



### Introduction to SOLVING LINEAR EQUATIONS

When two "things" are equal they create an equation.

The statement "Bob is a nice guy" could be written as the equation  $\text{Bob} = \text{nice guy}$

Equations have three parts: the left side, the right side, and the equals sign.

The equals sign means that the left side and the right side are, simply, the same.

In algebra, an equation is created by equating two expressions. An expression is a combination of letters, numbers, and math symbols.

$3x + 5$  is a two termed expression.  $20$  is a single termed expression.

When we equate the expressions, we create an equation:  $3x + 5 = 20$

To solve an equation means to find the value for the variable that will make the original equation true. In short, to solve means to find a number that can be put where the letter is. When the arithmetic is completed, both sides of the equation will look the same.

Example,

In  $3x + 5 = 20$ , if  $x$  is 5  
we would obtain  $3(5) + 5 = 20$   
 $15 + 5 = 20$   
 $20 = 20$

There are a series of steps that must be followed to solve an equation.

### STEPS FOR SOLVING A LINEAR EQUATION

1. Combine like terms on either side of the equation before you try to solve it.
2. When the equation has the letter (the variable) on one side, move any term without a letter to the opposite side by "doing the opposite operation". What you do to the left, you must do to right (stand up, sit down, fight-fight-fight!)
3. Finally, you want "x" as the answer. You may need to either divide or perhaps multiply by the reciprocal to change the number in front of x to a 1.

Examples:

1. Solve the equation:

$$3x + 5 = 17$$

Solution:

Keep "3x" on the left.

Move 5 to right by doing the opposite.

Subtract "5".

$$\begin{array}{rcl} 3x + 5 - 5 & = & 17 - 5 \\ 3x & = & 12 \end{array}$$

We want "1x".

Undo "3 times x" by doing the opposite.

Divide both sides by "3" ( the number in front of x )

$$\begin{array}{rcl} (3x)/3 & = & (12)/3 \\ 1x & = & 4 \end{array}$$

The solution is " x equals four".

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2. Solve the equation:

$$11x + 2 + 4x - 3x - 6 + 2x - 6x = -8 + 1 + 4$$

Combine like terms on each side:

$$\begin{array}{rcl} 11x + 2 + 4x - 3x - 6 + 2x - 6x & = & -8 + 1 + 4 \\ 11x + 4x - 3x + 2x - 6x + 2 - 6 & = & -8 + 1 + 4 \\ 8x & -4 & = -3 \end{array}$$

Keep "8x" on the left. Move "-4" to the right by doing the opposite.

Add "4".

$$\begin{array}{rcl} 8x - 4 + 4 & = & -3 + 4 \\ 8x & = & 1 \end{array}$$

We want "1x", so divide both sides by "8".

$$\begin{array}{rcl} (8x)/8 & = & (1)/8 \\ 1x & = & 1/8 \end{array}$$

The answer is "x equals one-eighth."

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3. Solve the equation:

$$(8/5)x - (4/5)x + 1 = (7/25)$$

Keep "x" on the left.

First combine like terms using the arithmetic of fractions.

Recall that  $(8/5) - (4/5)$  is  $(4/5)$

$$\begin{array}{l} (8/5)x - (4/5)x + 1 = (7/25) \\ (4/5)x + 1 = (7/25) \end{array}$$

Move the "1" to the right by subtracting "1".

Subtraction is the opposite of addition.

$$\begin{array}{l} (4/5)x + 1 - 1 = (7/25) - 1 \\ (4/5)x = (7/25) - 1 \end{array}$$

Obtain a common denominator, so you can combine the  $(7/25)$  and the  $(-1)$

Remember that  $1 = (25/25)$

$$\begin{array}{l} (4/5)x = (7/25) - 1 \\ (4/5)x = (7/25) - (25/25) \end{array}$$

Now subtract  $(7/25)$  and  $(25/25)$

Remember:  $(7/25) - (25/25)$  is  $(7 - 25)/(25)$  which is  $(-18/25)$

$$\begin{array}{l} (4/5)x = (7/25) - (25/25) \\ (4/5)x = (-18/25) \end{array}$$

To obtain "1x", multiply both sides of the equation by the reciprocal of  $(4/5)$ ; the number next to "x".

The reciprocal of  $(4/5)$  is  $(5/4)$ .

$$\begin{array}{l} (4/5)x = (-18/25) \\ (5/4)(4/5)x = (5/4)(-18/25) \end{array}$$

Cancel common factors.

$$(\frac{5}{4})(\frac{4}{5}) = 1$$

$$\begin{aligned}(\frac{5}{4})(-\frac{18}{25}) &= (\frac{1}{4})(-\frac{18}{5}) && \text{Reduce factors of "5".}\\&= (\frac{1}{4})(-\frac{18}{5}) && \text{Now reduce factors of "2"}\\&= (\frac{1}{2})(-\frac{9}{5})\end{aligned}$$

Multiply fractions: top times top and bottom times bottom.

$$\begin{aligned}(\frac{1}{2})(-\frac{9}{5}) &= \frac{(1 * -9)}{(2 * 5)}\\&= \frac{-9}{10}\\&= -\frac{9}{10}\end{aligned}$$

So after all of the work with fractions, our equation,

$$(\frac{5}{4})(\frac{4}{5})x = (\frac{5}{4})(-\frac{18}{25})$$

has become

$$1x = -\frac{9}{10}$$

The solution is "x equals a negative one-ninth".

## STEPS FOR SOLVING A LINEAR EQUATION

**with letters on both sides of the equation.**

1. Combine like terms on either side of the equation before you try to solve it.

2. Decide on which side of the equation you want all of the terms with letters.

Do you want the letters on the left? If so, move the terms with letters to the left and move the constants to the right side. Remember to move terms by "doing the opposite operation".

3. Finally, you want 1x as the answer. You may need to either divide or perhaps multiply by the reciprocal to change the number in front of x to a 1.

1. Example:

Solve the equation:

$$1x + 5x - 3 + 1x = 14 + 3x + 2x$$

Combine like terms.

$$\begin{aligned}1x + 5x - 3 + 1x &= 14 + 3x + 2x\\1x + 5x + 1x - 3 &= 14 + 3x + 2x\\7x - 3 &= 14 + 5x\end{aligned}$$

Decide on which side to have the letters.

I choose "left" in this problem.

So I move the terms with "x" to the left and the constant terms to the right.

$$\begin{aligned}7x - 3 &= 14 + 5x \\7x - 5x - 3 &= 14 + 5x - 5x \\2x - 3 &= 14 \\2x - 3 + 3 &= 14 + 3 \\2x &= 17\end{aligned}$$

We want "1x". Divide both sides by "2" .

$$\begin{aligned}2x &= 17 \\(2x)/2 &= (17)/2 \\1x &= 17/2\end{aligned}$$

The solution to the equation is " x equals seventeen halves".

2. Example:

Solve the equation:

$$4(7x - 1) + 3(2 - 5x) = 4(3x + 5)$$

Combine like terms (after you use the distributive law).

$$\begin{aligned}4(7x - 1) + 3(2 - 5x) &= 4(3x + 5) \\4(7x) + 4(-1) + 3(2) + 3(-5x) &= 4(3x) + 4(5) \\28x - 4 + 6 - 15x &= 12x + 20 \\28x - 15x - 4 + 6 &= 12x + 20 \\13x + 2 &= 12x + 20\end{aligned}$$

Decide on which side to have the letters.

I choose "left" in this problem.

So I move the terms with "x" to the left and the constant terms to the right.

$$\begin{aligned}13x + 2 &= 12x + 20 \\13x - 12x + 2 &= 12x - 12x + 20 \\1x + 2 - 2 &= 20 - 2 \\1x &= 18\end{aligned}$$

We want "1x" and that is what we have!

The solution is " x is 18 ".