MATH 150 PRELIMINARY NOTES 5

FUNCTIONS

Recall from your previous algebra classes that the set of all possible input values for a function is called the domain of the function, and the set of all output values for a function is called the range. Given y = f (x), the x variable is called the independent variable and represents an input value from the domain. The y variable is the dependent variable and represents the corresponding output value f (x) in the range.

DEFINITION: A function from a set of D to a set R is a rule that assigns a unique element f (x) in R to each element x in D.

Remember, for a relation to be a function, it must pass the vertical line test. The vertical line test states that for every value of x, there is exactly one y value.

DOMAINS

If we have defined a function y = f (x) with a formula and its domain is not stated explicitly, then we must assume that the domain is to be the largest set of x-values in which the formula gives real y-values. Some clues that we should look for when determining the domain is the following. **(1)** If the function has a denominator, then any value that makes the denominator zero cannot be included in the domain. **(2)** If the formula has an even root in it, then any value that makes the equation under the root sign negative cannot be included in the domain.

EXAMPLE 1: Find the domain for the following function.

$$f(x) = \sqrt{2x - 6}$$

SOLUTION: This function has an even root in it, so I will set 2x - 6 ≥ 0 and solve for x. These values will be the domain for this function.

$2x - 6 \geq 0 \rightarrow 2x \geq 6 \rightarrow x \geq 3$

The domain for this function is $[3, \infty)$.

EXAMPLE 2: Find the domain for the following function.

$$f(x) = \frac{1}{x^2 - 9}$$

SOLUTION: Since this function has a denominator, I will determine the x-values that make the denominator zero. These values will be excluded from the domain.

$x^2 - 9 = 0 \rightarrow (x - 3)(x + 3) = 0 \rightarrow x = \pm 3$

Therefore the domain for this function is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

EXAMPLE 3: Find the domain for the following function.

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

SOLUTION: Not only does this function have a denominator, but it also has an even root sign in it. So I will first determine what values of x make $4 - x^2$ equal zero.

$4 - x^2 = 0 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

These two values divide the number line up into three intervals, so I must determine which intervals produce positive values. The interval that does this will be the domain.

$(-\infty, -2)$	Let $x = -3$.	$4 - (-3)^2 = 4 - 9 = -5$	Negative
$(-2, 2)$	Let $x = 0$.	$4 - 0 = 4$	Positive
$(2, \infty)$	Let $x = 3$.	$4 - (3)^2 = 4 - 9 = -5$	Negative

So the domain for this function is the open interval $(-2, 2)$.

EXAMPLE 4: Find the domain for the function $f(x) = x^2 - 3x + 6$.

SOLUTION: Since this function does not have a denominator or contain an even root, then its domain is all real numbers, and is denoted by $(-\infty, \infty)$. Not only that, but this is a polynomial function, and the domain for all polynomial functions is $(-\infty, \infty)$.

FACT: The domain for all polynomial functions is $(-\infty, \infty)$.

EVEN FUNCTIONS AND ODD FUNCTIONS - SYMMETRY

DEFINITION: A function $y = f(x)$ is even if $f(-x) = f(x)$ for every number x in the domain of f.

If you are able to graph the function with your graphing calculator, then you can use the fact that an even function is symmetric with respect to the y-axis.

DEFINITION: A function $y = f(x)$ is odd if $f(-x) = -f(x)$ for every number x in the domain of f.

Again, an easy way to determine if a function is odd is to determine if the graph is symmetric with respect to the origin.

EXAMPLE 5: Determine whether the function is odd, even, or neither.

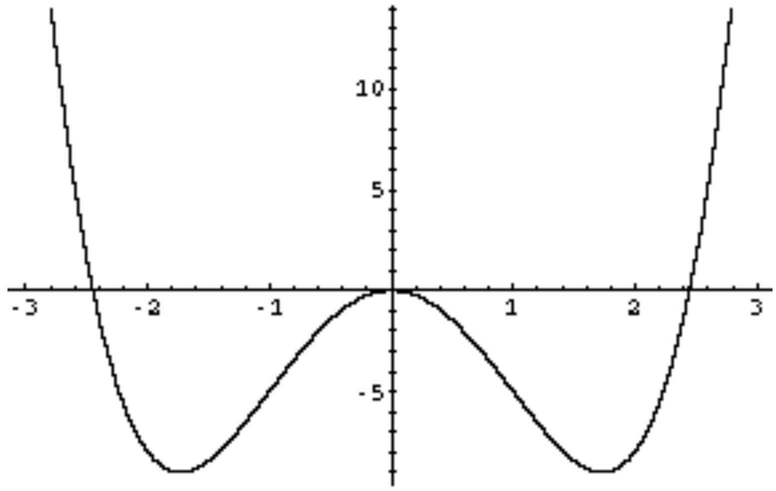
$$f(x) = x^4 - 6x^2$$

SOLUTION:

$f(-x) = (-x)^4 - 6(-x)^2 = x^4 - 6x^2 = f(x)$

Therefore, f (x) is an even function.

If you look at the graph of this function, you will notice that it is symmetric with respect to the y-axis. Therefore, f (x) is an even function.



$f(x) = x^4 - 6x^2$

EXAMPLE 6: Determine whether the function is odd, even, or neither.

$$f(x) = \frac{1}{x^3 + x}$$

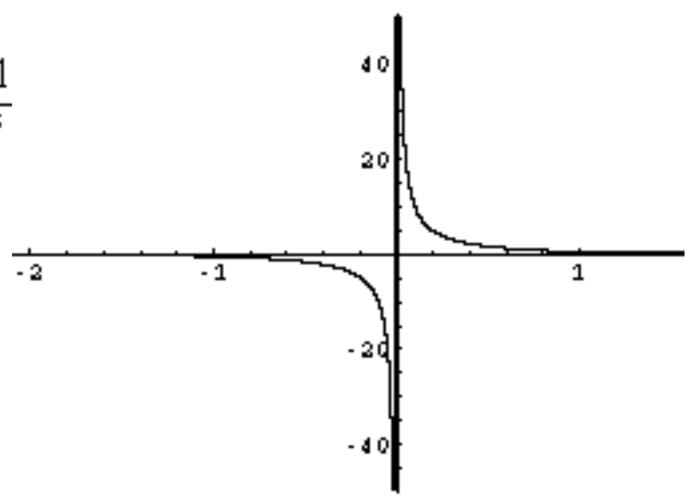
SOLUTION:

$$f(-x) = \frac{1}{(-x)^3 + (-x)} = \frac{1}{-x^3}$$

$$= \frac{1}{-(x^3 + x)} = -f(x)$$

Therefore, this function is odd.

If you look at the graph of this function, you should notice that it is symmetric with respect to the origin. Therefore, this function is odd.



$$f(x) = \frac{1}{x^3 + x}$$

EXAMPLE 7: Determine whether the function is odd, even, or neither.

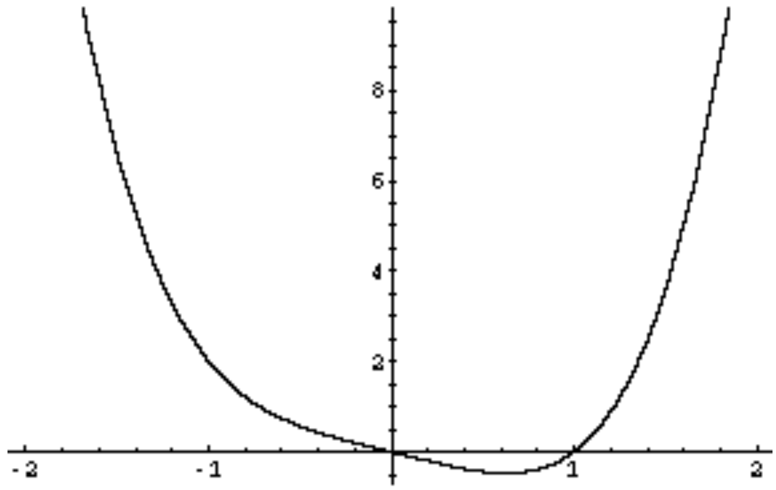
$f(x) = x^4 - x$

SOLUTION:

$f(-x) = (-x)^4 - (-x) = x^4 + x \neq f(x)$

This function is neither odd nor even.

If you look at the graph of this function, you should notice that it is not symmetric with respect to the y-axis or the origin. Therefore it is neither odd nor even.



$f(x) = x^4 - x$

SUM, DIFFERENCE, PRODUCTS, AND QUOTIENTS

Given two functions f and g and every x that belongs to the domains of both f and g, we can define new functions based on the four basic operations. Here are the new functions.

1. $(f + g)(x) = f(x) + g(x)$
2. $(f - g)(x) = f(x) - g(x)$
3. $(fg)(x) = f(x)\,g(x)$
4. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} \quad (g(x) \neq 0)$
5. If c is a constant, then $(cf)(x) = c\,f(x)$.

EXAMPLE 8: Given

$$f(x) = \sqrt{x+1} \quad \text{and} \quad g(x) = \sqrt{2-x},$$

find the domains of f, g, f + g, fg, and f/g.

SOLUTION:

$$f(x) = \sqrt{x+1} \qquad x+1 \geq 0 \;\rightarrow\; x \geq -1 \qquad D: \; [-1, \infty)$$

$$g(x) = \sqrt{2-x} \qquad 2-x \geq 0 \;\rightarrow\; 2 \geq x \qquad D: \; (-\infty, 2]$$

$$f + g = \sqrt{x+1} + \sqrt{2-x}$$

The domain of this function will be the interval where the two domains overlap.

Therefore the domain if the interval [-1, 2].

$$f \cdot g = \sqrt{(x+1)(2-x)}$$

Again, the domain will be the interval where the domains of f and g overlap, so it is the interval [-1, 2].

$$\frac{f}{g} = \frac{\sqrt{x+1}}{\sqrt{2-x}}$$

Again, the domain will be the overlapping interval, except since g is in the denominator, then $x \neq 2$. Therefore, the domain is [-1, 2).

COMPOSITE FUNCTIONS

If f and g are functions, then the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ consists of the numbers of x in the domain of g for which g (x) lies in the domain of f.

What this is saying is this- to find the domain of $f \circ g$, start with the domain of g and delete any x-values that are undefined in f.

EXAMPLE 9: Given

$$f(x) = x - 1 \quad \text{and} \quad g(x) = \frac{1}{x+2},$$

find (f ° g)(x) and the domains for f, g, and f ° g.

SOLUTION:

f (x) = x - 1 is a polynomial, so its domain is (-∞ , ∞).

$$g(x) = \frac{1}{x+2} \quad x \neq -2$$

So the domain for this function is (-∞ , -2) ∪ (-2, ∞).

$$(f \circ g)(x) = f(g(x)) = \frac{1}{x+2} - 1$$

Since g (x) is being plugged into f (x), then we will start with the domain of g and determine if there are any x-values that need to be deleted. The domain (-∞ , -2) ∪ (-2, ∞) works fine for the new function, so that is what the domain for f ° g is.

This set of preliminary notes contains three important topics from algebra. The first one was the concept of the domain of a function. This will be important when we talk about the continuity of a function, and start to look at the characteristics of a function. The next topic was the concept of odd and even functions. This concept will be useful later in the course when we start to talk about finding the area under a curve. The final topic was about creating new functions from two given functions. This concept will pop up now and then throughout the calculus sequence. Use these notes as a study guide, and if you have any questions on them, please feel free to contact me.

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