

How to find the domain of a function?

Let $y = f(x)$ be a function with an independent variable x and a dependent variable y .

If a function f provides a way to successfully produce a single value y using for that purpose a value for x then that chosen x -value is said to belong to the domain of f . If there is a requirement that a y -value produced by a function must be a real number, the following conditions are commonly checked:

1. **Denominators cannot equal 0.**
2. **Radicands (expressions under a radical symbol) of even roots (square roots, etc) cannot have a negative value.**
3. **Logarithms can only be taken of positive values.**
4. **In word problems physical or other real-life restrictions might be imposed, e.g. time is nonnegative, number of items is a nonnegative integer, etc.**

Examples

$$f(x) = \frac{1}{x}$$

(1) applies. Denominator = $x \neq 0$.

Domain = $\{x \mid x \neq 0\}$

$$f(x) = \frac{x}{(x-1)(x+3)}$$

(1) applies. Denominator = $(x-1)(x+3) \neq 0$. **Domain = $\{x \mid x \neq 1 \text{ and } x \neq -3\}$**

$$f(x) = \frac{x-1}{\sqrt{2-x}}$$

(1) and (2) apply: $2-x > 0$.

Domain = $\{x \mid x < 2\} = (-\infty, 2)$

$$f(x) = \frac{1}{\log(x^2-1)}$$

(1) applies: $\log(x^2-1) \neq 0$ so $x^2-1 \neq 1$ or $x \neq \pm\sqrt{2}$

(3) applies: $x^2-1 > 0 \rightarrow x^2 > 1 \rightarrow |x| > 1 \rightarrow x < -1 \text{ or } x > 1$

Combining (1) and (3) results in:

Domain = $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, -1) \cup (1, \sqrt{2}) \cup (\sqrt{2}, \infty)$

$f(x)$ = the age of the oldest person in a group of x people. (4) applies: **Domain = $\{x \text{ is an integer} \mid x > 0\}$**

Domains of selected function types

Polynomial

R = All real numbers

Rational

All real numbers except zeros of denominator

Absolute Value

R = All real numbers

Root $\sqrt[n]{x}$

R = All real numbers if n is odd OR All nonnegative real numbers if n is even

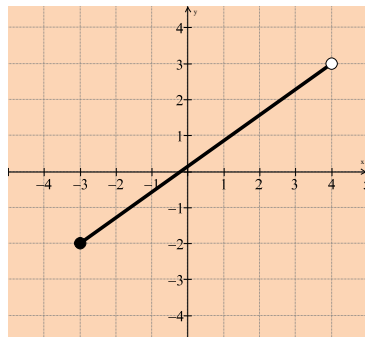
Exponential

R = All real numbers

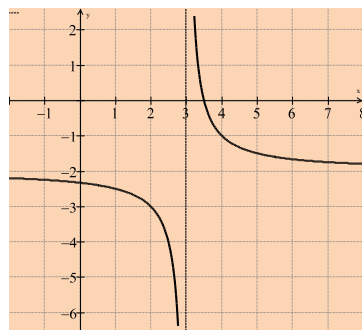
Logarithmic

All positive real numbers

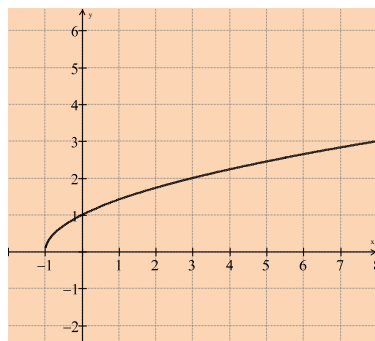
Finding domains from graphs



Domain = $\{x \mid -3 \leq x < 4\} = [-3, 4)$



Domain = $\{x \mid x \neq 3\}$



Domain = $\{x \mid x \geq -1\}$