12.7 Chi-Square Test for the Variance or Standard Deviation

When analyzing numerical data, sometimes you need to draw conclusions about the population variance or standard deviation. For example, recall that in the cereal-filling process described in Section 9.1, you assumed that the population standard deviation, σ, was equal to 15 grams. To see if the variability of the process has changed, you need to test whether the standard deviation has changed from the previously specified level of 15 grams.

Assuming that the data are normally distributed, you use the χ^2 **test for the variance or standard deviation** defined in Equation (12.10) to test whether the population variance or standard deviation is equal to a specified value.

χ^2 TEST FOR THE VARIANCE OR STANDARD DEVIATION

$$\chi^2_{STAT} = \frac{(n-1)S^2}{\sigma^2} \qquad \textbf{(12.10)}$$

where

$$n = \text{sample size}$$
$$S^2 = \text{sample variance}$$
$$\sigma^2 = \text{hypothesized population variance}$$

The test statistic χ^2_{STAT} follows a chi-square distribution with $n - 1$ degrees of freedom.

To apply the test of hypothesis, return to the cereal-filling example. You are interested in determining whether the standard deviation has changed from the previously specified level of 15 grams. Thus, you use a two-tail test with the following null and alternative hypotheses:

$$H_0: \sigma^2 = 225 \text{ (that is, } \sigma = 15 \text{ grams)}$$
$$H_1: \sigma^2 \neq 225 \text{ (that is, } \sigma \neq 15 \text{ grams)}$$

If you select a sample of 25 cereal boxes, you reject the null hypothesis if the computed χ^2_{STAT} test statistic falls into either the lower or upper tail of a chi-square distribution with $25 - 1 = 24$ degrees of freedom, as shown in Figure 12.18. From Equation (12.10), observe that the χ^2_{STAT} test statistic falls into the lower tail of the chi-square distribution if the sample standard deviation (S) is sufficiently smaller than the hypothesized σ of 15 grams, and it falls into the upper tail if S is sufficiently larger than 15 grams. From Table 12.18 (extracted from Table E.4), if you select a level of significance of 0.05, the lower and upper critical values are 12.401 and 39.364, respectively. Therefore, the decision rule is

$$\text{Reject } H_0 \text{ if } \chi^2_{STAT} < \chi^2_{\alpha/2} = 12.401 \text{ or if } \chi^2_{STAT} > \chi^2_{1-\alpha/2} = 39.364;$$

$$\text{otherwise, do not reject } H_0.$$

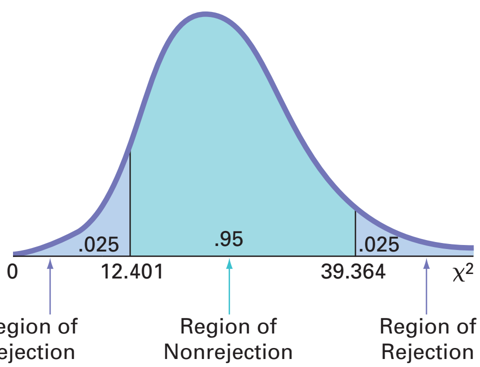


FIGURE 12.18
Determining the lower and upper critical values of a chi-square distribution with 24 degrees of freedom corresponding to a 0.05 level of significance for a two-tail test of hypothesis about a population variance or standard deviation

TABLE 12.18
Finding the Critical Values Corresponding to a 0.05 Level of Significance for a Two-Tail Test from the Chi-Square Distribution with 24 Degrees of Freedom

	Cumulative Area							
	.005	.01	$\boxed{.025}$	.05	.10	.90	.95	$\boxed{.975}$
	Upper-Tail Areas							
Degrees of Freedom	**.995**	**.99**	$\boxed{.975}$	**.95**	**.90**	**.10**	**.05**	$\boxed{.025}$
1	. . .	. . .	0.001	0.004	0.016	2.706	3.841	5.024
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076
$\boxed{24}$	9.886	10.856	$\boxed{12.401}$	13.848	15.659	33.196	36.415	$\boxed{39.364}$
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646

Source: *Extracted from Table E.4.*

Suppose that in the sample of 25 cereal boxes, the standard deviation, S, is 17.7 grams. Using Equation (12.10),

$$\chi^2_{STAT} = \frac{(n-1)S^2}{\sigma^2} = \frac{(25-1)(17.7)^2}{(15)^2} = 33.42$$

Because $\chi^2_{0.025} = 12.401 < \chi^2_{STAT} = 33.42 < \chi^2_{0.975} = 39.364$, or because the p-value $= 0.0956 > 0.05$ (see Figure 12.19), you do not reject H_0. You conclude that there is insufficient evidence that the population standard deviation is different from 15 grams.

FIGURE 12.19
Worksheet for testing the variance in the cereal-filling process

Figure 12.19 displays the **COMPUTE worksheet** *of the* **Chi-Square Variance workbook**. *Create this worksheet using the instructions in Section EG12.7.*

	A		B	
1	Cereal-Filling Analysis			
2				
3	Data			
4	Null Hypothesis	σ^2=	225	
5	Level of Significance		0.05	
6	Sample Size		25	
7	Sample Standard Deviation		17.7	
8				
9	Intermediate Calculations			
10	Degrees of Freedom		24	=B6 - 1
11	Half Area		0.025	=B5/2
12	Chi-Square Statistic		33.4176	=B10 * B7^2/B4
13				
14	Two-Tail Test			
15	Lower Critical Value		12.4012	=CHIINV(1 - B11, B10)
16	Upper Critical Value		39.3641	=CHIINV(B11, B10)
17	*p*-Value		0.0956	=IF(B12 - B15 < 0, 1 - CHIDIST(B12, B10), CHIDIST(B12, B10))
18	Do not reject the null hypothesis			=IF(B17 < B5/2, "Reject the null hypothesis", "Do not reject the null hypothesis")
19				

In testing a hypothesis about a population variance or standard deviation, you assume that the values in the population are normally distributed. Unfortunately, the test statistic discussed in this section is very sensitive to departures from this assumption (i.e., it is not a robust test). Thus, if the population is not normally distributed, particularly for small sample sizes, the accuracy of the test can be seriously affected.

Problems for Section 12.7

LEARNING THE BASICS

12.78 Determine the lower- and upper-tail critical values of χ^2 for each of the following two-tail tests:
a. $\alpha = 0.01, n = 26$
b. $\alpha = 0.05, n = 17$
c. $\alpha = 0.10, n = 14$

12.79 Determine the lower- and upper-tail critical values of χ^2 for each of the following two-tail tests:
a. $\alpha = 0.01, n = 24$
b. $\alpha = 0.05, n = 20$
c. $\alpha = 0.10, n = 16$

12.80 In a sample of $n = 25$ selected from an underlying normal population, $S = 150$. What is the value of χ^2_{STAT} if you are testing the null hypothesis H_0: $\sigma = 100$?

12.81 In a sample of $n = 16$ selected from an underlying normal population, $S = 10$. What is the value of χ^2_{STAT} if you are testing the null hypothesis H_0: $\sigma = 12$?

12.82 In Problem 12.81, how many degrees of freedom are there in the hypothesis test?

12.83 In Problems 12.81 and 12.82, what are the critical values from Table E.4 if the level of significance is $\alpha = 0.05$ and H_1 is as follows:
a. $\sigma \neq 12$?
b. $\sigma < 12$?

12.84 In Problems 12.81, 12.82, and 12.83, what is your statistical decision if H_1 is
a. $\sigma \neq 12$?
b. $\sigma < 12$?

12.85 If, in a sample of size $n = 16$ selected from a very left-skewed population, the sample standard deviation is $S = 24$, would you use the hypothesis test given in Equation (12.10) to test H_0: $\sigma = 20$? Discuss.

APPLYING THE CONCEPTS

12.86 A manufacturer of candy must monitor the temperature at which the candies are baked. Too much variation will cause inconsistency in the taste of the candy. Past records show that the standard deviation of the temperature has been 1.2°F. A random sample of 30 batches of candy is selected, and the sample standard deviation of the temperature is 2.1°F.
a. At the 0.05 level of significance, is there evidence that the population standard deviation has increased above 1.2°F?
b. What assumption do you need to make in order to perform this test?
c. Compute the *p*-value in (a) and interpret its meaning.

12.87 A market researcher for an automobile dealer intends to conduct a nationwide survey concerning car repairs. Among the questions included in the survey is the following: "What was the cost of all repairs performed on your car last year?" In order to determine the sample size necessary, the researcher needs to provide an estimate of the standard deviation. Using his past experience and judgment, he estimates that the standard deviation of the amount of repairs is $200. Suppose that a small-scale study of 25 auto owners selected at random indicates a sample standard deviation of $237.52.
a. At the 0.05 level of significance, is there evidence that the population standard deviation is different from $200?
b. What assumption do you need to make in order to perform this test?
c. Compute the *p*-value in part (a) and interpret its meaning.

12.88 The marketing manager of a branch office of a local telephone operating company wants to study characteristics of residential customers served by her office. In particular, she wants to estimate the mean monthly cost of calls within the local calling region. In order to determine the sample size necessary, she needs an estimate of the standard deviation. On the basis of her past experience and judgment, she estimates that the standard deviation is equal to $12. Suppose that a small-scale study of 15 residential customers indicates a sample standard deviation of $9.25.
a. At the 0.10 level of significance, is there evidence that the population standard deviation is different from $12?
b. What assumption do you need to make in order to perform this test?
c. Compute the *p*-value in (a) and interpret its meaning.

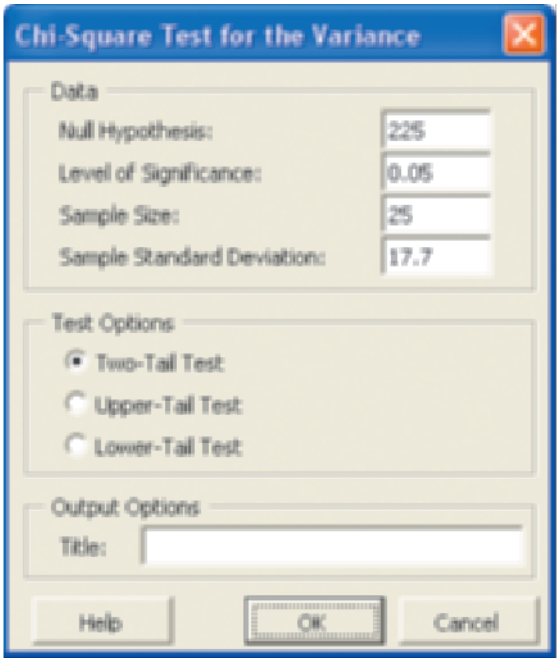
12.89 A manufacturer of doorknobs has a production process that is designed to provide a doorknob with a target diameter of 2.5 inches. In the past, the standard deviation of the diameter has been 0.035 inch. In an effort to reduce the variation in the process, various studies have resulted in a redesigned process. A sample of 25 doorknobs produced under the new process indicates a sample standard deviation of 0.025 inch.
a. At the 0.05 level of significance, is there evidence that the population standard deviation is less than 0.035 inch in the new process?
b. What assumption do you need to make in order to perform this test?
c. Compute the *p*-value in (a) and interpret its meaning.

EG12.7 CHI-SQUARE TEST for the VARIANCE or STANDARD DEVIATION EXCEL GUIDE

PHStat2 Use the **Chi-Square Test for the Variance** procedure to perform this chi-square test. For example, to perform the test for the Section 12.7 cereal-filling process example, select **PHStat ➔ One-Sample Tests ➔ Chi-Square**

Test for the Variance. In the procedure's dialog box (shown below):

1. Enter **225** as the **Null Hypothesis**.
2. Enter **0.05** as the **Level of Significance**.
3. Enter **25** as the **Sample Size.**
4. Enter **17.7** as the **Sample Standard Deviation**.
5. Select **Two-Tail Test**.
6. Enter a **Title** and click **OK**.



The procedure creates a worksheet similar to Figure 12.19.

In-Depth Excel Use the **CHIINV** and **CHIDIST** functions to help perform the chi-square test for the variance or standard deviation. Enter **CHIINV(1–** ***half area, degrees of freedom*****)** and enter **CHIINV(*****half area, degrees of freedom*****)** to compute the lower and upper critical values. Enter **CHIDIST(**χ^2 ***test statistic, degrees of freedom)*** to compute the p-value.

Use the **COMPUTE worksheet** of the **Chi-Square Variance workbook**, shown in Figure 12.19, as a template for performing the chi-square test. The worksheet contains the data for the cereal-filling process example. To perform the test for other problems, change the null hypothesis, level of significance, sample size, and sample standard deviation in the cell range B4:B7. (Open to the **COMPUTE_FORMULAS worksheet** to examine the details of all formulas used in the COMPUTE worksheet.)