## 1.6 How Long Does it Take to Solve Linear Equations.

### 1.6.1 Basic Priciples.

We saw in section 1.4.1 that for large $n$ the amount of time $T$ it takes to solve a linear system of $n$ equations and $n$ unknowns from scratch is approximately proportional to $n^3$, i.e.

$$T \approx Cn^3$$

where $C$ is a constant independent of $n$ but which does depend on the computer and software used. For example, in the next section there is an example where we use Mathematica to solve sets of linear equations of differing sizes. In each case we compute $C = T/n^3$. As one can see from the table below the value of $C$ obtained is fairly constant.

| $n$ | $T$ (sec) | $C$ |
|---|---|---|
| 1000 | 0.17 | $1.7 \times 10^{-10}$ |
| 2000 | 1.0 | $1.3 \times 10^{-10}$ |
| 3000 | 3.0 | $1.1 \times 10^{-10}$ |

We also saw in section 1.4.1 that if one has already done an $LU$ decomposition of the coefficient matrix then the amount of time it takes to do the back substituion to get the solution of a linear system with that coefficient matrix is approximately proportional to $n^2$ for large $n$, i.e.

$$T \approx Kn^2$$

where $K$ is a constant independent of $n$ but which does depend on the computer and software used. For example, in the next section there is an example where we use Mathematica to the back substitution for sets of linear equations of differing sizes. In each case we compute $K = T/n^2$. As one can see from the table below the value of $K$ obtained is fairly constant.

| $n$ | $T$ (sec) | $K$ |
|---|---|---|
| 1000 | 0.015 | $1.5 \times 10^{-8}$ |
| 2000 | 0.09 | $2.4 \times 10^{-8}$ |
| 3000 | 0.31 | $3.5 \times 10^{-8}$ |

One thing to note is that solving a system of linear equations is a lot faster if one already has available an $LU$ decomposition of the coefficient matrix.