Coevolution Tutorial

CEC 2005

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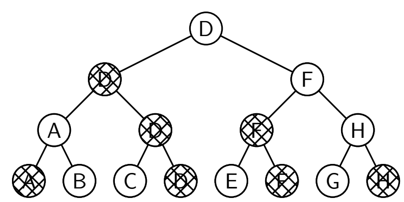
Outline

Definitions

- Coevolutionary Algorithm (CoEA) - an evolutionary algorithm (EA) in which evaluation is based on interactions between individuals.
  - Changes in the set of individuals used for evaluation can affect the ranking of individuals.
- Coevolutionary Computation (CoEC) - the subfield of computer science that is concerned with the study of CoEAs and their application to problem solving (predominantly) and modeling.
  - Encompasses both the algorithms and the problems they are applied to.

Examples

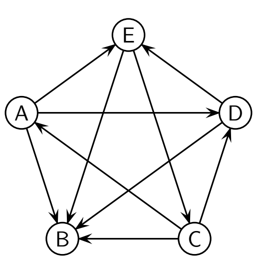
One population, single-elimination tournament



Individual	Fitness
D	4
F	3
A, H	2
B, C, E, G	1

Examples

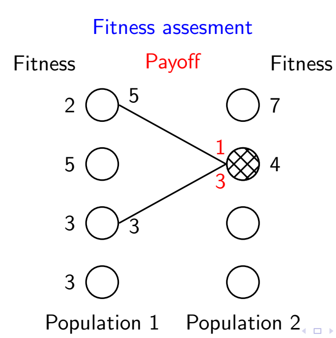
One population, full mixing



Individual	Fitness
A	1
B	4
C	1
D	2
D	2

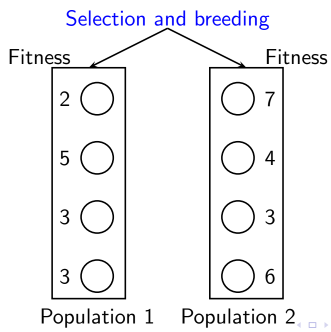
Examples

Two populations, random interacting individuals



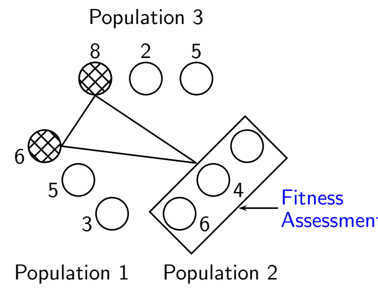
Examples

Two populations, random interacting individuals



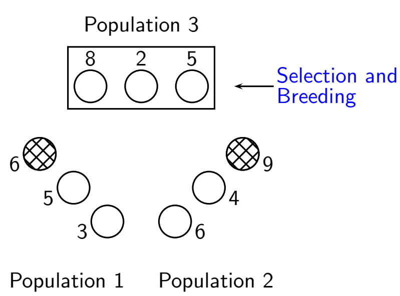
Examples

Three populations, cooperative



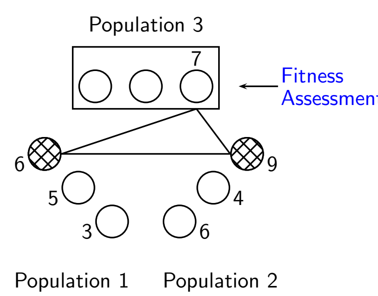
Examples

Three populations, cooperative



Examples

Three populations, cooperative



Motivation

- Simulation of processes from nature
- Problem decomposition for more efficient problem solving

Motivation

- Tackling domains in which performance of a potential solution can only be expressed by interaction with other potential solutions
  - A fitness function not based on interactions is extremely difficult / impossible to construct
  - Using the complete set of interactants is impossible
  - Using a large set of interactants is computationally expensive
  - Using a small fixed set of interactants is prone to overfitting or may not provide gradient
  - Using a small random set generally yields poor results due to sampling error

Motivation

- Domains where CoEAs may improve performance by:
  - Providing a dynamic gradient
  - Helping maintain diversity
  - Incrementally building complexity

Challenges

- Difficult to setup in order to obtain desired results
- Complex and sometimes unintuitive behaviors

Solution Concept

Solution Concept

A criterion specifying which locations in the search space are solutions and which are not (Ficici 2004).

- Engineers have a solution concept in mind when they apply an algorithm to a problem
- Algorithms *induce* a solution concept, by intention or not
  - Dynamically speaking, algorithms tend to be drawn to, or focus on, particular areas of a search space
- Key is to match algorithm's solution concept to the intended solution concept

★ Many coevolutionary algorithms fail because these are mismatched

Solution Concepts in Coevolution

Question:

What solution concept is implemented by traditional CoEAs?

- It is especially important in coevolution to have a well-defined notion of solution concept, but often there is none
- Recently, there have been several useful & well-defined coevolutionary solution concepts published:
  - Ideal Collaboration/Partnership (Wiegand 2004)
  - Maximum average/expected/cumulative payoff (Ficici 2005; de Jong 2005)
  - Nash equilibrium (Ficici & Pollack 2003)
  - Pareto-optimal set (de Jong & Pollack 2004)

Properties of Coevolutionary Problems

Question:

Are problems *inherently* coevolutionary?

Some problem properties lend themselves more toward coevolutionary systems than others:

- Underlying objectives
- Game-theoretic rewards
- Roles of problem elements

Underlying Objectives

Underlying Objective

An indicator of quality returning an element from an ordered set of scalar values, such as a real number. For any [coevolutionary] problem, a set of underlying objectives exists such that knowledge of the objective values of [a candidate provides as sufficient search information as] the outcomes of all possible [interactions] (de Jong 2004).

- Many coevolutionary problems have multiple underlying objectives
- CoEAs typically do not have direct access to these, but must infer them by testing different interactions
- Watson & Pollack (2001) describe a simple substrate for demonstrating problems with different underlying objectives
- Bucci *et al.* (2004) describe an algorithm that approximates a minimal set of underlying objectives

Game-Theoretic Rewards

One view of coevolutionary problems is one in which game-theoretic interactions are encoded directly into the problem

- Goal is to learn good players of some game
- Reward function of game is given as part of the problem
- Reward function may have various properties, e.g.:
  - Constant-sum / Variable-sum
  - Symmetry of reward
  - Other characteristics of player payoffs

Roles of Problem Elements

In coevolution, the role of elements of the problem may differ depending on the solution concept

- Object of learning/optimization may differ
  - Solving the problem may involve learning a variety of things
  - Important to understand which problem elements constitute candidate solutions and which serve supporting roles
- Different roles may be:

  Symmetric — Learn players for a game
  Asymmetric — Optimize sorters with challenging data

Categorizing Coevolutionary Problems

There are many ways to categorize coevolutionary problems based on their properties. For example:

Game-Theoretic Rewards: Use GT properties to classify problems based on the payoff from individual interactions

Roles of Problem Elements: Use the role of the solution to classify problems as *test-based* or *compositional*

Payoff from Individual Interactions

Competitive Interactions:

- *Competitor*: one of two or more striving to reach or obtain something that only one can possess (`www.m-w.com`)
- Example: constant-sum game (sufficient but not necessary)
- Can be approached with either one- or two-population algorithms
- More than two populations raises interesting denotational issues (e.g., "The enemy of my enemy is my friend.")

Competitive **problem** payoff, not competitive **algorithm**, because:

- Fitness is a function of payoffs from multiple interactions!
- Increase in (best/average) fitness values in one population **may** or **may not** cause decrease in fitness values in the opposite population

Payoff from Individual Interactions

Cooperative Interactions:

- To *cooperate*: to associate or act together with another or others for mutual benefit (`www.m-w.com`)
- Example: variable-sum game with symmetric payoff (sufficient but not necessary)
- Often useful problem decompositions lead to cooperative interactions among components

Payoff from Individual Interactions

Mixed Interactions:

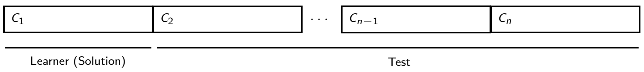
- Interactions can be a mixture of both competitive and cooperative rewards
- The majority of games are of this type
- Example: iterated prisoner's dilemma (Axelrod 1989)

Summary

- Constant-sum games are competitive
- Variable-sum games with symmetric payoff are cooperative
- Variable-sum games with asymmetric payoff can be any

Test-Based vs. Compositional

Let $C_1, C_2, ..., C_n$ be the n classes of elements involved in a problem specification



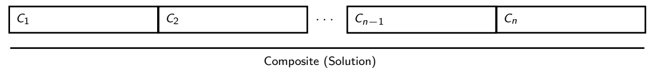
Test-based — *Solutions* specify elements from a single class

- Example: Sorting networks and challenging data
- Recall: Object of learning may differ
  - Some problem elements constitute candidate solutions and some serve supporting roles
  - Here, one class designated as the *learner* (or *candidate*), other class(es) designated as *tests* for the learner
  - For some test-based problems, roles can change during learning

Historically, called "competitive coevolution"

Test-Based vs. Compositional

Let $C_1, C_2, ..., C_n$ be the n classes of elements involved in a problem specification



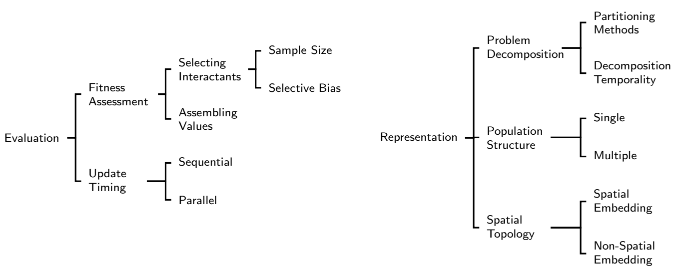
Compositional — *Solution* composite of elements from all classes

- Example: Team of cooperating agents
- Here, all elements of the problem serve a direct role in the assembled solution

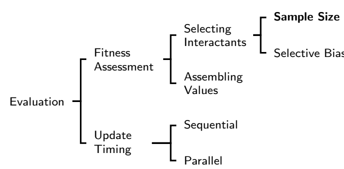
Historically, called "cooperative coevolution"

Mixed — *Solutions* specify elements from a subset of classes

Categorizing CoEAs



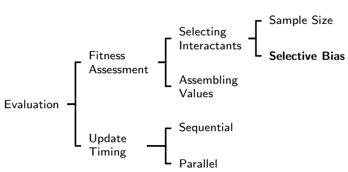
Categorizing CoEAs



- Same for all individuals
  - One
  - Few
  - All — Full mixing
- Variable
  - Single-elimination tournament

⋆ Popovici & De Jong 2005 AAAIFS; Panait & Luke 2002; Wiegand *et al.* 2001; Bull 2001; Bull 1997; Angeline & Pollack 1993

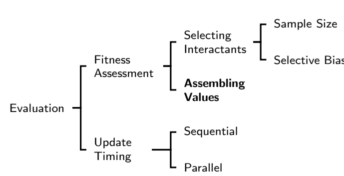
Categorizing CoEAs



- Random
- Fitness Based, e.g.:
  - *single-best collaboration* (Potter 1997)
  - *last elite opponent* (Sims 1994)
- Mixed
- Neighborhood (spatial embedding)
- Memory based, e.g.:
  - Hall of fame (Rosin & Belew 1997)

⋆ Popovici & De Jong 2005 AAAIFS; Wiegand *et al.* 2001

Categorizing CoEAs



- Single value
  - Best (mainly in traditional compositional settings)
  - Average (mainly in traditional test-based settings)
  - Competitive fitness sharing (Rosin & Belew 1995)
- n-uplu
  - Requires multi-objective-like selection methods (Ficici & Pollack 2001)

Categorizing CoEAs

- Evaluation
  - Fitness Assessment
    - Selecting Interactants
      - Sample Size
      - Selective Bias
    - Assembling Values
  - **Update Timing**
    - Sequential
    - Parallel

- Differences more complicated than might be expected; more than just separability is at stake (Jansen & Wiegand 2004)

Categorizing CoEAs

- Problem separability (Jansen & Wiegand 2004)
- Inter-population epistasis (Wiegand 2004; Bull 2001)
- Decompositional bias: matching decomposition of the representation to problem separability (Wiegand *et al.*)

- Representation
  - Problem Decomposition
    - **Partitioning Methods**
    - Decomposition Temporality
  - Population Structure
    - Single
    - Multiple
  - Spatial Topology
    - Spatial Embedding
    - Non-Spatial Embedding

Categorizing CoEAs

- Static (Potter 1997)
- Dynamic (Potter & De Jong 2000)
- Adaptive

- Representation
  - Problem Decomposition
    - Partitioning Methods
    - **Decomposition Temporality**
  - Population Structure
    - Single
    - Multiple
  - Spatial Topology
    - Spatial Embedding
    - Non-Spatial Embedding

Categorizing CoEAs

- In "symmetric" domains (i.e., with a single role) need to make a choice between one or two populations

- Representation
  - Problem Decomposition
    - Partitioning Methods
    - Decomposition Temporality
  - **Population Structure**
    - Single
    - Multiple
  - Spatial Topology
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    - Non-Spatial Embedding

Categorizing CoEAs

- Specialized choices about interaction methods and selection / survival
- Historically successful (Pagie1999;Cliff & Miller 1995; Hillis 1991)
- Helps maintain diversity of potential interactions (Williams & Mitchell 2005; Wiegand & Sarma 2004)

- Representation
  - Problem Decomposition
    - Partitioning Methods
    - Decomposition Temporality
  - Population Structure
    - Single
    - Multiple
  - **Spatial Topology**
    - Spatial Embedding
    - Non-Spatial Embedding

Why Study Dynamics?

- Coevolution seems a natural, highly adaptive search method for certain kinds of problems.
- Unfortunately, coevolutionary algorithms often disappoint engineers with poor performance and/or counterintuitive behavior
- Even modifications of the algorithms often lead to counterintuitive response on certain problems

⋆ Analyzing the dynamics (i.e., run-time behaviors) is key to understanding coevolution

Relativity & Its Effects

- The Red Queen — a character in Lewis Carroll's "Through the Looking Glass"
  - Imported first in biology and then in CoEC and interpreted in many different ways
  - A metaphor for relativity — the main feature of co-evolution: (internal) fitness is subjective
- Relativity's hope: to have an algorithm achieve a certain goal without specifically encoding it internally into fitness
- Relativity's caveats:
  - Generates intricate run-time behaviors
  - Makes it difficult to monitor progress towards the goal

E.g.: Wiegand 2004; de Jong & Pollack 2004; Watson & Pollack 2001; Pagie & Hogeweg 2000; Cliff & Miller 1995; Ridley 1993

Instrumenting CoEAs

- To understand how CoEA behaviors, many have attempted to measure their progress
- Metrics for the quality of individuals:
  - Contextual Dependence
    - subjective — the quality of an individual depends on the context in which it is evaluated
    - objective — context independent
  - Influence on the algorithm
    - internal — its values are used by the algorithm and influence its course
    - external — not internal

Instrumenting CoEAs

- Metrics for the quality of individuals
  - Traditional EA — internal objective metric as fitness.
  - CoEA — internal subjective metric as fitness; the context is a set of other evolving individuals.
  - Performance towards the goal — should be measured with an objective metric (in co-evolution, this will be external, as the internal measure is always subjective).

(Popovici & De Jong 2005 CEC; Watson & Pollack 2001)

Instrumenting CoEAs

- Metrics for the quality of individuals (Cont')
  - Other external metrics — designed to understand how an algorithm is functioning, but do not necessarily say anything about progress towards the goal.
    - External objective metrics
      - Ficici & Pollack 1998 — order theory
    - External subjective metrics:
      - Bader-Natal & Pollack 2004 — all of gen. ancestor contests
      - Stanley & Miikkulainen 2002 — dominance tournament
      - Funes & Pollack 2000 — information theory
      - Floreano & Nolfi 1997 — master tournament
      - Cliff & Miller 1995 — CIAO
  - Monitor best individual value / whole population average
  - Observe trends: increasing, decreasing, stagnating, noisy, repeated values, etc.

Instrumenting CoEAs

- Monitoring genotypic / phenotypic change
  - Examples:
    - Elite bitmaps
    - Ancestral Hamming maps (Cliff & Miller 1995)
    - Best-of-gen space trajectories (Popovici & De Jong 2004-5)
    - Trajectories for percentage of best in population (Wiegand 2004)
    - Dynamical system cob-web plots (Ficici *et al.* 2000)
  - Monitor best individual / whole population
  - Observe trends: cycling, converging, diverging, chaotic, etc.

Arms Race

Arms Race
Any competition where there is no absolute goal, only the relative goal of staying ahead of the other competitors (Wikipedia)

- Ideally, we hope that coevolution progresses by making parallel adaptive changes in responding strategies
- Example: Sorting networks learn to sort easy data, but the data learns to be more challenging, so the sorting network must learn to sort harder data, but the data ...
- In CoEC, we hope arms race behaviors produce a continuous increase in external objective progress toward the goal
- Unfortunately, there are several pathological dynamics that prevent arms race behaviors...

Cycling

- Visiting the same areas of the space multiple times in a repeated sequence
- Traced by observing changes in individuals at the genotypic / phenotypic level
- Generates some repeating patterns in the internal / external metrics
- Presumed related to intransitivities in the problem's definition

⋆ Popovici & De Jong 2005 CEC; Watson & Pollack 2001; Nolfi & Floreano 1997; Ficici & Pollack 1998; Juillé & Pollack 1998; Paredis 1997; Cliff & Miller 1995

Mediocre Stable States

- The external metric is "trapped" in a suboptimal value or set of values
- There may still be genotypic change occurring (e.g., cyclic or chaotic)
- Suggestions to replace the term with more specific ones

⋆ Ficici & Pollack 1998

Lack (Loss) of Gradient

- The distribution of fitness values in (at least) one population is (almost) flat
- Example: tests are too tough for learners; no learner solves any test
- May or may not be accompanied / caused by lack of diversity
- May be temporary (gradient may be regained due to changes in either population) or persistent
- Can cause genetic drift

⋆ Bucci *et al.* 2004; Wiegand & Sarma 2004; Watson & Pollack 2001; Juillé & Pollack 1998

Overspecialization

- Individuals improve on some of the underlying objectives but fail to do so on others
  - May be due to lack of representatives of those latter objectives
- Example: players discovering an opponents weaknesses and exploiting them but failing to learn the task in a general way
- Hard to detect

⋆ Bucci *et al.* 2004; Watson & Pollack 2001

Relative Overgeneralization

- Components that perform well in combination with a large number of other components are favored over components that perform very well (optimally) in combination with a small number of other components but poorly otherwise
- It is more likely to occur when the fitness results from averaging payoffs from many sampled interactions
- Depending on the goal, it can be good (e.g. when robustness, good cumulative / average performance is desired) or bad (when optimal possible payoff is desired)

⋆ Wiegand 2004

Monotonicity

- A good CoEA consistently refines solutions to the problem
- Ideally, we want algorithms that make *monotonic progress* toward the goal
- Monotonic solution concepts (Ficici 2005):
  - Use the solution concept to develop a preference relation over search space, creating a partial-ordering
  - A monotonic search process produces solution estimates that never contradict the partial ordering
  - A monotonic solution concept *guarantees* such operation
  - Algorithms that implement such solution concepts monotonically approach the solution
  - Example: Nash solution concept

Monotonicity

- Monotonic CoEAs and ideal evaluation (de Jong 2004):
  - Coevolution selects the tests used in evaluation adaptively
  - Ideal evaluation is one that uses a small set of tests but is equivalent to evaluating against all tests
  - At every point in time, only the relations between current candidate solutions need to be evaluated
  - A set of tests that reveal all relations between the existing candidate solutions is suficient.
  - Example: DELPHI
- Monotonic CoEAs and archives (de Jong 2005):
  - Monotonic CoEAs achieve reliable progress by using an archive of tests that guarantee monotonic progress
  - There is a distance function for which the distance to the solution concept decreases with every change to the archive
  - Example: MaxSolve

Empirical Analysis

- Component analysis: algorithm properties + problem properties → performance
  - Methods of interaction + cross-population epistasis in cooperative setups
    Bull 2001; Wiegand *et al.* 2001; Bull 1997; Potter 1997
  - Methods of interaction + noise in competitive setups
    Panait & Luke 2002
  - Methods of interaction + decompositional bias
    Wiegand *et al.* 2003
  - Different algorithms + asymmetry
    Olsson 2001
  - Different algorithms + intransitivity
    de Jong 2004
  - Memory & diversity maintenance + cycling
    Rosin & Belew 1997

Empirical Analysis

- Dynamics analysis
  - Run-time properties in isolation
    Cliff & Miller 1995; Stanley & Miikkulainen 2002
  - Run-time properties → performance
    Pagie & Mitchell 2002; Juillé & Pollack 1998
  - Algorithm properties → run-time properties
    Ficici & Pollack 2000
  - Problem properties → run-time properties → performance
    Watson &Pollack 2001
  - Algorithm properties + problem properties → run-time properties → performance
    Popovici & De Jong 2005 CEC; Popovici & De Jong 2005 AAAIFS

Theoretical Analysis

- Asymptotic run time analysis of the algorithms
  Jansen & Wiegand 2004;
- EGT, Dynamical systems analysis, & Markov Models
  Wiegand 2004; Ficici 2004; Schmitt 2003; Liekens 2002
- Category / Order theory
  Ficici 2004; Bucci *et al.* 2004
- Multiobjective techniques & Pareto dominance
  de Jong 2004
- Random walk & intransitivity
  Funes & Pujals 2005

Conclusions

- Coevolution can be useful for certain kinds of problems
- It can also behave in complex and counterintuitive ways
- The key to successful applications of coevolution revolves around the solution concept:

Understand your problem: What do you want to achieve?
Understand your algorithm: To what types of solutions is it drawn?
Match the two: Use the right tool for the right job

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