

# Classifying Real Numbers

## Warm Up

Start off each lesson by practicing prerequisite skills and math vocabulary that will make you more successful with today's new concept.

1. **Vocabulary** A \_\_\_\_\_ (Venn diagram, line plot) shows the relationship between sets.

2.  $\frac{2}{9}$

3.  $4\frac{3}{8}$

Write each fraction as a decimal.

Write each decimal as a fraction in simplest form.

4. 0.6

5. 5.75

## New Concepts

A **set** is a collection of objects. Each object in the set is called an element. A set is written by enclosing the elements within braces. There are three types of sets. A set with no elements is called the null or **empty set**. A set with a finite number of elements is a **finite set**. An **infinite set** has an infinite number of elements.

$\{12, 24, 36\}$	$\{1, 3, 5, \dots\}$	$\{\}$ or $\emptyset$
finite set	infinite set	null or empty set

The subsets of real numbers are infinite sets.

### Reading Math

The three dots inside the braces are called an ellipsis. An ellipsis shows that the numbers in the set continue on without end.

### Subsets of Real Numbers

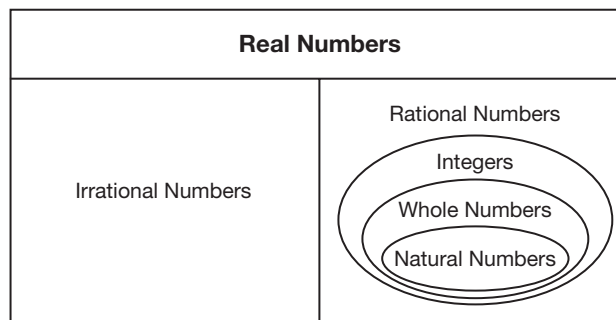
Subsets of Real Numbers	
<b>Natural Numbers</b>	The numbers used to count objects or things. $\{1, 2, 3, 4, \dots\}$
<b>Whole Numbers</b>	The set of natural numbers and zero. $\{0, 1, 2, 3, 4, \dots\}$
<b>Integers</b>	The set of whole numbers and the opposites of the natural numbers. $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
<b>Rational Numbers</b>	Numbers that can be written in the form $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$ . In decimal form, rational numbers either terminate or repeat. Examples: $\frac{1}{2}$ , $0.\overline{3}$ , $-\frac{2}{3}$ , $0.125$
<b>Irrational Numbers</b>	Numbers that cannot be written as the quotient of two integers. In decimal form, irrational numbers neither terminate nor repeat. Examples: $\sqrt[3]{5}$ , $\sqrt{2}$ , $-\sqrt{2}$ , $3\sqrt{3}$ , $\pi$ , $3\pi$
<b>Real Numbers</b>	The set including all rational and irrational numbers.



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The Venn diagram below shows how the sets of numbers are related.



### Math Language

The set of natural numbers is a **subset** of the set of real numbers because every natural number is a real number.

### Example 1 Identifying Sets

For each number, identify the subsets of real numbers to which it belongs.

a.  $\frac{1}{2}$

**SOLUTION**

{rational numbers, real numbers}

b. 5

**SOLUTION**

{natural numbers, whole numbers, integers, rational numbers, real numbers}

c.  $3\sqrt{2}$

**SOLUTION**

{irrational numbers, real numbers}

### Example 2 Identifying Sets for Real-World Situations

Identify the set of numbers that best describes each situation. Explain your choice.

a. the value of the bills in a person's wallet

**SOLUTION** The set of whole numbers best describes the situation. The wallet may contain no bills or any number of bills.

b. the balance of a checking account

**SOLUTION** The set of rational numbers best describes the situation. The balance could be positive or negative and may contain decimal amounts.

c. the circumference of a circular table when the diameter is a rational number

**SOLUTION** The set of irrational numbers describes the situation. Since circumference is equal to the diameter multiplied by pi, it will be an irrational number.

The **intersection of sets**  $A$  and  $B$ ,  $A \cap B$ , is the set of elements that are in  $A$  and  $B$ . The **union** of  $A$  and  $B$ ,  $A \cup B$ , is the set of all elements that are in  $A$  or  $B$ .

### Example 3 Finding Intersections and Unions of Sets

Find  $A \cap B$  and  $A \cup B$ .

a.  $A = \{2, 4, 6, 8, 10, 12\}$ ;  $B = \{3, 6, 9, 12\}$

**SOLUTION**

$$A \cap B = \{6, 12\}; A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$$

b.  $A = \{11, 13, 15, 17\}$ ;  $B = \{12, 14, 16, 18\}$

**SOLUTION**

$$A \cap B = \{\} \text{ or } \emptyset; A \cup B = \{11, 12, 13, 14, 15, 16, 17, 18\}$$

In some lessons, **Explorations** allow you to go into more depth with the mathematics by investigating math concepts with manipulatives, through patterns, and in a variety of other ways.

A set of numbers has **closure**, or is closed, under a given operation if the outcome of the operation on any two members of the set is also a member of the set. For example, the sum of any two natural numbers is also a natural number. Therefore, the set of natural numbers is closed under addition.

One example is all that is needed to prove that a statement is false.

An example that proves a statement false is called a **counterexample**.

### Example 4 Identifying a Closed Set Under a Given Operation

Determine whether each statement is true or false. Give a counterexample for false statements.

a. The set of whole numbers is closed under addition.

**SOLUTION**

Verify the statement by adding two whole numbers.

$$2 + 3 = 5$$

$$9 + 11 = 20$$

$$100 + 1000 = 1100$$

The sum is always a whole number.

The statement is true.

b. The set of whole number is closed under subtraction.

**SOLUTION**

Verify the statement by subtracting two whole numbers.

$$6 - 4 = 2$$

$$100 - 90 = 10$$

$$4 - 6 = -2$$

$4 - 6$  is a counterexample. The difference is not a whole number.

The statement is false.

## Lesson Practice

For each number, identify the subsets of real numbers to which it belongs.

(Ex 1)

a.  $-73$

b.  $\frac{5}{9}$

c.  $18\pi$

Identify the set of numbers that best describes each situation. Explain your choice.

(Ex 2)

d. the number of people on a bus

e. the area of a circular platform

f. the value of coins in a purse

Find  $C \cap D$  and  $C \cup D$ .

(Ex 3)

g.  $C = \{4, 8, 12, 16, 20\}$ ;  $D = \{5, 10, 15, 20\}$

h.  $C = \{6, 12, 18, 24\}$ ;  $D = \{7, 14, 21, 28\}$

**Verify** Determine whether each statement is true or false. Provide a counterexample for false statements.

(Ex 4)

i. The set of whole numbers is closed under multiplication.

j. The set of natural numbers is closed under division.

## Practice Distributed and Integrated

1. Multiply  $26.1 \times 6.15$ .

(SB 2)

2. Add  $\frac{4}{7} + \frac{1}{8} + \frac{1}{2}$ .

(SB 3)

3. Divide  $954 \div 0.9$ .

(SB 2)

4. Add  $\frac{3}{5} + \frac{1}{8} + \frac{1}{8}$ .

(SB 3)

5. Write  $\frac{3}{8}$  as a decimal.

(SB 5)

6. Write  $0.\overline{666}$  as a fraction.

(SB 6)

7. Add  $2\frac{1}{2} + 3\frac{1}{5}$ .

(SB 3)

8. Name a fraction equivalent to  $\frac{2}{5}$ .

(SB 7)

9. **Error Analysis** Two students determine the prime factorization of 72. Which student is correct? Explain the error.

(SB 12)

Student A
72
$= 9 \cdot 8$
$= 9 \cdot 4 \cdot 2$
$= 9 \cdot 2 \cdot 2 \cdot 2$

Student B
72
$= 9 \cdot 8$
$= 9 \cdot 4 \cdot 2$
$= 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$

10. Find the prime factorization of 144.

(SB 12)

11. Write 0.15 as a percent. If necessary, round to the nearest tenth.

(SB 5)

12. Write 7.2 as a percent. If necessary, round to the nearest tenth.

(SB 5)


The **italic numbers** refer to the lesson(s) in which the major concept of that particular problem is introduced. You can refer to the examples or practice in that lesson, if you need additional help.



- \*13. Use braces and digits to designate the set of natural numbers.  
(1)
- \*14. The set  $\{0, 1, 2, 3, \dots\}$  represents what set of numbers?  
(1)
- \*15. Represent the following numbers as being members of set  $K$ : 2, 4, 2, 0, 6, 0, 10, 8.  
(1)

- \*16. **Multiple Choice** Which of the following numbers is an irrational number?  
(1)

A 15                      B  $\sqrt{15}$                       C 15.15151515...                      D  $-\frac{15}{3}$

-  \*17. **Measurement** The surface area of a cube is defined as  $6s^2$ , where  $s$  is the length of the side of the cube. If  $s$  is an integer, then would the surface area of a cube be a rational or irrational number?  
(1)

18. **Verify** True or False: A right triangle can have an obtuse angle. Explain your answer.  
(SB 1)


19. **Anatomy** A baby's head is approximately one fourth of its total body length. If the baby's body measures 19 inches, what does the baby's head measure?  
(SB 3)


20. True or False: An acute triangle has 3 acute angles. Explain your answer.  
(SB 13)

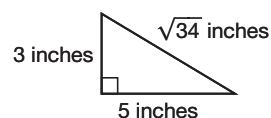
21. True or False: A trapezoid has two pairs of parallel sides. Explain your answer.  
(SB 14)

- \*22. **Track Practice** Tyrone ran 7 laps on the quarter-mile track during practice. Which subset of real numbers would include the distance Tyrone ran at practice?  
(1)

23. True or False: A parallelogram has two pairs of parallel sides. Explain your answer.  
(SB 14)

-  24. **Write** Use the divisibility test to determine if 1248 is divisible by 2. Explain your answer.  
(SB 4)

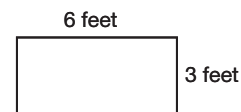
-  \*25. **Geometry** The diagram shows a right triangle. The length of the hypotenuse is a member of which subset(s) of real numbers?  
(1)




- \*26. **Multi-Step** The diagram shows a rectangle.  
(1)

a. Find the area of the rectangle.

b. The number of square feet is a member of which subset(s) of real numbers?



27. **Lunar Rover** The surface-speed record set by the lunar rover on the moon is 10.56 miles per hour. At that speed, how far would the rover travel in 3.5 hours?  
(SB 2)

-  28. **Write** Use the divisibility test to determine if 207 is divisible by 3. Explain your answer.  
(SB 4)

29. **Swimming** Vidiana and Jaime went swimming before school. Vidiana swam  $\frac{3}{5}$  mile and Jaime swam  $\frac{4}{7}$  mile. Write a comparison to show who swam farther. Use  $<$ ,  $>$ , or  $=$ .  
(SB 1)

- \*30. **Banking** Shayla is balancing her checkbook. Which subset of real numbers best describes her balance?  
(1)

In the **Practice**, you will review today's new concept as well as math you learned in earlier lessons. By practicing problems from many lessons every day, you will begin to see how math concepts relate and connect to each other and to the real world.

Also, because you practice the same topic in a variety of ways over several lessons, you will have "time to learn" the concept and will have multiple opportunities to show that you understand.

The mixed set of Practice is just like the mixed format of your state test. You'll be practicing for the "big" test every day!

The starred problems usually cover challenging or recently presented content. Because of that, it is suggested that these exercises be worked first, in case you might want help.

## Understanding Variables and Expressions

## Warm Up

1. **Vocabulary** When two numbers are multiplied, the result is called the \_\_\_\_\_. (*quotient, product*)  
(SB 2)

Add.

2.  $\frac{2}{5} + \frac{1}{3}$   
(SB 3)

3.  $654.1 + 78.39$   
(SB 2)

Multiply.

4.  $4.5(0.23)$   
(SB 2)

5.  $\frac{3}{8}\left(\frac{2}{9}\right)$   
(SB 3)

## New Concepts

A symbol, usually a letter, used to represent an unknown number is called a **variable**. In the algebraic expression  $4 + x$ ,  $x$  is a variable. The number 4 in this expression does not change value. A quantity whose value does not change is called a **constant**.

**Example 1** Identifying Variables and Constants

Identify the constants and the variables in each expression.

a.  $6 - 3x$

**SOLUTION**

The numbers 6 and 3 are constants because they never change. The letter  $x$  is a variable because it represents an unknown number.

b.  $71wz + 28y$

**SOLUTION**

The numbers 71 and 28 are constants because they never change. The letters  $w$ ,  $y$ , and  $z$  are variables because they represent unknown numbers.

**Math Reasoning**

**Connect** What other term can be used to describe the coefficient in the expression  $5mn$ ?

The expression  $4xy$  can also be written as  $4 \cdot x \cdot y$ . When two or more quantities are multiplied, each is a **factor** of the product. The numeric factor of a product including a variable is called the numeric coefficient, or simply the **coefficient**.

coefficient  
↓  
**4xy**  
↑↑↑  
factors



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**Example 2** Identifying Factors and Coefficients in Expressions

Identify the factors and coefficients in each expression.

**a.**  $7vw$

**SOLUTION**

The factors are 7,  $v$ , and  $w$ . The coefficient is 7.

**b.**  $-5rst$

**SOLUTION**

The factors are  $-5$ ,  $r$ ,  $s$ , and  $t$ . The coefficient is  $-5$ .

**c.**  $\frac{y}{3}$

**SOLUTION**

The factors are  $\frac{1}{3}$  and  $y$ . The coefficient is  $\frac{1}{3}$ .

**d.**  $cd$

**SOLUTION**

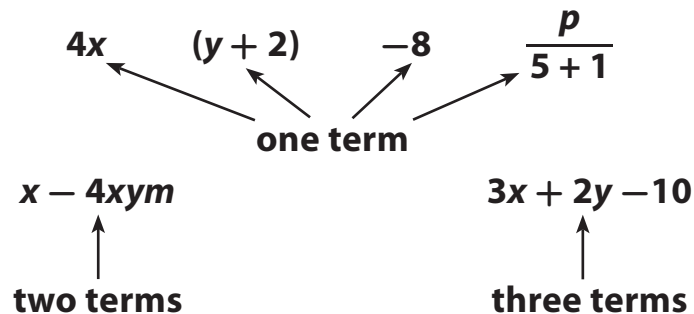
The factors are  $c$  and  $d$ . The expression  $cd$  has an implied coefficient of 1.

**Hint**

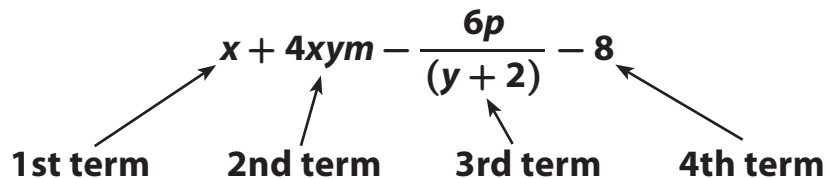
$$\frac{y}{3} = \frac{1}{3}y$$

$$cd = 1cd$$

Parts of an expression separated by  $+$  or  $-$  signs are called **terms of an expression**. A term that is in parentheses such as  $(y + 2)$  can include a plus or minus sign.



You can refer to a particular term of an expression by its placement within the expression. The terms of an expression are numbered from left to right, beginning with the first term.



**Example 3** Identifying Terms

Identify the terms in each expression.

**a.**  $6xy + 57w - \frac{24x}{5y}$

**SOLUTION**

The first term is  $6xy$ .

The second term is  $57w$ .

The third term is  $\frac{24x}{5y}$ .

**b.**  $m + 3mn - \frac{5t}{(d+8)} - 9$

**SOLUTION**

The first term is  $m$ .

The second term is  $3mn$ .

The third term is  $\frac{5t}{(d+8)}$ .

The fourth term is 9.

**Example 4** Application: Telecommunications

The local telephone company uses the expression below to determine the monthly charges for individual customers.

$$0.1m + 4.95$$

**a.** How many terms are in the expression?

**SOLUTION** There are two terms.

**b.** Identify the constant(s).

**SOLUTION** The constants are 0.1 and 4.95.

**c.** Identify the variable(s).

**SOLUTION** The variable is  $m$ .

**Lesson Practice**

**Identify the constants and variables in each expression.**

(Ex 1)

**a.**  $65qrs + 12x$

**b.**  $4gh - 71yz$

**Identify the factors and coefficients in each expression.**

(Ex 2)

**c.**  $17def$

**d.**  $\frac{uv}{4}$

**e.**  $-3st$

**f.**  $abc$

**Identify the terms in each expression.**

(Ex 3)

**g.**  $8v - 17yz + \frac{63b}{4gh}$

**h.**  $\frac{(4+2x)}{38q} + 18s - 47jkl$

**Bill's Bikes uses the expression below to calculate rental fees.**

(Ex 4)

$$6.50 + 3.25h - 0.75b$$

**i.** How many terms are in the expression?

**j.** Identify the constants.

**k.** Identify the variables.

Find the GCF of each pair of numbers.

1. 24, 32  
(SB 9)

2. 28, 42  
(SB 9)

Find the LCM of each group of numbers.

3. 9, 12  
(SB 10)

4. 3, 5, 6  
(SB 10)

Multiply or divide.

5.  $\frac{3}{4} \cdot \frac{8}{15}$   
(SB 3)

6.  $\frac{7}{15} \div \frac{21}{25}$   
(SB 3)

Identify the coefficients and variables in each expression.

\*7.  $rst - 12v$   
(2)

\*8.  $2xy + 7w - 8$   
(2)

\*9.  $47s + \frac{2}{5}t$   
(2)

Identify the following statements as true or false. Explain your choice.


\*10. **Verify** All whole numbers are natural numbers.  
(1)

11. **Verify** All integers are real numbers.  
(1)

12. **Verify** A number can be a member of the set of rational numbers and the set of irrational numbers.  
(1)


13. **Multi-Step** Use the following set of data.  
(SB 29)  
3, 6, 4, 3, 6, 5, 6, 7, 4, 3, 2, 4, 6  
a. What is the frequency of each number?  
b. Display the set of data in a line plot.


14. All natural numbers are members of which other subsets of real numbers?  
(1)

 15. **Measurement** Add  $7\frac{3}{8}$  meters +  $6\frac{1}{3}$  meters. Does the sum belong to the set of rational numbers, integers, or whole numbers?  
(1, SB 3)

16. Find the prime factorization of 153.  
(SB 12)

17. **Verify** True or False: An obtuse triangle can have more than one obtuse angle. Explain your choice.  
(SB 13)

 18. **Geometry** A line can be classified as a \_\_\_\_\_ angle.  
(SB 13)

 19. **Write** Use the divisibility test to determine if 2345 is divisible by 4. Explain your answer.  
(SB 4)

20. Write 0.003 as a percent. If necessary, round to the nearest tenth.  
(SB 5)

21. Use braces and digits to designate the set of whole numbers.  
(1)

22. The set  $\{1, 2, 3, \dots\}$  represents what set of numbers?  
(1)

\*23. **Multiple Choice** What is the second term in the expression

$$\sqrt{8} + \frac{gh}{5} + (3x + y) + 15gh$$

A  $(3x + y)$

B  $15gh$

C  $\sqrt{8}$

D  $\frac{gh}{5}$

\*24. **Astronomy** To calculate the amount of time it takes for a planet to travel around the sun, you use the following expression:  $\frac{2\pi r}{v}$ . Which values are constants, which are variables, and which are coefficients?

\*25. **Entertainment** Admission price for a matinee movie is \$5.75 for children and \$6.25 for adults. Brad uses the expression  $\$5.75c + \$6.25a$  to calculate the cost for his family. What are the variables in the expression?

\*26. **Error Analysis** The surface area of a rectangular prism is  $2lw + 2lh + 2wh$ . Two students determined the variables in the formula. Which student is correct? What was the error of the other student?

Student A	Student B
variables: $2lw$ , $2lh$	variables: $l$ , $w$ , $h$

\*27. **Cost Analysis** A large medical organization wants to put two cylindrical aquariums in the pharmacy area. It will cost the pharmacy 53 cents per cubic inch of aquarium. This is the formula for figuring out the cost:  $P = (\pi r^2 h)(\$0.53)$ .

a. Find the coefficients of the expression.

b. Find the variables of the expression.

28. **Multiple Choice** Which shape is not a parallelogram?

(SB 14) A square

B rectangle

C trapezoid

D rhombus

\*29. **Cycling** A bicycle shop uses the expression  $\$5 + \$2.25h$  to determine the charges for bike rentals. How many terms are in the expression?

30. **Attendance** The attendance clerk keeps records of students' attendance. Which subset of real numbers would include the number of students in attendance each school day?

# Simplifying Expressions Using the Product Property of Exponents

## Warm Up

1. **Vocabulary** In the term  $4x$ ,  $x$  is the \_\_\_\_\_. (*variable, coefficient*)  
(2)

Simplify.

2.  $(1.2)(0.7)$   
(SB 2)

4.  $\left(\frac{2}{3}\right)\left(\frac{6}{7}\right)$   
(SB 3)

3.  $(0.5)(11)(0.9)$   
(SB 2)

5.  $\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{15}{16}\right)$   
(SB 3)

## New Concepts

An exponent can be used to show repeated multiplication.

**base**  $\rightarrow 5^3 \leftarrow$  **exponent**

The **base of a power** is the number used as a factor. If the **exponent** is a natural number, it indicates how many times the base is used as a factor.

Words	Power	Multiplication	Value
five to the first power	$5^1$	5	5
five to the second power or five squared	$5^2$	$5 \cdot 5$	25
five to the third power or five cubed	$5^3$	$5 \cdot 5 \cdot 5$	125
five to the fourth power	$5^4$	$5 \cdot 5 \cdot 5 \cdot 5$	625

### Caution

Be careful not to multiply the base and the exponent when simplifying powers.

### Example 1 Simplifying Expressions with Exponents

Simplify each expression.

a.  $7^3$

**SOLUTION**

The exponent 3 indicates that the base is a factor three times.

$$\begin{aligned} 7^3 \\ &= 7 \cdot 7 \cdot 7 \\ &= 343 \end{aligned}$$

b.  $(0.3)^4$

**SOLUTION**

The exponent 4 indicates that the base is a factor four times.

$$\begin{aligned} (0.3)^4 \\ &= (0.3)(0.3)(0.3)(0.3) \\ &= 0.0081 \end{aligned}$$



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**Math Reasoning**

**Generalize** Examine the powers of 10. What pattern do you see?

c.  $\left(\frac{1}{2}\right)^5$

**SOLUTION**

The exponent 5 indicates that the base is a factor five times.

$$\begin{aligned}\left(\frac{1}{2}\right)^5 &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{1}{32}\end{aligned}$$

d.  $10^3$

**SOLUTION**

The exponent 3 indicates that the base is a factor three times.

$$\begin{aligned}10^3 &= 10 \cdot 10 \cdot 10 \\ &= 1000\end{aligned}$$

The product of powers whose bases are the same can be found by writing each power as repeated multiplication.

$$5^4 \cdot 5^5 = (5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) = 5^9$$

The sum of the exponents in the factors is equal to the exponent in the product.

Product Property of Exponents
<p>If <math>m</math> and <math>n</math> are real numbers and <math>x \neq 0</math>, then</p> $x^m \cdot x^n = x^{m+n}.$

**Example 2** Applying the Product Property of Exponents

Simplify each expression.

a.  $x^5 \cdot x^7 \cdot x^2$

**SOLUTION**

Since each of the factors has the same base, the exponents can be added to find the power of the product.

$$x^{5+7+2} = x^{14}$$

b.  $m^3 \cdot m^2 \cdot m^4 \cdot n^6 \cdot n^7$

**SOLUTION**

The first three factors have  $m$  as the base. The exponents can be added to find the product of those three factors. The last two factors have  $n$  as the base. The exponents can be added to find the product of the last two factors.

$$m^{3+2+4} \cdot n^{6+7} = m^9 n^{13}$$

**Math Reasoning**

**Estimate** Use the order of magnitude to estimate 1,127,000 times 108.

The **order of magnitude** is defined as the nearest power of ten to a given quantity. The order of magnitude can be used to estimate when performing calculations mentally.



**Example 3** Application: Speed of a Supercomputer

In 2006, the fastest supercomputer's performance topped out at about one PFLOPS. One PFLOPS is equal to  $10^3$  TFLOPS. Each TFLOPS is equal to  $10^{12}$  FLOPS. What was the computer's speed in FLOPS?

**SOLUTION****Understand**

$$1 \text{ PFLOPS} = 10^3 \text{ TFLOPS}$$

$$1 \text{ TFLOPS} = 10^{12} \text{ FLOPS}$$

Find the number of FLOPS in one PFLOPS.

**Plan**

Write an expression to find the number of FLOPS in one PFLOPS.

**Solve**

To find the speed in FLOPS, find the product of the number of TFLOPS,  $10^3$ , and the number of FLOPS in a TFLOPS,  $10^{12}$ .

$$10^3 \cdot 10^{12}$$

$$= 10^{3+12}$$

$$= 10^{15}$$

The computer performed at a speed of  $10^{15}$  FLOPS.

**Check**

$$10^3 \cdot 10^{12} \stackrel{?}{=} 10^{15}$$

$$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \stackrel{?}{=} 10^{15}$$

$$10^{15} = 10^{15} \quad \checkmark$$

**Lesson Practice**

Simplify each expression.

**a.**  $6^4$   
(Ex 1)

**b.**  $(1.4)^2$   
(Ex 1)

**c.**  $\left(\frac{2}{5}\right)^3$   
(Ex 1)

**d.**  $10^6$   
(Ex 1)

**e.**  $w^3 \cdot w^5 \cdot w^4$   
(Ex 2)

**f.**  $y^6 \cdot y^5 \cdot z^3 \cdot z^{11} \cdot z^2$   
(Ex 2)

**g.** If a supercomputer has a top speed of one EFLOPS which is equal to  $10^9$  GFLOPS, and if one GFLOPS is  $10^9$  FLOPS, what is the computer's speed in FLOPS?  
(Ex 3)

**Practice** Distributed and Integrated

Find the GCF for each pair of numbers.

**1.** 15, 35  
(SB 9)

**2.** 32, 48  
(SB 9)

Find the LCM for each group of numbers.

3. 8, 12  
(SB 10)

4. 2, 4, 7  
(SB 10)

Multiply or divide.

5.  $\frac{9}{16} \cdot \frac{12}{15}$   
(SB 3)

6.  $\frac{6}{15} \div \frac{24}{30}$   
(SB 3)

Identify the coefficients and variables in each expression.

7.  $6mn + 4b$   
(2)

8.  $5j - 9cd + 2$   
(2)

9.  $23t + \frac{4}{7}w$   
(2)

Identify the following statements as true or false. Explain your choice.

\*10. **Verify** All real numbers are integers.  
(1)

11. **Verify** All natural numbers are whole numbers.  
(2)

12. **Verify** All irrational numbers are real numbers.  
(2)

Complete the comparisons. Use  $<$ ,  $>$ , or  $=$ .

13.  $42.53 \bigcirc 42.35$   
(SB 1)

14.  $\frac{5}{9} \bigcirc \frac{7}{12}$   
(SB 1)

15. Add  $1\frac{1}{8} + 7\frac{2}{5}$ .  
(SB 3)



16. **Measurement** Use braces and digits to designate the set of integers. Which measurement can be described by the set of integers: temperature or volume?  
(1)

17. Find the prime factorization of 98.  
(SB 12)

\*18. **Error Analysis** Two students are trying to simplify the expression  $x^2 \cdot x^5$ . Which student is correct? Explain the error.  
(3)

	Student A	Student B
	$x^2 \cdot x^5$ $x^{2 \cdot 5} = x^{10}$	$x^2 \cdot x^5$ $x^{2+5} = x^7$

19. **Verify** True or False: A rhombus is always a square. Explain your choice.  
(SB 14)



20. **Write** Use the divisibility test to determine if 306 is divisible by 6. Explain your answer.  
(SB 4)

\*21. The expression  $3^6$  indicates the number of times 3 is used as a factor.  
(3)

a. Which number in the expression is the base?

b. Which number is the exponent?

c. What is the simplified value of this expression?

- \*22. **Multiple Choice** MFLOPS, TFLOPS, and PFLOPS are used to measure the speed of a computer. One PFLOP is equal to  $10^3$  TFLOPS. Each TFLOP is equal to  $10^6$  MFLOPS. How many MFLOPS are in a PFLOP?  
A  $10^{18}$                       B  $10^9$                       C  $10^6$                       D  $10^3$

- \*23. **Cooking** A cooking magazine advertises  $4^4$  recipes in every issue. How many recipes are in  $4^2$  issues?

- \*24. **Multi-Step** A business is worth  $10^6$  dollars this year. The business expects to be  $10^3$  more valuable in five years.

a. Simplify  $10^3$  to determine how many times more valuable the business will be.

b. What will the business be worth in five years? Express your answer in exponential form, then simplify your answer.

- \*25. **Population** The population of Bridgetown triples every decade. If the population in the year 2000 was 25,000, how many people will be living in Bridgetown in 2030?

26. **Multiple Choice** Which triangle is a right triangle?


(SB 13) A a triangle with angle measures of  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$

B a triangle with angle measures of  $40^\circ$ ,  $110^\circ$ , and  $30^\circ$

C a triangle with angle measures of  $55^\circ$ ,  $45^\circ$ , and  $80^\circ$

D a triangle with angle measures of  $60^\circ$ ,  $60^\circ$ , and  $60^\circ$

- \*27. **Bacteria** The population of a certain bacteria doubles in size every 3 hours. If a population begins with one bacterium, how many will there be after one day? Simplify the expression  $1 \cdot (2)^8$  to determine the population after one day.

-  28. **Geometry** You can calculate the area of a trapezoid using the following equation:  
(2)  $A = h \times \frac{b_1 + b_2}{2}$ . Identify the constant(s) in the equation.

- \*29. **Aquarium** A fish tank is in the shape of a cube. Each side measures 3 feet. What is the volume of the fish tank?  
(SB 26)

- \*30. **Remodeling** Vanessa is remodeling her bathroom. She uses the expression  $2l + 2w$  to determine the amount of wallpaper border she needs.

a. How many terms are in the expression?

b. What are the variables?

## Using Order of Operations

## Warm Up

1. **Vocabulary** A(n) \_\_\_\_\_ can be used to show repeated multiplication.

(3)  
Simplify.

2.  $28.75 + 13.5$   
(SB 2)

3.  $89.6 - 7.4$   
(SB 2)

4.  $\frac{2}{3} \cdot \frac{9}{16}$   
(SB 3)

5.  $4\frac{1}{5} \div 3\frac{1}{2}$   
(SB 3)

## New Concepts

To **simplify** an expression means to perform all indicated operations. Simplifying an expression could produce multiple answers without rules concerning the order in which operations are performed. Consider the example below.

**Method 1:**  $\frac{2 \cdot (3)^2}{6} = \frac{2 \cdot 9}{6} = \frac{18}{6} = 3$

**Method 2:**  $\frac{2 \cdot (3)^2}{6} = \frac{(2 \cdot 3)^2}{6} = \frac{6^2}{6} = \frac{36}{6} = 6$

To avoid confusion, mathematicians have agreed to use the order of operations. The **order of operations** is a set of rules for simplifying expressions. Method 1 followed the order of operations.

## Order of Operations

1. Work inside grouping symbols.
2. Simplify powers and roots.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

**Example 1** Simplifying Expressions with Parentheses

Simplify. Justify each step.

$$(10 \cdot 3) + 7 \cdot (5 + 4)$$

**SOLUTION**

Write the expression. Then use the order of operations to simplify.

$$(10 \cdot 3) + 7 \cdot (5 + 4)$$

$$= 30 + 7 \cdot 9$$

Simplify inside the parentheses.

$$= 30 + 63$$

Multiply.

$$= 93$$

Add.



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**Example 2** Simplifying Expressions with Exponents

Simplify each expression. Justify each step.

**a.**  $4^3 + 9 \div 3 - 2 \cdot (3)^2$

**SOLUTION** Write the expression. Then use the order of operations to simplify.

$$4^3 + 9 \div 3 - 2 \cdot (3)^2$$

$$= 64 + 9 \div 3 - 2 \cdot 9 \quad \text{Simplify exponents.}$$

$$= 64 + 3 - 18 \quad \text{Multiply and divide from left to right.}$$

$$= 49 \quad \text{Add and subtract from left to right.}$$

**b.**  $\frac{(2 \cdot 3 - 2)^2}{2}$

**SOLUTION** Write the expression. Then use the order of operations to simplify.

$$\frac{(2 \cdot 3 - 2)^2}{2}$$

$$= \frac{(6 - 2)^2}{2} \quad \text{Multiply inside the parentheses.}$$

$$= \frac{(4)^2}{2} \quad \text{Subtract inside the parentheses.}$$

$$= \frac{16}{2} \quad \text{Simplify the exponent.}$$

$$= 8 \quad \text{Divide.}$$

**Hint**

Remember to use the order of operations inside parentheses as well.

**Example 3** Comparing ExpressionsCompare the expressions. Use  $<$ ,  $>$ , or  $=$ .

$$(1.5 + 3) \div 9 + 3^3 \quad \bigcirc \quad \frac{(18 + 8)}{2} - 8 \div 4$$

**SOLUTION**

Use the order of operations to simplify the two expressions.

$$(1.5 + 3) \div 9 + 3^3 \qquad \frac{(18 + 8)}{2} - 8 \div 4$$

$$= (4.5) \div 9 + 3^3 \qquad = \frac{26}{2} - 8 \div 4$$

$$= 4.5 \div 9 + 27 \qquad = 13 - 8 \div 4$$

$$= 0.5 + 27 \qquad = 13 - 2$$

$$= 27.5 \qquad = 11$$

$$\text{Since } 27.5 > 11, (1.5 + 3) \div 9 + 3^3 \bigcirc \frac{(18 + 8)}{2} - 8 \div 4.$$

**Hint**

Remember to compare the original expressions in the inequality.

**Hint**

Remember that the formula for the area of a circle is  $\pi r^2$ .

**Example 4** Application: Comparing a Crop Circle to a Soccer Field

A crop circle in a wheat field has a diameter of 100 yards. Its area is  $3.14 \cdot \left(\frac{100}{2}\right)^2$  square yards. A World Cup soccer field is 70 yards by 110 yards. Its area is  $(70 \cdot 110)$  square yards. How much larger is the crop circle than the soccer field?

**SOLUTION**

Find each area and subtract to find the difference.

$$\text{Area of crop circle: } 3.14 \cdot \left(\frac{100}{2}\right)^2$$

$$\text{Area of soccer field: } (70 \cdot 110)$$

$$\text{Difference in area: } 3.14 \cdot \left(\frac{100}{2}\right)^2 - (70 \cdot 110)$$

Simplify the expression.

$$3.14 \cdot \left(\frac{100}{2}\right)^2 - (70 \cdot 110)$$

$$= 3.14 \cdot (50)^2 - (7700) \quad \text{Evaluate inside the parentheses.}$$

$$= 3.14 \cdot 2500 - 7700 \quad \text{Simplify the exponent.}$$

$$= 7850 - 7700 \quad \text{Multiply.}$$

$$= 150 \quad \text{Subtract.}$$

The crop circle is 150 yd<sup>2</sup> larger than the soccer field.

**Lesson Practice**

- a. Simplify  $45 - (2 + 4) \cdot 5 - 3$ . Justify each step.

(Ex 1)

- Simplify each expression. Justify each step.

(Ex 2)

b.  $9 \cdot 2^3 - 9 \div 3$

c.  $\frac{15 - 3^2 + 4 \cdot 2}{7}$

- d. Compare the expressions. Use  $<$ ,  $>$ , or  $=$ .

(Ex 3)

$$\frac{1}{4} + 3^2 + 6 \quad \bigcirc \quad 5 - 2 + 2 \cdot 4 + 3 \div 9$$

- e. Jonah is making a model of the moon using plastic foam. He uses the formula  $\frac{4}{3}\pi r^3$  to find the volume. The model moon's radius is  $\frac{3}{2}$  inches. What is the volume of the model moon? Give the answer in terms of  $\pi$ .

(Ex 4)

**Caution**

Do not forget to cube  $\frac{3}{2}$  in the expression for the model moon's volume.

Add, subtract, multiply, or divide.

1.  $2\frac{1}{4} + 4\frac{1}{2}$   
(SB 3)

2.  $5\frac{2}{5} - 3\frac{1}{4}$   
(SB 3)

3.  $1\frac{3}{4} + 4\frac{1}{8} - 2\frac{1}{2}$   
(SB 3)


4.  $4\frac{1}{3} \div 2\frac{1}{6}$   
(SB 3)

5.  $3.519 \div 0.3$   
(SB 2)

6.  $4.16 \cdot 2.3$   
(SB 2)

7. How many terms are in the algebraic expression  $14x^2 + 7x + \frac{x}{4}$ ?  
(2)

8. Find the prime factorization of 225.  
(SB 12)

 9. **Write** Use the divisibility test to determine if 124,302 is divisible by 3. Explain your answer.  
(SB 4)

10. Represent the following numbers as being members of set  $L$ :  $-15, 1, 7, 3, -8, 7, 0, 12, 6, 12$   
(1)

\*11. **Verify** True or False: All whole numbers are integers. Explain your answer.  
(1)

\*12. To which set(s) of numbers does  $\sqrt{5}$  belong?  
(1)

13. Write  $\frac{1}{6}$  as a percent. If necessary, round to the nearest tenth.  
(SB 5)

14. Write  $\frac{5}{9}$  as a percent. If necessary, round to the nearest tenth.  
(SB 5)

\*15. Compare  $3 \cdot 4^2 + 4^2$   $\bigcirc$   $3 \cdot (16 + 16)$  using  $<$ ,  $>$ , or  $=$ . Explain.  
(4)

16. **Multiple Choice** Which triangle is an obtuse triangle?  
(SB 13)

A a triangle with angle measures of  $45^\circ, 45^\circ$ , and  $90^\circ$

B a triangle with angle measures of  $40^\circ, 120^\circ$ , and  $20^\circ$


C a triangle with angle measures of  $55^\circ, 45^\circ$ , and  $80^\circ$

D a triangle with angle measures of  $60^\circ, 60^\circ$ , and  $60^\circ$

17. Display the following set of data in a line plot:  
(SB 29)

6, 7, 8, 4, 5, 4, 3, 4, 5, 3, 2, 6, 2, 7


18. **Verify** True or False: A square is a rectangle. Explain your choice.  
(SB 14)

 19. **Measurement** Subtract  $15\frac{1}{3}$  yards  $- 7\frac{4}{5}$  yards.  
(SB 3)

20. **Error Analysis** Two students determine the prime factorization of 108. Which student is correct? Explain the error.  
(SB 12)

Student A
$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Student B
$108 = 2^2 \cdot 3^3$

 21. **Write** Use the divisibility test to determine if 1116 is divisible by 9. Explain your answer.  
(SB 4)

22.  $\frac{n}{6} + 3xy - 19$

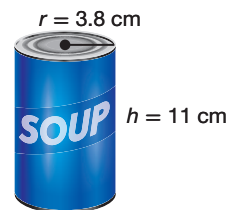
- Find the variables of the expression.
- Find the terms of the expression.

- \*23. **Biology** A survey found that there were 1100 gray wolves in Minnesota in 1976. By 2003, the number of gray wolves had increased to 2300. What was the average growth of the wolf population in one year? (Round to the nearest whole number.)

- \*24. **Multiple Choice** A bouquet is made from nine red roses that cost \$1.75 each and five white roses that cost \$1.50 each. Use the expression  $9 \cdot (\$1.75) + 5 \cdot (\$1.50)$  to find the cost of the bouquet.

- A \$31.00                      B \$23.25  
C \$25.25                      D \$21.75

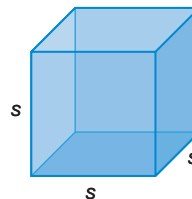
- \*25. A can of soup in the shape of a cylinder has a radius of 3.8 cm and a height of 11 cm. What is the surface area of the can to the nearest tenth? Use 3.14 for  $\pi$ .



- \*26. **Multi-Step** Two friends compare the amount of change they have in their pockets. Ashley has 12 nickels, 2 dimes, and 4 quarters. Beto has 10 nickels, 4 dimes, and 3 quarters. Who has more money?
- Write an expression to represent the value of Ashley's money. (Hint: Use 10¢ to represent the value of each dime, 5¢ for each nickel, and so on). Simplify the expression.
  - Write an expression to represent the value of Beto's money. Simplify the expression.
  - Compare the value of money that each friend has. Who has more?

- \*27. **School Supplies** Anthony had 10 packages of markers. Each package contained 8 markers. He gave 2 packages to each of the other 3 people in his group. Use the expression  $8(10 - 3 \cdot 2)$  to determine how many markers Anthony kept for himself.

- \*28. **Geometry** Use the cube shown to write a formula for the volume of any cube.



- \*29. **Temperature** The hottest day in Florida's history was 109°F, which occurred on June 29, 1931 in Monticello. Use the expression  $\frac{5}{9}(F - 32)$  to convert this temperature to degrees Celsius. Round your answer to the nearest tenth of a degree.

- \*30. **Billing** Each month Mrs. Li pays her phone company \$28 for phone service and \$0.07 per minute for long-distance calls. Use the expression  $28 + 0.07m$  to find the amount she was billed if her long-distance calls totaled 223 minutes.



# Finding Absolute Value and Adding Real Numbers

## Warm Up

1. **Vocabulary** The set of \_\_\_\_\_ (*integers, real numbers*) includes all <sup>(1)</sup> rational or irrational numbers.

Simplify.

2. <sub>(SB 2)</sub>  $54.2 - 27.38$

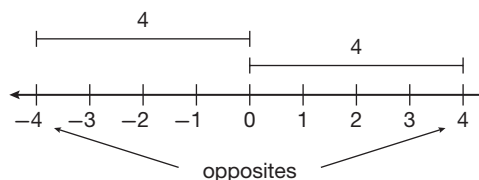
3. <sub>(SB 3)</sub>  $\frac{1}{2} + \frac{3}{8}$

4. <sub>(SB 2)</sub>  $1.09 + 76.9$

5. <sub>(SB 3)</sub>  $\frac{3}{4} - \frac{3}{8}$

## New Concepts

The **absolute value** of a number is the distance from the number to zero on a number line. The absolute value of 4 is written  $|4|$ .



$$|-4| = 4$$

$$|4| = 4$$

### Absolute Value

The absolute value of a number  $n$  is the distance from  $n$  to 0 on a number line.

#### Example 1 Finding the Absolute Value

Simplify.

a.  $|0|$

**SOLUTION**

The absolute value of 0 is 0.

c.  $\left|1 - \frac{3}{4}\right|$

**SOLUTION**

First simplify within the absolute-value bars. Then find the absolute value.

$$\left|1 - \frac{3}{4}\right| = \left|\frac{1}{4}\right| = \frac{1}{4}$$

b.  $|7.12|$

**SOLUTION**

The distance from 7.12 to 0 is 7.12. So the absolute value is 7.12.

d.  $-|11 - 2|$

**SOLUTION**

First simplify within the absolute-value bars. Then find the absolute value.

$$-|11 - 2| = -|9| = -9$$

### Reading Math

Read  $-|9|$  as the opposite of the absolute value of 9.

**Hint**

$$+ = 1$$

$$- = -1$$

$$+ + - = 0$$

**Exploration Modeling Real Number Addition**

Find the sum  $-5 + 3$ .

Model  $-5$  and  $3$  using algebra tiles.



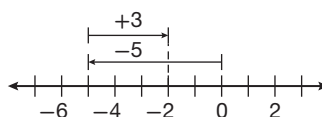
Group positive and negative tiles to make zero pairs.



Count the remaining tiles.

$$-5 + 3 = -2$$

Model  $-5 + 3$  on a number line.



Use algebra tiles to find the sum. Then model each problem on a number line.

**a.**  $4 + (-3)$

**b.**  $-2 + -6$

**c.**  $(-7) + 7$

**Generalize** Determine whether each statement is true or false. Provide a counterexample for false statements.

**d.** The sum of two positive numbers is always positive.

**e.** The sum of two negative numbers is always negative.

**f.** The sum of a positive and a negative number is always negative.

The sum of two numbers can also be found using the rules for adding real numbers. These rules apply to all real-number addends.

**Rules for Adding Real Numbers****Adding Numbers With the Same Sign**

To add numbers with the same sign, add their absolute values. The sum will have the same sign as the addends.

**Examples**  $3 + 2 = 5$   $-3 + (-2) = -5$

**Adding Numbers With Different Signs**

To add numbers with different signs, find the difference of their absolute values. The sum will have the sign of the addend with the greater absolute value.

**Examples**  $3 + (-2) = 1$   $(-3) + 2 = -1$



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**Example 2** Adding Real Numbers

Find the sum.

**a.**  $(-12) + 21$

**SOLUTION** Since the numbers have different signs, find the difference of their absolute values. The sum is positive because  $|-12| < |21|$ .

$$(-12) + 21 = 9$$

**c.**  $(3.2) + (-5.1)$

**SOLUTION** Since the numbers have different signs, find the difference of their absolute values. The sum is negative because  $|3.2| < |-5.1|$ .

$$(3.2) + (-5.1) = -1.9$$

**b.**  $(-19) + (-8)$

**SOLUTION** Since the numbers have the same sign, find the sum of their absolute values. The sum is negative because both addends are negative.

$$(-19) + (-8) = -27$$

**d.**  $\left(-\frac{3}{5}\right) + \left(-\frac{1}{5}\right)$

**SOLUTION** Since the numbers have the same sign, find the sum of their absolute values. The sum is negative because both addends are negative.

$$\left(-\frac{3}{5}\right) + \left(-\frac{1}{5}\right) = \left(-\frac{4}{5}\right)$$

**Example 3** Identifying Sets of Real Numbers Closed Under Addition

Determine whether each statement is true or false. Give a counterexample for false statements.

**a.** The set of integers is closed under addition.

**SOLUTION** The statement is true because the sum of any two integers will be an integer.

**b.** The set of real numbers is closed under addition.

**SOLUTION** The statement is true because the sum of any two real numbers will be a real number.

**Example 4** Application: Football

On the first down, the Cougars lost 4 yards. They gained 7 yards on the second down. Use addition to find the total number of yards lost or gained on the first two downs.

**SOLUTION** A loss of 4 yards can be expressed as  $-4$ .

$$(-4) + 7 = 3$$

The Cougars gained a total of 3 yards on the first two downs.

**Math Language**

A set of numbers is **closed** under a given operation if the outcome of the operation on any two members of the set is also a member of the set.

## Lesson Practice

**Simplify.**

(Ex 1)

a.  $|-3.4|$

b.  $\left|\frac{6}{7}\right|$

c.  $|14 + (-22)|$

d.  $-|7 + 16|$

**Find the sum.**

(Ex 2)

e.  $(-23.4) + 18.72$

f.  $\left(-\frac{2}{3}\right) + \left(-\frac{1}{6}\right)$

**Determine whether each statement is true or false. Give a counterexample for false statements.**

(Ex 3)

g. The set of rational numbers is closed under addition.

h. The set of positive integers is closed under addition.

(Ex 4) i. The temperature at 7:00 p.m. was  $34^{\circ}\text{F}$ . The temperature fell  $12^{\circ}\text{F}$  by midnight. Use addition to find the temperature at midnight.

## Practice Distributed and Integrated

**Add, subtract, multiply, or divide.**

1.  $1\frac{1}{6} + 3\frac{1}{3}$   
(SB 3)

2.  $2\frac{3}{8} - 1\frac{1}{4}$   
(SB 3)

3.  $3\frac{2}{3} + 1\frac{5}{8} - 1\frac{3}{4}$   
(SB 3)


4.  $3\frac{1}{3} \div 1\frac{3}{5}$   
(SB 3)

5.  $1.506 \div 0.2$   
(SB 2)

6.  $2.89 \cdot 1.2$   
(SB 2)

7. How many terms are in the algebraic expression  $2x^2 + 3x + 7$ ?  
(2)

8. Find the prime factorization of 150.  
(SB 12)

 9. **Write** Use the divisibility test to determine if 125,000 is divisible by 10. Explain your answer.  
(SB 4)

10. **Model** Represent the following numbers as being members of set  $L$ :  $-12, 0, -8, 4, -4, 4, 0, 8, 8, 12$ .  
(1)

11. **Verify** True or False: All integers are rational numbers. Explain your answer.  
(1)

12. **Error Analysis** Student A said that  $\frac{\sqrt{2}}{1}$  is a rational number. Student B said that it is an irrational number. Which student is correct? Explain your answer.  
(1)

13. Write  $\frac{5}{8}$  as a decimal and a percent.  
(SB 5)



14. **Measurement** Order the lengths 1.25 yards, 3 feet,  $1\frac{1}{3}$  yards from least to greatest.  
(SB 1)

15. Write 7% as a fraction in simplest form and as a decimal.  
(SB 5)

16. **Formulate** Write an equation using absolute values to represent the sentence.  
(5) "The distance from  $-11$  to  $0$  is  $11$ ."

17. **Multiple Choice** Which angle measures form an acute triangle?

(SB 13) A  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$

B  $40^\circ$ ,  $110^\circ$ , and  $20^\circ$

C  $55^\circ$ ,  $45^\circ$ , and  $80^\circ$

D  $30^\circ$ ,  $30^\circ$ , and  $120^\circ$

18. Estimate:  $\$1.48 + \$0.12 - \$0.27$ .

(SB 8)



19. **Write** Use the definition of absolute value to write  $|-5| = 5$  in words.

(5)

20. **Verify** True or False: A rectangle is a parallelogram. Explain your choice

(SB 14)



\*21. **Geometry** The hypotenuse squared ( $c^2$ ) can be determined by solving for  $a^2 + b^2$  in the Pythagorean Theorem. Using the order of operations, decide if the expression  $(a + b)$  should be determined before  $a^2$ ?

(4)

\*22. **Weather** One winter day the temperature rose  $29^\circ\text{F}$  from a low of  $-3^\circ\text{F}$  in the morning. What was the day's high temperature?

(5)

\*23. **Football** On the first down, the Tigers gained 8 yards. Then they were pushed back for a loss of  $13\frac{1}{2}$  yards on the second down. Write and solve an addition problem to find the total number of yards lost or gained on the first two downs.

(5)

\*24. **Multiple Choice** Which of these sets of numbers is closed under addition?

(5)

A integers

B rational numbers

C real numbers

D all of these

\*25. **Multi-Step** Airplane A took off from an airport that is 43 feet below sea level, and then climbed 20,512 feet to its cruising altitude. Airplane B took off at the same time from an airport that was 1924 feet above sea level, and then climbed 18,527 feet to its cruising altitude. Which airplane is currently cruising at a higher altitude?

(5)

\*26. **Banking** Martha had \$500 in her checking account. She made a withdrawal of \$34.65. Write and solve an addition problem to find Martha's balance after the withdrawal.

(5)

\*27. **Multi-Step** A china cup-and-saucer set sells for \$15.25 and a plate sells for \$25.

(4)

A woman buys 3 cup-and-saucer sets and 4 plates. If she pays a 5% sales tax, how much does she pay for her purchase?

a. Determine how much the woman spends before sales tax. Use the expression  $3 \cdot (\$15.25) + 4 \cdot (\$25)$  to solve.

b. How much does she pay with sales tax included? Round your answer to the nearest hundredth. Use the expression  $\$145.75 + (0.05) \cdot (\$145.75)$  to solve.

\*28. **Stocks** Stock in the ABC Company fell 12.67 points on Monday and 31.51 points on Tuesday. Determine the total change in the stock for the two days.

(5)

\*29. **Multiple Choice** Which expression correctly represents  $1.6^5$ ?

(3)

A  $1.6 \times 1.6 \times 1.6 \times 1.6 \times 1.6$

B  $1.6 + 1.6 + 1.6 + 1.6 + 1.6$

C  $0.6 \times 0.6 \times 0.6 \times 0.6 \times 0.6 + 1$

D  $1 \times 1 \times 1 \times 1 \times 1 + 0.6$

\*30. **Temperature** At midnight the temperature was  $-7^\circ\text{F}$ . By noon the temperature had risen  $23^\circ\text{F}$ . What was the temperature at noon?

(5)

## Subtracting Real Numbers

## Warm Up

1. **Vocabulary** The \_\_\_\_\_ of a number is the distance from the number to 0 on a number line.

Simplify.

2.  $86.9 - 18.94$   
(SB 2)

3.  $\frac{1}{3} + \frac{4}{9}$   
(SB 3)

4.  $41.06 + 83.7$   
(SB 2)

5.  $\frac{5}{6} - \frac{5}{12}$   
(SB 3)

## New Concepts

Two numbers with the same absolute value but different signs are called **opposites**. Another name for the opposite of a number is **additive inverse**. The sum of a number and its opposite is 0.

## Inverse Property of Addition

For every real number  $a$ ,  $a + (-a) = (-a) + a = 0$ .

**Example**  $5 + (-5) = 0$

Addition and subtraction are inverse operations. Subtracting a number is the same as adding the inverse of the number.

## Rules for Subtracting Real Numbers

To subtract a number, add its inverse. Then follow the rules for adding real numbers.

**Example**  $3 - 5 = 3 + (-5) = -2$

**Example 1** Subtracting Real Numbers

Find each difference.

a.  $(-12) - 21$

**SOLUTION**

$$(-12) - 21$$

$$(-12) + (-21) = -33$$

c.  $3.2 - (-5.1)$

**SOLUTION**

$$3.2 - (-5.1)$$

$$3.2 + (+5.1) = 8.3$$

b.  $(-19) - (-8)$

**SOLUTION**

$$(-19) - (-8)$$

$$(-19) + (+8) = -11$$

d.  $\left(-\frac{3}{5}\right) - \left(-\frac{1}{5}\right)$

**SOLUTION**

$$\left(-\frac{3}{5}\right) - \left(-\frac{1}{5}\right)$$

$$\left(-\frac{3}{5}\right) + \left(+\frac{1}{5}\right) = -\frac{2}{5}$$

## Math Reasoning

**Analyze** What is the meaning of  $-(-8)$ ?



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### Math Language

A set of numbers is **closed** under a given operation if the outcome of the operation on any two members of the set is also a member of the set.

### Example 2 Determining Closure Over Subtraction

Determine whether each statement is true or false. Give a counterexample for false statements.

- a. The set of integers is closed under subtraction.

#### SOLUTION

The statement is true because the difference of any two integers will be an integer.

- b. The set of real numbers is closed under subtraction.

#### SOLUTION

The statement is true because the difference of any two real numbers will be a real number.

### Example 3 Application: Dive Depth

Nayip collected a water sample at a depth of 23 meters from the surface. He descended another 12 meters to collect a plant sample. Where was Nayip in relation to the surface when he retrieved the plant sample?

#### SOLUTION

A depth of 23 meters can be written as  $(-23)$ .

$$\begin{aligned} &(-23) - 12 \\ &= (-23) + (-12) \\ &= -35 \end{aligned}$$

Nayip was at 35 meters below the surface when he collected the plant sample.

### Lesson Practice

Find each difference.

(Ex 1)

a.  $14 - (-22)$

b.  $(-7) - 16$

c.  $(-23.4) - 18.72$

d.  $\left(-\frac{2}{3}\right) - \left(-\frac{1}{6}\right)$

Determine whether each statement is true or false. Give a counterexample for false statements.

(Ex 2)

- e. The set of whole numbers is closed under subtraction.

- f. The set of rational numbers is closed under subtraction.

- g. On January 23, 1960, the Trieste dove to a record depth of 37,800 feet below sea level. The record set previously, on January 7th of the same year, was 13,800 feet less than the dive on January 23rd. What was the record set on January 7th in relation to sea level?

(Ex 3)

Add, subtract, multiply, or divide.

1.  $5\frac{1}{3} \div 2\frac{1}{3}$   
(SB 3)

2.  $40\frac{1}{8} - 21\frac{1}{4}$   
(SB 3)

3.  $5\frac{2}{3} + 2\frac{5}{6} + (-2\frac{1}{6})$   
(5)

4.  $1\frac{2}{3} \div 1\frac{1}{4} \cdot 1\frac{1}{2}$   
(SB 3)

5.  $0.74 \div 0.2 \cdot 0.3$   
(SB 2)

6.  $5.4 \cdot 0.3 \div 0.4$   
(SB 2)

7.  $1.24 \cdot 0.2 \div 0.1$   
(SB 2)

8.  $112.4 \div 3.2$   
(SB 2)

9. Find the prime factorization of 592.  
(SB 12)

10. Find the prime factorization of 168.  
(SB 12)

11. **Model** Display the following set of data in a line plot.  
(SB 29)

8, 6, 9, 7, 5, 4, 6, 7, 9, 8, 5, 6, 6, 8

 12. **Write** Use the divisibility test to determine if 2326 is divisible by 3. Explain your answer.  
(SB 4)

13. Write 6% as a fraction in simplest form and as a decimal.  
(SB 5)

 14. **Measurement** Write 1.25 feet as a fraction in simplest form and compare it to  $\frac{5}{3}$  feet. Which is greater?  
(SB 5)

15. Write  $\frac{3}{5}$  as a decimal and as a percent.  
(SB 5)

16. **Multiple Choice** What is the value of the expression below?  
(4)

$$\frac{(3 \cdot 20 + 2 \cdot 20) \cdot 6 - 20}{10^2}$$

A 2.8

B 58

C -14

D 5.8

17. Simplify  $\frac{(45 + 39 + 47 + 40 + 33 + 39 + 41)}{(2 \cdot 2)^2 - 12}$ .  
(4)

\*18. **Multiple Choice** Which of these differences will be negative?  
(6)


A  $-4.8 - (-5.2)$

B  $4.8 - 5.2$

C  $4.8 - 3.2$

D  $6.7 - (-7.8)$

\*19. **Football** Ryan's varsity football team is on its own 25-yard line. The quarterback stumbles for a loss of 15 yards. What line is Ryan's varsity football team on now?  
(6)

 \*20. **Geometry** If one angle in a triangle measures  $105.5^\circ$  and another measures  $38.2^\circ$ , what is the measurement of the third angle? Use the expression  $180 - 105.5 - 38.2$  to solve.  
(6)

\*21. **Temperature** On a winter day, a wind gust makes the temperature in Antarctica feel sixteen degrees colder than the actual temperature. If the temperature is  $-5^\circ\text{C}$ , how cold did it feel?  
(6)



- \*22. **Consumer Math** Leila issued a check for \$149.99 and deposited \$84.50 in her account. What is the net change in her account?

23. **Multiple Choice** Which triangle is an equiangular triangle?  
 (SB 13) A a triangle with angle measures of  $45^\circ$ ,  $45^\circ$ , and  $90^\circ$   
 B a triangle with angle measures of  $60^\circ$ ,  $60^\circ$ , and  $60^\circ$   
 C a triangle with angle measures of  $55^\circ$ ,  $35^\circ$ , and  $90^\circ$   
 D a triangle with angle measures of  $30^\circ$ ,  $30^\circ$ , and  $120^\circ$

24. **Boating** The tour boat can leave the dock only if the level of the lake is no more than 2 feet below normal. Before the recent rainfall, the level of the lake was  $5\frac{1}{3}$  feet below normal. After the recent rainfall, the level of the lake rose  $3\frac{1}{4}$  feet. Can the tour boat leave the dock? Explain.

25. **Geometry** The triangle inequality is a theorem from geometry stating that for any two real numbers  $a$  and  $b$ ,  $|a + b| \leq |a| + |b|$ . Verify the triangle inequality by simplifying  $|-18.5 + 4.75| \leq |-18.5| + |4.75|$ .

- \*26. **Error Analysis** Two students solved this problem. Which student is correct? Explain the error.

The elevator started on the second floor and went up 8 floors, then down 11 floors to the garage level, and then up 6 floors. Which floor is the elevator on now?

Student A	Student B
$2 + 8 + (-11) + 6 = 5$ The 5 <sup>th</sup> floor	$2 + 8 - (-11) + 6 = 27$ The 27 <sup>th</sup> floor

27. **Multi-Step** A bit is a binary digit and can have a value of either 0 or 1. A byte is a string of 8 bits.  
 (3) a. Write the number of bits in one byte as a power of 2.  
 b. Write 32 as a power of 2.  
 c. Write the number of bits in 32 bytes as a power of 2.

\*28.  $16c + (-4d) + \frac{8\pi}{15} + 21efg$

- (2) a. Find the coefficients of the expression.  
 b. Find the number of terms in the expression.  
 c. **Justify** Rewrite the expression so that there are no parentheses. Justify your change.

- \*29. **Multiple Choice** What subset of numbers does the number  $-9.0909090\overline{09}$  belong to?  
 (1) A integers B irrational numbers  
 C natural numbers D rational numbers

- \*30. **Oceanography** The Pacific Ocean has an average depth of 12,925 feet, while the Atlantic Ocean has an average depth of 11,730 feet. Find the difference in average depths.

# Simplifying and Comparing Expressions with Symbols of Inclusion

## Warm Up

1. **Vocabulary** A \_\_\_\_\_ is used to represent an unknown number.

Simplify.

2.  $-1.5 + 3^2 - (3 - 5)$

3.  $12 - 4 \cdot 0.5 + (3.4 - 1.7)$

4.  $\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{5}{6}$

## New Concepts

A mathematical expression can include numbers, variables, operations, and symbols of inclusion. Symbols of inclusion, such as fraction bars, absolute-value symbols, parentheses, braces, and brackets indicate which numbers, variables, and operations are parts of the same term. An example is shown below.

$$\left(\frac{2x}{3} + 3\frac{1}{5}\right) - 2y$$

The expression inside the parentheses is considered a single term. To simplify an expression with multiple symbols of inclusion, begin inside the innermost symbol of inclusion and work outward.

### Example 1 Expressions with Absolute-Value Symbols and Parentheses

Simplify each expression.

a.  $9 - |4 - 6|$

**SOLUTION**

$$9 - |4 - 6|$$

$$= 9 - |-2|$$

$$= 9 - 2$$

$$= 7$$

Subtract inside absolute-value symbols.

Simplify the absolute value.

Subtract.

b.  $5 \cdot 2 + [3 + (6 - 8)]$

**SOLUTION** Begin simplifying inside the innermost symbol of inclusion.

$$5 \cdot 2 + [3 + (6 - 8)]$$

$$= 5 \cdot 2 + [3 + (-2)]$$

$$= 5 \cdot 2 + 1$$

$$= 10 + 1$$

$$= 11$$

Subtract inside parentheses.

Add inside brackets.

Multiply.

Add.

### Math Language

( )	parentheses
[ ]	brackets
{ }	braces
$\frac{a}{b}$	fraction bar
x	absolute-value symbols



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It is important to follow the order of operations at all times, even when working inside symbols of inclusion.

#### Hint

Use the order of operations:

1. symbols of inclusion;
2. powers and roots;
3. multiply or divide;
4. add or subtract.

### Example 2 Simplifying Expressions with Brackets

Simplify.

$$3 + 5 \cdot [(9 - 3)^2 - 6]$$

#### SOLUTION

Begin inside the innermost symbol of inclusion and work outward.

$$3 + 5 \cdot [(9 - 3)^2 - 6]$$

$$= 3 + 5 \cdot [6^2 - 6] \quad \text{Simplify inside the parentheses.}$$

$$= 3 + 5 \cdot [36 - 6] \quad \text{Evaluate the exponent.}$$

$$= 3 + 5 \cdot 30 \quad \text{Subtract inside the brackets.}$$

$$= 3 + 150 \quad \text{Multiply.}$$

$$= 153 \quad \text{Add.}$$

To simplify a rational expression such as  $\frac{6 \cdot 3}{4 - 2}$ , the numerator and denominator must be simplified first.

### Example 3 Simplifying Expressions with Rational Numbers

Simplify. Justify each step.

$$[5 \cdot (4 + 2)^2] + \frac{4 \cdot 5}{2}$$

#### SOLUTION

Justify each step using the order of operations or mathematical properties.

$$[5 \cdot (4 + 2)^2] + \frac{4 \cdot 5}{2}$$

$$= [5 \cdot (6)^2] + \frac{4 \cdot 5}{2} \quad \text{Add inside the parentheses.}$$

$$= [5 \cdot 36] + \frac{4 \cdot 5}{2} \quad \text{Simplify the exponent.}$$

$$= 180 + \frac{4 \cdot 5}{2} \quad \text{Multiply inside the brackets.}$$

$$= 180 + \frac{20}{2} \quad \text{Simplify the numerator.}$$

$$= 180 + 10 \quad \text{Simplify the fraction.}$$

$$= 190 \quad \text{Add.}$$

### Example 4 Compare Expressions with Symbols of Inclusion

Compare the expressions. Use  $<$ ,  $>$ , or  $=$ .

$$12 + [5(7 - 5)^3 - 14] \quad \bigcirc \quad [(9 - 5)^2 + 7] - 3^3$$

**SOLUTION** Simplify each expression. Then compare.

$$\begin{array}{ll}
 12 + [5(7 - 5)^3 - 14] & [(9 - 5)^2 + 7] - 3^3 \\
 = 12 + [5(2)^3 - 14] & = [(4)^2 + 7] - 3^3 \\
 = 12 + [5 \cdot 8 - 14] & = [16 + 7] - 3^3 \\
 = 12 + [40 - 14] & = 23 - 3^3 \\
 = 12 + 26 & = 23 - 27 \\
 = 38 & = -4
 \end{array}$$

Since  $38 > -4$ ,  $12 + [5(7 - 5)^3 - 14] \otimes [(9 - 5)^2 + 7] - 3^3$ .

### Graphing Calculator



Enter the expression  $10 + 8 \div 2^2$  into your calculator. If the calculator follows the order of operations, the answer will be 12.

### Example 5 Application: Half Price Sale

Beatrice wants to buy 3 DVDs marked \$7 each and 4 CDs marked \$12 each. Everything in the store is on sale for half off the marked price. Beatrice has \$31.50 to spend and a coupon good for \$1 off each CD. Use the expression below to determine if Beatrice has enough money to buy all the items she wants.

$$\$31.50 - \left[ \frac{3 \cdot \$7}{2} + \frac{4 \cdot \$12}{2} - (4 \cdot \$1) \right]$$

**SOLUTION** Begin inside the innermost symbols of inclusion to simplify the expression.

$$\$31.50 - \left[ \frac{3 \cdot \$7}{2} + \frac{4 \cdot \$12}{2} - (4 \cdot \$1) \right]$$

$$= \$31.50 - \left[ \frac{3 \cdot \$7}{2} + \frac{4 \cdot \$12}{2} - \$4 \right]$$

Multiply inside the parentheses.

$$= \$31.50 - \left[ \frac{\$21}{2} + \frac{\$48}{2} - \$4 \right]$$

Simplify the numerators.

$$= \$31.50 - [\$10.50 + \$24 - \$4]$$

Simplify the fractions.

$$= \$31.50 - \$30.50$$

Simplify inside the brackets.

$$= \$1.00$$

Subtract.

Beatrice has enough money.

### Lesson Practice

**Simplify each expression.**

(Ex 1)

a.  $12 + |5 - 11|$

b.  $5(8 + 4) \div (15 - 5 - 4)$

c.  $5 + [6 \cdot (2^3 + 4)]$

d. **Justify** Simplify the expression. Justify each step.

(Ex 3)

$$4(1 + 2)^2 \div 6 + \frac{8 \cdot 3}{2}$$

- e. Compare the expressions. Use  $<$ ,  $>$ , or  $=$ .

(Ex 4)

$$(13 + 5) - [5 \cdot 2^2] \bigcirc [(7 + 11) - 5] - 2^3.$$

- f. **Health** Body Mass Index (BMI) is the relation of weight to height. The expression  $\left(\frac{W}{H^2}\right) \cdot 703$ , where  $W$  is weight in pounds and  $H$  is height in inches, is used to calculate BMI. Explain the steps that are necessary to simplify this expression.

## Practice Distributed and Integrated

Add, subtract, multiply, or divide.

1.  $(5 + 2)^2 - 50$

(4)

2.  $(3 - 5) + 7^2$

(4)

3.  $3\frac{1}{3} - 1\frac{1}{6} - 5\frac{1}{4}$

(6)

4.  $2\frac{1}{3} \cdot 3\frac{1}{4} \cdot 1\frac{1}{2}$

(SB 3)

5.  $(0.56 + 0.3) \cdot 0.2$

(4)

6.  $3.25 \cdot 0.4 + 0.1$

(4)

7.  $1.2 \div 0.1 \div 0.1$

(SB 2)

8.  $20.2 \cdot 0.1 \cdot 0.1$

(SB 2)

9. **Verify** True or False: All whole numbers are counting numbers. If true, explain your answer. If false, give a counterexample.

(1)

10. The set  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$  represents which set of numbers?

(1)

11. **Justify** True or False: An obtuse triangle has two obtuse angles. Explain your choice.

(SB 13)

12. Find the prime factorization of 207.

(SB 12)

13. Find the prime factorization of 37.

(SB 12)

14. **Write** Use the divisibility test to determine if 10,048 is divisible by 8. Explain.

(SB 4)

15. Write 0.345 as a fraction in simplest form and as a percent.

(SB 5)

16. Write 0.07% as a fraction in simplest form and as a decimal.

(SB 5)

- \*17. Evaluate  $(|-3| \cdot 4) + \left[\left(\frac{1}{2} + \frac{1}{4}\right) \div \frac{1}{3}\right]$ .

(7)

- \*18. Compare:  $\frac{1}{3} + \frac{1}{5} \cdot \frac{2}{15} \bigcirc \left(\frac{1}{3} + \frac{1}{5}\right) \cdot \frac{2}{15}$ .

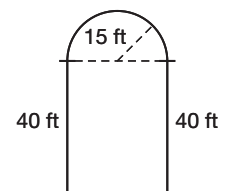
(7)

- \*19. **Temperature** The following two formulas are used to convert degrees Celsius ( $^{\circ}\text{C}$ ) to degrees Fahrenheit ( $^{\circ}\text{F}$ ) and vice versa:  $C = \frac{5}{9}(F - 32)$  and  $F = \frac{9}{5}C + 32$ . Explain how the equations are different.

(7)

- \*20. **Fencing** The diagram represents the fencing around a backyard. The fence is formed with parallel lines and a half-circle. Write and solve an equation to determine how many feet of fencing are needed. Round the answer to the nearest tenth.

(7)



- \*21. **Multiple Choice** Simplify  $[(10 - 8)^2 - (-1)] + (5 - 3)$ .  
 (7) A -38 B 7 C -80 D 37

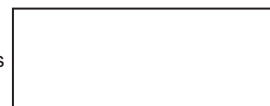
- \*22. **Manufacturing** A company produces two different types of 6-sided boxes. Box A is 12 inches long, 12 inches wide, and 12 inches tall. Box B is 16 inches long, 16 inches wide, and 6.75 inches tall. Both boxes have the same volume, but the company wants to know which box uses less material to produce.  
 (7) a. Write and solve an expression to find the surface area of Box A.  
 b. Write and solve an expression to find the surface area of Box B.  
 c. Compare the box sizes. Which box uses less material?

- \*23. **Multi-Step** A ball is dropped from a height of 25.6 feet. After it hits the ground, it bounces to 12.8 feet and falls back to the ground. Next it bounces to 6.4 feet and falls back to the ground. Then it bounces to 3.2 feet and falls back to the ground.  
 (6) a. Find the difference in heights between each consecutive bounce.  
 b. If the pattern continues, will the ball ever stop bouncing? Explain.



- \*24. **Geometry** What is the perimeter of the rectangle?  
 (6)

8.42 units



22.312 units



- \*25. **Measurement** A valley is 250 below sea level and a small hill is 78 feet above sea level. Solve  $|-250| + 78$  to determine the distance from the bottom of the valley to the top of the hill.  
 (5)

- \*26. **Transportation** In the last hour, 7 planes have landed at the airport and 11 planes have taken off. Use addition to find the change in the total number of planes at the airport in the last hour.  
 (5)

- \*27. **Error Analysis** The temperature in the morning was  $-18^{\circ}\text{F}$ . It increased by  $5^{\circ}$  by noon and dropped  $10^{\circ}$  in the evening. Two students determined the temperature in the evening. Which student is correct? Explain the error.  
 (4)

Student A
$-18 + 5 + 10 = -3$

Student B
$-18 + 5 + (-10) = -23$

- \*28. **Meteorology** The water level of the reservoir in Purcellville, Virginia was 2 feet below normal. After a heavy rainstorm, the water level increased to 5 feet above normal. Write and solve a subtraction problem to find the change in the water level caused by the rainstorm.  
 (6)

- \*29. **Multiple Choice** Which term in the expression  $\frac{\sqrt{9ny}}{nx} + a^2 - \frac{n}{4} + \frac{3\pi}{8}$  contains an irrational constant?  
 (1)

A  $\frac{\sqrt{9ny}}{nx}$

B  $\frac{3\pi}{8}$

C  $\frac{n}{4}$

D  $a^2$



- \*30. **Geometry** The measure of each interior angle of a hexagon is given by the expression  $\frac{180(6-2)^{\circ}}{6}$ . What is the measure of an interior angle of a hexagon?  
 (7)

## Using Unit Analysis to Convert Measures

## Warm Up

1. **Vocabulary** The amount of space a solid figure occupies is called the \_\_\_\_\_ (*area, volume*).

Simplify.

$$2. \frac{7}{12} \cdot \frac{36}{49}$$

$$3. \frac{8}{9} \cdot \frac{15}{36}$$

$$4. \frac{2}{5} \cdot \frac{15}{16} \cdot \frac{6}{7}$$

$$5. \frac{12}{13} \cdot \frac{1}{4} \cdot \frac{39}{48}$$

## New Concepts

Unit analysis is a process for converting measures into different units. A unit ratio, or conversion factor, compares 2 measures that name the same amount.

$$\frac{12 \text{ in.}}{1 \text{ ft}}$$

$$\frac{1 \text{ m}}{100 \text{ cm}}$$

$$\frac{3 \text{ ft}}{1 \text{ yd}}$$

Since the amounts used in a unit ratio are equal to each other, a unit ratio is always equal to 1. Since the product of 1 and a number is that number, a unit ratio multiplied by a measure will always name the same amount.

## Hint

A mile is equal to 1760 yards.

## Example 1 Converting Units of Length

A cheetah ran at a rate of 105,600 yards per hour. How fast did the cheetah run in miles per hour?

**SOLUTION** Find a unit ratio and multiply.

$$\frac{105,600 \text{ yd}}{1 \text{ hour}} = \frac{? \text{ mi}}{1 \text{ hour}}$$

Identify known and missing information.

$$105,600 \text{ yd} \rightarrow ? \text{ mi}$$

Write the conversion.

$$1 \text{ mi} = 1,760 \text{ yd}$$

Equate units.

$$\frac{1 \text{ mi}}{1760 \text{ yd}}$$

Write a unit ratio.

$$\frac{105,600 \text{ yd}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}}$$

Write the multiplication sentence.

$$= \frac{\cancel{105,600}^{60} \text{ yd}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{\cancel{1760}^1 \text{ yd}}$$

Cancel out common factors.

$$= \frac{60 \text{ mi}}{1 \text{ hr}}$$

Multiply.

$$\frac{105,600 \text{ yd}}{1 \text{ hr}} = \frac{60 \text{ mi}}{1 \text{ hr}}$$

Write the ratio of miles per hour.

The cheetah ran at a rate of 60 miles per hour.



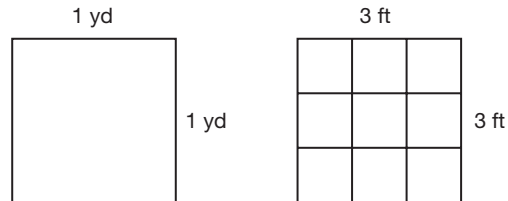
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### Exploration Using Unit Analysis

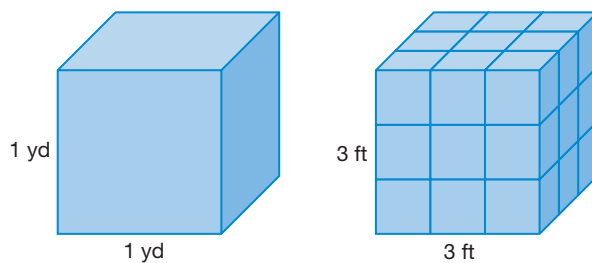
If a measure of length changes, then the unit analysis occurs in one dimension. If a measure of area changes, the units for the dimensions of both length and width must change.

Draw two congruent squares with side lengths of 3 inches on a sheet of paper. Label the sides of the first square 1 yard. Divide both the length and width of the second square into 3 equal sections. This will divide the square into 9 congruent smaller squares. Label the sides of the second square 3 feet.



- a. What is the area of the first square? Show your calculation.
- b. What is the area of the second square? Show your calculation.
- c. Write two unit ratios for converting between feet and yards.
- d. Which unit ratio should be used for converting yards to feet? Explain your choice.
- e. **Justify** Use a unit ratio to convert the area of the first square into square feet. Show your calculation.
- f. **Write** Why is it necessary to multiply by the unit ratio twice to convert square yards to square feet?

Extend the example of the area of squares to the volume of cubes. Draw two cubes, one with dimensions of 1 yard and one with dimensions of 3 feet.



#### Math Reasoning

**Write** How could a single unit ratio be used to perform the area conversion?

- g. What is the volume of the first cube? Show your calculation.
- h. What is the volume of the second cube? Show your calculation.
- i. **Justify** Use a unit ratio to convert the volume of the first cube into cubic feet. Show your calculation.
- j. **Write** Why is it necessary to multiply by the unit ratio three times to convert cubic yards to cubic feet?



### Math Reasoning

Each mat is 110 centimeters square. This means that the length and width are 110 centimeters each.

## Example 2 Converting Units of Area

A gym measures 8.5 meters by 14 meters. The owner bought mats to cover the floor. Each mat is 110 centimeters square. If 95 mats were purchased, are there enough mats to cover the floor?

**SOLUTION** Find the area and convert the unit of measure.

$$8.5 \text{ m} \cdot 14 \text{ m} = 119 \text{ m}^2$$

Find the area of the room.

$$119 \text{ m}^2 \rightarrow ? \text{ cm}^2$$

Write the conversion.

$$1 \text{ m} = 100 \text{ cm}$$

Equate units.

$$\frac{100 \text{ cm}}{1 \text{ m}}$$

Write a unit ratio.

$$119 \text{ m} \cdot \text{m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{100 \text{ cm}}{1 \text{ m}}$$

Write the multiplication sentence.

$$= 119 \cancel{\text{ m}} \cdot \cancel{\text{ m}} \cdot \frac{100 \text{ cm}}{1 \cancel{\text{ m}}} \cdot \frac{100 \text{ cm}}{1 \cancel{\text{ m}}}$$

Cancel out common factors.

$$= \frac{119 \cdot 100 \text{ cm} \cdot 100 \text{ cm}}{1} = 1,190,000 \text{ cm}^2$$

Multiply.

$$95(110 \text{ cm} \cdot 110 \text{ cm}) = 1,149,500 \text{ cm}^2$$

Find the area of 95 mats.

$$1,149,500 \text{ cm}^2 < 1,190,000 \text{ cm}^2$$

Compare the areas.

The area covered by 95 mats is less than the area of the floor, so there are not enough mats to cover the floor.

## Example 3 Application: Converting Units of Volume

A hose with a flow rate of 41,472 cubic inches per hour is filling up a pool. The volume of the pool is 1,104 cubic feet. How many hours will it take to fill the pool?

**SOLUTION** Find the volume and convert the unit of measure.

$$\frac{41,472 \text{ in}^3}{1 \text{ hour}} \rightarrow \frac{? \text{ ft}^3}{1 \text{ hour}}$$

Identify known and missing information.

$$1 \text{ ft} = 12 \text{ in.}$$

Equate units.

$$\frac{1 \text{ ft}}{12 \text{ in.}}$$

Write a unit ratio.

$$\frac{41,472 \text{ in.} \cdot \text{in.} \cdot \text{in.}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} \cdot \frac{1 \text{ ft}}{12 \text{ in.}}$$

Write the multiplication sentence.

$$= \frac{41,472 \cancel{\text{ in.}} \cdot \cancel{\text{ in.}} \cdot \cancel{\text{ in.}}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{\cancel{12 \text{ in.}}_1} \cdot \frac{1 \text{ ft}}{\cancel{12 \text{ in.}}_1} \cdot \frac{1 \text{ ft}}{\cancel{12 \text{ in.}}_1}$$

Cancel out common factors.

$$= \frac{24 \text{ ft} \cdot \text{ft} \cdot \text{ft}}{1 \text{ hr}} = \frac{24 \text{ ft}^3}{1 \text{ hr}}$$

Multiply.

$$1,104 \div 24 = 46 \text{ hours}$$

It will take 46 hours to fill the pool.

### Math Reasoning

**Analyze** Compare the process of converting feet to inches with the process of converting feet per minute to inches per minute.

Unit analysis can be used for more than just converting units of length. It can also be used to convert units of mass, density, temperature, capacity, or even money. In economics, the value of money is defined by what people are willing to exchange for it. This means that the value of a currency can change relative to the value of other currencies. An exchange-rate listing shows what a currency is worth compared to other currencies at that moment.

### Hint

Choose a unit conversion factor that cancels the units you want to change and replaces them with the units you want.

### Example 4 Foreign Travel: Converting Units of Currency

Jared and his family are going on a vacation to Europe. He takes \$225 with him. He needs to exchange this amount for its equivalent value in euros. If the current exchange rate is 1 euro = \$1.36, what is the value of Jared's \$225 in euros?

#### SOLUTION

Convert the unit of measure.

225 dollars  $\rightarrow$  ? euros

1 euro = 1.36 dollars

$$\frac{1 \text{ euro}}{1.36 \text{ dollars}}$$

$$225 \text{ dollars} \cdot \frac{1 \text{ euro}}{1.36 \text{ dollars}}$$

$$= 165.44 \text{ euros}$$

Write the conversion.

Equate units.

Write a unit ratio.

Write the multiplication sentence.

Multiply and cancel.

**Check** Since a euro is worth more than a dollar, Jared should have fewer euros than dollars after the exchange.

$$225 > 165.44 \quad \text{The answer is reasonable.}$$

### Lesson Practice

- a. (Ex 1) A Mourning Dove can reach speeds up to 35 miles per hour. How fast is this in feet per hour?
- b. (Ex 2) An interior wall measures 4.5 yards by 3.25 yards. What is the size of the wall in square feet?
- c. (Ex 3) Della has a small bag containing 50 cubic centimeters of potting soil. Her planter has a volume of 46,300 cubic millimeters. Does Della have enough soil to fill the planter? Explain.
- d. (Ex 4) Arthur just returned from London. He has 16 British pounds to convert to American dollars. If the exchange rate is 1 pound = \$2.016, what is the value of Arthur's 16 pounds in dollars?

## Practice Distributed and Integrated

Add, subtract, multiply, or divide.

1.  $4\frac{1}{3} \div 1\frac{1}{3} + 3\frac{1}{3}$

2.  $2\frac{3}{8} - 1\frac{3}{4} \div 1\frac{1}{2}$

3.  $2\frac{2}{3} + 1\frac{5}{6} - 6\frac{3}{4}$

4.  $3\frac{1}{3} \div 1\frac{1}{4} \cdot \frac{1}{2}$

5.  $0.37 \div 0.2 \cdot 0.1$

6.  $1.74 \cdot 0.3 \div 0.2$

7. Given the sets  $A = \{1, 3, 5\}$ ,  $B = \{0, 2, 4, 6\}$ , and  $C = \{1, 2, 3, 4\}$ , are the following statements true or false?

a.  $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

b.  $A \cap B = \{0, 1, 2, 3, 4, 5, 6\}$

c.  $B \cup C = \{2, 4\}$

d.  $A \cap C = \{1, 3\}$

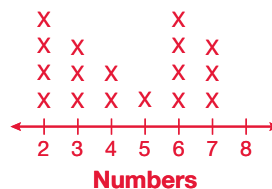
8. Compare the expressions using  $<$ ,  $>$ , or  $=$ . Explain.

$$8^2 \div 4 - 6^2 \bigcirc (6 \cdot 7 \cdot 5) \div 6 - 15$$

9. Draw a line plot for the frequency table.

Number	2	3	4	5	6	7
Frequency	4	3	2	1	4	3

Frequency of Numbers



10. Subtract  $78\frac{2}{5} - 14\frac{7}{10}$ .

11. Find the prime factorization of 484.

12. **Write** Use the divisibility test to determine if 22,993 is divisible by 5. Explain your answer.

13. Write 125% as a fraction in simplest form and as a decimal.

14. Convert 105 kilometers per hour to kilometers per minute.

\*15. Convert 74 square meters to square centimeters.

\*16. **Camping** Norman's camping tent has a volume of 72,576 cubic inches. What is the volume of the tent in cubic feet?

17. **Multiple Choice** Which of these differences will be positive?

A  $-\frac{1}{2} - \frac{1}{8}$

B  $\frac{9}{12} - 1$

C  $\frac{5}{7} - \frac{3}{10}$

D  $-\frac{14}{15} - \left(\frac{4}{15}\right)$

- 18. Error Analysis** Two students used unit analysis to convert a measurement of length.  
 (8) Which student is correct? Explain the error.

Student A	Student B
$1 \text{ cm} = 10 \text{ mm}$ $5540 \text{ mm} = 5540 \text{ mm} \times \frac{1 \text{ cm}}{10 \text{ mm}}$ $5540 \text{ mm} = 554 \text{ cm}$	$1 \text{ cm} = 10 \text{ mm}$ $5540 \text{ mm} = 5540 \text{ mm} \times \frac{10 \text{ mm}}{1 \text{ cm}}$ $5540 \text{ mm} = 55400 \text{ cm}$

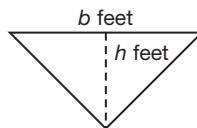
- \*19. Multiple Choice** Which one of the following ratios can be used to convert 120 cm into an equivalent measure in inches? (Hint: There are 2.54 cm in one inch.)

- A  $\frac{2.54 \text{ cm}}{1 \text{ in.}}$   
 B  $\frac{1 \text{ in.}}{2.54 \text{ cm}}$   
 C  $\frac{2.54 \text{ cm}}{1 \text{ in.}} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}}$   
 D  $\frac{1 \text{ in.}}{2.54 \text{ cm}} \cdot \frac{1 \text{ in.}}{2.54 \text{ in.}}$

- \*20. Weather Forecasting** One knot is exactly 1.852 kilometers per hour. The highest wind gust for the day was measured at 38 knots.

- a. How many km/hr was the recorded wind gust? (Hint: 1 knot = 1.852 km/hr)  
 b. How many mph was the recorded wind gust? (Hint: 1 mi = 1.609 km)

- \*21. Multi-Step** How can you find the area of the triangle in square inches?  
 (8)

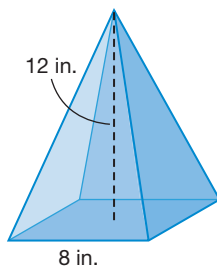


- a. Write a formula for computing the area of the triangle in units of square feet.  
 b. Write a new formula that computes the area of the triangle in units of square inches.  
 c. What is the area of the triangle in square inches if  $b = 3 \text{ ft}$  and  $h = 2.2 \text{ ft}$ ?
- \*22. Chemistry** Water has a density of 1 gram per cubic centimeter. What is the density of water in grams per cubic inch?  
 (8)
- \*23. Justify** True or False: A right triangle has one right angle, one obtuse angle, and one acute angle. If false, explain why.  
 (SB 13)

24. **Error Analysis** Student A and Student B simplified the expression  $\frac{24}{8} + (2 + 4)^2$ .  
(7) Which student is correct? Explain the error.

Student A	Student B
$\frac{24}{8} + (2 + 4)^2$	$\frac{24}{8} + (2 + 4)^2$
$3 + 6^2$	$3 + (2 + 16)$
$3 + 36 = 39$	$3 + 18 = 21$

25. **Geometry** A square pyramid has a base with edges of 8 inches and a height of 12 inches.



- a. Use the following formula to find the volume:  $V = \frac{1}{3}s^2h$ .  
b. **Analyze** Which term did you simplify first? Why?
- \*26. **Economics** Use the table of the Profit and Loss Report of ABC Company. What was the total profit or loss for the year? [( ) indicates a loss.]

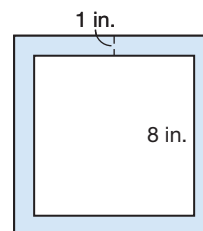
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Profit or Loss	\$6 million	(\$3.5 million)	(\$2 million)	\$5 million

- \*27. **Multi-Step** Two groups of students measured the length of a tabletop. Group A's measurement was  $56\frac{3}{4}$  inches. Group B's measurement was  $57\frac{3}{4}$  inches. If the actual length of the tabletop was  $57\frac{1}{2}$  inches, which group's measurement had the smaller error?
28. **Error Analysis** Two students simplified an expression containing an exponent.  
(3) Which student is correct? Explain the error.

Student A	Student B
$b^2 \cdot c^2 \cdot c \cdot b^2 \cdot b =$ $b^2 \cdot b^2 \cdot b \cdot c^2 \cdot c = b^5c^3$	$b^2 \cdot c^2 \cdot c \cdot b^2 \cdot b =$ $b^2 \cdot b^2 \cdot b \cdot c^2 \cdot c = b^4c^2$

29. **Speed** A giraffe can run 32 miles per hour. What is the speed in feet per hour?

30. **Geometry** A square photograph measuring 8 inches by 8 inches is positioned within a 1-inch-wide picture frame as shown.



- a. What is the area of the photograph?  
b. What is the combined area of the photograph and frame?  
c. What is the area of the frame alone?  
d. If the 1-inch-wide frame is replaced with a 2-inch-wide frame, how much more wall space will be needed to hang the framed photograph?

# Evaluating and Comparing Algebraic Expressions

## Warm Up

1. **Vocabulary** When two numbers are divided, the result is called the \_\_\_\_\_. (*quotient, product*).

Simplify.

$$2. x^5 b^2 \cdot 3b^3 x$$

$$3. \frac{4^3}{2^3} + 7^2$$

$$4. \frac{5^3}{3^2 + 4^2}$$

$$5. 32 \div [2 \cdot (8 - 7)]$$

## New Concepts

Any expression containing only numbers and operations is a **numeric expression**. An **algebraic expression** is an expression with variables and/or numbers that uses operations (e.g., +, −, ×, or ÷). An algebraic expression is also called a variable expression.

### Math Language

**Evaluate** means to substitute values for the variables and to simplify using the order of operations.

### Example 1 Evaluating Algebraic Expressions

Evaluate the expression when  $x = 3$  and  $a = 1$ .

$$3x - 4x + ax$$

**SOLUTION** Substitute 3 for  $x$  and 1 for  $a$  in the expression. Then simplify.

$$\begin{aligned} 3x - 4x + ax \\ &= 3 \cdot 3 - 4 \cdot 3 + 1 \cdot 3 \\ &= 9 - 12 + 3 \\ &= 0 \end{aligned}$$

When the variables in algebraic expressions have exponents, it is helpful to write the value in parentheses.

### Example 2 Evaluating Algebraic Expressions with Exponents

Evaluate the expression for  $y = 2$  and  $z = 4$ .

$$3(z - y)^2 - 4y^3$$

**SOLUTION** Substitute 2 for  $y$  and 4 for  $z$  in the expression. Then simplify.

$$\begin{aligned} 3(z - y)^2 - 4y^3 \\ &= 3(4 - 2)^2 - 4(2)^3 \\ &= 3(2)^2 - 4(2)^3 \\ &= 3 \cdot 4 - 4(8) \\ &= 12 - (32) \\ &= -20 \end{aligned}$$



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Two algebraic expressions are equivalent if they can be simplified to the same value. For example,  $2 \cdot 2 - 1$  is equivalent to  $15 \div 5$ . Both expressions have a simplified value of 3.

### Example 3 Comparing Algebraic Expressions

Compare the expressions when  $a = 4$  and  $b = 3$ . Use  $<$ ,  $>$ , or  $=$ .

$$3a^2 + 2b - 4b^3 \bigcirc 2a^2b^2$$

**SOLUTION** Simplify the expression on the left and then compare.

$$\begin{array}{rcl} 3a^2 + 2b - 4b^3 & & 2a^2b^2 \\ 3(4)^2 + 2(3) - 4(3)^3 & & 2(4)^2(3)^2 \\ = 3(16) + 2(3) - 4(27) & & = 2(16)(9) \\ = 48 + 6 - 108 & & = 288 \\ = 54 - 108 & & \\ = -54 & & \end{array}$$

Since  $-54 < 288$ ,  $3a^2 + 2b - 4b^3 < 2a^2b^2$  when  $a = 4$  and  $b = 3$ .

Many real-world situations can be described using math. Algebraic expressions can be used to represent relationships between quantities.

### Example 4 Application: Phone Charges

A cell phone company charges a \$20 monthly fee and then 45 cents per minute. The company uses the expression  $20 + 0.45m$  to find the total amount to charge for each month. How much would the company charge for 200 minutes?

**SOLUTION** Substitute 200 for  $m$  in the expression and simplify.

$$\begin{array}{rcl} 20 + 0.45m & & \\ = 20 + 0.45(200) & & \\ = 20 + 90 & & \\ = 110 & & \end{array}$$

The cell phone company would charge \$110.

#### Math Reasoning

**Analyze** In the expression  $20 + 0.45m$ , what does the variable  $m$  represent?

### Lesson Practice

Evaluate each expression for the given values.

a.  $3x - 4b + 2bx$ ;  $x = 10$ ,  $b = 2$   
(Ex 1)

b.  $2ab - 4a^2 + 10$ ;  $a = -1$ ,  $b = 8$   
(Ex 2)

c. Compare the expressions when  $x = 2$  and  $y = 5$ . Use  $<$ ,  $>$ , or  $=$ .  
(Ex 3)  $(6x^2 + y^3) - 3x^6 \bigcirc 8x^4 - y^3$

d. **Climate** The lowest recorded temperature is  $-89.4^\circ\text{C}$  in Antarctica.  
(Ex 4) The expression  $\frac{9}{5}C + 32$  can be used to convert Celsius measurements to Fahrenheit. What is the lowest recorded temperature in degrees Fahrenheit?

## Practice Distributed and Integrated

Add, subtract, multiply, or divide.

1.  $4\frac{1}{3} \div 2\frac{1}{3}$   
(SB 3)

2.  $42\frac{3}{8} - 21\frac{3}{4}$   
(SB 3)

3.  $1\frac{2}{3} + 2\frac{5}{6}$   
(SB 3)

4.  $2\frac{2}{3} \div 1\frac{3}{4}$   
(SB 3)

5.  $0.75 \div 0.2$   
(SB 2)

6.  $1.74 \div 0.3$   
(SB 2)

7.  $1.25 \cdot 0.2$   
(SB 2)

8.  $12.2 \times 3.2$   
(SB 2)

9. **Verify** True or False: A square is a rhombus. Explain your choice.  
(SB 14)

10. Simplify  $4[(6 - 4)^3 - 5]$ .  
(7)

11. Convert  $1.86 \text{ km}^2$  to  $\text{m}^2$ .  
(8)

\*12. Evaluate the expression  $14c + 28 - 12cd$  for the given values  $c = 4$  and  $d = 5$ .  
(9)

13. A straight angle measures \_\_\_\_\_.  
(SB 13)

14. Find the prime factorization of 125.  
(SB 12)

\*15. Find the value of the expression  $\frac{t-36}{36} + l$  if  $t = 72$  and  $l = 1$ .  
(9)

16. **Multiple Choice** Evaluate the expression  $14 + \frac{36}{9} \cdot (2 + 5)$ .  
(7)

A 126

B 21

C 42

D 140

17. Simplify  $(3 + 12) + (|-4| - 2)^3 + 1$ .  
(7)

\*18. **Flight** A rocket is fired upward at an initial speed of 112 feet per second (ft/sec). It travels at a speed of  $112 - 32t$  ft/sec, where  $t$  is the flight time in seconds.  
(9)

a. What is the rocket's speed after 1 second?

b. What is the rocket's speed after 2 seconds?

\*19. **Canoeing** Rachel wants to rent a canoe for 3 hours. Use the expression  $\$6.50 + \$1.75h$ , where  $h$  represents the number of hours, to calculate the cost of renting the canoe.  
(9)

\*20. **Error Analysis** Two students were asked to evaluate  $\frac{y^2}{-x}$  when  $x$  is 5 and  $y$  is  $-5$ .  
(9) Which student is correct? Explain the error.


Student A
$\frac{y^2}{-x} = \frac{-5^2}{-(5)} = \frac{-25}{-5} = 5$

Student B
$\frac{y^2}{-x} = \frac{(-5)^2}{-(5)} = \frac{25}{-5} = -5$

\*21. **Data Analysis** The variance of a set of data can be found with the expression  $\frac{s}{n}$ , where  $s$  is the sum of the squared deviation and  $n$  is the total number of data items in the set. What is the variance for a set of data with 12 items and a sum of the squared deviations equal to 30?  
(9)

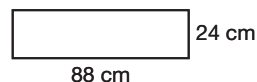


- \*22. **Sports** In volleyball, the statistic for total blocks at the net is calculated with the expression  $s + 0.5a$ , where  $s$  is the number of solo blocks and  $a$  is the number of assisted blocks. What is the total-blocks statistic for a player who has 80 solo blocks and 53 assisted blocks?


-  23. **Write** Use the divisibility test to determine if 224 is divisible by 6. Explain your answer.

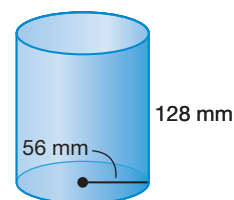
24. Write 35.2% as a fraction in simplest form and as a decimal.

- \*25. **Multi-Step** The rectangle has dimensions measured in centimeters. What is the ratio of the area of the rectangle in centimeters to the area of the rectangle in millimeters?



- Calculate the area of the rectangle.
- Find the area of the rectangle in square millimeters.
- Find the ratio of square centimeters to square millimeters for the rectangle.

-  \*26. **Geometry** A right circular cylinder has a base radius of 56 mm and a height of 128 mm. What is its volume in cubic centimeters? Round the answer to the nearest hundredth. Use 3.14 for  $\pi$ .



27. **Loans** The formula for long-term loans is  $F = P(1 + i)^{n \div 12}$ , where  $F$  is the future value of money,  $P$  is the present value,  $i$  is the interest rate, and  $n$  is the length of time the money is borrowed in months. When solving this equation for  $F$ , what step would you perform after adding 1 and  $i$ ?

28. **Golf** Below is Rickie's golf score for two golf tournaments. What is the difference in his final score for the 1st and 2nd tournament?

1st Tournament	1	-2	-3	2
2nd Tournament	-2	-1	1	-1

29. **Error Analysis** Two students were asked to evaluate  $(30 - 10)^2$ . Student A answered 400, and Student B answered -70. Which student is correct? Explain the error.

Student A	Student B
$(30 - 10)^2$ $= 20^2$ $= 400$	$(30 - 10)^2$ $= (30 - 10 \cdot 10)$ $= (30 - 100)$ $= -70$

- \*30. **Typing** Jared can type 35 words per minute. Use the expression  $35m$  to find the number of words he can type in 15 minutes.

# Adding and Subtracting Real Numbers

## Warm Up

1. **Vocabulary** Any real number that cannot be written as a quotient of <sup>(1)</sup> integers is called a(n) \_\_\_\_\_ number.

Simplify.

2. <sup>(4)</sup>  $(25 \div 5) - (30 \div 10)$   
 3. <sup>(5)</sup>  $-4 + (-9) + (-6)$   
 4. <sup>(4)</sup>  $(2.45 + 5.75) - (4.85 - 3.75)$   
 5. <sup>(3)</sup>  $(j^4k^5)(4kj^2)(3k^3)$

## New Concepts

When solving a problem containing addition and subtraction of signed numbers, begin by writing the problem as addition only. Next, group and add the terms with like signs. Then add the terms with unlike signs.

### Example 1 Adding and Subtracting Fractions and Decimals

Simplify.

a.  $-\frac{1}{5} + \frac{3}{5} - \frac{2}{5} - \left(-\frac{4}{5}\right)$

**SOLUTION**

$$\begin{aligned} &-\frac{1}{5} + \frac{3}{5} - \frac{2}{5} - \left(-\frac{4}{5}\right) \\ &= -\frac{1}{5} + \frac{3}{5} + \left(-\frac{2}{5}\right) + \frac{4}{5} \\ &= -\frac{1}{5} + \left(-\frac{2}{5}\right) + \frac{3}{5} + \frac{4}{5} \\ &= -\frac{3}{5} + \frac{7}{5} \\ &= -\frac{4}{5} \end{aligned}$$

Write the problem as addition.

Group the terms with like signs.

Add numbers with like signs.

Add.

b.  $3.16 + (-1.22) - 4.73 + 5.6$

**SOLUTION**

$$\begin{aligned} &3.16 + (-1.22) - 4.73 + 5.6 \\ &= 3.16 + (-1.22) + (-4.73) + 5.6 \\ &= 3.16 + 5.6 + (-1.22) + (-4.73) \\ &= 8.76 + (-5.95) \\ &= 2.81 \end{aligned}$$

Write the problem as addition.

Group the terms with the same signs.

Add numbers with like signs.

Add.

### Hint

Use the rules below for adding integers.

1. Like signs: Add and keep the sign.
2. Unlike signs: Subtract and keep the sign of the greater absolute value.



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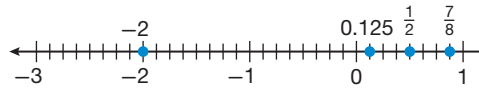
**Example 2** Ordering Rational Numbers

Order the numbers from least to greatest.

$$\frac{7}{8}, -2, 0.125, \frac{1}{2}$$

**SOLUTION**

Use a number line to order the numbers. Place each number on the number line.



To order these numbers from least to greatest, read the numbers on the number line from left to right.

$$-2, 0.125, \frac{1}{2}, \frac{7}{8}$$

**Example 3** Comparing Rational Expressions

Complete the comparison. Use  $<$ ,  $>$ , or  $=$ .

$$\frac{3}{8} + \left(-\frac{5}{8}\right) - \frac{1}{8} \bigcirc -2.75 + 6.25 - 3.75$$

**SOLUTION**

Simplify each expression. Then compare.

$$\begin{aligned} & \frac{3}{8} + \left(-\frac{5}{8}\right) - \frac{1}{8} \\ &= \frac{3}{8} + \left(-\frac{5}{8}\right) + \left(-\frac{1}{8}\right) \\ &= \frac{3}{8} + \left(-\frac{6}{8}\right) \\ &= -\frac{3}{8} \end{aligned}$$

$$\begin{aligned} & -2.75 + 6.25 - 3.75 \\ &= -2.75 + 6.25 + (-3.75) \\ &= -2.75 + (-3.75) + 6.25 \\ &= -6.50 + 6.25 \\ &= -0.25 \end{aligned}$$

$$\text{Since } -\frac{3}{8} < -0.25, \frac{3}{8} + \left(-\frac{5}{8}\right) - \frac{1}{8} \text{ } \textcircled{<} \text{ } -2.75 + 6.25 - 3.75.$$

**Hint**

Convert  $-\frac{3}{8}$  to a decimal or  $-0.25$  to a fraction to make comparing the values easier.

**Example 4** Application: Investing

Carly invested \$250 in two accounts. The table below shows the ending balance per quarter.

	1 <sup>st</sup> Quarter	2 <sup>nd</sup> Quarter	3 <sup>rd</sup> Quarter	4 <sup>th</sup> Quarter
Investment A	\$255.75	\$258.81	\$260.25	\$262.99
Investment B	\$260.66	\$274.22	\$268.92	\$290.07

- a. Which investment grew more?

**SOLUTION**

Find the differences and compare.

Investment A	Investment B
$\$262.99 - \$250 = \$12.99$	$\$290.07 - \$250 = \$40.07$

Investment B grew more.

- b. In which quarter did Investment B grow the most?

**SOLUTION**

$$\$260.66 - \$250.00 = \$10.66$$

$$\$274.22 - \$260.66 = \$13.56$$

$$\$268.92 - \$274.22 = -\$5.30$$

$$\$290.07 - \$268.92 = \$21.15$$

The greatest difference is \$21.15, which occurred in the 4th quarter.

**Lesson Practice**

**Simplify.**

(Ex 1)

a.  $\frac{4}{9} + \frac{2}{9} - \frac{5}{9}$

b.  $16.21 - 21.54 + 12.72$

- c. Order the numbers from least to greatest.

(Ex 2)

$$\frac{3}{4}, -1, 0.85, \frac{5}{8}$$

- d. Complete the comparison. Use  $<$ ,  $>$ , or  $=$ .

(Ex 3)

$$3.2 + (-2.8) - 5.2 \bigcirc \frac{7}{12} - \frac{5}{12} + \left(-\frac{11}{12}\right)$$

- e. Jonah ran a race in 32.68 seconds. Jarrod finished 1.92 seconds before Jonah. Gayle finished 3.01 seconds after Jonah. How many seconds did it take Gayle to run the race?

(Ex 4)

**Practice** Distributed and Integrated

**Add, subtract, multiply, or divide.**

1.  $\frac{1}{2} + \frac{3}{5}$

(SB 3)

2.  $15\frac{1}{3} - 7\frac{4}{5}$

(SB 3)

3.  $3\frac{2}{3} \cdot 2\frac{1}{4}$

(SB 3)

4.  $3\frac{2}{5} \div 1\frac{2}{3}$

(SB 3)

5.  $78\frac{2}{5} - 14\frac{7}{10}$

(SB 3)

6.  $2\frac{1}{3} \cdot 1\frac{1}{4}$

(SB 3)

7.  $10.2 \cdot 3.15$

(SB 2)

8.  $20.46 \div 2.2$

(SB 2)

9.  $12.3 \cdot 2.02$   
(SB 2)

10.  $0.8 \div 0.25$   
(SB 2)

\*11. Order from greatest to least:  $\frac{6}{7}, \frac{3}{5}, \frac{1}{7}, -\frac{4}{3}$ .  
(10)

12. A(n) \_\_\_\_\_ angle measures less than  $90^\circ$ .  
(SB 1)

13. Convert 8673 g to kg.  
(8)

14. Convert 26 mi to km. Round your answer to the nearest tenth.  
(8)

15. True or False:  $(2 + 5) - (3 \cdot 4) = 2 + 5 - 3 \cdot 4$ . Explain.  
(4)

\*16. **Multiple Choice** Simplify  $1.29 + 3.9 - 4.2 - 9.99 + 6.1$ .  
(10)

A -2.9                      B -1                      C 1                      D 2.9

\*17. **Error Analysis** Which student is correct? Explain the error.  
(10)

Student A	Student B
$1 - \left( \frac{1}{5} - \frac{2}{10} - \frac{1}{10} \right)$ $= 1 - \left( -\frac{1}{10} \right)$ $= 1\frac{1}{10}$	$1 - \left( \frac{1}{5} - \frac{2}{10} - \frac{1}{10} \right)$ $= 1 - \frac{1}{5} - \frac{2}{10} - \frac{1}{10}$ $= \frac{1}{2}$

\*18. **Time** A ship sailed northeast for  $2\frac{1}{4}$  hours. It then sailed east for  $1\frac{1}{3}$  hours.  
(10) How much longer did it sail northeast than east?


19. **Model** Draw a line plot for the frequency table.  
(SB 29)

Number	9	10	11	12	13	14
Frequency	4	3	2	1	0	4

\*20. **Multi-Step** A map shows streets  $\frac{1}{1000}$  of their size.  
(9)

a. Write an expression that represents the real length of a block if the length of the block on the map is  $b$ .

b. Find the actual length of a block that is 0.4 feet on the map.

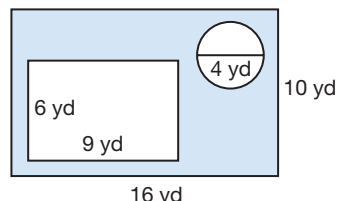
 \*21. **Geometry** A parallelogram has a base of  $z$  and a height of  $2z$ . Write an expression to find the area of the parallelogram. If  $z$  is equal to 12 cm, what is the area of the parallelogram?  
(9)

22. **Error Analysis** Two students used unit analysis to convert a measurement of area to a different unit. Which student is correct? Explain the error.  
(8)

Student A	Student B
$1 \text{ ft} = 12 \text{ in.}$ $9 \text{ ft}^2 = 9 \text{ ft}^2 \times \frac{12 \text{ in.}}{1 \text{ ft}}$ $9 \text{ ft}^2 = 108 \text{ in}^2$	$1 \text{ ft} = 12 \text{ in.}$ $9 \text{ ft}^2 = 9 \text{ ft}^2 \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{12 \text{ in.}}{1 \text{ ft}}$ $9 \text{ ft}^2 = 1,296 \text{ in}^2$

23. **Meteorology** When a weather system passes through, a barometer can be used to measure the change in atmospheric pressure in millimeters of mercury. What is the pressure difference in inches of mercury for a measured change of  $+4.5$  mm of mercury? Round the answer to the nearest thousandth.

- \*24. **Multi-Step** A plot of land contains a rectangular building that is 9 yards long and 6 yards wide and a circular building with a diameter of 4 yards.



- Write an expression for the area of the two buildings.
  - Write an expression and solve for how much area on the plot of land is not being taken up by the buildings. Round the answer to the nearest hundredth yard.
- \*25. **Error Analysis** Student A and Student B each added the numbers  $-4.8$  and  $3.6$  as shown below. Which student is correct? Explain the error.

Student A	Student B
$  \begin{array}{r}  -4.8 + 3.6 \\   -4.8  = 4.8 \\   +3.6  = 3.6 \\  4.8 \\  +3.6 \\  \hline  8.4 \\  8.4  \end{array}  $	$  \begin{array}{r}  -4.8 + 3.6 \\   -4.8  = 4.8 \\   +3.6  = 3.6 \\  4.8 \\  -3.6 \\  \hline  1.2 \\  -1.2  \end{array}  $

26. **Banking** Raul had \$500 in his checking account. He wrote checks for \$157.62 and \$43.96. Then he deposited \$225. Find Raul's balance after these three transactions.

27. **Write** Mutually exclusive means that two sets of numbers have no numbers in common. Name two subsets of real numbers that are mutually exclusive. Explain.

- \*28. **Stocks** Stock in the 123 Company fell 8.2 points on Monday and 5.3 points on Tuesday. On Wednesday the stock rose 9.1 points. Determine the total change in the stock for the three days.

- \*29. **Football** The Rams gained 4 yards on the first down, lost 6 yards on the second down, and gained 14 yards on the third down. How many total yards did the Rams gain on the three downs?

30. **Measurement** A kite flies 74 feet above the ground. The person flying the kite is 5 feet 6 inches tall. How far above the person is the kite?

## Generating Random Numbers

### Graphing Calculator Lab (Use with Investigation 1)

A set of random integers has no pattern. Some common methods for generating random integers include rolling a number cube or drawing numbers out of a hat. A graphing calculator can also be used to generate random integers.

Generate three random integers between 1 and 12.

1. Press **MATH** and then press the **▸** key three times to highlight PRB.
2. Select **5:randInt(** by pressing 5.
3. Identify the range of values.  
The lowest possible value is 1 and the highest is 12. So, press **1 [,] 12 [)]**.
4. Press **ENTER** to generate one integer between 1 and 12. An integer between 1 and 12 is 6.
5. Press **ENTER** two more times to generate two more integers between 1 and 12. Three integers between 1 and 12 are 6, 12, and 9.

```
MATH NUM CPX 123
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
randInt(1,12)
6
```

```
randInt(1,12)
6
12
9
```



Online Connection

[www.SaxonMathResources.com](http://www.SaxonMathResources.com)

### Lab Practice

Jared lost the number cubes to his favorite board game, but he does have his graphing calculator. According to the rules, the player should throw two number cubes and move the total number of spaces shown on the top faces of the cubes.

- a. What range of numbers does a single number cube generate? What would Jared enter into the calculator to simulate a number cube?
- b. How would Jared simulate rolling two number cubes? How would Jared know how many spaces to move?
- c. Simulate Jared taking three turns. What number of spaces will he move in each turn? What is the total number of spaces moved?



# Determining the Probability of an Event

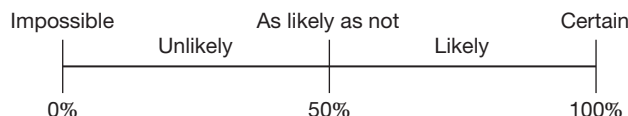
**Probability** is the measure of how likely a given event, or outcome, will occur. The probability of an event can be written as a fraction or decimal ranging from 0 to 1, or as a percent from 0% to 100%.

## Math Language

An **outcome** is a possible result of a probability experiment.

An **event** is an outcome or set of outcomes in a probability experiment.

## Range of Probability



Describe each of the events below as impossible, unlikely, as likely as not, likely, or certain.

1. Jake rolls a number less than 7 on a number cube.
2. February will have 30 days.
3. A tossed coin will land on tails.
4. Shayla correctly guesses a number between 1 and 100.

**Experimental probability** is the measure of how likely a given event will occur based on repeated trials.

$$\text{experimental probability} = \frac{\text{number of times an event occurs}}{\text{number of trials}}$$



## Online Connection

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## Materials

- small paper sacks
- small red, blue, green, and yellow paper squares

## Exploration Conducting Experiments to Find Probabilities

Place a small handful of colored squares into a paper sack. Draw out a square of paper without looking. Record the color in a frequency table like the one below.

Color	Tally	Frequency
Red		
Blue		
Green		
Yellow		

Repeat the experiment 50 times, replacing the square after each draw.

5. What is the experimental probability of drawing a red square? a blue square? a green square? a yellow square? Express each probability as a fraction and as a percent.
6. **Predict** Which color are you most likely to draw? Explain your reasoning.



Experimental probability is widely used in sports. In baseball, a player's batting average is the probability of a player getting a hit based on his previous at bats. It is typically expressed as a decimal to the thousandths place. For instance, if a player has made 3 hits after coming to bat 10 times, his batting average is .300.

7. **Sports** If a player gets 8 hits in 25 at bats, what is the probability that he will get a hit on his next at bat? Express the answer as a decimal number to the thousandths place.

In addition to sports, experimental probability is often used in banking, insurance, weather forecasting, and business.

8. **Quality Assurance** A piston manufacturer is concerned with the likelihood of defects, as this affects costs and profits. The manufacturer inspects 250 pistons and finds that 8 have defects.
- What is the probability a piston will have a defect? Express the probability as a percent.
  - If the same manufacturer produces 3000 pistons, about how many will likely have defects?
- c. **Evaluate** Pistons sell for \$35 and it costs \$25 in materials to make each piston. How much profit would the manufacturer likely make on 3000 pistons if defective ones cannot be sold?

#### Hint

Manufacturing costs must be paid for all pistons made, but only the ones that pass inspection can be sold.

A **random event** is an event whose outcome cannot be predicted. For example, drawing a card labeled 8 from a bin of cards, each labeled with a number from 1 to 100, represents a random event. An experiment could be conducted to determine the experimental probability of drawing a card labeled 8, however, it is not always practical to conduct an experiment to determine an experimental probability. In some instances it makes sense to perform a **simulation** of a random event using models such as number cubes, spinners, coins, or random number generators.

#### Graphing Calculator

For help with generating random numbers, see Graphing Calculator Lab 1 on page 52.



#### Exploration Using a Simulation to Find Probabilities

Saxon O's cereal is having a contest. Each box of cereal contains a prize piece and claims that 1 in 8 pieces is a winner. Conduct a simulation to determine the experimental probability of winning a prize piece within 50 boxes of cereal.

To simulate this problem, use the digits 1 through 8, with 1 representing a winning prize piece. Use your calculator to generate 50 random numbers.



9. According to your simulation, what is the probability of winning a prize in the Saxon O's contest? Express your answer as a fraction and as a percent.



10. **Verify** How does your answer in problem 9 compare to the likelihood stated on the cereal box?

## Investigation Practice

Describe each of the following events as impossible, unlikely, as likely as not, likely, or certain.

- a. Gavin rolls an even number on a number cube.
- b. In the northern hemisphere, the temperature will get above 90°F in the month of July.
- c. The first person that Sonya meets is a left-handed person.
- d. A player with a batting average of .875 gets a hit on his next at bat.

Jamie spun a game spinner and recorded the results in the table.

Outcome	Frequency
A	9
B	6
C	10

- e. What is the probability of landing on A? on B? on C? Express each probability as a fraction and as a percent.
- f. **Predict** Which letter will Jamie most likely spin? Explain your reasoning.
- g. **Sports** If a baseball player has 18 hits in 50 at bats, what is the probability that he will get a hit in his next at bat? Express your answer as a decimal number in the thousandths place.



- h. According to a survey at Johnson High School, 1 in 4 students has a part-time job. Conduct a simulation to determine the experimental probability of a student having a part-time job in a random group of 25 students.