上海 Shanghai American School
An International Community

Puxi High School Examinations
Semester 1, 2009 – 2010

IB Mathematics (HL)

Year 1

Paper 1

Wednesday, December 16th, 2009
12:45 pm – 3:15 pm

Time: 1 hour 15 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:_____

Instructions to the Candidate

- No food or drink to be brought into examination room.
- No cell phones/iPods are allowed during the examination at any time. You will be dismissed from the testing site if you are seen with one out. Please do not talk during the examination.
- If you have a problem please raise your hand and wait quietly for a teacher.
- Please do not open the examination booklet until directed to do so.
- Please ensure that you have the correct examination in front of you.
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- At the conclusion of your examination please refrain from speaking until you are outside the exam room as there may still be other examinations still in progress.
- Students are reminded that they are not permitted to leave the examination room early.

Special Instructions:

- Graphing Calculators NOT allowed.
- Be sure to answer ALL questions.
- Paper 1 has **8 pages** including the cover page. 100 points total.

  Section A: 9 short-response questions, 7 points each.
  Section B: 2 longer-response questions.

Paper 1—No calculator (1 hour, 15 minutes; 100 points total)

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations.

Section A: Short-response questions (9 questions, 7 points each)

1. Find the coefficient of x^7 in the expansion of $(3+2x)^{10}$, giving your answer as a whole number.

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2. An arithmetic sequence has 5 and 21 as its first and third terms, respectively.

   (a) Write down, in terms of n, an expression for the nth term, a_n.

   (b) Find the number of terms of the sequence which are less than 400.

3. A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

   (a) the common ratio;

   (b) the first term.

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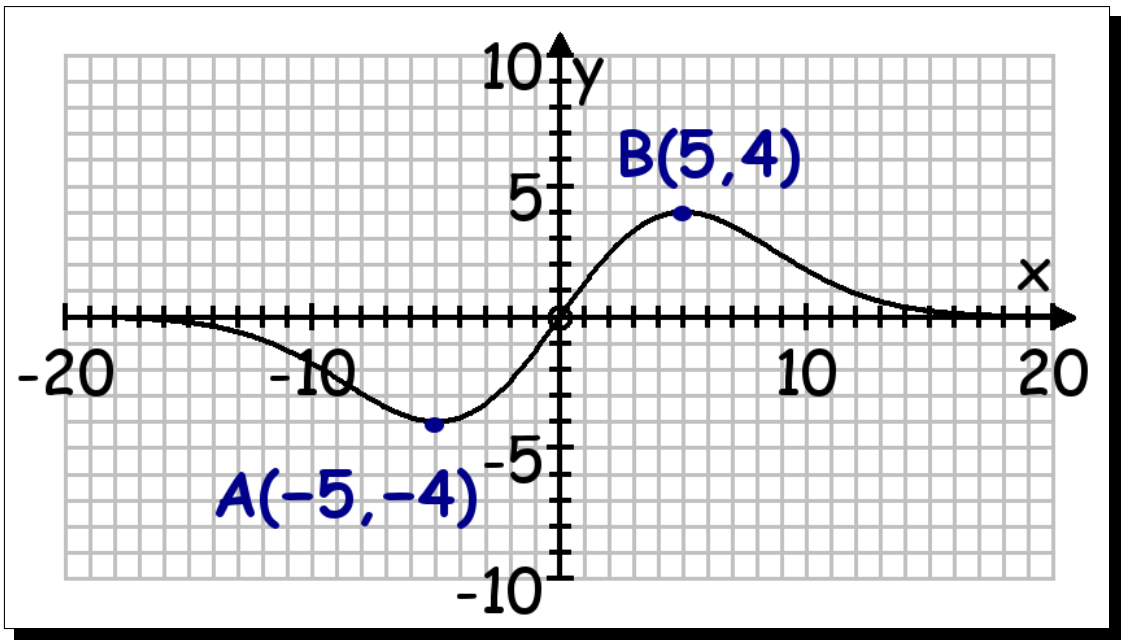
4. The diagram shows the graph of $y = f(x)$, with the x-axis as an asymptote.



   (a) On the same axes, draw the graph of $y = f(x+2) - 3$, indicating the coordinates of the images of the points A and B.

   (b) Write down the equation of the asymptote to the graph of $y = f(x+2) - 3$.

5. Let the complex number z be given by

$$z = 1 + \frac{i}{i - \sqrt{3}}.$$

Express z in the form $a + bi$, giving the **exact** values of the real constants a, b.

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6. Solve the inequality $|x - 2| \geq |2x + 1|$.

7. (a) Express $2\cos^2 x + \sin x$ in terms of $\sin x$ only.

   (b) Solve the equation $2\cos^2 x + \sin x = 2$ for x in the interval $0 \leq x \leq \pi$, giving your answers exactly.

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8. You are give the quadratic function $q(x) = 1 + 2x - x^2$.

   (a) Find the integer a such the slope of the line ℓ tangent to the graph of $y = q(x)$ at the point $(a, q(a))$ is -2.

   (b) Find the equation of ℓ and determine the x-intercept of ℓ.

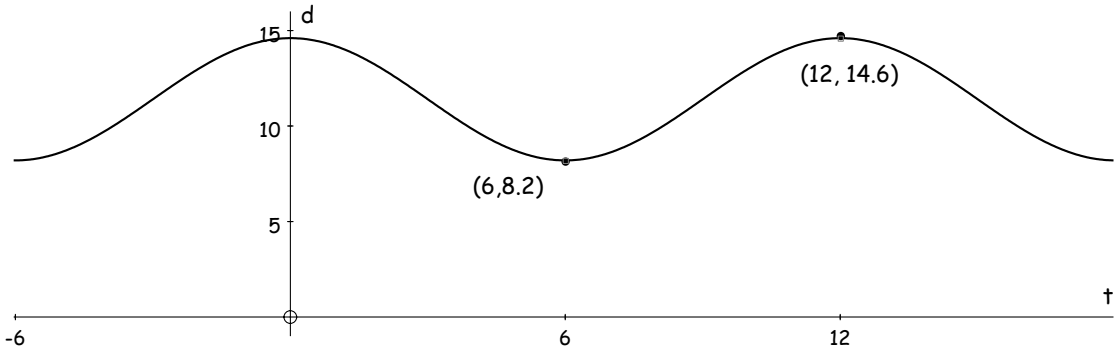
9. Given the quadratic function $q(x) = x^2 + (2-k)x + k^2$, determine the range of values of k for which $q(x) > 0$ for all real values of x.

Section B. Extended-response questions.

10. A formula for the depth d meters of water in a harbor at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point $(6, 8.2)$ is a minimum point and the point $(12, 14.6)$ is a maximum point.



(a) Find the value of

(i) **(3 points)** Q;

(ii) **(3 points)** P.

(b) **(6 points)** Find the **first** time in the 24-hour period when the depth of the water is 9.8 meters.

(c) (i) **(5 points)** Use the symmetry of the graph to find the **next** time when the depth of the water is 9.8 meters.

(ii) **(4 points)** Hence find the time intervals in the 24-hour period during which the water is less than 9.8 meters. deep.

11. You are given the function $f(x) = x^3 - 2x^2 - 11x + 12$.

(a) **(4 points)** Verify that this function has zeros $x = -3, 1, 4$

(b) **(6 points)** Compute the equation of the line ℓ tangent to the graph of $y = f(x)$ where $x = 1$.

(c) **(6 points)** Determine any additional points of intersection of ℓ with the graph of $y = f(x)$.

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Time: 1 hour 15 minutes

Teacher: Mr. Surowski

Testing Site: HS Gymnasium

Student Name:_____

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Special Instructions:

- Graphing Calculators are allowed.
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- Paper 1 has **9 pages** including the cover page. 100 points total.

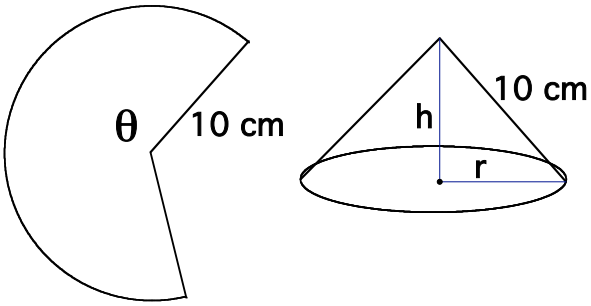
  Section A: 9 short-response questions, 7 points each.
  Section B: 2 longer-response questions.

Paper 2—Calculators Allowed (1 hour, 15 minutes; 100 points total)

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g., if graphs are used to find a solution, you should sketch these as part of your answer. Unless otherwise noted, answers should be given as exact values or should be correct to 3 significant figures.

Section A: Short response questions 9 questions, 7 points each)

1. The diagram to the right shows a circular sector of radius 10 cm and angle θ radians which is formed into a cone of slant height 10 cm. The vertical height h of the cone is equal to the radius r of its base. Find the angle θ radians.



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2. Solve the inequality $x^2 + \ln x < 9$.

3. The function f is defined on the domain $[0, \pi]$ by setting $f(\theta) = 4\cos\theta + 3\sin\theta$.

   (a) Express $f(\theta)$ in the form $R\cos(\theta - \alpha)$, where $0 < \alpha < \frac{\pi}{2}$.

   (b) Hence, or otherwise, write down the value of θ for which $f(\theta)$ takes on its maximum value.

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4. A jar contains 20 marbles: 8 are red, 6 are black, 4 are blue, and 2 are white. In how many ways can 6 marbles be simultaneously drawn from the jar and have at least 4 be black and none be white?

5. The polynomial $x^2 - 4x + 3$ is a factor of $x^3 + (a-4)x^2 + (3-4a)x + 3$. Calculate the value of the constant a.

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6. You are given the function $g(x) = \dfrac{2\cos^2 x - 1}{x}$

   (a) How many solutions of $g(x) = 0$ are there on the interval $]0, 4\pi]$?

   (b) Find the smallest solution of $g(x) = 0$ on the interval $]0, 4\pi]$.

7. Show that the solutions of the equation

$$x = 1 + \cfrac{1}{2 + \cfrac{1}{x}}$$

are given by $x = \sin(\pi/6) \pm \cos(\pi/6)$.

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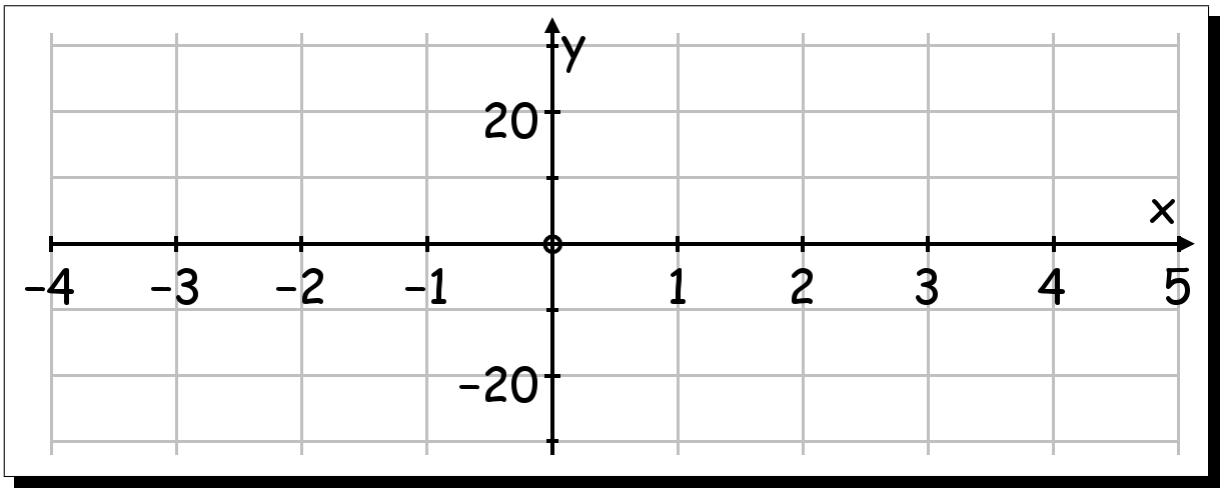
8. You are given the function $f(x) = x^2 - 4x + 5,\ x \leq 2$. Determine the inverse function f^{-1} explicitly, giving also its **domain** and **range**.

9. Consider the polynomial function $p(x) = x^3 - x^2 - 12x$. Determine those values of x for which the graph of $y = p(x)$ has a horizontal tangent line at the point (x, y). Graph this all on the coordinate axes given below.



Section B. Extended-response questions.

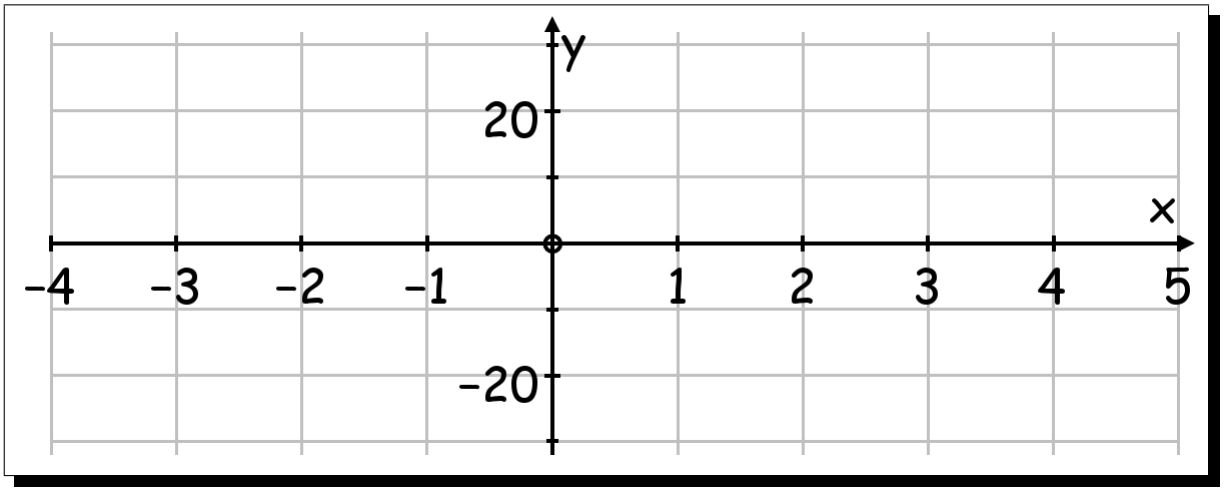
10. Consider the polynomial function $p(x) = x^3 - x^2 - 12x$.

    (a) **(3 points)** Determine the zeros of $p(x)$.

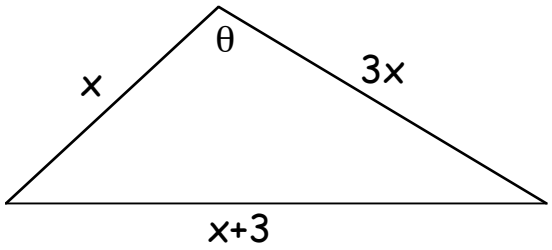
    (b) **(5 points)** Let a be the average of the non-negative zeros of $p(x)$. Find the equation of the line ℓ tangent to the graph of $y = p(x)$ where $x = a$.

    (c) **(5 points)** Show that the line ℓ computed in (b) passes through the point $(b, 0)$, where b is the unique negative zero of $p(x)$.

    (d) **(4 points)** Graph all of this activity below.



11. The area of the triangle shown below is 2.21 cm^2. The length of the shortest side is x cm and the other two sides are $3x$ and $(x+3)$ cm.



(a) **(4 points)** Using the formula for the area of the triangle, write down an expression for $\sin\theta$ in terms of x.

(b) **(4 points)** Using the cosine rule, write down and simplify an expression for $\cos\theta$ in terms of x.

(c) (i) **(4 points)** Using your answers to parts (a) and (b), show that

$$\left(\frac{3x^2-2x-3}{2x^2}\right)^2 = 1 - \left(\frac{4.42}{3x^2}\right)^2.$$

(ii) Hence find

(a) **(4 points)** the possible values of x;

(b) **(4 points)** the corresponding values of θ, **in radians**, using your answers to part (b), above.