the opposite of 3 groups of -4, or the opposite of -12, which is 12.

Lesson 2-3 Multiplying Rational Numbers 81

STOCK PRICES For Exercises 40 and 41, use the table that lists the closing prices of a company's stock over a one-week period.

Closing Stock Price ($)	
Day	**Price**
1	64.38
2	63.66
3	61.66
4	61.69
5	62.34

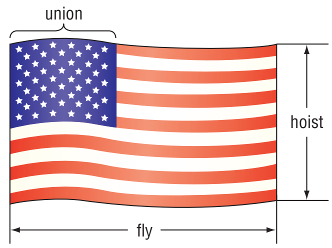
40. What was the change in price of 35 shares of this stock from day 2 to day 3? **−$70**

41. If you bought 100 shares of this stock on day 1 and sold half of them on day 4, how much money did you gain or lose on those shares? **−$134.50**

Evaluate each expression if $a = -2.7$, $b = 3.9$, $c = 4.5$, and $d = -0.2$.

42. $-5c^2$ **−101.25**
43. $-2b^2$ **−30.42**
44. $-4ab$ **42.12**
45. $-5cd$ **4.5**
46. $ad - 8$ **−7.46**
47. $ab - 3$ **−13.53**
★ 48. $d^2(b - 2a)$ **0.372**
★ 49. $b^2(d - 3c)$ **−208.377**

50. **CIVICS** In a United States flag, the length of the union is $\frac{2}{5}$ of the fly, and the width is $\frac{7}{13}$ of the hoist. If the fly is 6 feet, how long is the union? **$2\frac{2}{5}$ ft**



51. **COMPUTERS** The price of a computer dropped \$34.95 each month for 7 months. If the starting price was \$1450, what was the price after 7 months? **$1205.35**

52. **BALLOONING** The temperature drops about 2°F for every rise of 530 feet in altitude. Per Lindstrand achieved the altitude record of 64,997 feet in a hot-air balloon over Laredo, Texas, on June 6, 1988. About how many degrees difference was there between the ground temperature and the air temperature at that altitude? **Source:** *The Guinness Book of Records* **about −245°F**

ECOLOGY For Exercises 53 and 54, use the following information.
Americans use about 2.5 million plastic bottles every hour.
Source: www.savethewater.com

53. About how many plastic bottles are used in one day? **60 million**

54. About how many bottles are used in one week? **420 million**

55. **CRITICAL THINKING** An even number of negative numbers is multiplied. What is the sign of the product? Explain your reasoning.

55. Even; the product of two negative numbers is positive and all even numbers can be divided into groups of two.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How do consumers use multiplication of rational numbers?

Include the following in your answer:
- an explanation of why the amount of a coupon is expressed as a negative value, and
- an explanation of how you could use multiplication to find your total discount if you bought 3 CDs for \$13.99 each and there was a discount of \$1.50 on each CD.

More About. . .

Civics

The Marine Corps War Memorial in Washington, D.C., is dedicated to all Marines who have defended the United States since 1775. It is the most famous memorial that is centered around the flag.

Source: The United States National Park Service

Study Guide and Intervention, p. 87 (shown) and p. 88

Multiply Integers You can use the rules below when multiplying integers and rational numbers.

Multiplying Numbers with the Same Sign	The product of two numbers having the same sign is positive.
Multiplying Numbers with Different Signs	The product of two numbers having different signs is negative.

Example 1 **Find each product.**

a. −7(6)
The signs are different, so the product is negative.
$-7(6) = -42$

b. −18(−10)
The signs are the same, so the product is positive.
$-18(-10) = 180$

Example 2 **Simplify the expression (−2x)5y.**

$(-2x)5y$	$= (-2)(5)x \cdot y$	Commutative Property (×)
	$= (-2 \cdot 5)xy$	Associative Property
	$= -10xy$	Simplify.

Exercises

Find each product.

1. $11(4)$ **44**
2. $-5(-3)$ **15**
3. $(-24)(-2)$ **48**
4. $(60)(-3)$ **−180**
5. $(-2)(-3)(-4)$ **−24**
6. $8(-15)$ **−120**
7. $-15(3)$ **−45**
8. $(12)(-10)$ **−120**
9. $(-22)(-3)(2)$ **132**
10. $(5)(-5)(0)(4)$ **0**
11. $(-15)(45)$ **−675**
12. $(-12)(-23)$ **276**

Simplify each expression.

13. $4(-2x) - 8x$ **−16x**
14. $6(-2n) - 10n$ **−22n**
15. $6(3y - y)$ **12y**
16. $-3(3d + 2d)$ **−15d**
17. $-2x(2) + 2x(3y)$ **−4x + 6xy**
18. $4m(-2n) + 2d(-4e)$ **−8mn − 8de**
19. $-5(2x + x) - 3(-xy)$ **−15x + 3xy**
20. $(2)(-4x + 2x)$ **−4x**
21. $(-3)(-8n - 6m)$ **24n + 18m**

Skills Practice, p. 89 and Practice, p. 90 (shown)

Find each product.

1. $42(7)$ **294**
2. $-28(-17)$ **476**
3. $15(-34)$ **−510**
4. $\left(-\frac{3}{4}\right)\left(\frac{7}{8}\right)$ $-\frac{21}{32}$
5. $\left(-\frac{4}{5}\right)\left(-\frac{5}{6}\right)$ $\frac{2}{3}$
6. $\left(\frac{9}{10}\right)\left(\frac{5}{7}\right)$ $\frac{9}{14}$
7. $\left(-3\frac{1}{4}\right)\left(2\frac{1}{2}\right)$ $-\frac{65}{8}$ or $-8\frac{1}{8}$
8. $\left(-2\frac{2}{3}\right)\left(-1\frac{1}{6}\right)$ $\frac{28}{9}$ or $3\frac{1}{9}$
9. $\left(1\frac{1}{4}\right)\left(-1\frac{1}{5}\right)$ $-\frac{3}{2}$ or $-1\frac{1}{2}$
10. $(1.5)(8.8)$ **13.2**
11. $(6.8)(-1.3)$ **−8.84**
12. $(-0.2)(2.8)$ **−0.56**
13. $(-3.6)(-0.55)$ **1.98**
14. $6.3(-0.7)$ **−4.41**
15. $\frac{2}{3}(-4)(9)$ **−24**

Simplify each expression.

16. $5(-3a) + 18a$ **3a**
17. $-8(4c) + 12c$ **−20c**
18. $-9(2g - g)$ **−9g**
19. $7(2b - 4b)$ **−14b**
20. $-4x(2y) + (-3b)(-2d)$ **−8xy + 6bd**
21. $-5p(-3q) + (4m)(-6n)$ **15pq + (−24mn)**

Evaluate each expression if $a = -\frac{4}{5}$, $b = \frac{3}{4}$, $c = -3.4$, and $d = 0.7$.

22. $b^2\left(-\frac{2}{3}\right)$ $-\frac{3}{8}$
23. $4ab$ $-\frac{12}{5}$ or $-2\frac{2}{5}$
24. $5a^2(-b)$ $-\frac{12}{5}$ or $-2\frac{2}{5}$
25. $-6d^2$ **−2.94**
26. $cd - 3$ **−5.38**
27. $c^2(-5d)$ **−40.46**

28. **RECIPES** A recipe for buttermilk biscuits calls for $3\frac{1}{3}$ cups of flour. How many cups of flour do you need for $\frac{1}{2}$ the recipe? **$1\frac{2}{3}$ c**

COMPUTERS For Exercises 29 and 30, use the following information.
Leeza is downloading a file from a Web site at 47.3 kilobytes per second.

29. How many kilobytes of the file will be downloaded after one minute? **2838 kilobytes**

30. How many kilobytes will be downloaded after 4.5 minutes? **12,771 kilobytes**

CONSERVATION For Exercises 31 and 32, use the following information.
A county commission has set aside 640 acres of land for a wildlife preserve.

31. Suppose $\frac{2}{5}$ of the preserve is marshland. How many acres of the preserve are marshland? **256 acres**

32. If the forested area of the preserve is 1.5 times larger than the marshland, how many acres of the preserve are forested? **384 acres**

Reading to Learn Mathematics, p. 91 ELL

Pre-Activity **How do consumers use multiplication of rational numbers?**

Read the introduction to Lesson 2-3 at the top of page 79 in your textbook.
- How is the amount of the coupon shown on the sales slip? **−1.00**
- Besides the amount, how is the number representing the coupon different from the other numbers on the sales slip?
  It is negative.

Reading the Lesson

1. Complete: If two numbers have different signs, the one number is positive and the other number is **negative**.

2. Complete the table.

	Multiplication Example	Are the signs of the numbers the same or different?	Is the product positive or negative?
a.	(−4)(9)	different	negative
b.	(−2)(−13)	the same	positive
c.	5(−8)	different	negative
d.	6(3)	the same	positive

3. Explain what the term "additive inverse" means to you. Then give an example.
The product of any number and −1 is its additive inverse; $-\frac{2}{3} \cdot (-1) = \frac{2}{3}$.

Helping You Remember

4. Describe how you know that the product of −3 and −5 is positive. Then describe how you know that the product of 3 and −5 is negative.
Sample answer: The signs are the same; the signs are different.

Enrichment, p. 92

Compound Interest

In most banks, interest on savings accounts is compounded at set time periods such as three or six months. At the end of each period, the bank adds the interest earned to the account. During the next period, the bank pays interest on all the money in the bank, including interest. In this way, the account earns interest on interest.

Suppose Ms. Tanner has \$1000 in an account that is compounded quarterly at 5%. Find the balance after the first two quarters.

Use $I = prt$ to find the interest earned in the first quarter if $p = 1000$ and $r = 5\%$. Why is t equal to $\frac{1}{4}$?

First quarter: $I = 1000 \times 0.05 \times \frac{1}{4}$

$I = 12.50$

The interest, \$12.50, earned in the first quarter is added to \$1000. The principal becomes \$1012.50.

Second quar[illegible] $I = 1012.50 \times 0.05 \times \frac{1}{...}$

Standardized Test Practice

57. Which expression can be simplified as $-8xy$? **B**

(A) $2y - 4x$ (B) $-2x(4y)$ (C) $(-4)^2xy$ (D) $-4x(-2y)$

58. Find the value of m if $m = -2ab$, $a = -4$, and $b = 6$. **B**

(A) 8 (B) 48 (C) 12 (D) -48

Maintain Your Skills

Mixed Review

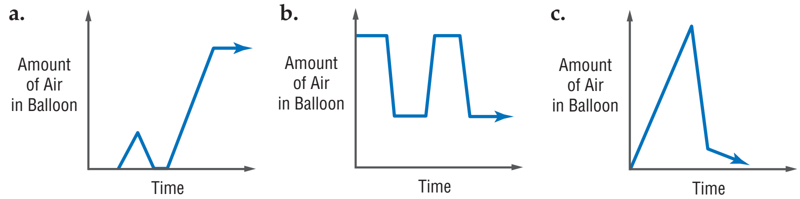
Find each sum or difference. *(Lesson 2-2)*

59. $-6.5 + (-5.6)$ **-12.1**
60. $\frac{4}{5} + \left(-\frac{3}{4}\right)$ **$\frac{1}{20}$**
61. $42 - (-14)$ **56**
62. $-14.2 - 6.7$ **-20.9**

Graph each set of numbers on a number line. *(Lesson 2-1)* **63–65. See margin.**

63. $\{\ldots, -3, -1, 1, 3, 5\}$
64. $\{-2.5, -1.5, 0.5, 4.5\}$
65. $\{-1, -\frac{1}{3}, \frac{2}{3}, 2\}$

66. Identify the graph below that best represents the following situation. Brandon has a deflated balloon. He slowly fills the balloon up with air. Without tying the balloon, he lets it go. *(Lesson 1-8)* **C**



Write a counterexample for each statement. *(Lesson 1-7)*

67. If $2x - 4 \geq 6$, then $x > 5$. **$x = 5$**
68. If $|a| > 3$, then $a > 3$. **$a = -4$**

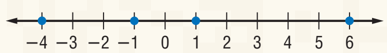
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each quotient.**
*(To review **division of fractions**, see pages 800 and 801.)*

69. $\frac{5}{8} \div 2$ **$\frac{5}{16}$**
70. $\frac{2}{3} \div 4$ **$\frac{1}{6}$**
71. $5 \div \frac{3}{4}$ **$6\frac{2}{3}$**
72. $1 \div \frac{2}{5}$ **$2\frac{1}{2}$**
73. $\frac{1}{2} \div \frac{3}{8}$ **$1\frac{1}{3}$**
74. $\frac{7}{9} \div \frac{5}{6}$ **$\frac{14}{15}$**
75. $\frac{4}{5} \div \frac{6}{5}$ **$\frac{2}{3}$**
76. $\frac{7}{8} \div \frac{2}{3}$ **$1\frac{5}{16}$**

Practice Quiz 1 — *Lessons 2-1 through 2-3*

1. Name the set of points graphed on the number line. *(Lesson 2-1)* **$\{-4, -1, 1, 6\}$**



2. Evaluate $32 - |x + 8|$ if $x = 15$. *(Lesson 2-1)* **9**

Find each sum or difference. *(Lesson 2-2)*

3. $-15 + 7$ **-8**
4. $27 - (-12)$ **39**
5. $-6.05 + (-2.1)$ **-8.15**
6. $-\frac{3}{4} - \left(-\frac{2}{5}\right)$ **$-\frac{7}{20}$**

Find each product. *(Lesson 2-3)*

7. $-9(-12)$ **108**
8. $(3.8)(-4.1)$ **-15.58**

9. Simplify $(-8x)(-2y) + (-3y)(z)$. *(Lesson 2-3)* **$16xy - 3yz$**

10. Evaluate $mn + 5$ if $m = 2.5$ and $n = -3.2$. *(Lesson 2-3)* **-3**

www.algebra1.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Have students write an explanation of why the product of two negative numbers is positive.

Getting Ready for Lesson 2-4

PREREQUISITE SKILL Students will learn about dividing rational numbers in Lesson 2-4. Use Exercises 69–76 to determine your students' familiarity with the division of fractions, which are part of the set of rational numbers.

Assessment Options

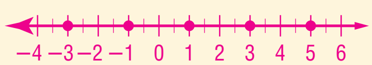
Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 2-1 through 2-3. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Answers

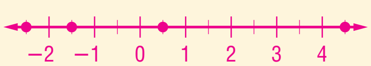
56. Sample answer: Multiplying lets consumers calculate quickly the total of several similar items. Answers should include the following.

- **Coupons are negative values because adding a negative number is the same as subtracting a positive number.**
- **Multiply 13.99 and −1.50 by three, then add the products.**

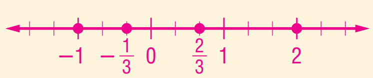
63.



64.



65.



2-4 Lesson Notes

1 Focus

5-Minute Check Transparency 2-4 Use as a quiz or review of Lesson 2-3.

Mathematical Background notes are available for this lesson on p. 66D.

How can you use the division of rational numbers to describe data?

Ask students:

- What is another word for mean as it is used in this problem? **average**
- How do you find the average or mean of a set of numbers? **Add the numbers and then divide the sum by the number of data items in the set.**
- If you add the numbers in the Change column in the table, will the sum be positive or negative? **negative**

TEACHING TIP

Additional practice for finding the mean is provided on pages 818 and 819.

2-4 Dividing Rational Numbers

What You'll Learn

- Divide integers.
- Divide rational numbers.

How can you use division of rational numbers to describe data?

Each year, many sea turtles are stranded on the Texas Gulf Coast. The number of sea turtles stranded from 1997 to 2000 and the changes in number from the previous years are shown in the table. The following expression can be used to find the *mean* change per year of the number of stranded turtles.

$$\text{mean} = \frac{(-127) + 54 + (-65)}{3}$$

Stranded Sea Turtles Texas Gulf Coast		
Year	Number of Turtles	Change
1997	523	——
1998	396	−127
1999	450	+54
2000	385	−65

Source: www.ridleyturtles.org

DIVIDE INTEGERS Since multiplication and division are inverse operations, the rule for finding the sign of the quotient of two numbers is similar to the rule for finding the sign of a product of two numbers.

Key Concept *Division of Integers*

- **Words** The quotient of two numbers having the *same sign* is positive. The quotient of two numbers having *different signs* is negative.
- **Examples** $(-60) \div (-5) = 12$ same signs → positive quotient
  $32 \div (-8) = -4$ different signs → negative quotient

Example 1 Divide Integers

Find each quotient.

a. $-77 \div 11$

$-77 \div 11 = -7$ negative quotient

b. $\frac{-51}{-3}$

$\frac{-51}{-3} = -51 \div (-3)$ Divide.

$= 17$ positive quotient

When simplifying fractions, recall that the fraction bar is a grouping symbol.

Example 2 Simplify Before Dividing

Simplify $\frac{-3(-12 + 8)}{7 + (-5)}$.

$\frac{-3(-12 + 8)}{7 + (-5)} = \frac{-3(-4)}{7 + (-5)}$ Simplify the numerator first.

$= \frac{12}{7 + (-5)}$ Multiply.

$= \frac{12}{2}$ or 6 same signs → positive quotient

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters
- Study Guide and Intervention, pp. 93–94
- Skills Practice, p. 95
- Practice, p. 96
- Reading to Learn Mathematics, p. 97
- Enrichment, p. 98
- Assessment, pp. 131, 133

Parent and Student Study Guide Workbook, p. 14

Prerequisite Skills Workbook, pp. 15–16, 19–20, 29–32, 39–40, 47–48, 51–54, 63–66, 75–76

School-to-Career Masters, p. 4

Transparencies

5-Minute Check Transparency 2-4
Answer Key Transparencies

Technology

AlgePASS: Tutorial Plus, Lesson 3
Interactive Chalkboard

DIVIDE RATIONAL NUMBERS The rules for dividing positive and negative integers also apply to division with rational numbers. Remember that to divide by any nonzero number, multiply by the reciprocal of that number.

Example 3 Divide Rational Numbers

Find each quotient.

a. $245.66 \div (-14.2)$

$245.66 \div (-14.2) = -17.3$ Use a calculator. different signs → negative quotient

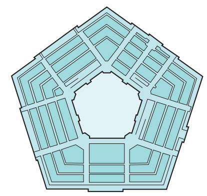
b. $-\frac{2}{5} \div \frac{1}{4}$

$-\frac{2}{5} \div \frac{1}{4} = -\frac{2}{5} \cdot \frac{4}{1}$ Multiply by $\frac{4}{1}$, the reciprocal of $\frac{1}{4}$.

$= -\frac{8}{5}$ or $-1\frac{3}{5}$ different signs → negative quotient

Example 4 Divide Rational Numbers to Solve a Problem

ARCHITECTURE **The Pentagon in Washington, D.C., has an outside perimeter of 4608 feet. Find the length of each outside wall.**



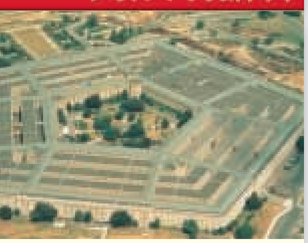
The Pentagon

To find the length of each wall, divide the perimeter by the number of sides.

$4608 \div 5 = 921.6$ same signs → positive quotient

The length of each outside wall is 921.6 feet.

More About. . .



Architecture

The Pentagon is one of the world's largest office buildings. It contains 131 stairways, 19 escalators, 13 elevators, 284 restrooms, and 691 drinking fountains.

Source: www.infoplease.com

You can use the Distributive Property to simplify fractional expressions.

Example 5 Simplify Algebraic Expressions

Simplify $\frac{24 - 6a}{3}$.

$\frac{24 - 6a}{3} = (24 - 6a) \div 3$ The fraction bar indicates division.

$= (24 - 6a)\left(\frac{1}{3}\right)$ Multiply by $\frac{1}{3}$, the reciprocal of 3.

$= 24\left(\frac{1}{3}\right) - 6a\left(\frac{1}{3}\right)$ Distributive Property

$= 8 - 2a$ Simplify.

Example 6 Evaluate Algebraic Expressions

Evaluate $\frac{ab}{c^2}$ if $a = -7.8$, $b = 5.2$, and $c = -3$. Round to the nearest hundredth.

$\frac{ab}{c^2} = \frac{(-7.8)(5.2)}{(-3)^2}$ Replace a with -7.8, b with 5.2, and c with -3.

$= \frac{-40.56}{9}$ Find the numerator and denominator separately.

≈ 4.51 Use a calculator. same signs → positive quotient

www.algebra1.com/extra_examples

2 Teach

DIVIDE INTEGERS

In-Class Examples Power Point®

1 Find each quotient.

a. $-60 \div (-5)$ **12**

b. $\frac{-108}{18}$ **−6**

2 Simplify $\frac{2(1 - 5)}{17 + (-13)}$. **−2**

DIVIDE RATIONAL NUMBERS

In-Class Examples Power Point®

3 Find each quotient.

a. $-112.23 \div 8.7$ **−12.9**

b. $-\frac{3}{8} \div \left(-\frac{1}{3}\right)$ $\frac{9}{8}$ **or** $1\frac{1}{8}$

4 **BASEBALL** The perimeter of a square baseball diamond is 360 feet. Find the length of one side of the diamond. **90 ft**

Teaching Tip You may also want to show Example 5 as the difference of two fractions with the same denominator.

$\frac{24 - 6a}{3} = \frac{24}{3} - \frac{6a}{3} = 8 - 2a$

5 Simplify $\frac{-39b + 65}{13}$. $-3b + 5$

6 Evaluate $\frac{wx}{y^2}$ if $w = 2$, $x = -9.1$ and $y = 4$. **−1.1375**

3 Practice/Apply

Study Notebook

Have students—

- *record the rules for dividing integers.*
- *include any other item(s) that they find helpful in mastering the skills in this lesson.*

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Use index cards to write each component of a division expression as an equation. Have students model the division and then rearrange the cards to make a multiplication sentence, setting the ÷ card aside.

$\boxed{-60}\ \boxed{\div}\ \boxed{-5}\ \boxed{=}\ \boxed{?} \rightarrow \boxed{?}\ \boxed{\times}\ \boxed{-5}\ \boxed{=}\ \boxed{-60}$

About the Exercises...

Organization by Objective

- **Divide Integers:** 17, 18, 37–44
- **Divide Rational Numbers:** 19–36, 45–54

Odd/Even Assignments

Exercises 17–54 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–31 odd, 35–41 odd, 45–51 odd, 55, 58–77

Average: 17–55 odd, 58–77

Advanced: 18–54 even, 56–73 (optional: 74–77)

4 Assess

Open-Ended Assessment

Writing Display an expression similar to that found in Example 6. Have one group of students evaluate the numerator while another evaluates the denominator. Have each group explain their results and then together find the quotient.

Getting Ready for Lesson 2-5

PREREQUISITE SKILL Students will learn about displaying and analyzing data in Lesson 2-5, including analyzing means, medians, and modes. Use Exercises 74–77 to determine your students' familiarity with finding mean, median, and mode.

Assessment Options

Quiz (Lessons 2-3 and 2-4) is available on p. 131 of the *Chapter 2 Resource Masters*.

Mid-Chapter Test (Lessons 2-1 through 2-4) is available on p. 133 of the *Chapter 2 Resource Masters*.

Check for Understanding

Concept Check

1. **Compare and contrast** multiplying and dividing rational numbers. **See margin.**
2. **OPEN ENDED** Find a value for x if $\frac{1}{x} > x$. **Sample answer: $\frac{1}{2}$**
3. **Explain** how to divide any rational number by another rational number. **To divide by a rational number, multiply by its reciprocal.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1, 3
10–12	2, 5
13–15	6
16	4

Find each quotient.

4. $96 \div (-6)$ **-16**
5. $-36 \div 4$ **-9**
6. $-64 \div 5$ **-12.8**
7. $64.4 \div 2.5$ **25.76**
8. $-\frac{2}{3} \div 12$ **$-\frac{1}{18}$**
9. $-\frac{2}{3} \div \frac{4}{5}$ **$-\frac{5}{6}$**

Simplify each expression.

10. $\frac{25+3}{-4}$ **-7**
11. $\frac{-650a}{10}$ **$-65a$**
12. $\frac{6b+18}{-2}$ **$-3b + (-9)$**

Evaluate each expression if $a = 3$, $b = -4.5$, and $c = 7.5$. Round to the nearest hundredth.

13. $\frac{2ab}{-ac}$ **1.2**
14. $\frac{cb}{4a}$ **-2.81**
15. $-\frac{a}{b} \div \frac{a}{c}$ **1.67**

Application

16. **ONLINE SHOPPING** During the 2000 holiday season, the sixth most visited online shopping site recorded 419,000 visitors. This is eight times as many visitors as in 1999. About how many visitors did the site have in 1999? **52,375**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
17–36	1, 3
37–44	5
45, 46, 55–57	4
47–54	6

Extra Practice

See page 824.

Find each quotient.

17. $-64 \div (-8)$ **8**
18. $-78 \div (-4)$ **19.5**
19. $-78 \div (-1.3)$ **60**
20. $108 \div (-0.9)$ **-120**
21. $42.3 \div (-6)$ **-7.05**
22. $68.4 \div (-12)$ **-5.7**
23. $-23.94 \div 10.5$ **-2.28**
24. $-60.97 \div 13.4$ **-4.55**
25. $-32.25 \div (-2.5)$ **12.9**
26. $-98.44 \div (-4.6)$ **21.4**
27. $-\frac{1}{3} \div 4$ **$-\frac{1}{12}$**
28. $-\frac{3}{4} \div 12$ **$-\frac{1}{16}$**
29. $-7 \div \frac{3}{5}$ **$-\frac{35}{3}$ or $-11\frac{2}{3}$**
30. $-5 \div \frac{2}{7}$ **$-\frac{35}{2}$ or $-17\frac{1}{2}$**
31. $\frac{16}{36} \div \frac{24}{60}$ **$\frac{10}{9}$ or $1\frac{1}{9}$**
32. $-\frac{24}{56} \div \frac{31}{63}$ **$-\frac{27}{31}$**
★33. $\frac{14}{32} \div \left(-\frac{12}{25}\right)$ **$-\frac{175}{192}$**
★34. $\frac{80}{25} \div \left(-\frac{2}{3}\right)$ **$-\frac{24}{5}$ or $-4\frac{4}{5}$**
35. Find the quotient of -74 and $-\frac{5}{3}$. **$\frac{222}{5}$ or $44\frac{2}{5}$**
36. Find the quotient of -156 and $-\frac{3}{8}$. **416**

Simplify each expression. **39. $-r + (-3)$ 40. $-h + (-5)$**

37. $\frac{81c}{9}$ **$9c$**
38. $\frac{105g}{5}$ **$21g$**
39. $\frac{8r+24}{-8}$
40. $\frac{7h+35}{-7}$
41. $\frac{40a-50b}{2}$ **$20a - 25b$**
42. $\frac{42c-18d}{3}$ **$14c - 6d$**
★43. $\frac{-8f+(-16g)}{8}$ **$-f + (-2g)$**
★44. $\frac{-5x+(-10y)}{5}$ **$-x + (-2y)$**

45. **CRAFTS** Hannah is making pillows. The pattern states that she needs $1\frac{3}{4}$ yards of fabric for each pillow. If she has $4\frac{1}{2}$ yards of fabric, how many pillows can she make? **2**

46. **BOWLING** Bowling centers in the United States made $2,800,000,000 in 1990. Their receipts in 1998 were $2,764,000,000. What was the average change in revenue for each of these 8 years? **Source:** U.S. Census Bureau **−$4,500,000**

Answer

1. **Sample answer: Dividing and multiplying numbers with the same signs both result in a positive answer while dividing or multiplying numbers with different signs results in a negative answer. However, when you divide rational numbers in fractional form, you must multiply a reciprocal.**

86 Chapter 2 Real Numbers

Evaluate each expression if $m = -8$, $n = 6.5$, $p = 3.2$, and $q = -5.4$. Round to the nearest hundredth.

47. $\frac{mn}{p}$ **−16.25**
48. $\frac{np}{m}$ **−2.6**
49. $mq \div np$ **2.08**
50. $pq \div mn$ **0.33**
51. $\frac{n+p}{m}$ **−1.21**
52. $\frac{m+p}{q}$ **0.89**
★ 53. $\frac{m-2n}{-n+q}$ **1.76**
★ 54. $\frac{p-3q}{-q-m}$ **1.45**

55. **BUSINESS** The president of a small business is looking at her profit/loss statement for the past year. The loss in income for the last year was $23,985. On average, what was the loss per month last year? **$1998.75**

More About. . .

Jewelry

The discovery of gold at Sutter's Mill early in 1848 brought more than 40,000 prospectors to California within two years.

Source: www.infoplease.com

JEWELRY For Exercises 56 and 57, use the following information.
The gold content of jewelry is given in karats. For example, 24-karat gold is pure gold, and 18-karat gold is $\frac{18}{24}$ or 0.75 gold. **56.** $\frac{5}{12}$; $\frac{7}{12}$

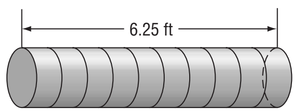
56. What fraction of 10-karat gold is pure gold? What fraction is not gold?
57. If a piece of jewelry is $\frac{2}{3}$ gold, how would you describe it using karats? **16-karat gold**
58. **CRITICAL THINKING** What is the least positive integer that is divisible by all whole numbers from 1 to 9? **2520**
59. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you use division of rational numbers to describe data?

Include the following in your answer:
- an explanation of how you could use the mean of a set of data to describe changes in the data over time, and
- reasons why you think the change from year to year is not consistent.

Standardized Test Practice

60. If the rod is cut as shown, how many inches long will each piece be? **D**



(A) 0.625 in. (B) 1.875 in.
(C) 5.2 in. (D) 7.5 in.

61. If $\frac{17}{3} = x$, then what is the value of $6x + 1$? **B**

(A) 32 (B) 33 (C) 44 (D) 35

Maintain Your Skills

Mixed Review

Find each product. *(Lesson 2-3)*

62. $-4(11)$ **−44**
63. $-2.5(-1.2)$ **3**
64. $\frac{1}{4}(-5)$ **$-1\frac{1}{4}$**
65. $1.6(0.3)$ **0.48**

Find each difference. *(Lesson 2-2)*

66. $8 - (-6)$ **14**
67. $15 - 21$ **−6**
68. $-7.5 - 4.8$ **−12.3**
69. $-\frac{5}{8} - \left(-\frac{1}{6}\right)$ **$-\frac{11}{24}$**

70. Name the property illustrated by $2(1.2 + 3.8) = 2 \cdot 5$. **Subs.**

Simplify each expression. If not possible, write *simplified*. *(Lesson 1-5)*

71. $8b + 12(b + 2)$ **$20b + 24$**
72. $6(5a + 3b - 2b)$ **$30a + 6b$**
73. $3(x + 2y) - 2y$ **$3x + 4y$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the mean, median, and mode for each set of data.
(To review ***mean***, ***median****, and* ***mode****, see pages 818 and 819.)*

74. 40, 34, 40, 28, 38 **36; 38; 40**
75. 3, 9, 0, 2, 11, 8, 14, 3 **6.25; 4.5; 3**
76. 1.2, 1.7, 1.9, 1.8, 1.2, 1.0, 1.5 **1.5; 1.5; 1.2**
77. 79, 81, 84, 75, 73, 80, 78 **79.3; 79.5; 84**

www.algebra1.com/self_check_quiz

Answer

59. Sample answer: You use division to find the mean of a set of data. Answers should include the following.
- You could track the mean number of turtles stranded each year and note if the value increases or decreases.
- Weather or pollution could affect the turtles.

Study Guide and Intervention, p. 93 (shown) and p. 94

Divide Integers The rules for finding the sign of a quotient are similar to the rules for finding the sign of a product.

Dividing Two Numbers with the Same Sign	The quotient of two numbers having the same sign is positive.
Dividing Two Numbers with Different Signs	The quotient of two numbers having different signs is negative.

Example 1 Find each quotient.

a. $-88 \div (-4)$
$-88 \div (-4) = 22$ same signs → positive quotient

b. $\frac{-64}{8}$
$\frac{-64}{8} = -8$ different signs → negative quotient

Example 2 Simplify $\frac{-4(-10+2)}{-3+(-1)}$.

$$\frac{-4(-10+2)}{-3+(-1)} = \frac{-4(-8)}{-3+(-1)} = \frac{32}{-3+(-1)} = \frac{32}{-4} = -8$$

Exercises

Find each quotient.

1. $-80 \div (-10)$ **8**
2. $-32 \div 16$ **−2**
3. $80 \div 5$ **16**
4. $18 \div (-3)$ **−6**
5. $-12 \div (-3)$ **4**
6. $8 \div (-2)$ **−4**
7. $-15 \div (-3)$ **5**
8. $121 \div (-11)$ **−11**
9. $-24 \div 1.5$ **−16**
10. $0 \div (-8)$ **0**
11. $-125 \div (-25)$ **5**
12. $-104 \div 4$ **−26**

Simplify.

13. $\frac{-2+(-4)}{(-2)+(-1)}$ **2**
14. $\frac{5(-10+(-2))}{-2+1}$ **60**
15. $\frac{-6(-6+2)}{-10+(-2)}$ **−2**
16. $\frac{-12(2+(-3))}{-4+1}$ **−4**
17. $\frac{-4(-8+(-4))}{-3+(-3)}$ **−8**
18. $\frac{4(-12+4)}{-2(8)}$ **2**

Skills Practice, p. 95 and Practice, p. 96 (shown)

Find each quotient.

1. $75 \div (-15)$ **−5**
2. $-323 \div (-17)$ **19**
3. $-88 \div 16$ **−5.5**
4. $65.7 \div (-9)$ **−7.3**
5. $-36.08 \div 8$ **−4.51**
6. $-40.05 \div (-2.5)$ **16.02**
7. $-9 \div \frac{3}{5}$ **−15**
8. $-\frac{5}{6} \div \left(-\frac{3}{8}\right)$ **$\frac{20}{9}$ or $2\frac{2}{9}$**
9. $\frac{14}{63} \div \left(-\frac{49}{54}\right)$ **$-\frac{12}{49}$**

Simplify each expression.

10. $\frac{168p}{-14}$ **$-12p$**
11. $\frac{25-5x}{5}$ **$5 - x$**
12. $\frac{3t+12}{-3}$ **$-t + (-4)$**
13. $\frac{18x+12y}{-6}$ **$-3x + (-2y)$**
14. $\frac{8k-12h}{4}$ **$2k - 3h$**
15. $\frac{-4c+(-16d)}{4}$ **$-c + (-4d)$**

Evaluate each expression if $p = -6$, $q = 4.5$, $r = 3.6$, and $s = -5.2$. Round to the nearest hundredth.

16. $\frac{qr}{p}$ **−2.7**
17. $\frac{rs}{q}$ **−4.16**
18. $ps \div qr$ **1.93**
19. $rs \div pq$ **0.69**
20. $\frac{p-q}{r}$ **−2.92**
21. $\frac{r+s}{q}$ **−0.36**

22. **EXERCISE** Ashley walks $2\frac{1}{2}$ miles around a lake three times a week. If Ashley walks around the lake in $\frac{3}{4}$ hour, what is her rate of speed? (*Hint*: Use the formula $r = \frac{d}{t}$, where r is rate, d is distance, and t is time.) **$3\frac{1}{3}$ mi/h**

23. **PUBLICATION** A production assistant must divide a page of text into two columns. If the page is $6\frac{3}{4}$ inches wide, how wide will each column be? **$3\frac{3}{8}$ in.**

ROLLER COASTERS For Exercises 24 and 25, use the following information.
The formula for acceleration is $a = \frac{f-s}{t}$, where a is acceleration, f is final speed, s is starting speed, and t is time.

24. The Hypersonic XLC roller coaster in Virginia goes from zero to 80 miles per hour in 1.8 seconds. What is its acceleration in miles per hour per second to the nearest tenth? **Source:** www.thrillride.com **about 44.4 mi/h per second**

25. What is the acceleration in feet per second per second? (*Hint:* Convert miles to feet and hours to seconds, then apply the formula for acceleration. 1 mile = 5280 feet) **about 65.2 ft/s per second**

Reading to Learn Mathematics, p. 97 ELL

Pre-Activity How can you use division of rational numbers to describe data?

Read the introduction to Lesson 2-4 at the top of page 84 in your textbook.

- What is meant by the term *mean*?
  the sum of a set of data items divided by the number of data items.
- In the expression $\frac{(-127) + 54 + (-65)}{3}$, will the numerator be positive or negative?
  negative

Reading the Lesson

1. Explain what the term *inverse operations* means to you.
Sample answer: Inverse operations are operations that undo one another.

2. Write *negative* or *positive* to describe the quotient. Explain your answer.

	Expression	Negative or Positive?	Explanation
a.	$\frac{35}{-7}$	negative	The signs of the two numbers are different.
b.	$\frac{-78}{-13}$	positive	The signs of the two numbers are the same.
c.	$\frac{(-5.6)(-2.4)}{1.92}$	positive	After multiplying, the signs of the numbers being divided are the same.

Helping You Remember

3. Explain how knowing the rules for multiplying rational numbers can help you remember the rules for dividing rational numbers.
Sample answer: Both rules state that the *answer* (product for multiplication, quotient for division) is positive if the signs are the same and negative if the signs are different.

Enrichment, p. 98

Other Kinds of Means

There are many different types of means besides the arithmetic mean. A mean for a set of numbers has these two properties:

a. It typifies or represents the set.
b. It is not less than the least number and it is not greater than the greatest number.

Here are the formulas for the arithmetic mean and three other means.

Arithmetic Mean

Add the numbers in the set. Then divide the sum by n, the number of elements in the set.

$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Geometric Mean

Multiply all the numbers in the set. Then find the nth root of their product.

$$\sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \cdots \cdot x_n}$$

Harmonic Mean

Divide the number of elements in the set by

Quadratic Mean

Add the squares of the numbers. Divide

2-5 Lesson Notes

1 Focus

5-Minute Check Transparency 2-5 Use as a quiz or review of Lesson 2-4.

Mathematical Background notes are available for this lesson on p. 66D. This lesson covers *univariate data*, which means data depending on only one random variable.

Building on Prior Knowledge

In Chapter 1, students learned how graphs can be used to visualize data. In this lesson, students will learn about other types of graphs for visualizing data.

How are line plots and averages used to make decisions?

Ask students:

- What was the most popular boys' name in all five decades? How can you tell? **The most popular boys' name in all five decades was Michael because it is in the first column for all five decades.**
- Were any girls' names as popular as Michael? How can you tell? **No, because no girl's name appears twice in any of the five columns.**

2-5 Statistics: Displaying and Analyzing Data

What You'll Learn

- Interpret and create line plots and stem-and-leaf plots.
- Analyze data using mean, median, and mode.

Vocabulary

- line plot
- frequency
- stem-and-leaf plot
- back-to-back stem-and-leaf plot
- measures of central tendency

How are line plots and averages used to make decisions?

How many people do you know with the same first name? Some names are more popular than others. The table below lists the top five most popular names for boys and girls born in each decade from 1950 to 1999.

Top Five First Names of America

Boys / Girls

1950-59	Michael	James	Robert	John	David
	Deborah	Mary	Linda	Patricia	Susan
1960-69	Michael	John	David	James	Robert
	Lisa	Deborah	Mary	Karen	Michelle
1970-79	Michael	Christopher	Jason	David	James
	Jennifer	Michelle	Amy	Melissa	Kimberly
1980-89	Michael	Christopher	Matthew	Joshua	David
	Jessica	Jennifer	Ashley	Sarah	Amanda
1990-99	Michael	Christopher	Matthew	Joshua	Nicholas
	Ashley	Jessica	Sarah	Brittany	Emily

Source: *The World Almanac*

To help determine which names appear most frequently, these data could be displayed graphically.

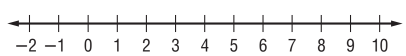
CREATE LINE PLOTS AND STEM-AND-LEAF PLOTS In some cases, data can be presented using a **line plot**. Most line plots have a number line labeled with a scale to include all the data. Then an × is placed above a data point each time it occurs to represent the **frequency** of the data.

Example 1 *Create a Line Plot*

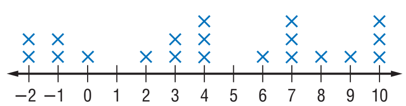
Draw a line plot for the data.

$-2 \quad 4 \quad 3 \quad 2 \quad 6 \quad 10 \quad 7 \quad 4 \quad -2 \quad 0 \quad 10 \quad 8 \quad 7 \quad 10 \quad 7 \quad 4 \quad -1 \quad 9 \quad -1 \quad 3$

Step 1 The value of the data ranges from −2 to 10, so construct a number line containing those points.



Step 2 Then place an × above a number each time it occurs.



Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters
- Study Guide and Intervention, pp. 99–100
- Skills Practice, p. 101
- Practice, p. 102
- Reading to Learn Mathematics, p. 103
- Enrichment, p. 104

Parent and Student Study Guide Workbook, p. 15
Prerequisite Skills Workbook, pp. 15–16, 61–62, 75–76

Transparencies

5-Minute Check Transparency 2-5
Answer Key Transparencies

Technology

Interactive Chalkboard

Line plots are a convenient way to organize data for comparison.

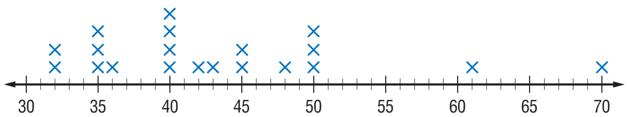
Example 2 Use a Line Plot to Solve a Problem

ANIMALS **The speeds (mph) of 20 of the fastest land animals are listed below.**

45	70	43	45	32	42	40	40	35	50
40	35	61	48	35	32	50	36	50	40

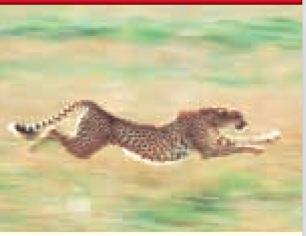
Source: *The World Almanac*

a. Make a line plot of the data.
The lowest value is 30, and the highest value is 70, so use a scale that includes those values. Place an × above each value for each occurrence.



b. Which speed occurs most frequently?
Looking at the line plot, we can easily see that 40 miles per hour occurs most frequently.

More About. . .



Animals

Whereas the cheetah is the fastest land animal, the fastest marine animal is the sailfish. It is capable of swimming 68 miles per hour.

Source: *The Top 10 of Everything*

Another way to organize and display data is by using a **stem-and-leaf plot**. In a stem-and-leaf plot, the greatest common place value is used for the *stems*. The numbers in the next greatest place value are used to form the *leaves*. In Example 2, the greatest place value is tens. Thus, 32 miles per hour would have a stem of 3 and a leaf of 2. A complete stem-and-leaf plot for the data in Example 2 is shown below.

Stem	Leaf
3	2 2 5 5 5 6
4	0 0 0 0 2 3 5 5 8
5	0 0 0
6	1
7	0 *3|2 = 32*

key

Study Tip

Stem-and-Leaf Plots

A key is included on stem-and-leaf plots to indicate what the stems and leaves represent when read.

Example 3 Create a Stem-and-Leaf Plot

Use the data below to make a stem-and-leaf plot.

108	104	86	82	80	72	70	62	64	68	84	64	98	96	98
103	87	65	83	79	97	96	112	62	80	62	83	76	66	97

The greatest common place value is tens, so the digits in the tens place are the stems.

Stem	Leaf
6	2 2 2 4 4 5 6 8
7	0 2 6 9
8	0 0 2 3 3 4 6 7
9	6 6 7 7 8 8
10	3 4 8
11	2 *10|3 = 103*

A **back-to-back stem-and-leaf plot** can be used to compare two related sets of data.

www.algebra1.com/extra_examples

2 Teach

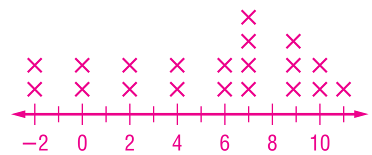
CREATE LINE PLOTS AND STEM-AND-LEAF PLOTS

In-Class Examples

Power Point®

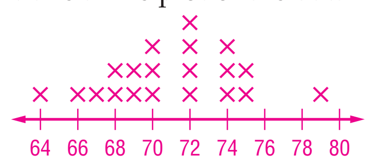
Reading Tip Explain that the *frequency* of the data is how many times a given number occurs. For example, in Example 1, the number 4 occurs 3 times, so its frequency is 3.

1 Draw a line plot for the data.
11 −2 10 −2 7 2 7 4 9 0
6 9 7 2 0 4 10 7 6 9



2 **TRAFFIC** The highway patrol did a radar survey of the speeds of cars along a stretch of highway for 1 minute. The speeds (in miles per hour) of the 20 cars that passed are listed below.
72 70 72 74 68 69 70
72 74 75 79 75 74 72
70 64 69 66 68 67

a. Make a line plot of the data.



b. Which speed occurs most frequently? **72 mph**

In-Class Examples

Power Point®

3 Use the data below to make a stem-and-leaf plot.

85	115	126	92	104	107
78	131	114	92	85	116
100	121	123	131	88	97
99	116	79	90	110	129
108	93	84	75	70	132

Stem	Leaf
7	0 5 8 9
8	4 5 5 8
9	0 2 2 3 7 9
10	0 4 7 8
11	0 4 5 6 6
12	1 3 6 9
13	1 1 2 11|5 = 115

4 **WEATHER** Monique wants to compare the monthly average high temperatures of Dallas and Atlanta before she decides to which city she wants to move. The table shows the monthly average high temperatures (°F) for both cities.

Monthly Average High Temperature	
Dallas	**Atlanta**
54 59 68 77	50 55 64 72
83 91 95 95	75 85 88 87
87 78 66 57	81 72 63 54

a. Make a stem-and-leaf plot to compare the data.

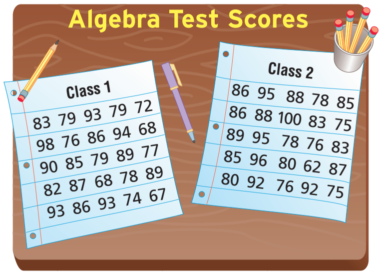
Dallas	Stem	Atlanta
9 7 4	5	0 4 5
8 6	6	3 4
8 7	7	2 2 5
7 3	8	1 5 7 8
5 5 1	9	
8|6 = 68		7|2 = 72

b. What is the difference between the highest average temperatures in each city? **7°F**

c. Which city has higher average temperatures? **Dallas has a greater number of average high temperatures above 80°F.**

Example 4 Back-to-Back Stem-and-Leaf Plot

Mrs. Evans wants to compare recent test scores from her two algebra classes. The table shows the scores for both classes.



a. Make a stem-and-leaf plot to compare the data.
To compare the data, we can use a back-to-back stem-and-leaf plot. Since the data represent similar measurements, the plot will share a common stem.

Class 1	Stem	Class 2
8 8 7	6	2
9 9 9 8 7 6 4 2	7	5 5 6 6 8 8
9 9 7 6 6 5 3 2	8	0 0 3 3 5 5 6 6 7 8 8 9
8 4 3 3 3 0	9	2 2 5 5 6
7|6 = 67	10	0 6|2 = 62

b. What is the difference between the highest score in each class?
100 − 98 or 2 points

c. Which class scored higher overall on the test?
Looking at the scores of 80 and above, we see that class 2 has a greater number of scores at or above 80 than class 1.

ANALYZE DATA When analyzing data, it is helpful to have one number that describes the set of data. Numbers known as **measures of central tendency** are often used to describe sets of data because they represent a centralized, or middle, value. Three of the most commonly used measures of central tendency are the mean, median, and mode.

When you use a measure of central tendency to describe a set of data, it is important that the measure you use best represents all of the data.

- Extremely high or low values can affect the mean, while not affecting the median or mode.
- A value with a high frequency can cause the mode to be misleading.
- Data that is clustered with a few values separate from the cluster can cause the median to be too low or too high.

Example 5 Analyze Data

Which measure of central tendency best represents the data?

Stem	Leaf
7	7 8 9
8	2 2 2 2 2 3 4 4 6
9	
10	8
11	6 8 7|9 = 0.79

Determine the mean, median, and mode.

The mean is about 0.88. Add the data and divide by 15.

The median is 0.82. The middle value is 0.82.

The mode is 0.82. The most frequent value is 0.82.

Either the median or the mode best represent the set of data since both measures are located in the center of the majority of the data. In this instance, the mean is too high.

Study Tip

Look Back
To review **finding mean, median,** and **mode,** see pages 818 and 819.

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Analyzing data in a graphical form such as a line plot or stem-and-leaf plot may be more intuitive for some students than finding the mean, median, and mode. Suggest that students compare their results when finding mean, median, and mode to the way the data looks in either of the plots. Do the measures of central tendency make sense when compared to the plots?

Example 6 Determine the Best Measure of Central Tendency

PRESIDENTS **The numbers below show the ages of the U.S. Presidents since 1900 at the time they were inaugurated. Which measure of central tendency best represents the data?**

42	51	56	55	51	54	51	60	62
43	55	56	61	52	69	64	46	54

The mean is about 54.6. Add the data and divide by 18.

The median is 54.5. The middle value is 54.5.

The mode is 51. The most frequent value is 51.

The mean or the median can be used to best represent the data. The mode for the data is too low.

Check for Understanding

Concept Check

1. **Explain** why it is useful to find the mean, median, and mode of a set of data.
2. Mitchell says that a line plot and a line graph are the same thing. **Find a counterexample** to show that he is incorrect. **See pp. 117A–117B.**
3. **OPEN ENDED** Write a set of data for which the median is a better representation than the mean. **Sample answer: 13, 14, 14, 28**

1. They describe the data as a whole.

Guided Practice

4. Use the data to make a line plot. **See pp. 117A–117B.**

22 19 14 15 14 21 19 16 22 19 10 15 19 14 19

GUIDED PRACTICE KEY	
Exercises	**Examples**
4, 5	1, 2
6, 7, 9, 10, 12, 13	5, 6
8, 11	3, 4

For Exercises 5–7, use the list that shows the number of hours students in Mr. Ricardo's class spent online last week.

7	4	7	11	3	1	5	10	10	0	9	4	0	14	13	4
11	3	1	12	0	9	13	14	7	6	10	5	12	0	6	5

5. Make a line plot of the data. **See pp. 117A–117B.**
6. Which value occurs most frequently? **0**
7. Does the mean, median, or mode best represent the data? Explain. **See pp. 117A–117B.**
8. Use the data to make a stem-and-leaf plot. **See pp. 117A–117B.**

68 66 68 88 76 71 88 93 86 64 73 80 81 72 68

For Exercises 9 and 10, use the data in the stem-and-leaf plot.

Stem	Leaf
9	3 5 5
10	2 2 5 8
11	5 8 8 9 9 9
12	0 1 7 8 9 $9\|3 = 9.3$

9. What is the difference between the least and greatest values? **3.6**
10. Which measure of central tendency best describes the data? Explain. **Median; most of the data clusters higher, near the median.**

Application

BUILDINGS **For Exercises 11–13, use the data below that represents the number of stories in the 25 tallest buildings in the world.**

88	88	110	88	80	69	102	78	70	54	80	85	
83	100	60	90	77	55	73	55	56	61	75	64	105

11. Make a stem-and-leaf plot of the data. **See pp. 117A–117B.**
12. Which value occurs most frequently? **88**
13. Does the mode best describe the set of data? Explain.

13. The mode is not the best measure as it is higher than most of the values.

ANALYZE DATA

In-Class Examples

Power Point®

5 Which measure of central tendency best represents the data?

Stem	Leaf
4	1 1 2 4 4 4 5 8
5	0
6	2 5 7
7	3 9
8	1 $6\|2 = 6.2$

The mean is about 5.5.
The median is 4.8.
The mode is 4.4.
Either the median or the mode best represent the data. The mean is too high.

6 **POLITICS** The number of electoral votes for the 12 most populous states in the 2000 Presidential election are listed below. Which measure of central tendency best represents the data?

21 22 18 23 15 25
14 32 13 33 13 54

The mean is about 23.6.
The median is 21.5.
The mode is 13.
Either the mean or median can be used to represent the data. The mode is too low.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- create examples of a line plot and a stem-and-leaf plot, along with the data used to create the plots. They can even use the same set of data to create both plots.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

Unlocking Misconceptions

Mean and Median Mean and median are two mathematical terms that are often confused. Remind students that the mean is the arithmetic average of a set of data and the median is the number that is in the middle of the set of data. The mean and median can be very similar if the values in the data set are evenly spread between the lowest and highest value. But a few very high or very low values in a data set can cause the mean and median to have significantly different values.

Practice and Apply

Homework Help

For Exercises	See Examples
14–18	1, 2
20–22, 28, 29, 32, 33, 35, 36	3, 4
19, 23–27, 30, 31, 34, 37	5, 6

Extra Practice
See page 824.

Use each set of data to make a line plot. 14–15. See margin.

14. 43 36 48 52 41 54 45 48 49 52 35 44 53 46 38 41 53

15. 1.0 −1.5 1.5 2.0 −1.5 2.1 −2.0 2.4 1.5 −1.4
2.5 1.4 −1.2 1.3 1.0 2.2 2.3 −1.2 −1.5 2.1

BASKETBALL **For Exercises 16–19, use the table that shows the seeds, or rank, of the NCAA men's basketball Final Four from 1991 to 2001.**

Year	Seeds
1991	1 1 2 3
1992	1 2 4 6
1993	1 1 1 2
1994	1 1 2 3
1995	1 2 2 4
1996	1 1 4 5
1997	1 1 1 4
1998	1 2 3 3
1999	1 1 1 4
2000	1 5 8 8
2001	1 1 2 3

Source: www.espn.com

16. Make a line plot of the data. **See margin.**

17. How many of the teams in the Final Four were *not* number 1 seeds? **23**

18. How many teams were seeded higher than third? (*Hint*: Higher seeds have lesser numerical value.) **29**

19. Which measure of central tendency best describes the data? Explain. **Sample answer: Median; most of the data are near 2.**

Use each set of data to make a stem-and-leaf plot. 20–21. See margin.

20. 6.5 6.3 6.9 7.1 7.3 5.9 6.0 7.0 7.2 6.6 7.1 5.8

21. 31 30 28 26 22 34 26 31 47 32 18 33 26 23 18 29

WEATHER **For Exercises 22–24, use the list of the highest recorded temperatures in each of the 50 states.**

112	100	128	120	134	118	106	110	109	112
100	118	117	116	118	121	114	114	105	109
107	112	114	115	118	117	118	125	106	110
122	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: *The World Almanac*

22. Make a stem-and-leaf plot of the data. **See pp. 117A–117B.**

23. Which temperature occurs most frequently? **118**

24. Does the mode best represent the data? Explain. **No; the mode is higher than most of the data.**

25. **RESEARCH** Use the Internet or another source to find the total number of each CD sold over the past six months to reach number one. Which measure of central tendency best describes the average number of top selling CDs sold? Explain. **See students' work.**

GEOLOGY **For Exercises 26 and 27, refer to the stem-and-leaf plot that shows the magnitudes of earthquakes occurring in 2000 that measured at least 5.0 on the Richter scale.**

Stem	Leaf
5	1 2 2 3 4 8 8 9 9
6	1 1 2 3 4 5 6 7 8
7	0 1 1 2 2 3 5 5 5 6 8 8
8	0 0 2 $5\|1 = 5.1$

Source: National Geophysical Data Center

26. What was the most frequent magnitude of these earthquakes? **7.5**

27. Which measure of central tendency best describes this set of data? Explain. **Mean or median; both are centrally located and the mode is too high.**

About the Exercises…

Organization by Objective

- **Create Line Plots and Stem-and-Leaf Plots:** 14–16, 20–22, 28, 32, 35
- **Analyze Data:** 17–19, 23–25, 27, 29–31, 33–34, 36–37, 39–41

Odd/Even Assignments

Exercises 14–15 and 20–21 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 25 involves research on the Internet or other reference materials.

Assignment Guide

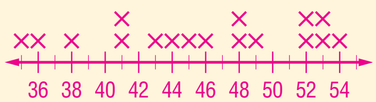
Basic: 15–19, 21–27, 38, 42–64

Average: 15, 21, 25–34, 38–64

Advanced: 14, 20, 35–56 (optional: 57–64)

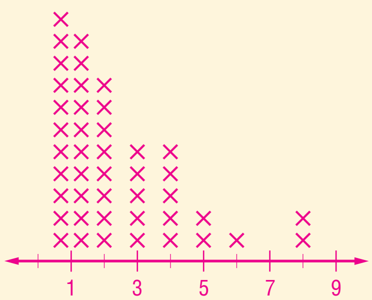
Answers

14.



15. See below right.

16.



20.

Stem	Leaf
5	8 9
6	0 3 5 6 9
7	0 1 1 2 3

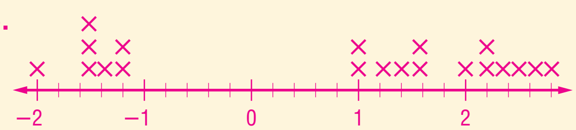
$5|8 = 5.8$

21.

Stem	Leaf
1	8 8
2	2 3 6 6 6 8
3	9
4	0 1 1 2 3 4 7

$1|8 = 18$

15.



OLYMPICS For Exercises 28–31, use the information in the table that shows the number of medals won by the top ten countries during the 2000 Summer Olympics in Sydney, Australia.

Sydney Olympics Total Medals by Country

Country	Gold	Silver	Bronze	Total
United States	40	24	33	97
Russia	32	28	28	88
China	28	16	15	59
Australia	16	25	17	58
Germany	13	17	26	56
France	13	14	11	38
Italy	14	8	13	35
Cuba	11	11	7	29
Britain	11	10	7	28
Korea	8	10	10	28

Source: www.espn.com

28. Make a line plot showing the number of gold medals won by the countries. **See pp. 117A–117B.**

29. How many countries won fewer than 25 gold medals? **7**

30. What was the median number of gold medals won by a country? **13.5**

31. Is the median the best measure to describe this set of data? Explain. **Sample answer: Yes; most of the data are near the median.**

CARS For Exercises 32–34, use the list of the fuel economy of various vehicles in miles per gallon.

25	28	29	30	24	28	29	31	34	30
33	47	34	43	33	36	37	29	30	30
29	26	29	22	23	19	18	20	23	21
20	20	19	16	18	21	20	19	28	20

Source: United States Environmental Protection Agency

32. Make a stem-and-leaf plot of the data. **See pp. 117A–117B.**

33. How many of the vehicles get more than 25 miles per gallon? **22**

34. Which measure of central tendency would you use to describe the fuel economy of the vehicles? Explain your reasoning. **Sample answer: Mean; the median is too high, and the modes are either too high or too low.**

EDUCATION For Exercises 35–37, use the table that shows the top ten public libraries in the United States by population served.

Top Libraries

Location	Number of Branches	Location	Number of Branches
Brooklyn, NY	59	Los Angeles County, CA	84
Broward County, FL	34	Miami, FL	30
Chicago, IL	77	New York, NY	85
Houston, TX	37	Philadelphia P	52
Los Angeles, CA	67	Queens Borough, NY	62

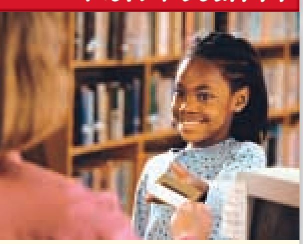
35. Make a stem-and-leaf plot to show the number of library branches. **See pp. 117A–117B.**

36. Which interval has the most values? **30–39**

37. What is the mode of the data? **no mode**

38. **CRITICAL THINKING** Construct a set of twelve numbers with a mean of 7, a median of 6, and a mode of 8. **Sample answer: 4, 4, 4, 5, 5, 5, 7, 8, 8, 8, 8, 18**

More About. . .



Education

In 1848, the Boston Public Library became the first public library to allow users to borrow books and materials.

Source: The Boston Public Library

www.algebra1.com/self_check_quiz

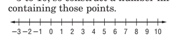
Study Guide and Intervention, p. 99 (shown) and p. 100

Create Line Plots and Stem-and-Leaf Plots One way to display data graphically is with a **line plot**. A line plot is a number line labeled with a scale that includes all the data and ×s placed above a data point each time it occurs in the data list. The ×s represent the **frequency** of the data. A **stem-and-leaf plot** can also be used to organize data. The greatest *common* place value is called the stem, and the numbers in the next greatest place value form the leaves.

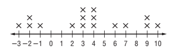
Example 1 **Draw a line plot for the data.**

−3 3 4 7 9 10 −2 3
6 4 3 9 −1 −2 4 2

Step 1 The value of the data ranges from −3 to 10, so construct a number line containing those points.



Step 2 Then place an × above the number each time it occurs.



Example 2 **Use the data below to create a stem-and-leaf plot.**

62 74 89 102 92 65 68 98 78 65
78 80 83 93 87 89 104 109 104
68 97 68 64 98 93 90 102 104

The greatest common place value is tens, so the digits in the tens place are the stems. Thus 62 would have a stem of 6 and 104 would have a stem of ten. The stem-and-leaf plot is shown below.

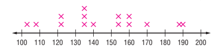
Stem	Leaf
6	2 4 5 5 8 8 8
7	4 8 8
8	0 3 7 9 9
9	0 2 3 3 7 8 8
10	2 2 4 4 4 9 $\quad 6|2 = 62$

Exercises

Use the table at the right for Exercises 1–3.

Weights of Junior Varsity Wrestlers (pounds)							
170	160	135	135	160	122	188	154
108	135	140	122	103	190	154	

1. Make a line plot representing the weights of the wrestlers shown in the table at the right.



2. How many wrestlers weigh over 140 lb? **7**

3. What is the greatest weight? **190 lb**

Use each set of data to make a stem-and-leaf plot.

4. 32 45 41 29 30 30 31 34 38
36 32 34 41 40 42 41 29 30

Stem	Leaf
2	9 9
3	0 0 0 1 2 2 4 4 6 8
4	0 1 1 1 2 5 $\quad 2|9 = 29$

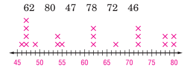
5. 102 104 99 109 108 112 115 120
112 114 98 94 96 101 100 102

Stem	Leaf
9	4 6 8 9
10	0 1 2 2 4 8 9
11	2 2 4 5
12	0 $\quad 9|4 = 94$

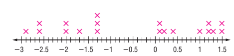
Skills Practice, p. 101 and Practice, p. 102 (shown)

Use each set of data to make a line plot.

1. 72 47 62 78 49 67 80
54 47 72 55 62 47 54
62 80 47 78 72 46



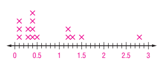
2. −2 −1.3 1.5 0.1 −1.7 0.4 1.5
1 −1.3 −2 −2.9 0.1 1.3 1.2
−2.6 1.2 0.2 −1.3 −2.6



HEALTH For Exercises 3 and 4, use the list that shows the grams of saturated fat in a serving of a variety of grains such as bread, cereal, crackers, and pasta.

0.3	1.2	0.1	0.3	0.4
0.4	0.5	0.1	0.4	0.4
0.1	1.2	2.8	1.3	1.5

3. Make a line plot of the data.



4. Which measure of central tendency best describes the data? Explain. **The mode and median, both 0.4. The mean is too high because of the value 2.8.**

Use each set of data to make a stem-and-leaf plot.

5. 41 53 22 50 41 27 36 57 20 31
28 52 41 33 28 27 41 52 22 30

Stem	Leaf
2	0 2 2 7 7 8 8
3	0 1 3 6
4	1 1 1 1
5	0 2 2 3 7 $\quad 3|3 = 33$

6. 4.1 7.3 6.9 5.7 4.8 7.3 5.6
6.0 4.4 7.5 4.6 7.9 5.1 7.7

Stem	Leaf
4	1 4 6 8
5	1 6 7
6	0 9
7	3 3 5 7 9 $\quad 7|7 = 7.7$

EMPLOYMENT For Exercises 7–10, use the lists that show survey results of students' time spent on the Internet and on the telephone for a month.

Internet								Telephone							
42	19	28	8	35	42	20	18	36	52	40	28	43	24	8	53
51	4	7	29	14	22	6		41	26	48	35	58	8	4	

7. Make a stem-and-leaf plot to compare the data.

8. Which value appears most frequently in each set of data? **Internet: 42; telephone: 8**

9. Is the mode the best measure to compare the data? Explain. **No; it is too high for the Internet and too low for the telephone.**

10. Overall, did students spend more time on the Internet or the telephone? **telephone**

Internet	Stem	Telephone
8 7 6 4	0	4 8 8
9 8 4	1	
9 8 2 0	2	4 6 8
5	3	5 6
2 2	4	0 1 3 8
1	5	2 3 8
$5|1 = 51$		$2|6 = 26$

Reading to Learn Mathematics, p. 103

ELL

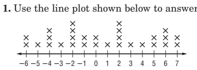
Pre-Activity **How are line plots and averages used to make decisions?**

Read the introduction to Lesson 2-5 at the top of page 88 in your textbook.

- What was the number one name for boys in all five decades? **Michael**
- Look at the decade in which you were born. Is your name or the names of any of the other students in your class in the top five for that decade? **See students' work.**

Reading the Lesson

1. Use the line plot shown below to answer the questions.



a. What are the data points for the line plot? **−6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7**

b. What do the three ×'s above the 6 represent? **6 occurs in the data set 3 times.**

2. Explain what is meant by the frequency of a data number. **how many times the data number occurs**

3. Use the stem-and-leaf plot shown at the right.

Stem	Leaf
72	0 1 1 2 5
73	2 2 2 7 9 9
74	1 3 3
75	6 6 8 9
76	0 1 8 8 8 $\quad 74|2 = 742$

a. How is the number 758 represented on the plot? ***75|8***

b. Explain how you know there are 23 numbers in the data. **There are 23 leaves.**

Helping You Remember

4. Describe how you would explain the process of finding the median and mode from a stem-and-leaf plot to a friend who missed Lesson 2-5. **Sample answer: To find the median, count the number of leaves, then find the number in the middle. To find the mode, identify the leaf that occurs most frequently and use the stem and key to identify the data number.**

Enrichment, p. 104

Runs Created

In *The 1978 Bill James Baseball Abstract*, the author introduced the "runs created" formula.

$R = \frac{(h + w)t}{(b + w)}$ where for each player h = number of hits
w = number of walks,
t = number of total bases,
b = number of at-bats, and
R = approximate number of runs a team scores due to this player's actions

1. As of June 29, 2001, Roberto Alomar of the Cleveland Indians and Seattle Mariners player Ichiro Suzuki were tied with the highest American League batting average at .351. Find the number of runs created by each player using the data below.

		w	t		Runs Created

4 Assess

Open-Ended Assessment

Speaking Draw several line plots and stem-and-leaf plots on the chalkboard. Make sure that some of your plots have skewed data. Ask students to comment on how they think the data in the plots will affect the measures of central tendency.

Getting Ready for Lesson 2-6

PREREQUISITE SKILL Students will learn about simple probability and odds in Lesson 2-6. Probability and odds are often expressed as fractions in simplest form. Use Exercises 57–64 to determine your students' familiarity with simplifying fractions.

Teaching Tip You may want to teach box-and-whisker plots after this lesson. These can be found in Lesson 13-5.

Answer

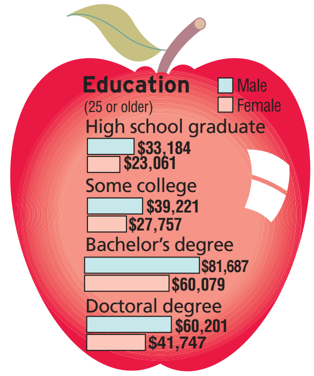
42. Sample answer: They can be used in marketing or sales to sell the most products to a specific group. Answers should include the following.
    - a line plot showing the number of males with the names from the beginning of the lesson
    - By finding out the most popular names you can use the popular names on more of your items.

39. High school: \$10,123; College: \$11,464; Bachelor's Degree: \$18,454; Doctoral Degree: \$21,608

40. Sample answer: The higher the education, the higher the income.

41. Sample answer: Because the range in salaries is often very great with extreme values on both the high end and low end.

SALARIES For Exercises 39–41, refer to the bar graph that shows the median income of males and females based on education levels.



Source: *USA TODAY*

39. What are the differences between men's and women's salaries at each level of education?

40. What do these graphs say about the difference between salaries and education levels?

41. Why do you think that salaries are usually represented by the median rather than the mean?

42. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

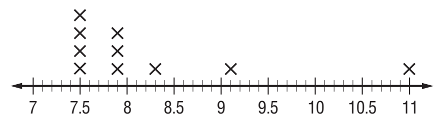
    How are line plots and averages used to make decisions? See margin.

    Include the following in your answer:
    - a line plot to show how many male students in your class have the most popular names for the decade in which they were born, and
    - a convincing argument that explains how you would use this information to sell personalized T-shirts.

Standardized Test Practice

For Exercises 43 and 44, refer to the line plot.

Wingspan (in.) of Ten Largest Butterflies



43. What is the average wingspan for these types of butterflies? **C**

    (A) 7.6 in. (B) 7.9 in.
    (C) 8.2 in. (D) 9.1 in.

44. Which sentence is *not* true? **C**

    (A) The difference between the greatest and least wingspan is 3.5 inches.
    (B) Most of the wingspans are in the 7.5 inch to 8.5 inch interval.
    (C) Most of the wingspans are greater than 8 inches.
    (D) The mode of the data is 7.5 inches.

Maintain Your Skills

Mixed Review

Find each quotient. *(Lesson 2-4)*

45. $56 \div (-14)$ **−4** 46. $-72 \div (-12)$ **6** 47. $-40.5 \div 3$ **−13.5** 48. $102 \div 6.8$ **15**

Simplify each expression. *(Lesson 2-3)*

49. $-2(6x) - 5x$ **$-17x$** 50. $3x(-7y) - 4x(5y)$ **$-41xy$** 51. $5(3t - 2t) - 2(4t)$ **$-3t$**

52. Write an algebraic expression to represent the amount of money in Kara's savings account if she has d dollars and adds x dollars per week for 12 weeks. *(Lesson 1-1)* **$d + 12x$**

Evaluate each expression if $x = 5$, $y = 16$, and $z = 9$. *(Lesson 1-2)*

53. $y - 3x$ **1** 54. $xz \div 3$ **15** 55. $2x - x + (y \div 4)$ **9** 56. $\frac{x^2 - z}{2y}$ **$\frac{1}{2}$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write each fraction in simplest form.**
*(To review **simplifying fractions**, see pages 798 and 799.)*

57. $\frac{12}{18}$ **$\frac{2}{3}$** 58. $\frac{54}{60}$ **$\frac{9}{10}$** 59. $\frac{21}{30}$ **$\frac{7}{10}$** 60. $\frac{42}{48}$ **$\frac{7}{8}$**

61. $\frac{32}{64}$ **$\frac{1}{2}$** 62. $\frac{28}{52}$ **$\frac{7}{13}$** 63. $\frac{16}{36}$ **$\frac{4}{9}$** 64. $\frac{84}{90}$ **$\frac{14}{15}$**

94 Chapter 2 Real Numbers

Interpreting Statistics

The word *statistics* is associated with the collection, analysis, interpretation, and presentation of numerical data. Sometimes, when presenting data, *notes* and *unit indicators* are included to help you interpret the data.

Headnotes give information about the table as a whole.

Footnotes give information about specific items within the table.

If the numerical data are too large, *unit indicators* are used to save space.

Public Elementary and Secondary School Enrollment, 1994–1998
[(in thousands) 44,111 represents 44,111,000.]
As of fall year, Kindergarten includes nursery schools.

Grade	1994	1995	1996	1997	1998, prel.
Pupils enrolled	**44,111**	**44,840**	**45,611**	**46,127**	**46,535**
Kindergarten and grades 1 to 8	31,898	32,341	32,764	33,073	33,344
Kindergarten	4047	4173	4202	4198	4171
First	3593	3671	3770	3755	3727
Second	3440	3507	3600	3689	3682
Third	3439	3445	3524	3597	3696
Fourth	3426	3431	3454	3507	3592
Fifth	3372	3438	3453	3458	3520
Sixth	3381	3395	3494	3492	3497
Seventh	3404	3422	3464	3520	3530
Eighth	3302	3356	3403	3415	3480
Unclassified[1]	494	502	401	442	460
Grades 9 to 12	12,213	12,500	12,847	13,054	13,191
Ninth	3604	3704	3801	3819	3856
Tenth	3131	3237	3323	3376	3382
Eleventh	2748	2826	2930	2972	3018
Twelfth	2488	2487	2586	2673	2724
Unclassified[1]	242	245	206	214	211

[1] Includes ungraded and special education.
Source: U.S. Census Bureau

Suppose you need to find the number of students enrolled in the 9th grade in 1997. The following steps can be used to determine this information.

Step 1 Locate the number in the table. The number that corresponds to 1997 and 9th grade is 3819.

Step 2 Determine the unit indicator. The *unit indicator* is thousands.

Step 3 If the unit indicator is not 1 unit, multiply to find the data value. In this case, multiply 3819 by 1000.

Step 4 State the data value. The number of students enrolled in the 9th grade in 1997 was 3,819,000.

Reading to Learn

Use the information in the table to answer each question. **1–2. See margin.**

1. Describe the data.
2. What information is given by the footnote?
3. How current is the data? **1999**
4. What is the unit indicator? **thousands**
5. How many acres of state parks and recreation areas does New York have? **1,016,000 acres**
6. Which of the states shown had the greatest number of visitors? How many people visited that state's parks and recreation areas in 1999? **California; 76,736,000 visitors**

State Parks and Recreation Areas for Selected States, 1999

State	Acreage (1000)	Visitors (1000)[1]
United States	**12,916**	**766,842**
Alaska	3291	3855
California	1376	76,736
Florida	513	14,645
Indiana	178	18,652
New York	1016	61,960
North Carolina	158	13,269
Oregon	94	38,752
South Carolina	82	9563
Texas	628	21,446

Source: U.S. Census Bureau
[1] Includes overnight visitors.

Answers

1. Sample answer: The data shows the acreage and number of visitors in thousands for selected state parks and recreation areas as of 1999.
2. Sample answer: The footnote indicates that the number of visitors includes those staying overnight.

Getting Started

Ask students if they have ever used abbreviations when writing. What are some of the abbreviations that students use often? Why do they use them? How do they indicate to others what the abbreviations mean? How do they learn the meaning of abbreviations when they read them?

Teach

Reading Graphs Many people make important decisions in their lives based on how they analyze the statistics involved. In order to properly analyze statistics, you have to understand what the statistics mean. This is especially true when statistics are presented in graphical or tabular form.

Make sure students understand why unit indicators are used. As well as saving space, they also clarify a table. On the board, rewrite some of the data without the unit indicator so students can see how all the extra zeroes make the table more difficult to read.

Assess

Study Notebook

Ask students to summarize what they have learned about interpreting statistics.

ELL English Language Learners may benefit from writing key concepts from this activity in their Study Notebooks in their native language and then in English.

2-6 Lesson Notes

1 Focus

5-Minute Check Transparency 2-6 Use as a quiz or a review of Lesson 2-5.

Mathematical Background notes are available for this lesson on p. 66D.

Why is probability important in sports?

Ask students:

- Who is more likely to make a free throw, a player who makes 75% of her shots, or one who makes 50% of her shots? **the one who makes 75%**
- If you are a coach, which player would you rather have making a potentially game-tying shot? **the player with the higher free throw average**
- Why do you think probability is important in sports? **Probability can help a coach decide which player or players to use when making a shot is important.**

2-6 Probability: Simple Probability and Odds

What You'll Learn

- Find the probability of a simple event.
- Find the odds of a simple event.

Vocabulary

- probability
- simple event
- sample space
- equally likely
- odds

Why is probability important in sports?

A basketball player is at the free throw line. Her team is down by one point. If she makes an average of 75% of her free throws, what is the probability that she will tie the game with her first shot?

PROBABILITY One way to describe the likelihood of an event occurring is with probability. The **probability** of a **simple event**, like a coin landing heads up when it is tossed, is a ratio of the number of favorable outcomes for the event to the total number of possible outcomes of the event. The probability of an event can be expressed as a fraction, a decimal, or a percent.

Suppose you wanted to find the probability of rolling a 4 on a die. When you roll a die, there are six possible outcomes, 1, 2, 3, 4, 5, or 6. This list of all possible outcomes is called the **sample space**. Of these outcomes, only one, a 4, is favorable. So, the probability of rolling a 4 is $\frac{1}{6}$, $0.1\overline{6}$, or about 16.7%.

Key Concept — *Probability*

The probability of an event a can be expressed as

$$P(a) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}.$$

Study Tip

Reading Math
$P(a)$ is read the probability of a.

Example 1 Find Probabilities of Simple Events

a. Find the probability of rolling an even number on a die.

There are six possible outcomes. Three of the outcomes are favorable. That is, three of the six outcomes are even numbers.

Sample space: 1, 2, 3, 4, 5, 6 — 3 even numbers → $\frac{3}{6}$ ← 6 total possible outcomes

So, $P(\text{even number}) = \frac{3}{6}$ or $\frac{1}{2}$.

b. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find *P*(blue).

There are 7 blue chips and 28 total chips.

$P(\text{blue chip}) = \frac{7}{28}$ ← number of favorable outcomes / ← number of possible outcomes

$= \frac{1}{4}$ or 0.25 Simplify.

The probability of selecting a blue chip is $\frac{1}{4}$ or 25%.

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters
- Study Guide and Intervention, pp. 105–106
- Skills Practice, p. 107
- Practice, p. 108
- Reading to Learn Mathematics, p. 109
- Enrichment, p. 110
- Assessment, p. 132

Parent and Student Study Guide Workbook, p. 16
Prerequisite Skills Workbook, pp. 17–18, 37–38, 67–70, 99–100

Transparencies
5-Minute Check Transparency 2-6
Answer Key Transparencies

Technology
Interactive Chalkboard

c. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find P(red or yellow).

There are 5 ways to pick a red chip and 6 ways to pick a yellow chip. So there are 5 + 6 or 11 ways to pick a red or a yellow chip.

$$P(\text{red or yellow}) = \frac{11}{28} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$\approx 0.39 \quad \text{Divide.}$$

The probability of selecting a red chip or a yellow chip is $\frac{11}{28}$ or about 39%.

d. A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find P(not green).

There are 5 + 7 + 6 or 18 chips that are not green.

$$P(\text{not green}) = \frac{18}{28} \quad \begin{matrix}\leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes}\end{matrix}$$

$$\approx 0.64 \quad \text{Divide.}$$

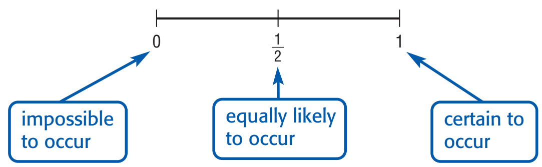
The probability of selecting a chip that is not green is $\frac{9}{14}$ or about 64%.

Study Tip

Reading Math

Inclusive means that the end values are included.

Notice that the probability that an event will occur is somewhere between 0 and 1 inclusive. If the probability of an event is 0, that means that it is impossible for the event to occur. A probability equal to 1 means that the event is certain to occur. There are outcomes for which the probability is $\frac{1}{2}$. When this happens, the outcomes are **equally likely** to occur or not to occur.



ODDS

Another way to express the chance of an event occurring is with **odds**.

Key Concept *Odds*

The odds of an event occurring is the ratio that compares the number of ways an event can occur (successes) to the number of ways it cannot occur (failures).

Study Tip

Odds

Odds are usually written in the form *number of successes : number of failures.*

Example 2 *Odds of an Event*

Find the odds of rolling a number less than 3.

There are 6 possible outcomes, 2 are successes and 4 are failures.

Sample space: 1, 2, 3, 4, 5, 6

2 numbers less than 3

4 numbers not less than 3

$\frac{2}{4}$ or $\frac{1}{2}$

So, the odds of rolling a number less than three are $\frac{1}{2}$ or 1:2.

www.algebra1.com/extra_examples

2 Teach

PROBABILITY

Tips for New Teachers

Some students confuse the numbers on a die with the probability of rolling a particular number. Once they see that the probability of rolling 6 is $\frac{1}{6}$, they are more likely to think, for example, that the probability of rolling 5 is $\frac{1}{5}$. Consider using dice with symbols on the sides rather than numbers to teach probability.

In-Class Example Power Point®

1

a. Find the probability of rolling a number greater than two on a die. $\frac{4}{6}$ or $\frac{2}{3}$

b. A class contains 6 students with black hair, 8 with brown hair, 4 with blonde hair, and 2 with red hair. Find P(black). $\frac{3}{10}$ or 0.3

c. A class contains 6 students with black hair, 8 with brown hair, 4 with blonde hair, and 2 with red hair. Find P(red or brown). $\frac{1}{2}$ or 0.5

d. A class contains 6 students with black hair, 8 with brown hair, 4 with blonde hair, and 2 with red hair. Find P(not blonde). $\frac{4}{5}$ or 0.8

DAILY INTERVENTION

Unlocking Misconceptions

Odds and Probability Odds and probability are very often confused and the terms are mistakenly used interchangeably in the media. One characteristic to remember is that a probability will never be greater than one but odds can be greater than one. Remind students to read problems and label their answers very carefully when working with odds and probability.

ODDS

In-Class Examples

Power Point®

Teaching Tip To point out the difference between probability and odds, lead students to understand why the probability of rolling a number less than three is $\frac{1}{3}$, while the odds of rolling a number less than three is $\frac{1}{2}$.

2 Find the odds of rolling a number greater than two. $\frac{2}{1}$ or 2:1

3 A card is selected at random from a standard deck of 52 cards. What are the odds against selecting a 2 or a 3? $\frac{11}{2}$

4 **TRAVEL** Melvin is waiting to board a flight to Washington, D.C. According to the airline, the flight he is waiting for is on time 80% of the times it flies. What are the odds that the plane will be on time? 4:1

Answer

1. Sample answers: impossible event: a number greater than 6; certain event: a number from 1 to 6; equally likely event: even number

The odds *against* an event occurring are the odds that the event will *not* occur.

Study Tip

In this text, a *standard deck of cards* always indicates 52 cards in 4 suits of 13 cards each.

Example 3 Odds Against an Event

A card is selected at random from a standard deck of 52 cards. What are the odds against selecting a 3?

There are four 3s in a deck of cards, and there are 52 − 4 or 48 cards that are not a 3.

odds against a 3 = $\frac{48}{4}$ ← number of ways to *not* pick a 3 / ← number of of ways to pick a 3

The odds against selecting a 3 from a deck of cards are 12:1.

Example 4 Probability and Odds

WEATHER **A weather forecast states that the probability of rain the next day is 40%. What are the odds that it will rain?**

The probability that it will rain is 40%, so the probability that it will not rain is 60%.

odds of rain = 40:60 or 2:3

The odds that it will rain tomorrow are 2:3.

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an impossible event, a certain event, and an equally likely event when a die is rolled. See margin.
2. **Describe** how to find the odds of an event occurring if the probability that the event will occur is $\frac{3}{5}$.
3. **FIND THE ERROR** Mark and Doug are finding the probability of picking a red card from a standard deck of cards.

Mark	Doug
P(red card) = $\frac{26}{26}$ or $\frac{1}{1}$	P(red card) = $\frac{26}{52}$ or $\frac{1}{2}$

Who is correct? Explain your reasoning. Doug; Mark determined the odds in favor of picking a red card.

2. The probability is $\frac{3}{5}$, which means there are 3 favorable outcomes and 5 − 3 or 2 unfavorable outcomes. Thus, the odds are 3:2.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7,	1
12, 13	
8–11	2, 3

A card is selected at random from a standard deck of cards. Determine each probability.

4. $P(5)$ $\frac{1}{13}$
5. $P(\text{red } 10)$ $\frac{1}{26}$
6. $P(\text{odd number})$ $\frac{4}{13}$
7. $P(\text{queen of hearts or jack of diamonds})$ $\frac{1}{26}$

Find the odds of each outcome if the spinner is spun once.

8. multiple of 3 3:7
9. even number less than 8 3:7
10. odd number or blue 7:3
11. red or yellow 6:4

Application

NUMBER THEORY **One of the factors of 48 is chosen at random.**

12. What is the probability that the chosen factor is not a multiple of 4? $\frac{2}{5}$
13. What is the probability that the number chosen has 4 and 6 as two of its factors? $\frac{3}{10}$

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Give groups of students marbles or colored cubes. Ask them to model probabilities and then odds. Have students take turns in the group modeling probabilities and odds until all group members understand both concepts and the difference between them.

Practice and Apply

Homework Help

For Exercises	See Examples
14–35, 51, 54, 56	1
36–47, 52, 53, 55	2, 3
48, 49	4

Extra Practice

See page 824.

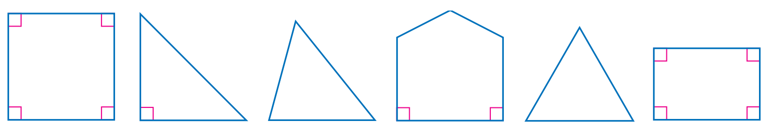
One coin is randomly selected from a jar containing 70 nickels, 100 dimes, 80 quarters, and 50 1-dollar coins. Find each probability.

14. P(quarter) $\frac{4}{15} \approx 27\%$
15. P(dime) $\frac{1}{3} \approx 33\%$
16. P(nickel or dollar) $\frac{2}{5} = 40\%$
17. P(quarter or nickel) $\frac{1}{2} = 50\%$
18. P(value less than \$1.00) $\frac{5}{6} \approx 83\%$
19. P(value greater than \$0.10) $\frac{13}{30} \approx 43\%$
20. P(value at least \$0.25) $\frac{13}{30} \approx 43\%$
21. P(value at most \$1.00) 1

Two dice are rolled, and their sum is recorded. Find each probability.

22. P(sum less than 7) $\frac{5}{12} \approx 42\%$
23. P(sum less than 8) $\frac{7}{12} \approx 58\%$
24. P(sum is greater than 12) $0 = 0\%$
25. P(sum is greater than 1) $1 = 100\%$
26. P(sum is between 5 and 10) $\frac{5}{9} \approx 56\%$
27. P(sum is between 2 and 9) $\frac{25}{36} \approx 69\%$

One of the polygons is chosen at random. Find each probability.



28. P(triangle) $\frac{1}{2} = 50\%$
29. P(pentagon) $\frac{1}{6} \approx 17\%$
30. P(not a triangle) $\frac{1}{2} = 50\%$
31. P(not a quadrilateral) $\frac{2}{3} \approx 67\%$
32. P(more than three sides) $\frac{1}{2} = 50\%$
33. P(more than one right angle) $\frac{1}{2} = 50\%$

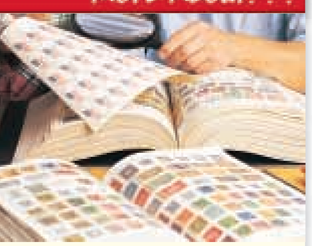
34. $\frac{1}{30} \approx 3\%$

34. If a person's birthday is in April, what is the probability that it is the 29th?
35. If a person's birthday is in July, what is the probability that it is after the 16th? $\frac{15}{31} \approx 48\%$

Find the odds of each outcome if a computer randomly picks a letter in the name *The United States of America*.

36. the letter a 3:21 or 1:7
37. the letter t 4:20 or 1:5
38. a vowel 11:13
39. a consonant 13:11
40. an uppercase letter 4:20 or 1:5
41. a lowercase vowel 9:15 or 3:5

More About. . .



Stamp Collecting

Stamp collecting can be a very inexpensive hobby. Most stamp collectors start by saving stamps from letters, packages, and postcards.

Source: United States Postal Service

STAMP COLLECTING **Lanette collects stamps from different countries. She has 12 from Mexico, 5 from Canada, 3 from France, 8 from Great Britain, 1 from Russia, and 3 from Germany. Find the odds of each of the following if she accidentally loses one stamp.**

42. the stamp is from Canada 5:27
43. the stamp is from Mexico 12:20 or 3:5
44. the stamp is not from France 29:3
45. the stamp is not from a North American country 15:17
46. the stamp is from Germany or Russia 4:28 or 1:7
47. the stamp is from Canada or Great Britain 13:19

48. If the probability that an event will occur is $\frac{3}{7}$, what are the odds that it will occur? 3:4
49. If the probability that an event will occur is $\frac{2}{3}$, what are the odds against it occurring? 1:2

www.algebra1.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- add a description comparing and contrasting probability and odds.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR To solve this problem, students must be able to differentiate between probability and odds. Remind students that for probability to equal 1, or 100%, the event must always occur. Are all the cards in a standard deck of cards red?

About the Exercises...

Organization by Objective
- **Probability:** 14–35, 51, 54, 57
- **Odds:** 36–50, 52, 53, 55, 56, 58

Odd/Even Assignments

Exercises 14–49 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–49 odd, 51–53, 59–82

Average: 15–49 odd, 51–56, 59–82

Advanced: 14–50 even, 54–74 (optional: 75–82)

All: Practice Quiz 2 (1–10)

Teacher to Teacher

Shawntay Moore — Jupiter Community H.S., Jupiter, FL

"I have my students count the number of jelly beans in a package or cup and then find the probability and odds of choosing a specific color. I also have students collect the data and represent their findings through charts and graphs."

Study Guide and Intervention, p. 105 (shown) and p. 106

Probability The **probability** of a **simple event** is a ratio that tells how likely it is that the event will take place. It is the ratio of the number of favorable outcomes of the event to the number of possible outcomes of the event. You can express the probability either as a fraction, as a decimal, or as a percent.

Probability of a Simple Event	For an event a, $P(a) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$.

Example 1 **Mr. Babcock chooses 5 out of 25 students in his algebra class at random for a special project. What is the probability of being chosen?**

$P(\text{being chosen}) = \frac{\text{number of students chosen}}{\text{total number of students}}$

The probability of being chosen is $\frac{5}{25}$ or $\frac{1}{5}$.

Example 2 **A bowl contains 3 pears, 4 bananas, and 2 apples. If you take a piece of fruit at random, what is the probability that it is *not* a banana?**

There are 3 + 4 + 2 or 9 pieces of fruit.

There are 3 + 2 or 5 pieces of fruit that are not bananas.

$P(\text{not banana}) = \frac{\text{number of other pieces of fruit}}{\text{total number of pieces of fruit}} = \frac{5}{9}$

The probability of *not* choosing a banana is $\frac{5}{9}$.

Exercises

A card is selected at random from a standard deck of 52 cards. Determine each probability.

1. $P(10)$ $\frac{1}{13}$
2. $P(\text{red } 2)$ $\frac{1}{26}$
3. $P(\text{king or queen})$ $\frac{2}{13}$
4. $P(\text{black card})$ $\frac{1}{2}$
5. $P(\text{ace of spades})$ $\frac{1}{52}$
6. $P(\text{spade})$ $\frac{1}{4}$

Two dice are rolled and their sum is recorded. Find each probability.

7. $P(\text{sum is } 1)$ 0
8. $P(\text{sum is } 6)$ $\frac{5}{36}$
9. $P(\text{sum is less than } 4)$ $\frac{1}{12}$
10. $P(\text{sum is greater than } 11)$ $\frac{1}{36}$
11. $P(\text{sum is less than } 15)$ 1
12. $P(\text{sum is greater than } 8)$ $\frac{5}{18}$

A bowl contains 4 red chips, 3 blue chips, and 8 green chips. You choose one chip at random. Find each probability.

13. $P(\text{not a red chip})$ $\frac{11}{15}$
14. $P(\text{red or blue chip})$ $\frac{7}{15}$
15. $P(\text{not a green chip})$ $\frac{7}{15}$

A number is selected at random from the list {1, 2, 3, ..., 10}. Find each probability.

16. $P(\text{even number})$ $\frac{1}{2}$
17. $P(\text{multiple of } 3)$ $\frac{3}{10}$
18. $P(\text{less than } 4)$ $\frac{3}{10}$
19. A computer randomly chooses a letter from the word *COMPUTER*. Find the probability that the letter is a vowel. $\frac{3}{8}$

Skills Practice, p. 107 and Practice, p. 108 (shown)

One chip is randomly selected from a jar containing 13 blue chips, 8 yellow chips, 15 brown chips, and 6 green chips. Find each probability.

1. $P(\text{brown})$ $\frac{5}{14} \approx 36\%$
2. $P(\text{green})$ $\frac{1}{7} \approx 14\%$
3. $P(\text{blue or yellow})$ $\frac{1}{2} = 50\%$
4. $P(\text{not yellow})$ $\frac{17}{21} \approx 81\%$

A card is selected at random from a standard deck of 52 cards. Find each probability.

5. $P(\text{heart})$ $\frac{1}{4} = 25\%$
6. $P(\text{black card})$ $\frac{1}{2} = 50\%$
7. $P(\text{jack})$ $\frac{1}{13} \approx 8\%$
8. $P(\text{red jack})$ $\frac{1}{26} \approx 4\%$

Two dice are rolled and their sum is recorded. Find each probability.

9. $P(\text{sum less than } 6)$ $\frac{5}{18} \approx 28\%$
10. $P(\text{sum less than } 2)$ $0 = 0\%$
11. $P(\text{sum greater than } 10)$ $\frac{1}{12} \approx 8\%$
12. $P(\text{sum greater than } 9)$ $\frac{1}{6} \approx 17\%$

Find the odds of each outcome if a computer randomly picks a letter in the name *The Badlands of North Dakota*.

13. the letter *d* 3:21 or 1:7
14. the letter *a* 4:20 or 1:5
15. the letter *h* 2:22 or 1:11
16. a consonant 16:8 or 2:1

CLASS PROJECTS **For Exercises 17–20, use the following information.**

Students in a biology class can choose a semester project from the following list: animal behavior (4), cellular processes (2), ecology (6), health (7), and physiology (3). Find each of the odds if a student selects a topic at random.

17. the topic is ecology 6:16 or 3:8
18. the topic is animal behavior 4:18 or 2:9
19. the topic is not cellular processes 20:2 or 10:1
20. the topic is not health 15:7

SCHOOL ISSUES For Exercises 21 and 22, use the following information.

A news team surveyed students in grades 9–12 on whether to change the time school begins. One student will be selected at random to be interviewed on the evening news. The table gives the results.

Grade	9	10	11	12
No change	6	2	5	3
Hour later	10	7	9	8

21. What is the probability the student selected will be in the 9th grade? $\frac{8}{25} = 32\%$
22. What are the odds the student selected wants no change? 16:34 or 8:17

Reading to Learn Mathematics, p. 109 ELL

Pre-Activity **Why is probability important in sports?**

Read the introduction to Lesson 2-6 at the top of page 96 in your textbook.

Look up the definition of the word *probability* in a dictionary. Rewrite the definition in your own words.

Sample answer: the likelihood of something happening

Reading the Lesson

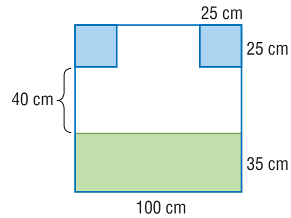
1. Write whether each statement is *true* or *false*. If false, replace the underlined word or number to make a true statement.
   a. <u>Probability</u> can be written as a fraction, a decimal, or a percent. true
   b. The <u>sample</u> space of flipping one coin is heads or tails. true
   c. The probability of an impossible event is <u>1</u>. false; 0
   d. The odds against an event occurring are the odds that the event <u>will</u> occur. false; will not
2. Explain why the probability of an event cannot be greater than 1 while the odds of an event can be greater than 1.
   Sample answer: To find the probability of an event, you compare a part of the sample space to the whole sample space. When you find the odds of an event, you compare the number of favorable outcomes to the number of unfavorable outcomes. In some situations, there may be more favorable than unfavorable outcomes.

Helping You Remember

3. Probabilities are usually written as fractions, decimals, or percents. Odds are usually written with a colon (for example, 1:3). How can the spelling of the word *colon* help remember this?
   Sample answer: The word *colon* has the letter "o" as its only vowel, and the word *odds* also has the letter "o" as its only vowel.

50. **CONTESTS** Every Tuesday, Mike's Submarine Shop has a business card drawing for a free lunch. Four coworkers from InvoAccounting put their business cards in the bowl for the drawing. If there are 80 cards in the bowl, what are the odds that one of the coworkers will win a free lunch? **1:19**

GAMES **For Exercises 51–53, use the following information.**
A game piece is randomly placed on the board shown at the right by blindfolded players.



51. What is the probability that a game piece is placed on a shaded region? $\frac{19}{40} = 47.5\%$
52. What are the odds against placing a game piece on a shaded region? **21:19**
53. What are the odds that a game piece will be placed within the green rectangle? **7:13**

More About. . .

Baseball

The record for the most home runs in a single season is 84. It was set by Joshua Gibson of the Homestead Grays in 1934.

Source: National Baseball Hall of Fame

BASEBALL **For Exercises 54–56, use the following information.**
The stem-and-leaf plot shows the number of home runs hit by the top major league baseball players in the 2000 season. **Source:** www.espn.com

Stem	Leaf
3	0 0 0 0 1 1 1 1 1 1 1 2 2 2 3
	3 4 4 4 5 5 5 6 6 6 7 7 8 8 9
4	0 1 1 1 1 2 2 3 3 3 4 4 7 7 9
5	0 $3 \mid 0 = 30$

54. What is the probability that one of these players picked at random hit more than 35 home runs? $\frac{12}{23} \approx 52\%$
55. What are the odds that a randomly selected player hit less than 45 home runs? **42:4 or 21:2**
56. If a player batted 439 times and hit 38 home runs, what is the probability that the next time the player bats he will hit a home run? $\frac{38}{439} \approx 9\%$

CONTESTS **For Exercises 57 and 58, use the following information.**
A fast-food restaurant is holding a contest in which the grand prize is a new sports car. Each customer is given a game card with their order. The contest rules state that the odds of winning the grand prize are 1:1,000,000.

57. For any randomly-selected game card, what is the probability that it is the winning game card for the grand prize? $\frac{1}{1{,}000{,}001}$
58. Do your odds of winning the grand prize increase significantly if you have several game cards? Explain. **See margin.**

WebQuest

You can use real-world data to find the probability that a person will live to be 100. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

59. **CRITICAL THINKING** Three coins are tossed, and a tail appears on at least one of them. What is the probability that at least one head appears? $\frac{6}{7} \approx 86\%$

60. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 117A–117B.**

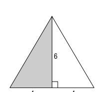
    Why is probability important in sports?

    Include the following in your answer:
    - examples of two sports in which probability is used and an explanation of each sport's importance, and
    - examples of methods other than probability used to show chance.

Enrichment, p. 110

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the probability that it will hit the shaded region? This can be determined by comparing the area of the shaded region to the area of the entire board. This ratio indicates what fraction of the tosses should hit in the shaded region.



$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)} = \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R, then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following:

$$P(S) = \frac{\text{area of subregion } S}{\text{area or region } R}$$

Standardized Test Practice

61. If the probability that an event will occur is $\frac{12}{25}$, what are the odds that the event will *not* occur? **B**

(A) 12:13 (B) 13:12 (C) 13:25 (D) 25:12

62. What is the probability that a number chosen at random from the domain $\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$ will satisfy the inequality $3x + 2 \le 17$? **C**

(A) 20% (B) 27% (C) 73% (D) 80%

Maintain Your Skills

Mixed Review

63. **WEATHER** The following data represents the average daily temperature in Fahrenheit for Sacramento, California, for two weeks during the month of May. Organize the data using a stem-and-leaf plot. *(Lesson 2-5)* **See margin.**

58.3	64.3	66.7	65.1	68.7	67.0	69.3
70.0	72.8	77.4	77.4	73.2	75.8	65.5

Evaluate each expression if $a = -\frac{1}{3}$, $b = \frac{2}{5}$, and $c = \frac{1}{2}$. *(Lesson 2-4)*

64. $b \div c$ $\mathbf{\frac{4}{5}}$
65. $2a \div b$ $\mathbf{-\frac{5}{3}}$ **or** $\mathbf{-1\frac{2}{3}}$
66. $\frac{ab}{c}$ $\mathbf{-\frac{4}{15}}$

Find each sum. *(Lesson 2-2)*

67. $4.3 + (-8.2)$ $\mathbf{-3.9}$
68. $-12.2 + 7.8$ $\mathbf{-4.4}$
69. $-\frac{1}{4} + \left(-\frac{3}{8}\right)$ $\mathbf{-\frac{5}{8}}$
70. $\frac{7}{12} + \left(-\frac{5}{6}\right)$ $\mathbf{-\frac{1}{4}}$

Find each absolute value. *(Lesson 2-1)*

71. $|4.25|$ **4.25**
72. $|-8.4|$ **8.4**
73. $\left|-\frac{2}{3}\right|$ $\mathbf{\frac{2}{3}}$
74. $\left|\frac{1}{6}\right|$ $\mathbf{\frac{1}{6}}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
(To review ***evaluating expressions****, see Lesson 1-2.)*

75. 6^2 **36**
76. 17^2 **289**
77. $(-8)^2$ **64**
78. $(-11.5)^2$ **132.25**
79. 1.6^2 **2.56**
80. $\left(\frac{5}{12}\right)^2$ $\mathbf{\frac{25}{144}}$
81. $\left(-\frac{4}{9}\right)^2$ $\mathbf{\frac{16}{81}}$
82. $\left(-\frac{16}{15}\right)^2$ $\mathbf{\frac{256}{225}}$

Practice Quiz 2

Lessons 2-4 through 2-6

Find each quotient. *(Lesson 2-4)*

1. $-136 \div (-8)$ **17**
2. $15 \div \left(-\frac{3}{8}\right)$ **−40**
3. $(-46.8) \div 4$ **−11.7**

Simplify each expression. *(Lesson 2-4)*

4. $\frac{3a + 9}{3}$ $\mathbf{a + 3}$
5. $\frac{4x + 32}{4}$ $\mathbf{x + 8}$
6. $\frac{15n - 20}{-5}$ $\mathbf{-3n + 4}$

7. State the scale you would use to make a line plot for the following data. Then draw the line plot. *(Lesson 2-5)* **See margin.**

1.9	1.1	3.2	5.0	4.3	2.7	2.5	1.1	1.4	1.8	1.8	1.6
4.3	2.9	1.4	1.7	3.6	2.9	1.9	0.4	1.3	0.9	0.7	1.9

Determine each probability if two dice are rolled. *(Lesson 2-6)*

8. $P(\text{sum of } 10)$ $\mathbf{\frac{1}{12}}$
9. $P(\text{sum} \ge 6)$ $\mathbf{\frac{13}{18}}$
10. $P(\text{sum} < 10)$ $\mathbf{\frac{5}{6}}$

4 Assess

Open-Ended Assessment

Writing Have students write a short essay comparing and contrasting probability and odds. Have students use examples in their comparisons.

Getting Ready for Lesson 2-7

PREREQUISITE SKILL Students will learn about square roots and real numbers in Lesson 2-7. Finding square roots is the opposite of squaring a number. Use Exercises 75–82 to determine your students' familiarity with squaring rational numbers.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 2-4 through 2-6. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 2-5 and 2-6) is available on p. 132 of the *Chapter 2 Resource Masters*.

Answers

58. No; even with 100 game cards the odds of winning are only 100:999,901. It would require several hundred thousand cards to significantly increase the odds of winning.

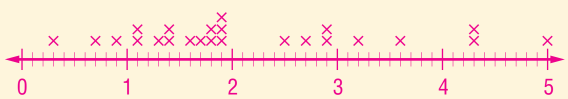
63.

Stem	Leaf
5	8.3
6	4.3 5.1 5.5 6.7 7.0 8.7 9.3
7	0.0 2.8 3.2 5.8 7.4

5|8.3 = 58.3

Answer

7. Sample answer: scale 0–5.0



Lesson 2-6 Probability: Simple Probability and Odds 101

Algebra Activity

A Follow-Up of Lesson 2-6

Investigating Probability and Pascal's Triangle

Collect the Data

- If a family has one child, you know that the child is either a boy or a girl. You can make a simple table to show this type of family.

1 boy	1 girl
B	G

You can see that there are 2 possibilities for a one-child family.

- If a family has two children, the table below shows the possibilities for two children, including the order of birth. For example, BG means that a boy is born first and a girl second.

2 boys, 0 girls	1 boy, 1 girl	0 boys, 2 girls
BB	BG GB	GG

There are 4 possibilities for the two-child family: BB, BG, GB, or GG.

Analyze the Data

3 boys	2 boys, 1 girl	1 boy, 2 girls	3 girls
BBB	BBG **BGB** **GBB**	BGG **GBG** **GGB**	GGG

1. Copy and complete the table that shows the possibilities for a three-child family.
2. Make your own table to show the possibilities for a four-child family. **See pp. 117A–117B.**
3. List the total number of possibilities for a one-child, two-child, three-child, and four-child family. How many possibilities do you think there are for a five-child family? a six-child family? Describe the pattern of the numbers you listed. **3. 2, 4, 8, 16; 32; 64; The pattern represents powers of 2.**
4. Find the probability that a three-child family has 2 boys and 1 girl. **$\frac{3}{8}$**
5. Find the probability that a four-child family has 2 boys and 2 girls. **$\frac{6}{16}$ or $\frac{3}{8}$**

Make a Conjecture

6. Blaise Pascal was a French mathematician who lived in the 1600s. He is known for this triangle of numbers, called Pascal's triangle, although the pattern was known by other mathematicians before Pascal's time.

				1					Row 0
			1		1				Row 1
		1		2		1			Row 2
	1		3		3		1		Row 3
1		4		6		4		1	Row 4

Explain how Pascal's triangle relates to the possibilities for the make-up of families. (*Hint*: The first row indicates that there is 1 way to have 0 children.) **See margin.**

7. Use Pascal's triangle to find the probability that a four-child family has 1 boy. **use Row 5; $\frac{4}{16}$ or $\frac{1}{4}$**

Algebra Activity

A Follow-Up of Lesson 2-6

Getting Started

Objective Investigate the use of Pascal's triangle for predicting probabilities.

Teach

Explain to students that this activity is not so much about finding probability as they learned in Lesson 2-6, as it is about counting possible outcomes. To translate these outcomes into probabilities, you must read the table carefully. Each column in the table shows the total number of outcomes for the given situation. So, to find the probability that a 2-child family will have one boy and one girl, look at how many possible outcomes there are in this column: 2. There are four total possible outcomes, so the probability is $\frac{2}{4}$ or $\frac{1}{2}$.

Assess

In **Exercise 7**, students should correctly extend the pattern in Pascal's triangle. A given number in the table is the sum of the two numbers above and to the left and right. If a given number has only one number above it, the number is 1.

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

Answer

6. Sample answer: Each number in each row shows the number of ways to have boys and girls for a given number of children.

Resource Manager

Teaching Algebra with Manipulatives

- p. 49 (student recording sheet)

102 Chapter 2 Real Numbers

2-7 Square Roots and Real Numbers

What You'll Learn

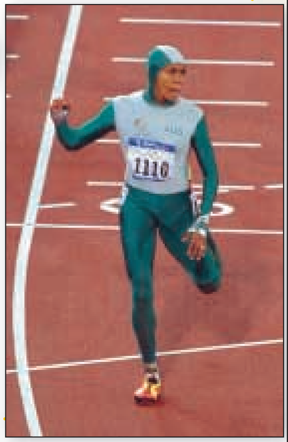
- Find square roots.
- Classify and order real numbers.

How can using square roots determine the surface area of the human body?

In the 2000 Summer Olympics, Australian sprinter Cathy Freeman wore a special running suit that covered most of her body. The surface area of the human body may be found using the formula below, where height is measured in centimeters and weight is in kilograms.

Surface Area $= \sqrt{\dfrac{\text{height} \times \text{weight}}{3600}}$ square meters

The symbol $\sqrt{}$ designates a square root.



Vocabulary

- square root
- perfect square
- radical sign
- principal square root
- irrational numbers
- real numbers
- rational approximations

SQUARE ROOTS A **square root** is one of two equal factors of a number. For example, one square root of 64 is 8 since $8 \cdot 8$ or 8^2 is 64. Another square root of 64 is -8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, whose square root is a rational number is called a **perfect square**.

The symbol $\sqrt{}$, called a **radical sign**, is used to indicate a nonnegative or **principal square root** of the expression under the radical sign.

Study Tip

Reading Math

$\pm\sqrt{64}$ is read *plus or minus the square root of 64.*

$\sqrt{64} = 8$ ← $\sqrt{64}$ indicates the *principal* square root of 64.

$-\sqrt{64} = -8$ ← $-\sqrt{64}$ indicates the *negative* square root of 64.

$\pm\sqrt{64} = \pm 8$ ← $\pm\sqrt{64}$ indicates *both* square roots of 64.

Note that $-\sqrt{64}$ is not the same as $\sqrt{-64}$. The notation $-\sqrt{64}$ represents the negative square root of 64. The notation $\sqrt{-64}$ represents the square root of -64, which is not a real number since no real number multiplied by itself is negative.

Example 1 *Find Square Roots*

Find each square root.

a. $-\sqrt{\dfrac{49}{256}}$

$-\sqrt{\dfrac{49}{256}}$ represents the negative square root of $\dfrac{49}{256}$.

$\dfrac{49}{256} = \left(\dfrac{7}{16}\right)^2 \rightarrow -\sqrt{\dfrac{49}{256}} = -\dfrac{7}{16}$

2-7 Lesson Notes

1 Focus

5-Minute Check Transparency 2-7 Use as a quiz or a review of Lesson 2-6.

Mathematical Background notes are available for this lesson on p. 66D.

Building on Prior Knowledge

In Chapter 1, students reviewed squares and other powers. In this lesson, they should recognize that finding square roots is the opposite of finding squares.

How can using square roots determine the surface area of the human body?

Ask students:

- What is the opposite of addition? **subtraction**
- What is the opposite of multiplication? **division**
- How might the name of the operation "square root" help you determine its opposite operation? **Sample answer: The term "square" might indicate that finding the square root is the opposite of finding the square.**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 111–112
- Skills Practice, p. 113
- Practice, p. 114
- Reading to Learn Mathematics, p. 115
- Enrichment, p. 116
- Assessment, p. 132

Graphing Calculator and Spreadsheet Masters, p. 25

Parent and Student Study Guide Workbook, p. 17

Prerequisite Skills Workbook, pp. 75–76

Transparencies

5-Minute Check Transparency 2-7

Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

SQUARE ROOTS

In-Class Example PowerPoint®

1 Find each square root.

a. $\pm\sqrt{\frac{16}{9}}$ **$\pm\frac{4}{3}$**

b. $\sqrt{0.0144}$ **0.12**

CLASSIFY AND ORDER NUMBERS

In-Class Example PowerPoint®

2 Name the set or sets of numbers to which each real number belongs.

a. $\sqrt{17}$
Because $\sqrt{17} = 4.1231056\ldots$, which is neither a repeating nor terminating decimal, this number is irrational.

b. $\frac{1}{6}$
Because 1 and 6 are integers and $1 \div 6 = 0.1666\ldots$ is a repeating decimal, the number is a rational number.

c. $\sqrt{169}$
Because $\sqrt{169} = 13$, this number is a natural number, a whole number, an integer, and a rational number.

d. -327
This number is an integer and a rational number.

---

b. $\pm\sqrt{0.81}$

$\pm\sqrt{0.81}$ represents the positive and negative square roots of 0.81.

$0.81 = 0.9^2$ and $0.81 = (-0.9)^2$

$\pm\sqrt{0.81} = \pm0.9$

CLASSIFY AND ORDER NUMBERS Recall that rational numbers are numbers that can be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

As you have seen, the square roots of perfect squares are rational numbers. However, numbers such as $\sqrt{3}$ and $\sqrt{24}$ are the square roots of numbers that are not perfect squares. Numbers like these cannot be expressed as a terminating or repeating decimal.

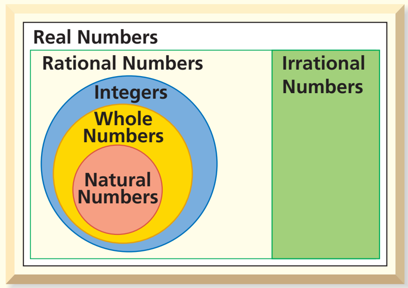
$$\sqrt{3} = 1.73205080\ldots$$

$$\sqrt{24} = 4.89897948\ldots$$

Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**. Irrational numbers and rational numbers together form the set of **real numbers**.

Concept Summary *Real Numbers*

The set of real numbers consists of the set of rational numbers and the set of irrational numbers.



Study Tip

Common Misconception
Pay close attention to the placement of a negative sign when working with square roots. $\sqrt{-121}$ is undefined for real numbers since no real number multiplied by itself can result in a negative product.

Example 2 *Classify Real Numbers*

Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727\ldots$ is a repeating decimal, this number is a rational number.

b. $\sqrt{121}$

Because $\sqrt{121} = 11$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

Because $\sqrt{56} = 7.48331477\ldots$, which is not a repeating or terminating decimal, this number is irrational.

d. $-\frac{36}{4}$

Because $-\frac{36}{4} = -9$, this number is an integer and a rational number.

104 Chapter 2 Real Numbers

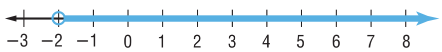
In Lesson 2-1 you graphed rational numbers on a number line. However, the rational numbers alone do not complete the number line. By including irrational numbers, the number line is complete. This is illustrated by the **Completeness Property** which states that each point on the number line corresponds to exactly one real number.

Recall that inequalities like $x < 7$ are open sentences. To solve the inequality, determine what replacement values for x make the sentence true. This can be shown by the solution set {all real numbers less than 7}. Not only does this set include integers like 5 and -2, but it also includes rational numbers like $\frac{3}{8}$ and $-\frac{12}{13}$ and irrational numbers like $\sqrt{40}$ and π.

Example 3 Graph Real Numbers

Graph each solution set.

a. $x > -2$



The heavy arrow indicates that all numbers to the right of -2 are included in the graph. The circle at -2 indicates -2 is *not* included in the graph.

b. $a \le 4.5$

The heavy arrow indicates that all points to the left of 4.5 are included in the graph. The dot at 4.5 indicates that 4.5 *is* included in the graph.

To express irrational numbers as decimals, you need to use a rational approximation. A **rational approximation** of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number. For example, a rational approximation of $\sqrt{2}$ is 1.41 when rounded to the nearest hundredth.

Example 4 Compare Real Numbers

Replace each ● with <, >, or = to make each sentence true.

a. $\sqrt{19}$ ● $3.\overline{8}$

Find two perfect squares closest to $\sqrt{19}$ and write an inequality.

$16 < 19 < 25$ — 19 is between 16 and 25.

$\sqrt{16} < \sqrt{19} < \sqrt{25}$ — Find the square root of each number.

$4 < \sqrt{19} < 5$ — $\sqrt{19}$ is between 4 and 5.

Since $\sqrt{19}$ is between 4 and 5, it must be greater than $3.\overline{8}$.

So, $\sqrt{19} > 3.\overline{8}$.

b. $7.\overline{2}$ ● $\sqrt{52}$

You can use a calculator to find an approximation for $\sqrt{52}$.

$\sqrt{52} = 7.211102551\ldots$

$7.\overline{2} = 7.222\ldots$

Therefore, $7.\overline{2} > \sqrt{52}$.

www.algebra1.com/extra_examples

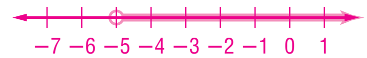
In-Class Examples

Power Point®

3 Graph each solution set.

a. $y \le 8$

b. $z > -5$



4 Replace each ● with <, >, or = to make each sentence true.

a. 14 ● $\sqrt{196}$ The numbers are equal, so $14 = \sqrt{196}$.

b. $\sqrt{48}$ ● $6.\overline{9}$ $\sqrt{48} = 6.9282032\ldots$ so $\sqrt{48} < 6.\overline{9}$

Lesson 2-7 Square Roots and Real Numbers 105

Tips for New Teachers

Estimating Square Roots

You can use perfect squares to estimate square roots that are irrational. To determine where $\sqrt{27}$ lies on a number line, ask students between which two perfect squares 27 lies. Since $25 < 27 < 36$, $\sqrt{25} < \sqrt{27} < \sqrt{36}$ or $5 < \sqrt{27} < 6$. Also, since 27 is closer to 25 than 36, you know that $\sqrt{27}$ is closer to 5 than 6. In fact, $\sqrt{27}$ is about 5.2. Estimating square roots mentally can help students master problems like Example 5 more easily.

In-Class Examples

Power Point®

5 Write $\frac{12}{5}$, $\sqrt{6}$, $2.\overline{4}$, and $\frac{61}{25}$ in order from least to greatest.

$\frac{12}{5}$, $\frac{61}{25}$, $2.\overline{4}$, $\sqrt{6}$

6 For what value of x is $\sqrt{x} < 1 < \frac{1}{\sqrt{x}}$ true? **C**

A -5
B 0
C $\frac{1}{5}$
D 5

Answer

2. Rational numbers are numbers that when written as decimals terminate or repeat. Irrational numbers do not terminate nor do they repeat.

You can write a set of real numbers in order from greatest to least or from least to greatest. To do so, find a decimal approximation for each number in the set and compare.

Example 5 Order Real Numbers

Write $2.\overline{63}$, $-\sqrt{7}$, $\frac{8}{3}$, $\frac{53}{-20}$ in order from least to greatest.

Write each number as a decimal.

$2.\overline{63} = 2.6363636\ldots$ or about 2.636.

$-\sqrt{7} = -2.64575131\ldots$ or about -2.646.

$\frac{8}{3} = 2.66666666\ldots$ or about 2.667.

$\frac{53}{-20} = -2.65$

$-2.65 < -2.646 < 2.636 < 2.667$

The numbers arranged in order from least to greatest are $\frac{53}{-20}$, $-\sqrt{7}$, $2.\overline{63}$, $\frac{8}{3}$.

You can use rational approximations to test the validity of some algebraic statements involving real numbers.

Standardized Test Practice

A B C D

Example 6 Rational Approximation

Multiple-Choice Test Item

For what value of x is $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ true?

Ⓐ $\frac{1}{2}$ Ⓑ 0 Ⓒ -2 Ⓓ 3

Read the Test Item

The expression $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ is an open sentence, and the set of choices $\left\{\frac{1}{2}, 0, -2, 3\right\}$ is the replacement set.

Solve the Test Item

Replace x in $\frac{1}{\sqrt{x}} > \sqrt{x} > x$ with each given value.

Ⓐ $x = \frac{1}{2}$

$\frac{1}{\sqrt{\frac{1}{2}}} \overset{?}{>} \sqrt{\frac{1}{2}} \overset{?}{>} \frac{1}{2}$ Use a calculator.

$1.41 > 0.71 > 0.5$ ✓ True

Ⓑ $x = 0$

$\frac{1}{\sqrt{0}} \overset{?}{>} \sqrt{0} \overset{?}{>} 0$

False; $\frac{1}{\sqrt{0}}$ is not a real number.

Ⓒ $x = -2$

$\frac{1}{\sqrt{-2}} \overset{?}{>} \sqrt{-2} \overset{?}{>} -2$

False; $\frac{1}{\sqrt{-2}}$ and $\sqrt{-2}$ are not real numbers.

Ⓓ $x = 3$

$\frac{1}{\sqrt{3}} \overset{?}{>} \sqrt{3} \overset{?}{>} 3$ Use a calculator.

$\cancel{0.58 > 1.73 > 3}$ False

The inequality is true for $x = \frac{1}{2}$, so the correct answer is A.

The Princeton Review

Test-Taking Tip

You could stop when you find that A is a solution. But testing the other values is a good check.

Standardized Test Practice

A B C D

Example 6 Advise students to examine the answer choices before substituting them into the inequality. Choice B can be eliminated because $0 = \sqrt{0}$, making $\frac{1}{\sqrt{x}}$ an undefined term. Since C is negative and the square root of a negative number is undefined for the real number set, it can be eliminated. Therefore, the only choices remaining to evaluate are A and D.

Check for Understanding

Concept Check

1. Sometimes; the square root of a number can be negative, such as $\sqrt{16} = 4$ and $\sqrt{16} = -4$.

1. **Tell** whether the square root of any real number is *always*, *sometimes* or *never* positive. Explain your answer.
2. **OPEN ENDED** Describe the difference between rational numbers and irrational numbers. Give examples of both. **See margin.**
3. **Explain** why you cannot evaluate $\sqrt{-25}$ using real numbers. **There is no real number that can be multiplied by itself to result in a negative product.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1
8–11	2
12, 13	3
14–16	4
17, 18	5
19	6

Find each square root. If necessary, round to the nearest hundredth.

4. $-\sqrt{25}$ **−5**
5. $\sqrt{1.44}$ **1.2**
6. $\pm\sqrt{\frac{16}{49}}$ **$\pm\frac{4}{7}$**
7. $\sqrt{32}$ **5.66**

Name the set or sets of numbers to which each real number belongs.

8. $-\sqrt{64}$ **integers, rationals**
9. $\frac{8}{3}$ **rationals**
10. $\sqrt{28}$ **irrationals**
11. $\frac{56}{7}$ **naturals, whole, integers, rationals**

Graph each solution set. **12–13. See margin.**

12. $x < -3.5$
13. $x \geq -7$

Replace each ● with <, >, or = to make each sentence true.

14. $0.3 \bullet \frac{1}{3}$ **<**
15. $\frac{2}{9} \bullet 0.\overline{2}$ **=**
16. $\frac{1}{6} \bullet \sqrt{6}$ **<**

Write each set of numbers in order from least to greatest.

17. $\frac{1}{8}, \sqrt{\frac{1}{8}}, 0.\overline{15}, -15$ **$-15, \sqrt{\frac{1}{8}}, \frac{1}{8}, 0.\overline{15}$**
18. $\sqrt{30}, 5\frac{4}{9}, 13, \frac{1}{\sqrt{30}}$ **$\frac{1}{\sqrt{30}}, 5\frac{4}{9}, \sqrt{30}, 13$**

Standardized Test Practice

19. For what value of a is $-\sqrt{a} < -\frac{1}{\sqrt{a}}$ true? **C**

(A) $\frac{1}{3}$ (B) -4 (C) 2 (D) 1

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
20–31, 50, 51	1
32–49	2
52–57	3
58–63	4
64–69	5

Extra Practice

See page 825.

32–49. See pp. 117A–117B.

Find each square root. If necessary, round to the nearest hundredth.

20. $\sqrt{49}$ **7**
21. $\sqrt{81}$ **9**
22. $\sqrt{5.29}$ **2.3**
23. $\sqrt{6.25}$ **2.5**
24. $-\sqrt{78}$ **−8.83**
25. $-\sqrt{94}$ **−9.70**
26. $\pm\sqrt{\frac{36}{81}}$ **$\pm\frac{2}{3}$**
27. $\pm\sqrt{\frac{100}{196}}$ **$\pm\frac{5}{7}$**
28. $\sqrt{\frac{9}{14}}$ **0.80**
29. $\sqrt{\frac{25}{42}}$ **0.77**
30. $\pm\sqrt{820}$ **±28.64**
31. $\pm\sqrt{513}$ **±22.65**

Name the set or sets of numbers to which each real number belongs.

32. $-\sqrt{22}$
33. $\frac{36}{6}$
34. $\frac{1}{3}$
35. $-\frac{5}{12}$
36. $\sqrt{\frac{82}{20}}$
37. $-\sqrt{46}$
38. $\sqrt{10.24}$
39. $\frac{-54}{19}$
40. $-\frac{3}{4}$
41. $\sqrt{20.25}$
42. $\frac{18}{3}$
43. $\sqrt{2.4025}$
44. $\frac{-68}{35}$
45. $\frac{6}{11}$
46. $\sqrt{5.5696}$
47. $\sqrt{\frac{78}{42}}$
48. $-\sqrt{9.16}$
★ 49. π

www.algebra1.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 2.
- copy the concept summary for Real Numbers, along with an explanation of how to find both square roots of a number.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Square Roots:** 20–31, 50
- **Classify and Order Numbers:** 32–49, 52–69

Odd/Even Assignments

Exercises 20–69 are structured so that students practice the same concepts whether they are assigned odd or even problems.

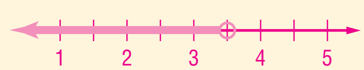
Assignment Guide

Basic: 21–47 odd, 51–55 odd, 59, 61, 65–69 odd, 73, 77–88

Average: 21–69 odd, 70–73, 77–88

Advanced: 20–68 even, 73–88

Answers

12.


13.


DAILY INTERVENTION

Differentiated Instruction

Intrapersonal This is the last lesson of this chapter. The chapter contains numerous concepts which may have been unfamiliar to students. Encourage each student to use their Foldables and the vocabulary list on p. 110 of the student edition to pinpoint any concepts with which students still feel unsure.

Study Guide and Intervention, p. 111 (shown) and p. 112

Square Roots A **square root** is one of two equal factors of a number. For example, the square roots of 36 are 6 and −6, since $6 \cdot 6$ or 6^2 is 36 and $(-6)(-6)$ or $(-6)^2$ is also 36. A rational number like 36, whose square root is a rational number, is called a **perfect square**.

The symbol $\sqrt{\ }$ is a **radical sign**. It indicates the nonnegative, or **principal**, square root of the number under the radical sign. So $\sqrt{36} = 6$ and $-\sqrt{36} = -6$. The symbol $\pm\sqrt{36}$ represents both square roots.

Example 1 Find $-\sqrt{\frac{25}{49}}$.

$-\sqrt{\frac{25}{49}}$ represents the negative square root of $\frac{25}{49}$.

$\frac{25}{49} = \left(\frac{5}{7}\right)^2 \rightarrow -\sqrt{\frac{25}{49}} = -\frac{5}{7}$

Example 2 Find $\pm\sqrt{0.16}$.

$\pm\sqrt{0.16}$ represents the positive and negative square roots of 0.16.

$0.16 = 0.4^2$ and $0.16 = (-0.4)^2$

$\pm\sqrt{0.16} = \pm 0.4$

Exercises

Find each square root.

1. $\sqrt{64}$ **8**
2. $-\sqrt{81}$ **−9**
3. $\sqrt{16.81}$ **4.1**
4. $\pm\sqrt{100}$ **±10**
5. $-\sqrt{\frac{4}{25}}$ $-\frac{2}{5}$
6. $-\sqrt{121}$ **−11**
7. $\pm\sqrt{\frac{25}{144}}$ $\pm\frac{5}{12}$
8. $-\sqrt{\frac{25}{16}}$ $-\frac{5}{4}$
9. $\pm\sqrt{\frac{121}{100}}$ $\pm\frac{11}{10}$
10. $-\sqrt{3600}$ **−60**
11. $-\sqrt{6.25}$ **−2.5**
12. $\pm\sqrt{0.0004}$ **±0.02**
13. $\sqrt{\frac{144}{196}}$ $\frac{6}{7}$
14. $-\sqrt{\frac{36}{49}}$ $-\frac{6}{7}$
15. $\pm\sqrt{1.21}$ **±1.1**

Skills Practice, p. 113 and Practice, p. 114 (shown)

Find each square root. If necessary, round to the nearest hundredth.

1. $\sqrt{324}$ **18**
2. $-\sqrt{62}$ **−7.87**
3. $\pm\sqrt{25}$ **±5**
4. $-\sqrt{84}$ **−9.17**
5. $\pm\sqrt{\frac{4}{289}}$ $\pm\frac{2}{17}$
6. $-\sqrt{\frac{7}{12}}$ **−0.76**
7. $-\sqrt{0.081}$ **−0.28**
8. $\pm\sqrt{3.06}$ **±1.75**

Name the set or sets of numbers to which each real number belongs.

9. $\sqrt{93}$ **irrational**
10. $-\sqrt{0.0625}$ **rational**
11. $\frac{8}{7}$ **rational**
12. $-\frac{144}{3}$ **integer, rational**

Graph each solution set.

13. $x < -0.5$

14. $x \geq -3.5$

Replace each ● with <, >, or = to make each sentence true.

15. $0.\overline{93}$ ● $\sqrt{0.93}$ **<**
16. $8.\overline{17}$ ● $\sqrt{66}$ **>**
17. $\frac{5}{6}$ ● $\frac{\sqrt{5}}{6}$ **>**

Write each set of numbers in order from least to greatest.

18. $\sqrt{0.03}, \frac{\sqrt{2}}{8}, 0.\overline{17}$ $0.\overline{17}, \sqrt{0.03}, \frac{\sqrt{2}}{8}$
19. $-\frac{84}{30}, -\sqrt{8}, -\frac{\sqrt{7}}{8}$ $-\sqrt{8}, -\frac{84}{30}, -\frac{\sqrt{7}}{8}$
20. $-\sqrt{8.5}, -\frac{\sqrt{35}}{2}, -2\frac{19}{20}$ $-\frac{\sqrt{35}}{2}, -2\frac{19}{20}, -\sqrt{8.5}$

21. **SIGHTSEEING** The distance you can see to the horizon is given by the formula $d = \sqrt{1.5h}$, where d is the distance in miles and h is the height in feet above the horizon line. Mt. Whitney is the highest point in the contiguous 48 states. Its elevation is 14,494 feet. The lowest elevation, at −282 feet, is located near Badwater, California. With a clear enough sky and no obstructions, could you see from the top of Mt. Whitney to Badwater if the distance between them is 135 miles? Explain. **Yes; you can see about 149 miles from the top of Mt. Whitney to an elevation of −282 feet.**

22. **SEISMIC WAVES** A *tsunami* is a seismic wave caused by an earthquake on the ocean floor. You can use the formula $s = 3.1\sqrt{d}$, where s is the speed in meters per second and d is the depth of the ocean in meters, to determine the speed of a tsunami. If an earthquake occurs at a depth of 200 meters, what is the speed of the tsunami generated by the earthquake? **about 43.8 m/s**

Reading to Learn Mathematics, p. 115 ELL

Pre-Activity **How can using square roots determine the surface area of the human body?**

Read the introduction to Lesson 2-7 at the top of page 103 in your textbook.

The expression $\sqrt{3600}$ is read, "the square root of 3600." How would you read the expression $\sqrt{64}$?

the square root of 64

Reading the Lesson

Complete each statement below.

1. The symbol $\sqrt{\ }$ is called a **radical sign** and is used to indicate a nonnegative or principal square root of the expression under the symbol.
2. A **rational approximation** of an irrational number is a rational number that is close to, but not equal to, the value of the irrational number.
3. The positive square root of a number is called the **principal** square root of the number.
4. A number whose positive square root is a rational number is a **perfect square**.
5. Write each of the following as a mathematical expression that uses the $\sqrt{\ }$ symbol.
   a. the positive square root of 1600 $\sqrt{1600}$
   b. the negative square root of 729 $-\sqrt{729}$
   c. the principal square root of 3025 $\sqrt{3025}$
6. The irrational numbers and rational numbers together form the set of **real** numbers.

Helping You Remember

7. Use a dictionary to look up several words that begin with "ir-". What does the prefix "ir-" mean? How can this help you remember the meaning of the word *irrational*?
Sample answer: The prefix "ir-" means *not*. So an irrational number is a number that is not a rational number.

---

50. **PHYSICAL SCIENCE** The time it takes for a falling object to travel a certain distance d is given by the equation $t = \sqrt{\frac{d}{16}}$, where t is in seconds and d is in feet. If Krista dropped a ball from a window 28 feet above the ground, how long would it take for the ball to reach the ground? **1.32 s**

51. **LAW ENFORCEMENT** Police can use the formula $s = \sqrt{24d}$ to estimate the speed s of a car in miles per hour by measuring the distance d in feet a car skids on a dry road. On his way to work, Jerome skidded trying to stop for a red light and was involved in a minor accident. He told the police officer that he was driving within the speed limit of 35 miles per hour. The police officer measured his skid marks and found them to be $43\frac{3}{4}$ feet long. Should the officer give Jerome a ticket for speeding? Explain.
No; Jerome was traveling at about 32.4 mph.

Graph each solution set. **52–57. See margin.**

52. $x > -12$
53. $x \leq 8$
54. $x \geq -10.2$
55. $x < -0.25$
★ 56. $x \neq -2$
★ 57. $x \neq \pm\sqrt{36}$

Replace each ● with <, >, or = to make each sentence true.

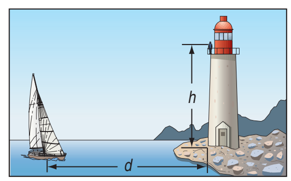
58. $5.\overline{72}$ ● $\sqrt{5}$ **>**
59. $2.\overline{63}$ ● $\sqrt{8}$ **<**
60. $\frac{1}{7}$ ● $\frac{1}{\sqrt{7}}$ **<**
61. $\frac{2}{3}$ ● $\frac{2}{\sqrt{3}}$ **<**
★ 62. $\frac{1}{\sqrt{31}}$ ● $\frac{\sqrt{31}}{31}$ **=**
★ 63. $\frac{\sqrt{2}}{2}$ ● $\frac{1}{2}$ **>**

Write each set of numbers in order from least to greatest.

64. $\sqrt{0.42}, 0.\overline{63}, \frac{\sqrt{4}}{3}$ $0.\overline{63}, \sqrt{0.42}, \frac{\sqrt{4}}{3}$
65. $\sqrt{0.06}, 0.\overline{24}, \frac{\sqrt{9}}{12}$ $0.\overline{24}, \sqrt{0.06}, \frac{\sqrt{9}}{12}$
66. $-1.\overline{46}, 0.2, \sqrt{2}, -\frac{1}{6}$
67. $-4.\overline{83}, 0.4, \sqrt{8}, -\frac{3}{8}$
68. $-\sqrt{65}, -6\frac{2}{5}, -\sqrt{27}$
69. $\sqrt{122}, 7\frac{4}{9}, \sqrt{200}$

TOURISM **For Exercises 70–72, use the following information.**

The formula to determine the distance d in miles that an object can be seen on a clear day on the surface of a body of water is $d = 1.4\sqrt{h}$, where h is the height in feet of the viewer's eyes above the surface of the water.



70. A charter plane is used to fly tourists on a sightseeing trip along the coast of North Carolina. If the plane flies at an altitude of 1500 feet, how far can the tourists see? **about 54.2 mi**

71. Dillan and Marissa are parasailing while on vacation. Marissa is 135 feet above the ocean while Dillan is 85 feet above the ocean. How much farther can Marissa see than Dillan? **about 3.4 mi**

72. The observation deck of a lighthouse stands 120 feet above the ocean surface. Can the lighthouse keeper see a boat that is 17 miles from the lighthouse? Explain. **No; the lighthouse keeper can only see about 15.3 mi.**

73. **CRITICAL THINKING** Determine when the following statements are all true for real numbers q and r. **They are true if q and r are positive and $q > r$.**
   a. $q^2 > r^2$
   b. $\frac{1}{q} < \frac{1}{r}$
   c. $\sqrt{q} > \sqrt{r}$
   d. $\frac{1}{\sqrt{q}} < \frac{1}{\sqrt{r}}$

More About. . .

Tourism

Built in 1758, the Sambro Island Lighthouse at Halifax Harbor is the oldest operational lighthouse in North America.

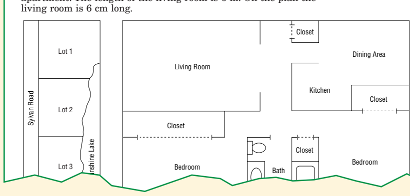
Source: Canadian Coast Guard

66. $-1.\overline{46}, -\frac{1}{6}, 0.2, \sqrt{2}$
67. $-4.\overline{83}, -\frac{3}{8}, 0.4, \sqrt{8}$
68. $-\sqrt{65}, -6\frac{2}{5}, -\sqrt{27}$
69. $7\frac{4}{9}, \sqrt{122}, \sqrt{200}$

Enrichment, p. 116

Scale Drawings

The map at the left below shows building lots for sale. The scale ratio is 1:2400. At the right below is the floor plan for a two-bedroom apartment. The length of the living room is 6 m. On the plan the living room is 6 cm long.



GEOMETRY For Exercises 74–76, use the table.

Squares		
Area (units^2)	Side Length	Perimeter
1	1	4
4	2	8
9	3	12
16	4	16
25	5	20

74. Copy and complete the table. Determine the length of each side of each square described. Then determine the perimeter of each square.

75. Describe the relationship between the lengths of the sides and the area.

75. The length of the side is the square root of the area.

76. Write an expression you can use to find the perimeter of a square whose area is a units^2. $4\sqrt{a}$

77. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

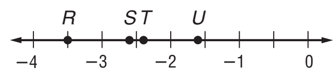
How can using square roots determine the surface area of the human body?

Include the following in your answer:

- an explanation of the order of operations that must be followed to calculate the surface area of the human body,
- a description of other situations in which you might need to calculate the surface area of the human body, and
- examples of real-world situations involving square roots.

Standardized Test Practice

78. Which point on the number line is closest to $-\sqrt{7}$? B



(A) R (B) S

(C) T (D) U

79. Which of the following is a true statement? B

(A) $-\frac{6}{3} > \frac{3}{6}$ (B) $-\frac{3}{6} > -\frac{6}{3}$ (C) $-\frac{3}{6} < -\frac{6}{3}$ (D) $\frac{6}{3} < \frac{3}{6}$

Maintain Your Skills

Mixed Review **Find the odds of each outcome if a card is randomly selected from a standard deck of cards.** *(Lesson 2-6)*

80. red 4 1:25

81. even number 5:8

82. against a face card 10:3

83. against an ace 12:1

84. **AUTO RACING** Jeff Gordon's finishing places in the 2000 season races are listed below. Which measure of central tendency best represents the data? Explain. *(Lesson 2-5)* Sample answer: Mean; the median and mode are too low.

34	10	28	9	8	8	25	4	1	11	14	10	32	14	8	1	4
10	5	3	33	23	36	23	4	1	6	9	5	39	4	2	7	7

Simplify each expression. *(Lesson 2-3)* 88. $-7xy + 14xz$

85. $4(-7) - 3(11)$ -61

86. $3(-4) + 2(-7)$ -26

87. $1.2(4x - 5y) - 0.2(-1.5x + 8y)$ $5.1x - 7.6y$

88. $-4x(y - 2z) + x(6z - 3y)$

4 Assess

Open-Ended Assessment

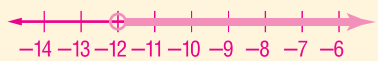
Modeling Arrange 16 square tiles into a 4-by-4 grid and show how the side length models $\sqrt{16}$. Now give students other square numbers of tiles and have them model the square root.

Assessment Options

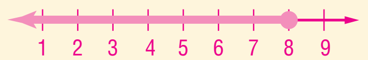
Quiz (Lesson 2-7) is available on p. 132 of the *Chapter 2 Resource Masters*.

Answers

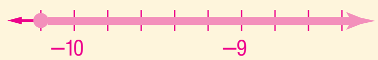
52.



53.

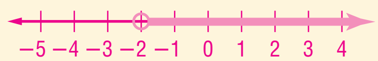


54.



55.

56.



57.

77. Sample answer: By using the formula

$$\text{Surface Area} = \sqrt{\frac{\text{height} \times \text{weight}}{3600}},$$

you need to use square roots to calculate the quantity. Answers should include the following.

- You must multiply height by weight first. Divide that product by 3600. Then determine the square root of that result.
- Sample answers: exposure to radiation or chemicals; heat loss; scuba suits
- Sample answers: determining height, distance

Chapter 2 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 2 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 2 is available on p. 130 of the *Chapter 2 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 2 Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 69)	irrational number (p. 104)	probability (p. 96)
additive inverses (p. 74)	line plot (p. 88)	radical sign (p. 103)
back-to-back stem-and-leaf plot (p. 89)	measures of central tendency (p. 90)	rational approximation (p. 105)
Completeness Property (p. 105)	natural number (p. 68)	rational number (p. 68)
coordinate (p. 69)	negative number (p.68)	real number (p. 104)
equally likely (p. 97)	odds (p. 97)	sample space (p. 96)
frequency (p. 88)	opposites (p. 74)	simple event (p. 96)
graph (p. 69)	perfect square (p. 103)	square root (p. 103)
infinity (p. 68)	positive number (p. 68)	stem-and-leaf plot (p. 89)
integers (p. 68)	principal square root (p. 103)	whole number (p. 68)

State whether each sentence is *true* or *false*. If false, replace the underlined term or number to make a true sentence.

1. The absolute value of −26 is 26. **true**
2. Terminating decimals are rational numbers. **true**
3. The principal square root of 144 is 12. **true**
4. $-\sqrt{576}$ is an irrational number. **false; rational number**
5. 225 is a perfect square. **true**
6. −3.1 is an integer. **false; sample answer: −3**
7. 0.666 is a repeating decimal. **false; sample answer: $0.\overline{6}$ or 0.666...**
8. The product of two numbers with different signs is negative. **true**

Lesson-by-Lesson Review

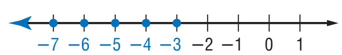
2-1 Rational Numbers on the Number Line

See pages 68–72.

Concept Summary

- A set of numbers can be graphed on a number line by drawing points.
- To evaluate expressions with absolute value, treat the absolute value symbols as grouping symbols.

Example Graph {…, −5, −4, −3}.



The bold arrow means that the graph continues indefinitely in that direction.

Exercises **Graph each set of numbers.** *See Example 2 on page 69.* **9–11. See margin.**

9. {5, 3, −1, −3}
10. $\left\{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, \ldots\right\}$
11. {integers less than −4 and greater than or equal to 2}

Evaluate each expression if $x = -4$, $y = 8$, and $z = -9$. *See Example 4 on page 70.*

12. $32 - |y - 3|$ **27**
13. $3|x| - 7$ **5**
14. $4 + |z|$ **13**
15. $46 - y|x|$ **14**

www.algebra1.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables*.

Have students review their Foldables to be sure they have included notes on every lesson in this chapter.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

2-2 Adding and Subtracting Rational Numbers

See pages 73–78.

Concept Summary

- To add rational numbers with the *same* sign, add their absolute values. The sum has the same sign as the addends.
- To add rational numbers with *different* signs, subtract the lesser absolute value from the greater absolute value. The sum has the same sign as the number with the greater absolute value.
- To subtract a rational number, add its additive inverse.

Examples

1 Find $-4 + (-3)$.

$-4 + (-3)$

$= -(|-4| + |-3|)$ Both numbers are negative, so the sum is negative.

$= -(4 + 3)$

$= -7$

2 Find $12 - 18$.

$12 - 18$ To subtract 18, add its inverse.

$= 12 + (-18)$

$= -(|-18| - |12|)$ The absolute value of 18 is greater, so the result is negative.

$= -(18 - 12)$

$= -6$

Exercises **Find each sum or difference.** *See Examples 1–3 on pages 73–75.*

16. $4 + (-4)$ **0**
17. $2 + (-7)$ **-5**
18. $-0.8 + (-1.2)$ **-2**
19. $-3.9 + 2.5$ **-1.4**
20. $-\frac{1}{4} + \left(-\frac{1}{8}\right)$ **$-\frac{3}{8}$**
21. $\frac{5}{6} + \left(-\frac{1}{3}\right)$ **$\frac{1}{2}$**
22. $-2 - 10$ **-12**
23. $9 - (-7)$ **16**
24. $1.25 - 0.18$ **1.07**
25. $-7.7 - (-5.2)$ **-2.5**
26. $\frac{9}{2} - \left(-\frac{1}{2}\right)$ **5**
27. $-\frac{1}{8} - \left(-\frac{2}{3}\right)$ **$\frac{13}{24}$**

2-3 Multiplying Rational Numbers

See pages 79–83.

Concept Summary

- The product of two numbers having the same sign is positive.
- The product of two numbers having different signs is negative.

Example

Multiply $\left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right)$.

$\left(-2\frac{1}{7}\right)\left(3\frac{2}{3}\right) = \frac{-15}{7} \cdot \frac{11}{3}$ Write as improper fractions.

$= \frac{-55}{7}$ or $-7\frac{6}{7}$ Simplify.

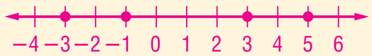
Exercises **Find each product.** *See Examples 1 and 3 on pages 79 and 80.*

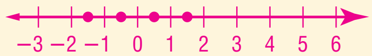
28. $(-11)(9)$ **-99**
29. $12(-3)$ **-36**
30. $-8.2(4.5)$ **-36.9**
31. $-2.4(-3.6)$ **8.64**
32. $\frac{3}{4} \cdot \frac{7}{12}$ **$\frac{7}{16}$**
33. $\left(-\frac{1}{3}\right)\left(-\frac{9}{10}\right)$ **$\frac{3}{10}$**

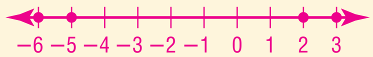
Simplify each expression. *See Example 2 on page 80.*

34. $8(-3x) + 12x$ **$-12x$**
35. $-5(-2n) - 9n$ **n**
36. $-4(6a) - (-3)(-7a)$ **$-45a$**

Answers

9.


10.


11.


Chapter 2 Study Guide and Review 111

2-4 Dividing Rational Numbers

See pages 84–87.

Concept Summary

- The quotient of two positive numbers is positive.
- The quotient of two negative numbers is positive.
- The quotient of a positive number and a negative number is negative.

Example Simplify $\frac{-3(4)}{-2-3}$.

$\frac{-3(4)}{-2-3} = \frac{-12}{-2-3}$ Simplify the numerator.

$= \frac{-12}{-5}$ Simplify the denominator.

$= 2\frac{2}{5}$ same signs → positive quotient

Exercises **Find each quotient.** *See Examples 1–3 on pages 84 and 85.*

37. $\frac{-54}{6}$ **−9**
38. $-\frac{74}{8}$ **−9.25**
39. $21.8 \div (-2)$ **−10.9**
40. $-7.8 \div (-6)$ **1.3**
41. $-15 \div \left(\frac{3}{4}\right)$ **−20**
42. $\frac{21}{24} \div \frac{1}{3}$ **$\frac{21}{8}$ or $2\frac{5}{8}$**

Simplify each expression. *See Example 5 on page 85.*

43. $\frac{14-28x}{-7}$ **$-2+4x$**
44. $\frac{-5+25x}{5}$ **$-1+5x$**
45. $\frac{-4x+24y}{4}$ **$-x+6y$**

Evaluate each expression if $x = -4$, $y = 2.4$, and $z = 3$. *See Example 6 on page 85.*

46. $xz - 2y$ **−16.8**
47. $-2\left(\frac{2y}{z}\right)$ **−3.2**
48. $\frac{2x-z}{4} + 3y$ **4.45**

2-5 Statistics: Displaying and Analyzing Data

See pages 88–94.

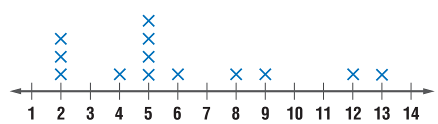
Concept Summary

- A set of numerical data can be displayed in a line plot or stem-and-leaf plot.
- A measure of central tendency represents a centralized value of a set of data. Examine each measure of central tendency to choose the one most representative of the data.

Examples **1** **Draw a line plot for the data.**

2 8 6 4 5 9 13 12 5 2 5 5 2

The value of the data ranges from 2 to 13. Construct a number line containing those points. Then place an × above a number each time it occurs.



112 Chapter 2 Real Numbers

2 SCHOOL Melinda's scores on the 25-point quizzes in her English class are 20, 21, 12, 21, 22, 22, 22, 21, 20, 20, and 21. Which measure of central tendency best represents her grade?

mean:	20.2	Add the data and divide by 11.
median:	21	The middle value is 21.
mode:	21	The most frequent value is 21.

The median and mode are both representative of the data. The mean is less than most of the data.

Exercises

49. Draw a line plot for the data. Then make a stem-and-leaf plot.
See Examples 1–3 on pages 88 and 89. **See margin.**

28	17	16	18	19	21	26	15
19	19	16	14	21	12	26	17
30	17	13	18	14	22	20	12
19	19	15	12	15	21	15	17

50. BUSINESS Of the 42 employees at Pirate Printing, four make \$6.50 an hour, sixteen make \$6.75 an hour, six make \$6.85 an hour, thirteen make \$7.25 an hour, and three make \$8.85 an hour. Which measure best describes the average wage? Explain. *See Examples 5 and 6 on pages 90 and 91.* **See margin.**

51. HOCKEY Professional hockey uses a point system based on wins, losses and ties, to determine teams' rank. The stem-and-leaf plot shows the number of points earned by each of the 30 teams in the National Hockey League during the 2000–2001 season. Which measure of central tendency best describes the average number of points earned? Explain.
See Example 5 on page 90. **Sample answer: Median; it is closest in value to most of the data.**

Stem	Leaf
11	1 1 8
10	0 3 6 9
9	0 0 0 2 3 5 6 6 8
8	0 8 8
7	0 1 1 2 3
6	0 6 6 8
5	2 9 $11|1 = 111$

2-6 Probability: Simple Probability and Odds

See pages 96–101.

Concept Summary

- The probability of an event a can be expressed as
  $P(a) = \dfrac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$.
- The odds of an event can be expressed as the ratio of the number of successful outcomes to the number of unsuccessful outcomes.

Examples

1 Find the probability of randomly choosing the letter *I* in the word *MISSISSIPPI*.

$P(\text{letter I}) = \dfrac{4}{11}$ ← number of favorable outcomes / ← number of possible outcomes

≈ 0.36

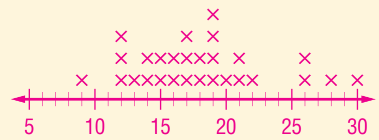
The probability of choosing an I is $\frac{4}{11}$ or about 36%.

Answers

49.

Stem	Leaf
1	2 2 2 3 4 4 5 5 5 5 6 6 7 7 7 7 8 8 9 9 9 9 9
2	0 1 1 1 2 6 6 8
3	0

$1|2 = 12$



50. Sample answer: Mean; the median and mode are too low.

Chapter 2 Study Guide and Review 113

Answers

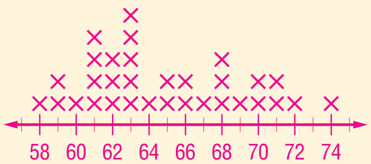
64. rationals

65. naturals, wholes, integers, rationals

66. irrationals

Answers (p. 115)

31.



32. Sample answer: The median and mode can be used to best represent the data. The mean is too high.

• Extra Practice, see pages 823–825.
• Mixed Problem Solving, see page 854.

2 **Find the odds that you will randomly select a letter that is *not* S in the word *MISSISSIPPI*.**

number of successes : number of failures = 7:4

The odds of not selecting an S are 7:4.

Exercises **Find the probability of each outcome if a computer randomly chooses a letter in the word *REPRESENTING*.** *See Example 1 on pages 96 and 97.*

52. $P(S)$ $\frac{1}{12}$ **53.** $P(E)$ $\frac{1}{4}$ **54.** $P(\text{not N})$ $\frac{5}{6}$ **55.** $P(\text{R or P})$ $\frac{1}{4}$

Find the odds of each outcome if you randomly select a coin from a jar containing 90 pennies, 75 nickels, 50 dimes, and 30 quarters.
See Examples 2 and 3 on pages 97 and 98.

56. a dime **10:39** **57.** a penny **18:31** **58.** *not* a nickel **34:15** **59.** a nickel or a dime **25:24**

2-7 Square Roots and Real Numbers

See pages 103–109.

Concept Summary

- A square root is one of two equal factors of a number.
- The symbol $\sqrt{}$ is used to indicate the nonnegative square root of a number.

Example **Find $\sqrt{169}$.**

$\sqrt{169}$ represents the square root of 169.

$169 = 13^2 \rightarrow \sqrt{169} = 13$

Exercises **Find each square root. If necessary, round to the nearest hundredth.**
See Example 1 on page 103.

60. $\sqrt{196}$ **14** **61.** $\pm\sqrt{1.21}$ **±1.1** **62.** $-\sqrt{160}$ **−12.65** **63.** $\pm\sqrt{\frac{4}{225}}$ $\pm\frac{2}{15}$

Name the set or sets of numbers to which each real number belongs.
See Example 2 on page 104. **64–66. See margin.**

64. $\frac{16}{25}$ **65.** $\frac{\sqrt{64}}{2}$ **66.** $-\sqrt{48.5}$

Replace each ● with <, >, or = to make each sentence true. *See Example 4 on page 105.*

67. $\frac{1}{8}$ ● $\frac{1}{\sqrt{49}}$ **<** **68.** $\sqrt{\frac{2}{3}}$ ● $\frac{4}{9}$ **>** **69.** $\sqrt{\frac{3}{4}}$ ● $\sqrt{\frac{1}{3}}$ **>**

70. WEATHER Meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$ to estimate the amount of time t in hours a storm of diameter d will last. Suppose the eye of a hurricane, which causes the greatest amount of destruction, is 9 miles in diameter. To the nearest tenth of an hour, how long will the worst part of the hurricane last?
See Example 1 on pages 103 and 104. **1.8 h**

Chapter 2

Practice Test

Vocabulary and Concepts

Choose the correct term to complete each sentence.

1. The (*absolute value*, *square*) of a number is its distance from zero on a number line.
2. A number that can be written as a fraction where the numerator and denominator are integers and the denominator does not equal zero is a (*repeating*, *rational*) number.
3. The list of all possible outcomes is called the (*simple event*, *sample space*).

Skills and Applications

Evaluate each expression.

4. $-|x| - 38$ if $x = -2$ **-40**
5. $34 - |x + 21|$ if $x = -7$ **20**
6. $-12 + |x - 8|$ if $x = 1.5$ **-5.5**

Find each sum or difference.

7. $-19 + 12$ **-7**
8. $-21 - (-34)$ **13**
9. $16.4 + (-23.7)$ **-7.3**
10. $6.32 - (-7.41)$ **13.73**
11. $-\frac{7}{16} + \frac{3}{8}$ **$-\frac{1}{16}$**
12. $-\frac{7}{12} - \left(-\frac{5}{9}\right)$ **$-\frac{1}{36}$**

Find each quotient or product.

13. $-5(19)$ **-95**
14. $-56 \div (-7)$ **8**
15. $96 \div (-0.8)$ **-120**
16. $(-7.8)(5.6)$ **-43.68**
17. $-\frac{1}{8} \div -5$ **$\frac{1}{40}$**
18. $-\frac{15}{32} \div \frac{3}{4}$ **$-\frac{5}{8}$**

Simplify each expression. **21. $28mn - 12cd$**

19. $5(-3x) - 12x$ **$-27x$**
20. $7(6h - h)$ **$35h$**
21. $-4m(-7n) + (3d)(-4c)$
22. $\frac{36k}{4}$ **$9k$**
23. $\frac{9a + 27}{-3}$ **$-3a - 9$**
24. $\frac{70x - 30y}{-5}$ **$-14x + 6y$**

Find each square root. If necessary, round to the nearest hundredth.

25. $-\sqrt{64}$ **-8**
26. $\sqrt{3.61}$ **1.9**
27. $\pm\sqrt{\frac{16}{81}}$ **$\pm\frac{4}{9}$**

Replace each ● with <, >, or = to make each sentence true.

28. $\frac{1}{\sqrt{3}}$ ● $\frac{1}{3}$ **>**
29. $\sqrt{\frac{1}{2}}$ ● $\frac{8}{11}$ **<**
30. $\sqrt{0.56}$ ● $\frac{\sqrt{3}}{2}$ **<**

STATISTICS **For Exercises 31 and 32, use the following information.** **31–32. See margin.**
The height, in inches, of the students in a health class are 65, 63, 68, 66, 72, 61, 62, 63, 59, 58, 61, 74, 65, 63, 71, 60, 62, 63, 71, 70, 59, 66, 61, 62, 68, 69, 64, 63, 70, 61, 68, and 67.

31. Make a line plot of the data.
32. Which measure of central tendency best describes the data? Explain.

33. **STANDARDIZED TEST PRACTICE** During a 20-song sequence on a radio station, 8 soft-rock, 7 hard-rock, and 5 rap songs are played at random. Assume that all of the songs are the same length. What is the probability that when you turn on the radio, a hard-rock song will be playing? **B**

(A) $\frac{1}{4}$ (B) $\frac{7}{20}$ (C) $\frac{2}{5}$ (D) $\frac{13}{20}$ (E) $\frac{7}{10}$

www.algebra1.com/chapter_test

Chapter 2 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 2 can be found on p. 130 of the *Chapter 2 Resource Masters*.

Chapter Tests There are six Chapter 2 Tests and an Open-Ended Assessment task available in the *Chapter 2 Resource Masters*.

Chapter 2 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	117–118
2A	MC	average	119–120
2B	MC	average	121–122
2C	FR	average	123–124
2D	FR	average	125–126
3	FR	advanced	127–128

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 2 can be found on p. 129 of the *Chapter 2 Resource Masters*. A sample scoring rubric for these tasks appears on p. A28.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction Where will you use integers in the world outside your algebra classroom? Are integers common or does the outside world more often use fractions or decimals?

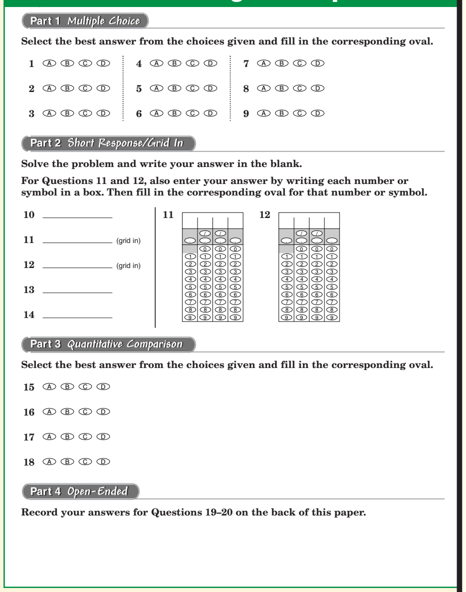
Ask Students Golf is an example of a game that uses integers in its scoring. Have students investigate the game of golf and how it is scored. Then have them report on how integers are used to keep score in a golf tournament. How do the integers indicate which player is the winner?

Chapter 2 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 2 Resource Masters*.

Standardized Test Practice
Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 11 and 12, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 ______
11 ______ (grid in)
12 ______ (grid in)
13 ______
14 ______

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

15 Ⓐ Ⓑ Ⓒ Ⓓ
16 Ⓐ Ⓑ Ⓒ Ⓓ
17 Ⓐ Ⓑ Ⓒ Ⓓ
18 Ⓐ Ⓑ Ⓒ Ⓓ

Part 4 Open-Ended

Record your answers for Questions 19–20 on the back of this paper.

Additional Practice

See pp. 135–136 in the *Chapter 2 Resource Masters* for additional standardized test practice.

Teaching Tip Exercises 10–14 are all short answer questions. However, only Exercises 11 and 12 might appear as grid-in questions.

Chapter 2 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

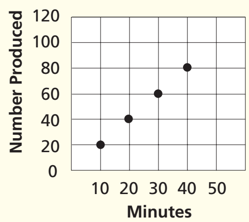
1. Darryl works 9 days at the State Fair and earns \$518.40. If he works 8 hours each day, what is his hourly pay? (Prerequisite Skill) **B**

Ⓐ \$6.48 Ⓑ \$7.20
Ⓒ \$30.50 Ⓓ \$57.60

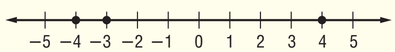
2. The graph below shows how many toy trains are assembled at a factory at the end of 10-minute intervals. What is the best prediction for the number of products assembled per hour? (Prerequisite Skill) **C**

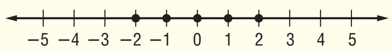
Ⓐ 80
Ⓑ 100
Ⓒ 120
Ⓓ 130

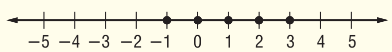
Toy Train Production

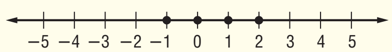


3. Which graph shows the integers greater than −2 and less than or equal to 3? (Lesson 2-1) **C**

Ⓐ


Ⓑ


Ⓒ


Ⓓ


The Princeton Review — Test-Taking Tip

Question 1 If you don't know how to solve a problem, eliminate the answer choices you know are incorrect and then guess from the remaining choices. Even eliminating only one answer choice greatly increases your chance of guessing the correct answer.

4. Which number is the greatest? (Lesson 2-1) **D**

Ⓐ $|-4|$ Ⓑ $|4|$
Ⓒ $|7|$ Ⓓ $|-9|$

5. What is $-3.8 + 4.7$? (Lesson 2-2) **A**

Ⓐ 0.9 Ⓑ −0.9
Ⓒ 8.5 Ⓓ −8.5

6. Simplify $3(-2m) - 7m$. (Lesson 2-3) **D**

Ⓐ $-12m$ Ⓑ $-m$
Ⓒ $-2m$ Ⓓ $-13m$

7. Which statement about the stem-and-leaf plot is *not* true? (Lesson 2-5) **D**

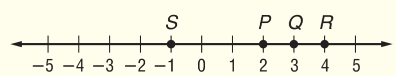
Stem	Leaf
3	1 1 5 6 8 8
4	2 2 2 4
5	0 0
6	0 3 7 8 9 9
7	4 7|4 = 74

Ⓐ The greatest value is 74.
Ⓑ The mode is 42.
Ⓒ Seven of the values are greater than 50.
Ⓓ The least value is 38.

8. There are 4 boxes. If you choose a box at random, what are the odds that you will choose the one box with a prize? (Lesson 2-6) **A**

Ⓐ 1:3 Ⓑ 1:4
Ⓒ 3:1 Ⓓ 3:4

9. Which point on the number line is closest to $\sqrt{10}$? (Lesson 2-7) **B**



Ⓐ point P Ⓑ point Q
Ⓒ point R Ⓓ point S

The Princeton Review ***Log On*** **for Test Practice**

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

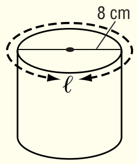
TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Ethan needs to wrap a label around a jar of homemade jelly so that there is no overlap. Find the length of the label. (Prerequisite Skill) **25.12 cm**



11. Evaluate $\frac{5-1}{4+12 \div 3 \times 2}$. (Lesson 1-2) **1/3**

12. Find the solution of $4m - 3 = 9$ if the replacement set is {0, 2, 3, 5}. (Lesson 1-3) **3**

13. Write an algebraic expression for *2p plus three times the difference of m and n*. (Lesson 1-6) **$3(m - n) + 2p$**

14. State the hypothesis in the statement *If $3x + 3 > 24$, then $x > 7$.* (Lesson 1-7) **$3x + 3 > 24$**

Part 3 Quantitative Comparison

Compare the quantity in Column A to the quantity in Column B. Then determine whether:

- (A) **the quantity in Column A is greater,**
- (B) **the quantity in Column B is greater,**
- (C) **the quantities are equal, or**
- (D) **the relationship cannot be determined from the information given.**

15. $x > 0$

Column A	Column B
$\lvert x \rvert$	$\lvert -x \rvert$

C (Lesson 2-1)

16. $x > y > 0$

Column A	Column B
$\frac{1}{x}$	$\frac{1}{y}$

B (Lesson 2-5)

www.algebra1.com/standardized_test

17. $\frac{1}{n} > 1$

Column A	Column B
1	n

A (Lesson 2-6)

18. $a^2 = 49$

Column A	Column B
a	7

D (Lesson 2-7)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

19. Mia has created the chart below to compare the three cellular phone plans she is considering. (Lessons 2-2 and 2-3)

Plan	Monthly Fee	Cost/Minute
A	$5.95	$0.30
B	$12.95	$0.10
C	$19.99	$0.08

a. Write an algebraic expression that Mia can use to figure the monthly cost of each plan. Use C for the total monthly cost, m for the cost per minute, x for the monthly fee, and y for the minutes used per month. **a–b. See margin.**

b. If Mia uses 150 minutes of calls each month, which plan will be least expensive? Explain.

20. The stem-and-leaf plot lists the annual profit for seven small businesses. (Lesson 2-5) **a–b. See margin.**

Stem	Leaf
3	2 9
4	1 1 3 5
5	0

a. Explain how the absence of a key could lead to misinterpreting the data.

b. How do the keys below affect how the data should be interpreted?

$3|2 = 3.2$ $\quad$ $3|2 = 0.32$

Evaluating Open-Ended Assessment Questions

Open-Ended Assessment questions are graded by using a multilevel rubric that guides you in assessing a student's knowledge of a particular concept.

Goal: Identify what is wrong with a stem-and-leaf plot, and compare cellular phone service plans.

Sample Scoring Rubric: The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.

Score	Criteria
4	A correct solution that is supported by well-developed, accurate explanations
3	A generally correct solution, but may contain minor flaws in reasoning or computation
2	A partially correct interpretation and/or solution to the problem
1	A correct solution with no supporting evidence or explanation
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given

Answers

19a. $C = x + my$

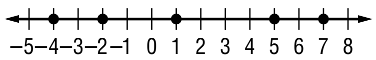
19b. **Plan B is the least expensive. It costs $27.95 for 150 minutes. Plan A costs $50.95 and Plan C costs $31.99.**

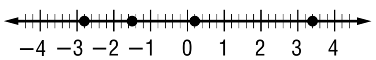
20a. **Without a key, you cannot determine what the values are.**

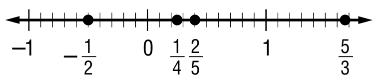
20b. **If the key is $3|2 = 3.2$, then the data are ten times as great as they would be if the key is $3|2 = 0.32$.**

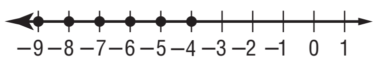
Pages 70–72, Lesson 2-1

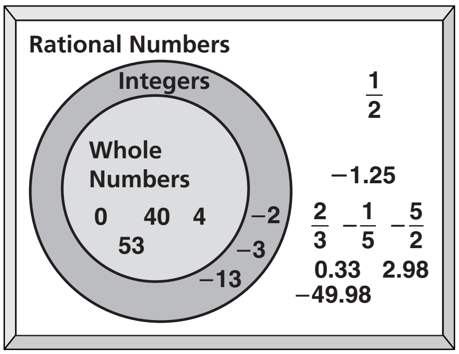
1. always
2. Sample answer: Absolute value is how far from zero a number is.
3. Sample answer: Describing distances such as north versus south, or left versus right.

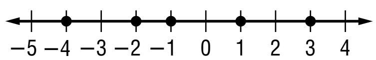
6. 

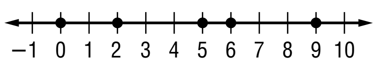
7. 

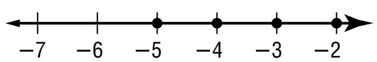
8. 

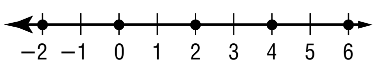
9. 

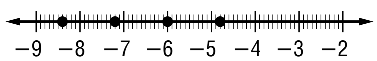
17. 

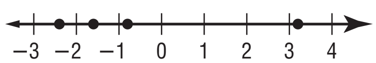
24. 

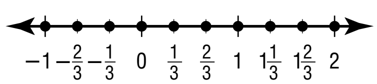
25. 

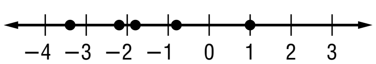
26. 

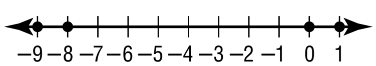
27. 

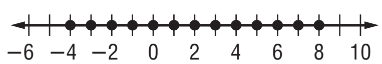
28. 

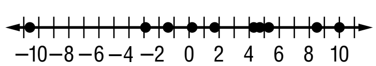
29. 

30. 

31. 

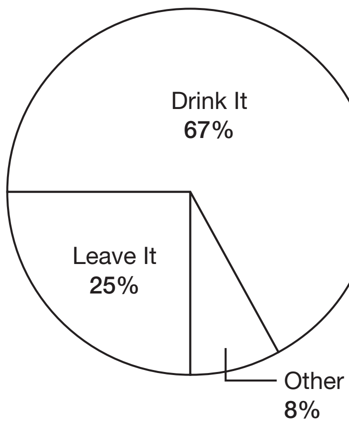
32. 

33. 

42. 

Page 78, Lesson 2-2

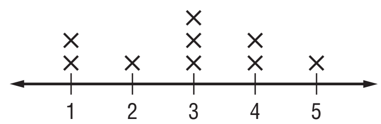
71. **Cereal Milk**



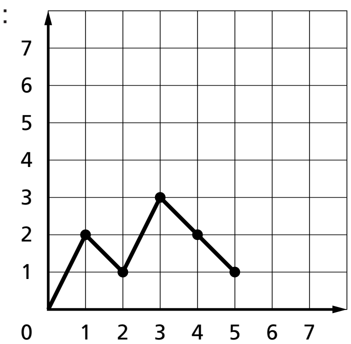
Pages 91–93, Lesson 2-5

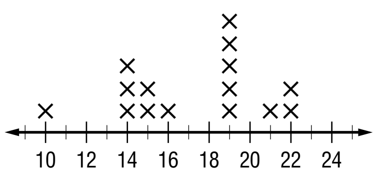
2. Sample answer:

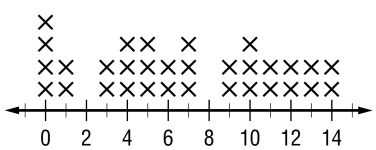
Line Plot:



Line Graph:



4. 

5. 

7. The mean and the median both represent the data accurately as they are fairly central.

8.

Stem	Leaf
6	4 6 8 8 8
7	1 2 3 6
8	0 1 6 8 8
9	3

6|4 = 64

11.

Stem	Leaf
5	4 5 5 6
6	0 1 4 9
7	0 3 5 7 8
8	0 0 3 5 8 8 8
9	0
10	0 2 5
11	0

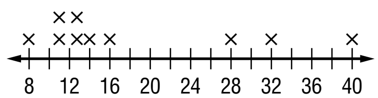
5|4 = 54

22.

Stem	Leaf
10	0 0 4 5 5 6 6 7 8 9 9
11	0 0 0 0 1 1 2 2 2 2 3 3 4 4 4 4 4 5 6 7 7 7 8 8 8 8 8 8 9
12	0 0 0 0 1 1 2 8
13	4

10|0 = 100

28.



32.

Stem	Leaf
1	6 8 8 9 9 9
2	0 0 0 0 0 1 1 2 2 3 3 4 6 8 8 8 9 9 9 9 9
3	0 0 0 0 1 3 3 4 4 6 7
4	3 7

1|6 = 16

35.

Stem	Leaf
3	0 4 7
4	
5	2 9
6	2 7
7	7
8	4 5

3|0 = 30

Page 100–101, Lesson 2-6

60. Sample answer: Probabilities are often used for strategy like placing a certain pitcher against a batter who has a low probability of hitting a pitch from that pitcher. Answers should include the following.

- baseball: using the probability that a team can get base runners out; basketball: the probability that a player can make a basket from a certain place on the court; auto racing: the probability that a set of tires will hold out for the remainder of a race
- Odds in favor of an event and odds against an event are frequently used.

Page 102, Algebra Activity

2.

4 boys	3 boys, 1 girl	2 boys, 2 girls	1 boy, 3 girls	4 girls
BBBB	BBBG	BBGG	BGGG	GGGG
	BBGB	BGBG	GBGG	
	BGBB	BGGB	GGBG	
	GBBB	GBBG	GGGB	
		GBGB		
		GGBB		

Page 107, Lesson 2-7

32. irrationals
33. naturals, wholes, integers, rationals
34. rationals
35. rationals
36. irrationals
37. irrationals
38. rationals
39. rationals
40. rationals
41. rationals
42. naturals, wholes, integers, rationals
43. rationals
44. rationals
45. rationals
46. rationals
47. irrationals
48. irrational
49. irrational