Energy of a Macroscopic System

Objective: To calculate spring potential energy; to apply the energy principle to a particle-spring system; to define thermal energy; to qualitatively relate thermal energy to temperature; to describe thermal energy transfer; to apply the energy principle to a macroscopic system.

Spring Potential Energy

The potential energy stored in a spring is

$$U_s = \frac{1}{2}k_x s^2 + U_0 \tag{1}$$

where U_0 is the energy stored in the spring when it is unstretched. This term is necessary to satisfy the condition required by relativity that the potential energy is 0 when the particles are infinitely separated.

Ball and spring model of a solid

We have modeled a solid as balls (atoms) connected by springs (bonds). These balls are always moving (i.e. vibrating). To find the energy of a solid at an instant of time, we have to add up the kinetic energy of each particle and the potential energy of each spring at that instant. This energy (sum of kinetic and potential energies of all microscopic particles and springs in a macroscopic system) is called thermal energy. But, for 10^{23} atoms, that's a monumental task even for a supercomputer.

It turns out that the temperature and the average energy of atoms are related. Thus, a hot object has a greater thermal energy than a cold object. Thus, a measurable macroscopic property, temperature, is related to a microscopic property, average atomic energy. Such relationships are powerful!

Joule found that doing 4.2 J of work on 1 gram of water raised its temperature 1 K. A change in temperature of 1 kelvin is equivalent to a change in temperature of 1 Celsius degree, although a temperature in K is NOT EQUAL to the equivalent temperature in °C. The amount of energy required to change the temperature of 1 gram of a substance 1 kelvin is called the heat capacity of the substance. Thus, water has a heat capacity of 4.2 J/(g·K). Thus,

$$\Delta E_{therm} = mC\Delta T \tag{2}$$

Thermal energy transfer

When you put your hand on a block of ice, it "feels" cold. Why is that? The atoms in your hand which are moving more vigorously (i.e. have a greater thermal energy) interact with atoms in the ice at the surface of contact. As a result, the atoms in the ice move more vigorously and the atoms in your hand move less vigorously. The ice gains thermal energy, and your hand loses thermal energy. Thus, there was a transfer of thermal energy. This transfer of thermal energy is called heat, Q.

There are other methods of transferring thermal energy. For instance, radiation, the transfer of photons from surroundings to a system, is a form of thermal energy transfer as a result of a temperature difference between two bodies.

We'll also see later on that tansfering thermal energy does not always result in a change in temperature of the system. The system may also undergo a phase change.

A macroscopic system can lose or gain energy as a result of work (transfer of mechanical energy as a result of external forces acting through displacements) and heat (transfer of thermal energy as a result of a temperature difference between a system and its surroundings and the atomic interactions at the boundary). Thus, the energy principle gives

$$\Delta E_{sys} = Q + W \tag{3}$$

If thermal energy is transferred to the system, then Q is positive. If thermal energy is transferred from the system to its surroundings, then Q is negative.

If the total work done by external forces (surroundings) on the system is positive, then W is positive. If the total work done by external forces on the system is negative (i.e. the system did positive work on the surroundings), then W is negative.

Positive Q and positive W both correspond to a deposit of energy to the system. Negative Q and negative W both correspond to the system losing energy.

Power

Power is the rate of transferring energy. Thus, its units are J/s, called watts, W.

Application

1. A spring of spring constant 100 N/m is initially at its unstretched position. You stretch it 0.050 m. What is the elastic potential energy stored in the spring after stretching it?

2. For the spring in the previous question, suppose you let it relax a bit so that it is stretched a distance of 0.025 m from its unstretched position. How much energy is now stored in the spring?

3. A spring is used to launch a 0.020-kg hot wheels car around a loop and up a ramp. If the spring has a spring constant of 150 N/m and is compressed 0.050 m, what will be the maximum height of the car on the ramp if friction is negligible? Does your answer depend on the angle of the ramp?

4. You want your Hot-wheels car to just barely jump a wall 0.20 m high when leaving a ramp 0.10 m high at an angle of 30°. If you use your spring launcher to start the car on a flat, level track, what distance should you compress the spring. Use the values for mass and spring constant that are given in the previous question. Again, assume that friction is approximately negligible.

5. In one pot, you have 2.0 kg of water. In another identical pot, you have 1.0 kg of water. Which pot of water will have a higher heat capacity? Which pot will require more energy to raise the temperature of the water 1 K? (neglect the pot, just compare the water)

6. 1.5 kg of water and 1.5 kg of aluminum absorb the same amount of energy via a transfer of thermal energy. Which ones temperature will change the most? (Assume that no phase change occurs during the process.)

7. If an amount of energy Q_A is transferred to object A, it will cause each gram of A to rise in temperature by 3 C°. If the same amount of energy is transferred to object B, then the temperature of each gram of B will rise by 4 C°. Which object has the greater heat capacity?

8. A 0.010 kg aluminum strip at room temperature (20 °C) is brought into contact with a 0.030 kg iron strip at 40 °C. What will be the equilibrium temperature of the strips? What assumptions did you make regarding the systems when doing your calculation?