

## Lecture 1 : Real Numbers, Absolute Value.

### Real Numbers

We can represent the real numbers by the set of points on a line. The origin corresponds to the number 0. Numbers to the right of 0 are positive or  $> 0$  and numbers to the left of 0 are negative or  $< 0$ .



The set of Real numbers is denoted by  $\mathbb{R}$  and contains all of the following number types:

- The natural numbers (or counting numbers) denoted by  $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ .
- The integers (or whole numbers) denoted by  $\mathbb{Z} = \{\dots - 3, -2, -1, 0, 1, 2, 3, \dots\}$ .
- The rational numbers (or fractions) denoted by  $\mathbb{Q} =$  the set of all numbers of the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers.
- The irrational numbers. These are numbers which do not fit any of the above descriptions, in particular, they are not rational. Numbers such as  $\pi$  and  $\sqrt{2}$  fit this description.

Letting  $A \subset B$  denote that the set  $A$  is a subset of  $B$ , we see that  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ .

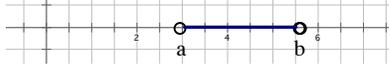
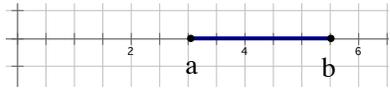
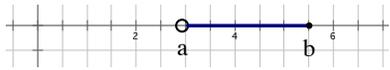
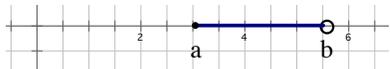
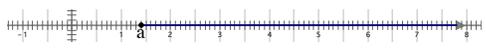
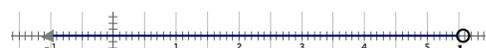
## Intervals

An important property of the real numbers is that they are totally ordered, so we can compare any two real numbers,  $a$  and  $b$ , and make a statement of the form  $a \leq b$  or  $b \leq a$ , with strict inequality if  $a \neq b$ .

Given any two points on the real line,  $a$  and  $b$ , we call the set of points between  $a$  and  $b$  an interval.

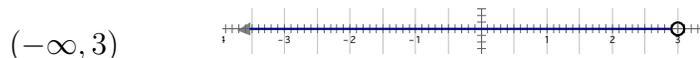
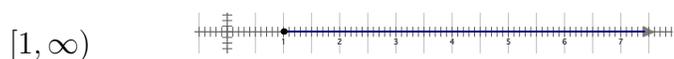
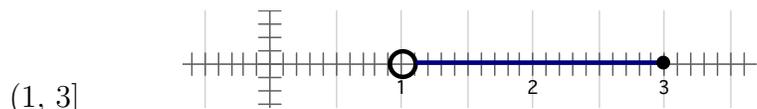
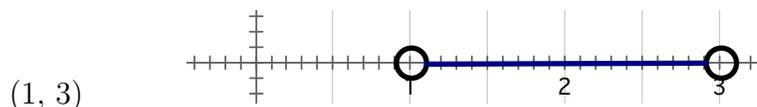
When discussing these intervals, it is important to indicate whether we are including one or both endpoints.

We also have the concept of intervals that extend to the ends of the number line  $+\infty$  and  $-\infty$ . We list the notations for each type of interval and their graphs below. (The statement  $\{x|a \leq x \leq b\}$  should be read as “the set of all  $x$  such that  $a$  is less than or equal to  $x$  which is less than or equal to  $b$ ” (the symbol  $|$  denotes ”such that”). We can represent intervals in three ways:

Notation	Set Description	Graphical Representation
$(a, b)$	$\{x a < x < b\}$	
$[a, b]$	$\{x a \leq x \leq b\}$	
$(a, b]$	$\{x a < x \leq b\}$	
$[a, b)$	$\{x a \leq x < b\}$	
$[a, \infty)$	$\{x a \leq x < \infty\}$	
$(a, \infty)$	$\{x a < x < \infty\}$	
$(-\infty, b]$	$\{x -\infty < x \leq b\}$	
$(-\infty, b)$	$\{x -\infty < x < b\}$	
$(-\infty, \infty)$	$\{x -\infty < x < \infty\}$	entire number line.

**Example** Sketch the following intervals

$$(1, 3), \quad [1, 3], \quad [1, \infty), \quad (-\infty, 3).$$



**Example** Graph the intervals  $[1, 7)$ ,  $(0, \infty)$ ,  $(-\infty, -1]$ ,  $[-1, 3]$ .

## Absolute Value

The absolute value of a real number  $a$  is denoted by  $|a|$  and it is the distance from  $a$  to the origin 0 on the number line. The absolute value is always positive. We can give a formula for the absolute value of the number, which depends on whether  $a$  is positive or negative. Because of this we have to make two statements to describe the formula.

**Definition** If  $a$  is a real number, the **absolute value** of  $a$  is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

**Example** Evaluate  $|2|$ ,  $|-10|$ ,  $|5 - 9|$ ,  $|9 - 5|$ .

### Algebraic properties of the Absolute Value

1.  $|a| \geq 0$  for all real numbers  $a$ .
2.  $|a| = |-a|$  for all real numbers  $a$ .
3.  $|ab| = |a||b|$ , the absolute value of the product of two numbers is the product of the absolute values.
4.  $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ , the absolute value of the quotient of two numbers is the quotient of the absolute values.

**Example** Evaluate  $\left|\frac{7-10}{21}\right|$  and  $||-2| - |-4||$ .

### Distance Between Two Points on The Real Line.

If  $a$  and  $b$  are real numbers, then the distance between the points  $a$  and  $b$  on the real line is

$$d(a, b) = |b - a|.$$

**Example** Find the distance between the numbers  $-2$  and  $4$ .