3.5 Graphing Linear Equations in Three Variables

Goals
- Graph linear equations in three variables and evaluate linear functions of two variables.
- Use functions of two variables to model real-life situations.

Your Notes

VOCABULARY

Three-dimensional coordinate system A coordinate system determined by three mutually perpendicular axes

z-axis A vertical line through the origin and perpendicular to the *xy*-coordinate plane in a three-dimensional coordinate system

Ordered triple A set of three numbers of the form (x, y, z) that represents a point in space

Octants The three axes of a three-dimensional coordinate system determine three planes that divide space into 8 parts called octants.

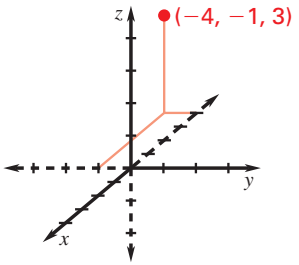
Linear equation in three variables An equation of the form $ax + by + cz = d$ where x, y, and z are variables and a, b, and c are not all zero

Function of two variables A relationship in which one variable depends on two other variables

Example 1 ***Plotting Points in Three Dimensions***

Plot (−4, −1, 3) in a three-dimensional coordinate system.

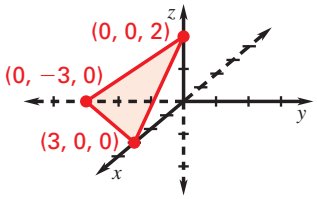
To plot (−4, −1, 3), find the point (−4 , −1) in the *xy*-plane. The point (−4, −1, 3) lies 3 units above .



Your Notes

Example 2 ***Graphing a Linear Equation in Three Variables***

Sketch the graph of $2x - 2y + 3z = 6$.



Find points at which the graph intersects the axes. Let $x = 0$ and $y = 0$, and solve for z to get $z =$ 2. This tells you that the z-intercept is 2, so plot (0, 0, 2). In a similar way, you can find that the x-intercept is 3 and the y-intercept is −3. After plotting the intercepts, connect the points with lines to form the triangular region of the plane.

Example 3 ***Evaluating a Function of Two Variables***

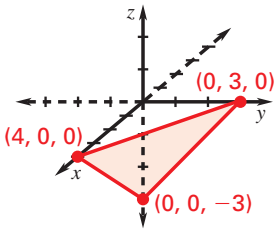
Write the linear equation $2x - 2y + 3z = 6$ as a function of x and y. Then evaluate the function when $x = 1$ and $y = 1$. Interpret the result geometrically.

$2x - 2y + 3z = 6$	**Write original equation.**
$3z = 6 - 2x + 2y$	**Isolate z-term.**
$z = \frac{1}{3}(6 - 2x + 2y)$	**Solve for z.**
$f(x, y) = \frac{1}{3}(6 - 2x + 2y)$	**Replace z with $f(x, y)$.**

$f(1, 1) = \frac{1}{3}(6 - 2(1) + 2(1)) = 2$. This tells you that the graph of f contains the point (1, 1, 2).

✔ ***Checkpoint*** **Complete the following exercise.**

1. Graph $3x + 4y - 4z = 12$. Then write the equation as a function of x and y and evaluate $f(0, 2)$.



$f(x, y) = -\frac{1}{4}(12 - 3x - 4y)$

$f(0, 2) = -1$

Your Notes

Example 4 ***Modeling a Real-Life Situation***

You are painting the inside of your house and plan to use two colors of paint: white and blue. White costs $12 per gallon and blue costs $15 per gallon. You also buy supplies totaling $40.

a. Write a model for the total amount you spend as a function of the number of gallons of white and blue paint you use.

b. Evaluate the model for 3 gallons of white paint and 2 gallons of blue paint.

Solution

a. **Verbal Model**

$$\boxed{\text{Total cost}} = \boxed{\text{White paint cost}} \cdot \boxed{\text{White paint amount}} + \boxed{\text{Blue paint cost}} \cdot \boxed{\text{Blue paint amount}} + \boxed{\text{Supplies}}$$

Labels

Total Cost = C	**(dollars)**
White paint cost = 12	**(dollars per gallon)**
White paint amount = w	**(gallons)**
Blue paint cost = 15	**(dollars per gallon)**
Blue paint amount = b	**(gallons)**
Supplies = 40	**(dollars)**

Algebraic Model $C = 12w + 15b + 40$

b. Substitute $w =$ 3 and $b =$ 2 into the function to find the total cost.

$C = 12w + 15b + 40$	**Write original function.**
$= 12(3) + 15(2) + 40$	**Substitute for w and b.**
$= 106$	**Simplify.**

The total cost is $ 106 .

Homework