SOLVING LINEAR EQUATIONS

Lesson 1: The connection between linear functions and linear equations

LESSON 1: OPENER

On a business trip, you rent a car. The rental company charges $24.95 plus 5¢ per mile for each mile you drive.

1. Write a function rule to describe the relationship between the cost of the rental, ***r***, and the number of miles you drove, ***m***.

   r = 24.95 + 0.05m

2. Use the function rule you wrote in question 1 to determine the cost of the rental if you drove 200 miles.

   r = 24.95 + 0.05(200)
   r = 24.95 + 10
   r = 34.95

   If you drove 200 miles, the cost of the rental would be $34.95.

3. Use the function rule you wrote in question 1 to determine the distance you drove if the cost of the rental is $37.45.

   37.45 = 24.95 + 0.05m
   12.50 = 0.05m
   250 = m

   If the cost of the rental is $37.45, you must have driven 250 miles.

LESSON 1: CORE ACTIVITY

1. Write an equation that you could solve to find the distance you drove if the cost of the rental is $35.80.

   35.80 = 24.95 + 0.05m

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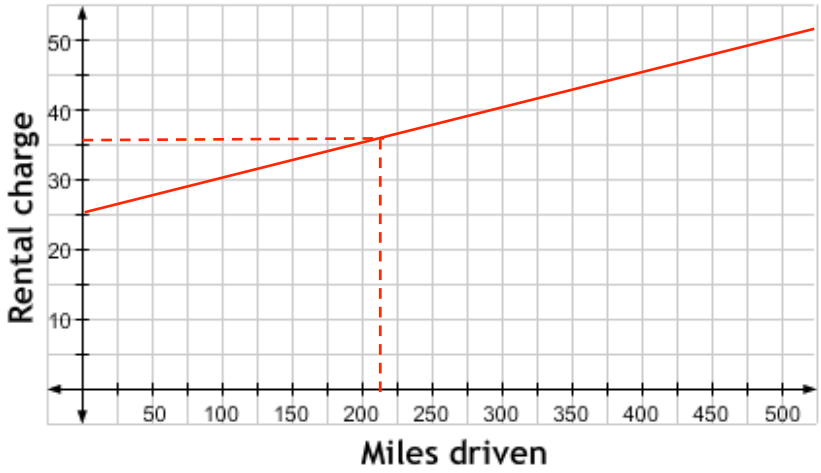
2. Show how to find the solution to your equation in question 1 using each of these four methods.

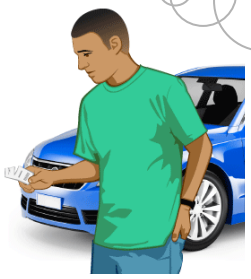
a. Using a table

Miles driven, ***m***	Rental charge, ***r***
212	35.55
213	35.60
214	35.65
215	35.70
216	35.75
→ 217	35.80
218	35.85
219	35.90
220	35.95
221	36.00
222	36.05
223	36.10

b. Using a graph



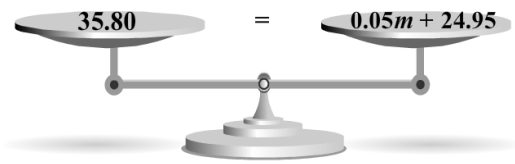
c. Undoing



If the total charge is $35.80, how can I work backward to find the miles driven?

I had to pay $24.95 for the car, even if I didn't drive any miles. So, of the $35.80 total, only 35.80 – 24.95, or $10.85, was the charge for the miles I drove. The rental company charged me a nickel for each mile I drove. Since there are 20 nickels in a dollar, I was charged 20 times 10.85 nickels for the miles I drove, or 217 nickels. That means I drove 217 miles.

d. Algebraic operations



$$35.80 = 0.05m + 24.95$$
$$35.80 - 24.95 = 0.05m + 24.95 - 24.95$$
$$10.85 = 0.05m$$
$$\frac{10.85}{0.05} = \frac{0.05m}{0.05}$$
$$217 = m$$

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3. Do all four of your methods result in the same solution? If not, can you explain the differences?

   All solutions should be the same, although the solution from the graph may be slightly different than the rest because it involves estimating the location of a point.

4. Describe the connections between using "undoing" to solve an equation and using algebraic operations to solve an equation. How are the two methods similar? How are the two methods different?

   Using algebraic operations is the same as "undoing" except that the steps are shown using algebraic operations instead of words.

LESSON 1: CONSOLIDATION ACTIVITY

1. **Objective:** Create sets of matching cards. Each set of matching cards will have five cards: a problem card that describes the situation, an equation card that represents the problem with an algebraic equation, and three solution cards, each showing a different way to solve the equation.

   Materials: Your teacher will give you and your partner pages from which to cut out the cards. There are four problem cards (labeled 1-4), four equation cards (labeled A-D), and twelve solution cards (labeled a-m).

   Instructions: Work with your partner to find four sets of five matching cards. When you both agree on a set of matching cards, tape together the cards that form the set. So that you can more easily check your answers, tape each set with the situation card on the left, the equation card second from the left, and the solution cards on the right, as shown here.

Groupings:
1-B-b-g-i
2-D-a-h-j
3-C-d-f-k
4-A-c-e-m

2. Complete a journal entry explaining in your own words what each of these mathematical ideas means.

Vocabulary term	My understanding of what the term means	An example that shows the meaning of the term
a. Function rule b. Equation c. Solving an equation	Student responses will vary	Student responses will vary

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LESSON 1: HOMEWORK

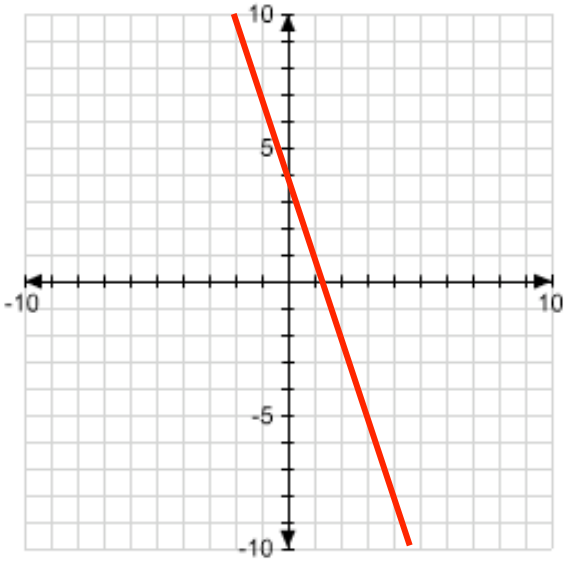
Notes or additional instructions based on whole-class discussion of homework assignment:

1. Complete the table for the function ***y*** = -3***x*** + 4. Then, create a graph of the function.

a.

x	***y***
-2	10
-1	7
0	4
1	1
2	-2
3	-5
4	-8

b.



2. Use your responses from question 1 to solve these equations.
   a. $-8 = -3x + 4$ x = 4
   b. $1 = -3x + 4$ x = 1
   c. $8 = -3x + 4$ $x = -1\frac{1}{3}$

3. a. Describe how you could use algebraic operations to solve the equation 1 = -3***x*** + 4.

   Subtract 4 from each side and then divide each side by -3.

   b. How could you use the table from question 1 to determine whether your solution to the equation 1 = -3***x*** + 4 is correct? How could you tell by looking at the graph?

   Look in the table for when y = 1 and x will be the corresponding x-value. On the graph, look for when y = 1 and find the corresponding x-value that places the point on the line.

   c. How is solving the equation using algebraic operations related to solving the equation using "undoing"?

   Student responses will vary.

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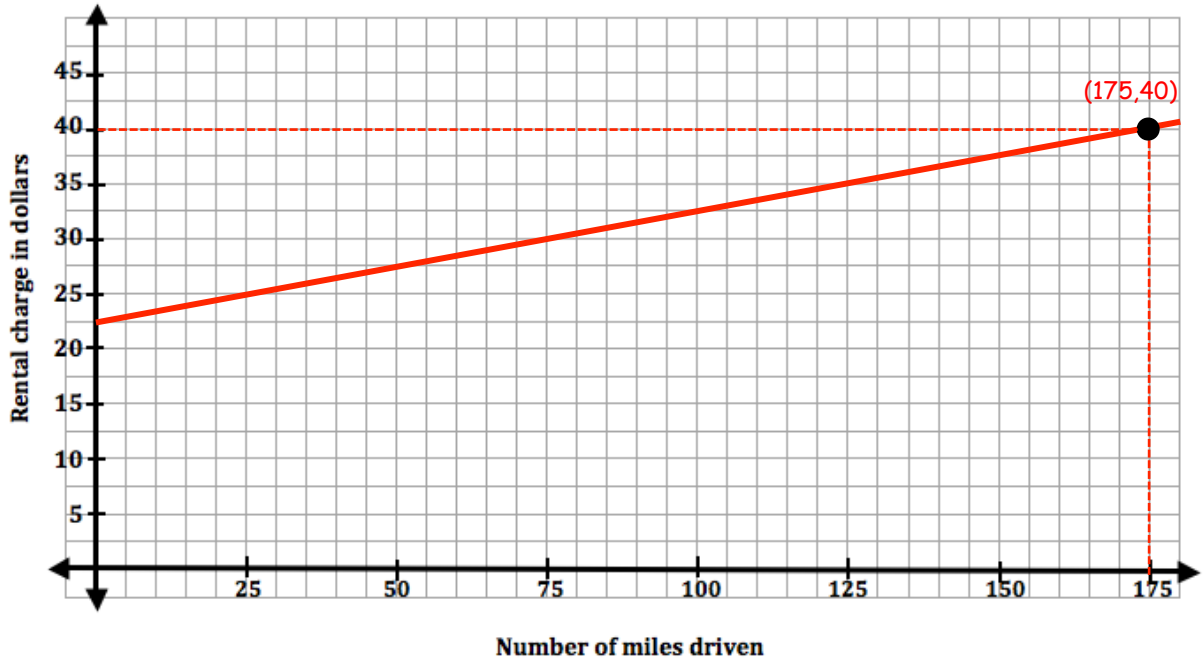
4. Suppose a car rental company charges $22.50 plus 10¢ per mile for each mile you drive.

   a. Write a function rule that models this situation.
   $y = 22.50 + 0.10x$

   b. Write an equation you could solve to find how many miles you drove if the total rental charge was $40.00.
   $40 = 22.5 + 0.10x$

   c. Solve this equation using all four methods you explored in Lesson 1. A grid has been provided to help you with the graphical solution method. (Use a separate sheet of notebook paper, if needed.)



$40 = 22.5 + .1x$

$17.5 = .1x$

$175 = x$

x	155	165	175	185	195
y	38	39	40	41	42

If the total charge is $40, then $22.50 of that is for renting the car. Subtracting $22.50 from the total charge of $40 leaves $17.50 of the total charge to cover mileage. At $0.10 per mile, the mileage would be 17.50/0.1, which is 175 miles.

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LESSON 1: STAYING SHARP

Practicing skills & concepts

1. A scale like the one shown is balanced when it has 18 1-pound bricks on one side and 3 equally sized pieces of gold on the other. How much does each piece of gold weigh?

Answer with supporting work:

18 bricks = 3 gold pieces

6 bricks = 1 gold pieces

6 bricks = 6 pounds = 1 gold piece ⇒ 1 gold piece weighs 6 pounds

2. A line is described by the equation $y = \frac{3}{5}x + \frac{4}{7}$ in slope-intercept form. Identify the slope and y-intercept of the line.

Slope = 3/5

y-intercept = $(0, {}^{4}/_{7})$

Preparing for upcoming lessons

3. Show whether the values $x = 5$ and $y = 6$ make the equation $7x - 3y = 19$ true.

$7(5) - 3(6) = 35 - 18 = 17$

$17 \neq 19$

The values x = 5 and y = 6 do not make the equation true.

4. Show whether the values $x = 4$ and $y = 5$ make the inequality $7x - 3y < 19$ true.

$7(4) - 3(5) = 28 - 15 = 13$

$13 < 19$

x = 4 and y = 5 make the inequality true.

Reviewing ideas from earlier grades

5. A square has a perimeter of 64 inches.

a. What is the length of one side of the square?

64 ÷ 4 = 16 inches

b. What is the area of the square?

16 • 16 = 256 inches^2

6. Find each sum or difference.

a. $\frac{1}{7} + \frac{2}{7}$ = 3/7

b. $\frac{11}{3} - \frac{4}{3}$ = 7/3

c. $\frac{14}{5} + \frac{-6}{5}$ = 8/5

Why would finding the sum $\frac{3}{5} + \frac{2}{7}$ be more complicated than finding the sums in parts a or c?

Because you would need to find a common denominator first.

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Lesson 2: Solving equations with tables

LESSON 2: OPENER

1. Make a table of values for $y = 4x + 8$.

x	y
0	8
1/2	10
1	12
3/2	14
2	16

2. Explain how you could use your table to solve the equation $4x + 8 = 14$. If you are unable to use your table to solve the equation, explain how you could modify your table to show the solution.

The solution is the x-value when the y-value is 14.

x = 1.5

LESSON 2: CORE ACTIVITY

1. Use the table feature on your graphing calculator to solve the equation from the *Opener*, $4x + 8 = 14$. Then describe the process you used.
   Student responses will vary

2. Use the table feature on your graphing calculator to solve the following equations. For each equation, record the table from your calculator and describe the process you used to find your solution.

	a. $3x + 6 = -6$	b. $8x + 10 = 0$
Table	x / y 0 / 6 -1 / 3 -2 / 0 -3 / -3 -4 / -6 -5 / -9 Student descriptions will vary.	x / y 0 / 10 -0.25 / 8 -0.5 / 6 -0.75 / 4 -1 / 2 -1.25 / 0 -1.5 / -2 Student descriptions will vary.
Solution	x = -4	x = -1.25

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<table>
<tr><td></td><td>c. $5x - 7 = 5 - x$</td><td>d. $4(x + 3) = 2x + 5$</td></tr>
<tr><td>Table</td><td><table><tr><td>x</td><td>Y1</td><td>Y2</td></tr><tr><td>0</td><td>-7</td><td>5</td></tr><tr><td>1</td><td>-2</td><td>4</td></tr><tr><td>2</td><td>3</td><td>3</td></tr><tr><td>3</td><td>8</td><td>2</td></tr><tr><td>4</td><td>13</td><td>1</td></tr></table>
Student descriptions will vary.</td><td><table><tr><td>x</td><td>Y1</td><td>Y2</td></tr><tr><td>-2.5</td><td>2</td><td>0</td></tr><tr><td>-3</td><td>0</td><td>-1</td></tr><tr><td>-3.5</td><td>-2</td><td>-2</td></tr><tr><td>-4</td><td>-4</td><td>-3</td></tr><tr><td>-4.5</td><td>-6</td><td>-4</td></tr></table>
Student descriptions will vary.</td></tr>
<tr><td>Solution</td><td>x = 2</td><td>x = -3.5</td></tr>
</table>

3. How does using a table to solve $8x + 10 = 0$ compare to using a table to solve $3x + 6 = -6$? What did you do differently?

   Student responses will vary, but they will probably mention the need to change the Δx value to 0.01 (or perhaps 0.25) and search between x = -1 and x = -2 to find the solution.

4. How does using a table to solve $5x - 7 = 5 - x$ compare to using a table to solve the first two equations in question 2? What did you do differently?

   Student responses will vary, but they will probably mention entering both sides of the equation as separate functions and looking for the x-value when the outputs are equal.

LESSON 2: REVIEW END-OF-UNIT ASSESSMENT

Today you will review the end-of-unit assessment.

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LESSON 2: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve the following equations using tables.

a. $-2x + 4 = 7$

x	-2	-1.5	-1	-.5	0
y	8	7	6	5	4

x = 1.5

b. $2x + 7 = 4 + x$

x	-3.5	-3	-2.5	-2
y1	0	1	2	3
y2	.5	1	1.5	2

x = -3

c. $3(x - 1) = x + 2$

x	2	2.5	3
y1	3	4.5	6
y2	4	4.5	5

x = 2.5

2. Carlton built the following table to solve the equation $5x - 3 = 68$. How would you advise him to use his calculator so that it displays the solution to the equation?

x	1	2	3	4	5	6	7	8
$5x - 3$	2	7	12	17	22	27	32	37

The value of interest is greater than the values shown on the table. Carlton needs to scroll to larger x values. Start at x = 14 and use $\Delta x = 0.1$. Then Carlton will see that y = 68 when x = 14.2. The solution to the equation is x = 14.2.

3. Sydney built the following table to solve the equation $2x + 6 = 20$. How would you advise her to use her calculator so that it displays the solution to the equation?

x	2	4	6	8	10	12	14	16
$2x + 6$	10	14	18	22	26	30	34	38

The value of interest falls between x = 6 and x = 8. Set $\Delta x = 1$. Then Sydney will see that y = 20 when x = 7. The solution to the equation is x = 7.

4. Suppose a movie rental company charges $9.99 per month plus $1.25 for every movie rented.

a. Write a function rule that represents the relationship between the number of movies rented in a month and the rental charge for that month.

y = 9.99 + 1.25m

b. Write an equation that you could solve to find how many movies you rented if the total rental charge for one month was $23.74.

23.74 = 9.99 + 1.25m

c. Solve this equation using a table.

m = 11

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5. Mauricio built the following table to solve the equation $2(x - 1) = 4x + 1$:

x	$2(x - 1)$	$4x + 1$
-4	-10	-15
-3	-8	-11
-2	-6	-7
-1	-4	-3
0	-2	1
1	0	5

However, Mauricio is confused because he is not sure how to use the table to find the solution. Explain to Mauricio what he should do to find the solution to the equation using this table. What is the solution?

He should look for when both outputs, 2(x-1) and 4x + 1, are equal. When he finds this, the solution will be the corresponding x-value. He needs to look between x = -2 and x = -1, so he should change Δx. Δx = 0.5 will work. x = -1.5.

6. Janice is using a table to solve the equation $2x - 5 = x + 4$. Here is the table she constructs:

x	$2x - 5$	$x + 4$
7	9	11
8	11	12
9	13	13
10	15	14
11	17	15

Janice claims that the solution to the equation is 13. Is Janice correct? If so, explain how the table shows that she is correct. If not, how can you use the table to find the correct solution?

Janice is not correct. She needs to find the x-value where the y-values are the same. The solution is x = 9.

7. Think about your performance on the Unit 4 end-of-unit assessment.

   a. Are you pleased with your performance on the end-of-unit assessment? Circle one: Yes / No

   Responses will vary

   b. Does your performance reflect your understanding of the topics in Unit 4? Circle one: Yes / No

   Responses will vary

   If you answered "No", why do you think this?

   Responses will vary

   c. Based on your answers to parts a and b, do you need to revise the goal you wrote at the end of the last unit? If so, write your new goal below along with any enabling goals that will help you reach your goal.

   Responses will vary.

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LESSON 2: STAYING SHARP

Practicing skills & concepts

1. Sofia raised $140 for charity. Her mom gave her some money to get started, and her friends donated the rest of the money. Dominic gave her $75, Hannah gave her $10, and Julie gave her $30. How much did her mom give her?

Answer with supporting work:

140 – 75 – 10 – 30 = 25

Her mom gave her $25

2. The ***x***-intercept of a line is the point at which it crosses the ***x***-axis. It always has a ***y***-coordinate of 0.

The ***y***-intercept of a line is the point at which it crosses the ***y***-axis. It always has an ***x***-coordinate of 0.

Find the coordinates of the ***x***- and ***y***-intercepts of the line with the equation $4x + 3y = 60$.

Answer with supporting work:

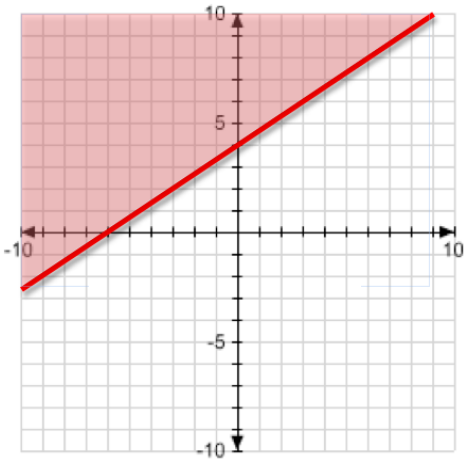
x-intercept: $4x + 3(0) = 60 \Rightarrow 4x = 60 \Rightarrow x = 15$

y-intercept: $4(0) + 3y = 60 \Rightarrow 3y = 60 \Rightarrow y = 20$

Preparing for upcoming lessons

3. Graph the line with equation $y = \frac{2}{3}x + 4$ on the coordinate grid provided. (It may help to make a table of points for the line.)

4. Shade the area above the line you graphed. Identify the coordinates of one point in the shaded region of the graph and one point in the unshaded region of the graph.



Point in shaded region: (-6,3)

Point in unshaded region: (5,-1)

Student responses may vary

Reviewing ideas from earlier grades

5. A square has a perimeter of 20 cm. A new square is created that has double the perimeter of the original square. How does the **area** of the new square compare to the **area** of the original square?

The area of the new square is 4 times the area of the old square.

6. Find the product of each pair of fractions. Simplify all products to lowest terms.

a. $\frac{2}{7} \cdot \frac{3}{5} = 6/35$

b. $\frac{2}{7} \cdot \frac{7}{3} = 14/21 = 2/3$

c. $\frac{2}{9} \cdot \frac{3}{5} = 6/45 = 2/15$

d. $\frac{2}{3} \cdot \frac{3}{2} = 6/6 = 1$

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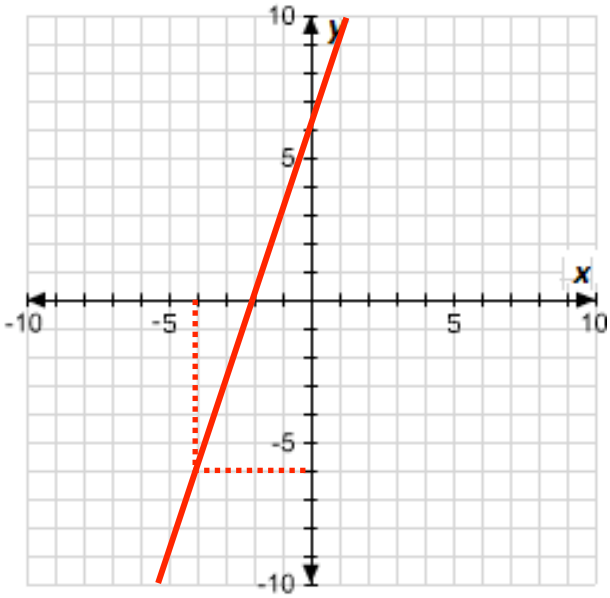
Lesson 3: Solving equations with graphs

LESSON 3: OPENER

1. Graph $y = 3x + 6$.

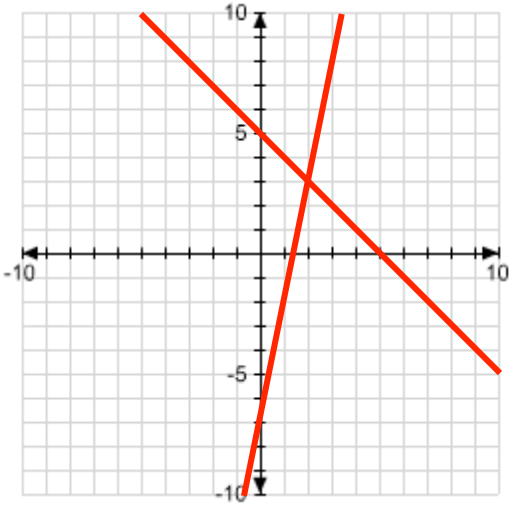


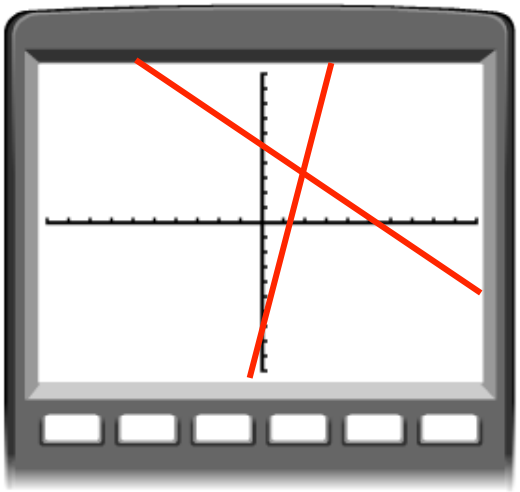
2. Explain how you could use your graph to solve the equation $3x + 6 = -6$.

Look for the x-value when y = -6. The x-value that corresponds to y = -6 is x = -4. So, x = -4 is the solution to the equation.

LESSON 3: CORE ACTIVITY

1. Graph the function represented by each side of the equation $5x - 7 = 5 - x$. First, graph the functions by hand, using the grid provided. Then, use the graphing feature on your calculator to graph the two functions. Sketch the graph produced by the calculator to the right of the graph you constructed by hand.





2. What are the coordinates of the point of intersection for the graphs in question 1? What does the x-coordinate of the intersection point mean in the context of the equation? What does the y-coordinate of the intersection point mean in the context of the equation?

x = 2 represents the solution to 5x – 7 = 5 – x. y = 3 represents the value of each side of the equation 5x – 7 = 5 – x when 2 is substituted for x.

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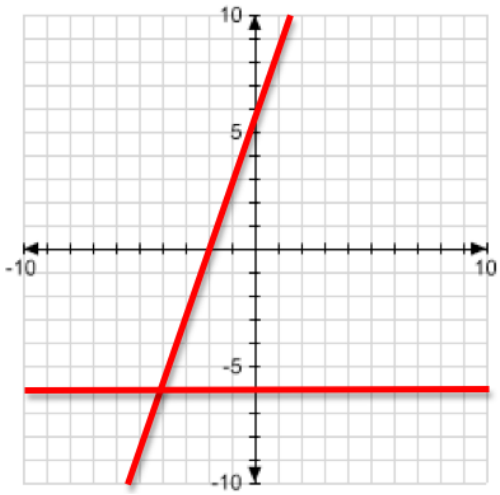
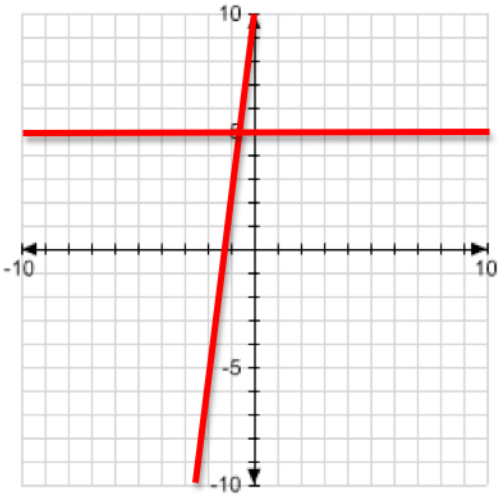
3. Solve $3x + 6 = -6$ by graphing $y_1 = 3x + 6$ and $y_2 = -6$ on your calculator. Sketch your graph and explain how the graph can be used to find the solution to the equation.



The solution is the x-value where the two lines intersect.

x = -4

4. Solve the equation $8x + 10 = 5$ by using the graphing capabilities on your calculator. Sketch your graph and explain how you used the graph to solve the equation.



The solution is the x-value where the two lines intersect.

x = -0.625

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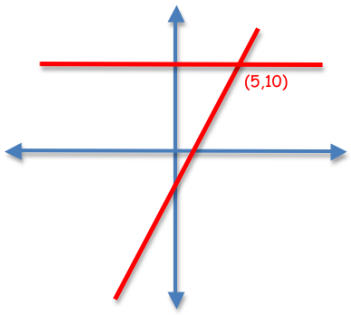
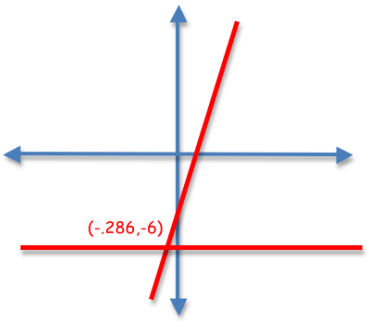
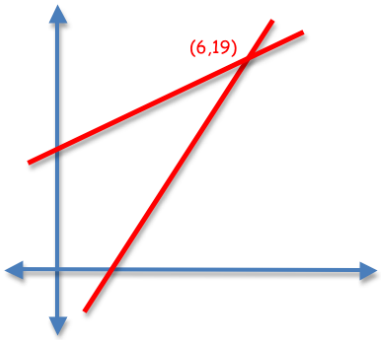
LESSON 3: CONSOLIDATION ACTIVITY

1. Work with a partner to solve each of the following equations as follows:
   - One partner solves equation 1 using the **table** feature on the graphing calculator. Sketch the calculator screen and identify the solution.
   - The other partner solves equation 1 using the **graphing** feature on the graphing calculator. Sketch the calculator screen and identify the solution.
   - Compare solutions with your partner, checking for agreement and accuracy. Which method was more efficient for solving this equation? Write your responses in the last column of the table.
   - Alternate methods so that each partner solves three equations by using a table and three equations by graphing.

Equation	Table Solution	Graph Solution	Comparison
a. $3x - 5 = 10$	<table><tr><td>x</td><td>y1</td><td>y2</td></tr><tr><td>2</td><td>1</td><td>10</td></tr><tr><td>3</td><td>4</td><td>10</td></tr><tr><td>4</td><td>7</td><td>10</td></tr><tr><td>5</td><td>10</td><td>10</td></tr></table>		Responses will vary
b. $7x - 4 = -6$	<table><tr><td>x</td><td>y1</td><td>y2</td></tr><tr><td>0</td><td>-4</td><td>-6</td></tr><tr><td>1</td><td>3</td><td>-6</td></tr><tr><td>2</td><td>10</td><td>-6</td></tr><tr><td>3</td><td>17</td><td>-6</td></tr></table>		Responses will vary
c. $4x - 5 = 2x + 7$	<table><tr><td>x</td><td>y1</td><td>y2</td></tr><tr><td>4</td><td>11</td><td>15</td></tr><tr><td>5</td><td>15</td><td>17</td></tr><tr><td>6</td><td>19</td><td>19</td></tr><tr><td>7</td><td>23</td><td>21</td></tr></table>		Responses will vary

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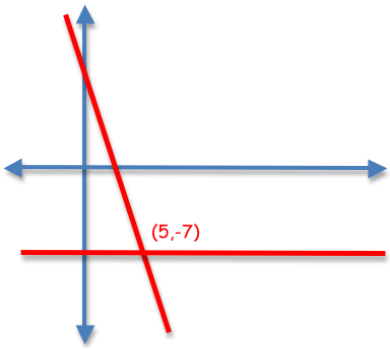
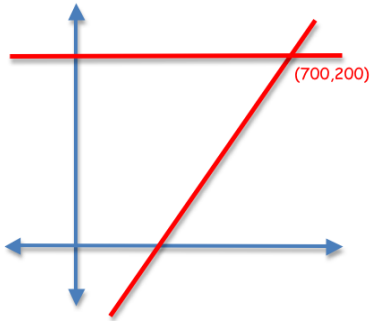
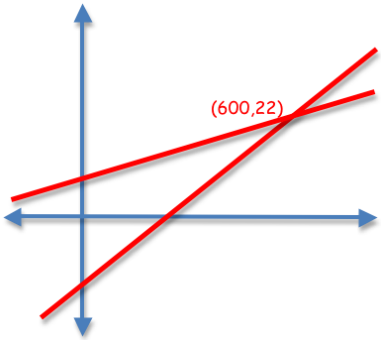
Equation	Table Solution	Graph Solution	Comparison
d. $8 - 3x = -7$			Responses will vary
e. $200 = 0.5x - 150$			Responses will vary
f. $0.03x + 4 = 0.07x - 20$			Responses will vary

x	y1	y2
3	-1	-7
4	-4	-7
5	-7	-7
6	-10	-7

x	y1	y2
600	150	200
700	200	200
800	250	200

x	y1	y2
500	19	15
600	22	22
700	25	29

2. You compared using a table and using a graph for solving each equation.

   a. For which equations was using a table more useful for solving? Why?

   Student responses will vary

   b. For which equations was using a graph more useful for solving? Why?

   Student responses will vary

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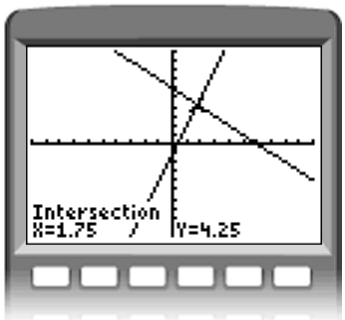
LESSON 3: HOMEWORK

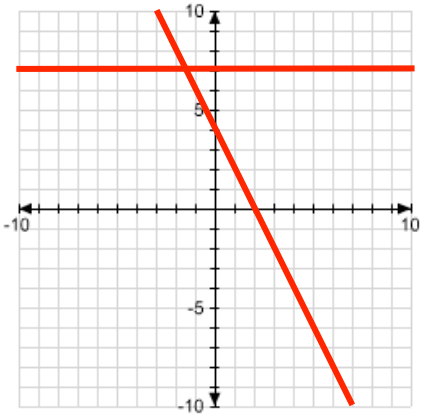
Notes or additional instructions based on whole-class discussion of homework assignment:

1. Joni is solving the equation $3x - 1 = 6 - x$ using the graphing capabilities on her calculator. Here is what the calculator screen displays when she finds the intersection of the graphs of $Y_1 = 3x - 1$ and $Y_2 = 6 - x$. What is the solution to the equation? How do you know?



You are looking for an x-value that gives both equations the same y-value. The solution is 1.75, because when x = 1.75, both equations graphed have a y-value of 4.25.

2. Solve -2x + 4 = 7 by using a graph. Recall that you solved this equation using a table in the homework from Lesson 2. How does solving this equation by graphing compare to solving using a table? Which method do you prefer for solving this equation? Why?

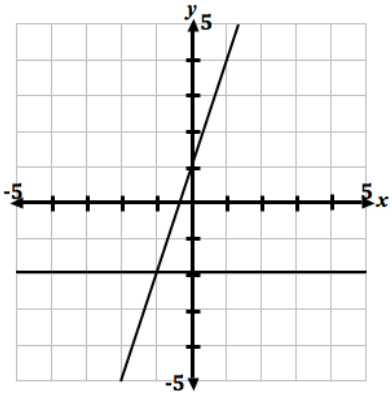


x = -1.5

Student responses will vary

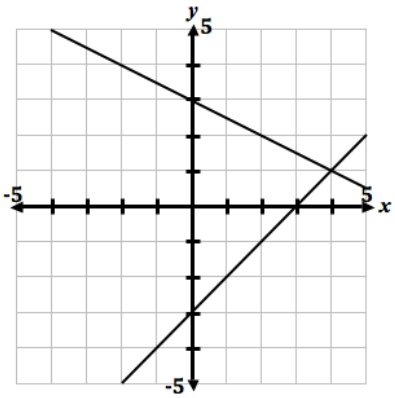
3. Write an equation for the solution shown in each graph. Then, write the solution to each equation.

a.



$-2 = 1 + 3x \Rightarrow x = -1$

b.



$-3 + x = 3 - \frac{1}{2}x \Rightarrow x = 4$

4. Alexis uses the graphing capability on her graphing calculator to solve the equation $4(x + 1) = 5 + 6x$. She finds that the intersection point is (-0.5,2). She says that the solution to the equation is 2. Is Alexis correct? If so, explain how you know. If not, determine the solution and explain how you know your solution is correct.

She should be looking for the x-value at which the two intersect, not the y-value. The solution is $x = -\frac{1}{2}$

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LESSON 3: STAYING SHARP

Practicing skills & concepts	**1.** What number could you substitute in place of the star to make this equation true? $5 \cdot \star + 9 = 39$ $\star = 6$	**2.** Consider the following questions. a. Would it be easier to find the slope or the ***x***-intercept of the line described by $y = 2x + 3$? Explain. The slope, because it is actually the coefficient of x in this equation format. b. Would it be easier to find the slope or the ***x***-intercept of the line described by $8x - 4y = 12$? Explain. x-intercept ⇒ simply substitute 0 for y and solve for x. This is much quicker than solving for y to find the slope.
Preparing for upcoming lessons	**3.** Show whether the point (7,–2) satisfies the equation $3x + 4y = 29$. Answer with supporting work: $3(7) + 4(-2) = 21 - 8 = 13$ $13 \neq 29$ No, (7,-2) does not satisfy $3x + 4y = 29$.	**4.** Show whether the point (3,5) satisfies the inequality $y > 2x - 4$. Answer with supporting work: $2(3) - 4 = 6 - 4 = 2$ $5 > 2$ Yes, (3,5) satisfies $y > 2x - 4$
Reviewing ideas from earlier grades	**5.** A rectangle has a perimeter of 26 inches. All of its side lengths are doubled to create a new rectangle. a. How does the **perimeter** of the new rectangle compare to the **perimeter** of the original rectangle? The new perimeter is two times the original perimeter. b. How does the **area** of the new rectangle compare to the **area** of the original rectangle? The new area is four times the original area.	**6.** Find a common denominator that can be used to add the fractions below. Then, find the sum. $\frac{5}{6} + \frac{3}{8}$ $\frac{5}{6} \bullet \frac{4}{4} + \frac{3}{8} \bullet \frac{3}{3} = \frac{20}{24} + \frac{9}{24} = 29/24$

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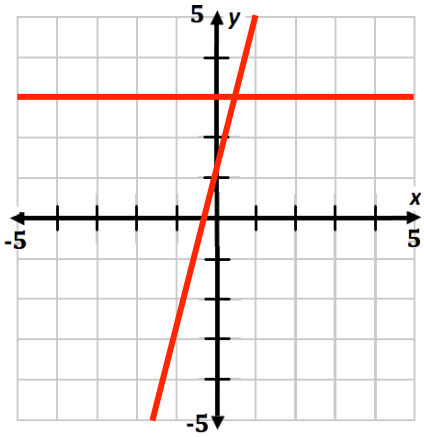
Lesson 4: Solving one-step equations

LESSON 4: OPENER

1. Solve the equation $4x + 1 = 3$ without a calculator, using either a table or a graph. If you decide to solve by graphing, a coordinate grid has been provided.



x	y
0	1
.5	3
1	5
1.5	7
2	9

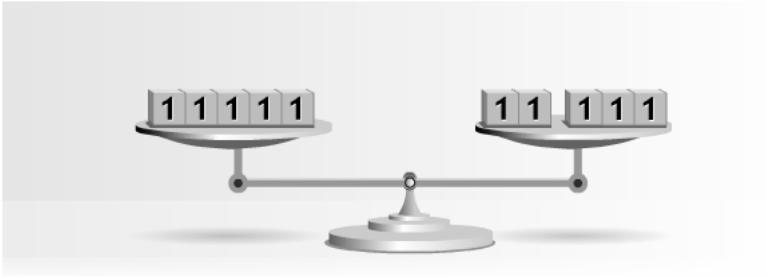
x = 0.5

2. What method did you choose? What challenges did you have in using that method for this equation?

Student responses will vary; using a graph is challenging because the intersection is not at a whole value for x.

LESSON 4: CORE ACTIVITY

1. When you add weight to the balance scale, what must you do to each side of the scale to keep it balanced?



$5 = 2 + 3$

You must add the same number of weights to each side.

2. Solve the equation $4 = 1 + x$. What property of equality did you use to solve the problem?

$4 - 1 = 1 - 1 + x$

$3 = x$ Subtraction Property of Equality was used.

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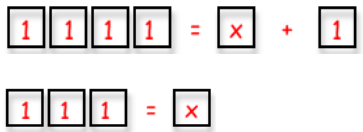
3. Build a model for the equation 4 = 1 + ***x*** using algebra tiles. Make a sketch of your model and explain how you would use the algebra tiles to find the solution to the equation.



Remove one unit from each side.

4. Solve the equation $x + 2 = 7$ using the subtraction method and the additive inverse method.

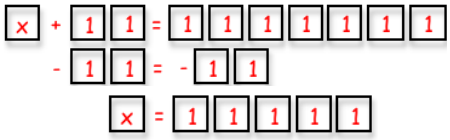
Subtraction method	Additive inverse method
$x + 2 = 7$ $x + 2 - 2 = 7 - 2$ $x = 7 - 2$ $x = 5$	$x + 2 = 7$ $x + 2 + (-2) = 7 + (-2)$ $x + 0 = 7 + (-2)$ $x = 5$

Now compare the methods. Describe how the methods are similar and how they are different.

They are different because you are subtracting in one method and adding in the other. They are similar, though, because adding -2 is the same as subtracting 2, and both methods lead to the same solution.

Build a model using algebra tiles for each equation in questions 5-6. Sketch each step of the model and record each step using symbols.

5. ***x*** + 2 = 7

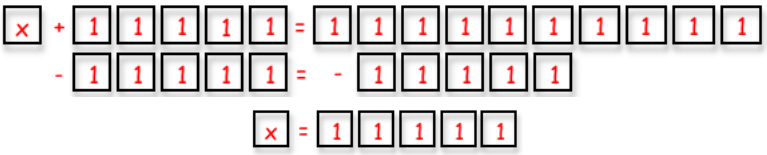


x + 2 = 7

x + 2 - 2 = 7 - 2

x = 5

6. ***x*** + 5 = 10



x + 5 = 10

x + 5 - 5 = 10 - 5

x = 5

Use algebra tiles to model solving each equation in questions 7-8. Record each step using symbols.

7. 8 = ***x*** – 3

8 = x - 3

8 + 3 = x - 3 + 3

11 = x

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8. $5 - x = 7$

   $5 - x = 7$

   $5 - 5 - x = 7 - 5$

   $-x = 2$

   $x = -2$

9. Use algebraic operations to solve the equation $15d = 300$. Describe the process you used to find the solution. How is this process different from the way you solved the equations in questions 1-8?

   "Undo" multiplication by dividing each side by 15. This is the same as multiplying each side by 1/15:

   $15d = 300$

   $15d \div 15 = 300 \div 15$

   $d = 20$

   Questions 1-8 only involved undoing addition and subtraction. In this problem we could not subtract 15 to isolate the variable, since 15 is multiplied to ***d***, not added. Undoing multiplication calls for the inverse operation to multiplication, which is division.

LESSON 4: CONSOLIDATION ACTIVITY

Materials: Your teacher will give you and your partner pages from which to cut out the cards. There are 6 equation cards and 12 solution method cards.

Instructions: Work with a partner to solve the equation on each of the equation cards as follows:

- Sort the cards into two groups: equation cards and solution method cards.
- One partner draws an equation card, and both partners record the equation in the table below.
- Each partner draws a solution method card and solves the equation (in the space in the table below) using the method written on the drawn card.
- Partners explain their solutions to each other and compare their solutions. Record in the table any differences you found between your solutions and how you resolved them.
- Repeat the above steps for each of the remaining five equation cards.

Equation on card	Solution	Comparison with partner
	Method from card: _____ Responses will vary	
	Method from card: _____ Responses will vary	
	Method from card: _____	

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Equation on card	Solution	Comparison with partner
	Responses will vary	
	Method from card: _____ Responses will vary	
	Method from card: _____ Responses will vary	
	Method from card: _____ Responses will vary	

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LESSON 4: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve each of these equations using algebraic operations.

a. $7 = 12 + x$

$7 - 12 = 12 - 12 + x$

$-5 = x$

b. $-14 = -2x$

$-14 \div -2 = -2x \div -2$

$7 = x$

c. $3x = 10$

$3x \div 3 = 10 \div 3$

$x = 10/3$

d. $x - 45 = -135$

$x - 45 + 45 = -135 + 45$

$x = -90$

e. $x + 7 = -3$

$x + 7 - 7 = -3 - 7$

$x = -10$

f. $\frac{1}{2}x = -3$

$(1/2)x \bullet 2 = -3 \bullet 2$

$x = -6$

g. $-4 = 16x$

$-4 \div 16 = 16x \div 16$

$-1/4 = x$

h. $0.25x = 2.75$

$0.25x \div 0.25 = 2.75 \div 0.25$

$x = 11$

i. $3 - x = 7$

$3 - 3 - x = 7 - 3$

$-x = 4$

$-1 \bullet -x = -1 \bullet 4$

$x = -4$

2. Billy is solving the equation $-3x = 12$. His work is shown below along with his justification.

$-3x = 12$	This is the original equation.
$-3x + 3 = 12 + 3$	I added the same thing to both sides of the equation.
$x = 15$	The -3 and +3 cancel out on the left side, so all that's left is the x and 12 + 3 is 15. So, x must be 15.

Billy's solution is incorrect. Identify Billy's mistake and rework the problem correctly.

-3x means -3 times x, not x minus 3. To undo multiplication, you must divide. So, Billy should divide each side by -3 to get 1x on the left side.

$-3x = 12$

$-3x \div -3 = 12 \div -3$

$x = -4$

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LESSON 4: STAYING SHARP

Practicing skills & concepts

1. Use the input-output table for the function rule $y = 3x - 5$ to find the solution for x in the equation $4 = 3x - 5$.

Answer with reasoning:

To solve 4 = 3x - 5 you need to find the x-value when y = 4, so x = 3

x	y
-2	-11
-1	-8
0	-5
1	-2
2	1
3	4
4	7

2. Bret is examining a line with equation $y = 3x - 7$. He says that the y-intercept of the line is 7. Is he correct? Explain.

No, the y-intercept is at -7 because the y intercept occurs when x = 0.

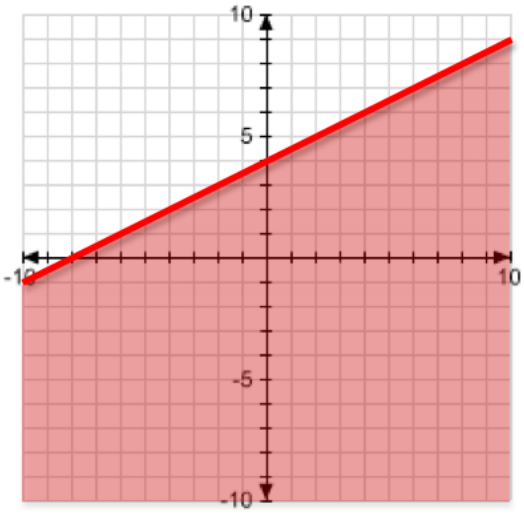
y = 3(0) - 7

y = -7

Preparing for upcoming lessons

3. Graph the equation $4y - 2x = 16$ on the coordinate grid provided.

4. Shade the area below the line you graphed. Identify the coordinates of one point in the shaded region of the graph and one point in the unshaded region of the graph.



Point in shaded region: (3,3)

Point in unshaded region: (0,8)

Student responses will vary

Reviewing ideas from earlier grades

5. What is the volume of a cube that has a side length of 2 cm?

Volume = $(2\text{cm})^3 = 8\ \text{cm}^3$

6. Find a common denominator you can use to subtract the fractions shown. Then, find the difference.

$$\frac{7}{4} - \frac{1}{10}$$

$$\frac{7}{4} \bullet \frac{5}{5} - \frac{1}{10} \bullet \frac{2}{2} = \frac{35}{20} - \frac{2}{20} = 33/20$$

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Lesson 5: Solving two-step equations

LESSON 5: OPENER

Consider the equation $5x + 7 = 22$.

1. What two operations are represented in this equation?

   Addition and multiplication

2. How would you "undo" each of those operations?

   Subtraction and division

3. Which operation should you "undo" first? Why?

   Undo the "last" thing that was done, addition

LESSON 5: CORE ACTIVITY

1. Fill in the table to show how the equation $2x + 5 = 11$ is built and solved.

Action indicated by equation	Result	Equation
Choose an input for x		$x = 3$
Multiply by 2		$2x = 6$
Add 5	11	$2x + 5 = 11$

2. In your own words, describe how you would use inverse operations to solve the equation $2x + 5 = 11$.

   Subtract 5 from each side and then divide each side by 2.

3. Build a model for the equation $2x + 5 = 11$ using algebra tiles. Make a sketch showing the tile representation. Describe how you would use the algebra tiles to help you find the solution. Then, find the solution.

Subtract 5 tiles from each side and the divide each side by 2.

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4. Solve the following equations using algebraic (or inverse) operations. If you need to, you can sketch algebra tile models to help you solve the problems.

a. $3\boldsymbol{b} - 4 = 5$

3b – 4 + 4 = 5 + 4

3b = 9

3b ÷ 3 = 9 ÷ 3

b = 3

b. $0.4\boldsymbol{y} + 9.2 = 10$

0.4y + 9.2 – 9.2 = 10 – 9.2

0.4y = .8

0.4y/0.4 = .8/.4

y = 2

c. $5 + \frac{a}{4} = 2$

5 – 5 + a/4 = 2 – 5

a/4 = -3

(a/4) • (4) = -3 • 4

a = -12

d. $10 = -\boldsymbol{m} + 8$

10 – 8 = -m + 8 – 8

2 = -m

2/-1 = -m/-1

m = -2

e. $\frac{5}{7}x + \frac{1}{7} = 3$

(5/7)x + (1/7) – (1/7) = 3 – (1/7)

(5/7)x = 20/7

(5/7)x ÷ (5/7) = (20/7) ÷ (5/7)

x = 4

f. $\frac{t+1}{3} = 2$

((t + 1)/3) • 3 = 2 • 3

t + 1 = 6

t + 1 – 1 = 6 – 1

t = 5

5. Which properties of equality did you use to solve question 4c? In what order did you use them? Justify your reasoning.

Answers will vary depending on the solution method used. For the solution as shown above:

Subtraction Property of Equality (first step)

Multiplication Property of Equality (third step)

LESSON 5: CONSOLIDATION ACTIVITY

In this activity you will work with your partner to build and then solve two-step equations.

1. On the left side of the paper, build two-step equations involving any of the following actions: addition, subtraction, multiplication, or division. The example shows a two-step equation involving multiplication and subtraction. In table 4, one of the steps has been completed for you.
2. Write the resulting equation in the first space in the table on the right side of the page.
3. Once you have written all four equations, fold your paper along the dashed line and exchange papers with your partner. (Your partner is not allowed to look at the side of the paper where you constructed the equation!)
4. Your partner will solve the equations you constructed by filling out the table in the right-hand column.
5. Then, you and your partner will compare the actions you used to construct the equation to the "undoing" that your partner used to solve the equation.

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Ex.

Action	Equation
Choose an input for x	$x = 13$
Multiply by 2	$2x = 26$
Subtract 9	$2x - 9 = 17$

Equation	"Undoing" Process	Action
$2x - 9 = 17$	$2x - 9 + 9 = 17 + 9$	Add 9 to both sides.
$2x = 26$	$\frac{2x}{2} = \frac{26}{2}$	Divide by 2.
$x = 13$	$x = 13$	The solution is 13.

1.

Action	Equation
Choose an input for x	Student responses/work will vary

Equation	"Undoing" Process	Action
	Student responses/work will vary	

2.

Action	Equation
Choose an input for x	Student responses/work will vary

Equation	"Undoing" Process	Action
	Student responses/work will vary	

3.

Action	Equation
Choose an input for x	Student responses/work will vary

Equation	"Undoing" Process	Action
	Student responses/work will vary	

4.

Action	Equation
Choose an input for x	Student responses/work will vary
Multiply by a	

Equation	"Undoing" Process	Action
	Student responses/work will vary	

5. After you have finished the activity, discuss with your partner the connection between the "action" used to build an equation and the "undoing process" used to solve the equation. Record your observations below.

Responses will vary.

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LESSON 5: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve each of these equations using algebraic operations.

a. $2x - 10 = 4$

$2x - 10 + 10 = 4 + 10$

$2x = 14$

$2x/2 = 14/2$

$x = 7$

b. $5 - 2x = 9$

$5-5-2x = 9 - 5$

$-2x = 4$

$-2x/-2 = 4/-2$

$x = -2$

c. $6x + 3 = 5$

$6x + 3 - 3 = 5 - 3$

$6x = 2$

$6x/6 = 2/6$

$x = 1/3$

d. $13 - x = 6$

$13 - 13 - x = 6 - 13$

$-x = -7$

$-x/-1 = -7/-1$

$x = 7$

e. $12 - 3x = 11$

$12 - 12 - 3x = 11 - 12$

$-3x = -1$

$-3x/-3 = -1/-3$

$x = 1/3$

f. $4x - 3 = -11$

$4x - 3 + 3 = -11 + 3$

$4x = -8$

$4x/4 = -8/4$

$x = -2$

g. $15 - 2x = 1$

$15 - 15 - 2x = 1 -15$

$-2x = -14$

$-2x/-2 = -14/-2$

$x = 7$

h. $3(x + 1) = 11$

$3x + 3 = 11$

$3x + 3 - 3 = 11 - 3$

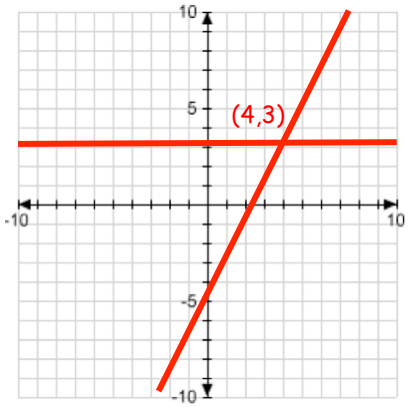
$3x = 8$

$3x/3 = 8/3$

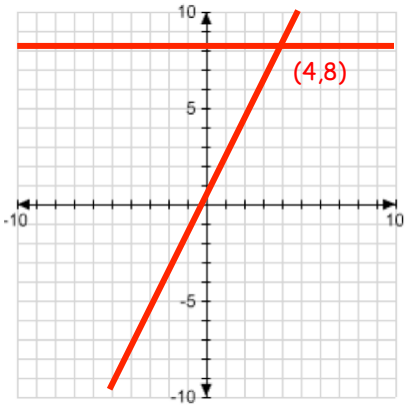
$x = 8/3$

2. Construct graphs that you could use to solve each equation by graphing each side of the equation as a function. Then, label the intersection points on each graph.

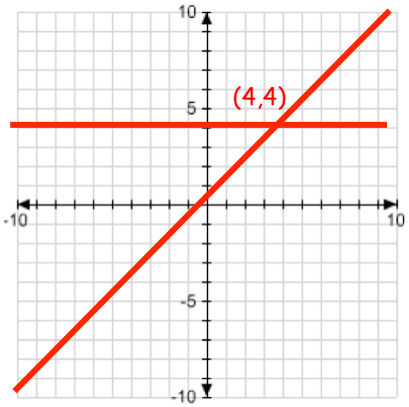
a. $2x - 5 = 3$ $x = 4$



b. $2x = 8$ $x = 4$



c. $x = 4$ $x = 4$



3. Use the graphs you constructed in question 2 to answer the following questions.

a. What do you notice about the x-coordinate of each of the intersection points?
They are all $x = 4$

b. What does this tell you about the three equations?
They are algebraically equivalent equations.

c. Using algebraic operations, show how the equations in problem 2 are equivalent equations.

$2x - 5 = 3$ $\Rightarrow 2x = 8$ $\Rightarrow x = 4$

$2x - 5 + 5 = 3 + 5$ $2x/2 = 8/2$

$2x = 8$ $x = 4$

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4. Solve the following equations for ***x***.

a. ***ax*** +5 = 20

$ax + 5 = 20$

$ax = 5 - 5 = 20 - 5$

$ax = 15$

$x = \frac{15}{a}$

b. 8 – ***bx*** =-15

$8 - bx = -15$

$8 - 8 - bx = -15 - 8$

$-bx = -23$

$x = \frac{23}{b}$

c. 12 = ***rx*** +24

$12 = rx + 24$

$12 - 24 = rx + 24 - 24$

$-12 = rx$

$-\frac{12}{r} = x$

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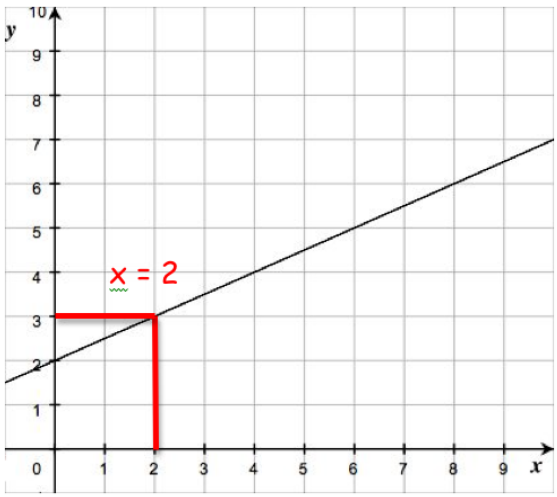
LESSON 5: STAYING SHARP

Practicing skills & concepts

1. Use the graph of the function rule $y = 2 + 0.5x$ to solve the equation $3 = 2 + 0.5x$ for x.



2. Find the x- and y-intercepts of the line with the equation $4x - 9y = 30$.

Answer with supporting work:

x-intercept:

$\Rightarrow 4x - 0 = 30 \Rightarrow x = 7.5$

y-intercept:

$\Rightarrow 0 - 9y = 30 \Rightarrow y = -30/9$

Preparing for upcoming lessons

3. Identify whether each point makes the inequality $3x + 4y > 24$ true.

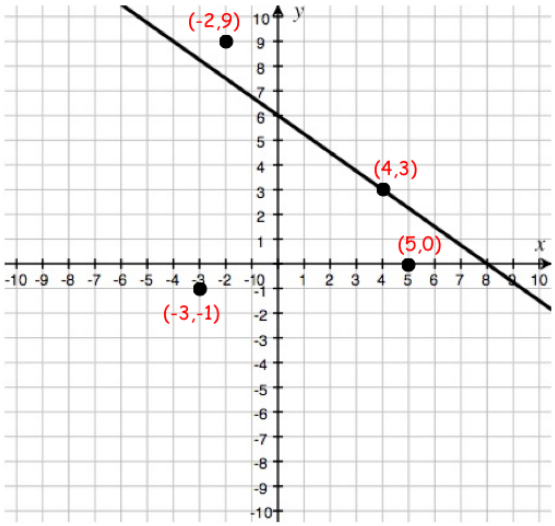
a. (5, 0) $\Rightarrow$ $3(5) + 4(0) = 15$
No $15 < 24$

b. (4, 3) $\Rightarrow$ $3(4) + 4(3) = 24$
No $24 = 24$

c. (-2, 9) $\Rightarrow$ $3(-2) + 4(9) = 30$
Yes $30 > 24$

d. (-3, -1) $\Rightarrow$ $3(-3) + 4(-1) = -13$
No $-13 < 24$

4. The graph of the equation $3x + 4y = 24$ is shown. Plot the points from question 3 on the coordinate grid.



Reviewing ideas from earlier grades

5. What is the ratio of the volume of a cube with a side length of 2 cm to the volume of a cube with a side length of 4 cm?

$2^3 : 4^3 = 8 : 64 = 1 : 8$

6. By what number would you multiply $-\frac{3}{8}$ to get a product of 1?

Any number multiplied by its reciprocal is equal to 1.
Multiply -3/8 by -8/3 to get 1.

$-3/8 \bullet -8/3 = 1$

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Lesson 6: Solving multi-step equations

LESSON 6: OPENER

Justify each step in the solution of the equation $2x + 6 = -4$. Use the answer choices provided.

Subtraction Property of Equality	**Addition Property of Equality**	**Multiplication Property of Equality**
Multiplication of real numbers	**Addition of real numbers**	**Division Property of Equality**

$$2x+6=-4$$

Addition Property of Equality

$$2x+6+(-6)=-4+(-6)$$

Addition of real numbers

$$2x=-10$$

Multiplication Property of Equality

$$\frac{1}{2}\cdot 2x=\frac{1}{2}\cdot -10$$

Multiplication of real numbers

$$x=-5$$

LESSON 6: CORE ACTIVITY

1. Complete a journal entry about algebraic properties and how they can be used to solve equations.

Algebraic property	My understanding of what the property means	How I would use this property to solve this equation
a. Addition Property of Equality	Student responses will vary	$a+6=9$ $a+6+(-6)=9+(-6)$ $a=3$
b. Subtraction Property of Equality		$a+6=9$ $a+6-6=9-6$ $a=3$
c. Multiplication Property of Equality		$4x=24$ $\frac{1}{4}\cdot 4x=\frac{1}{4}\cdot 24$ $x=6$
d. Division Property of Equality		$4x=24$ $\frac{4x}{4}=\frac{24}{4}$ $x=6$

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2. Use algebraic properties to solve the equation $3(\boldsymbol{x} + 2) + \boldsymbol{x} = 16$

$(3 \cdot x) + (3 \cdot 2) + x = 16$
$3x + 6 + x = 16$
$3x + x + 6 = 16$
$(3 + 1)x + 6 = 16$
$4x + 6 = 16$
$4x + 6 - 6 = 16 - 6$
$4x = 10$
$4x/4 = 10/4$
$x = 10/4$
$x = 5/2$

3. Use algebraic operations to solve the equation $8 - 3\boldsymbol{y} = 7\boldsymbol{y} - 2 - 5\boldsymbol{y}$.

$8 - 3y = 2y - 2$
$8 - 3y - 2y = 2y - 2y - 2$
$8 - 5y = -2$
$8 - 8 - 5y = -2 - 8$
$-5y = -10$
$-5y/-5 = -10/-5$
$y = 2$

4. Use algebraic operations to solve the equation $4(\boldsymbol{k} - 3) + 7 = 2\boldsymbol{k} - (\boldsymbol{k} + 8)$.

$4k - 12 + 7 = 2k - k - 8$
$4k - 5 = 1k - 8$
$4k - 1k - 5 = 1k - 1k - 8$
$3k - 5 = -8$
$3k - 5 + 5 = -8 + 5$
$3k/3 = -3/3$
$k = -1$

LESSON 6: CONSOLIDATION ACTIVITY

1. Selling shirts outside the ballpark, Marcos gets paid \$50.00 a day, plus \$3.00 for every shirt he sells. He also has to pay \$1.00 tax for every shirt he sells. He models his earnings for the day as $\boldsymbol{e} = 3\boldsymbol{s} + 50 - 1\boldsymbol{s}$, where $\boldsymbol{e}$ represents total earnings and $\boldsymbol{s}$ represents the number of shirts he sells.

   To figure out the number of shirts he has to sell to earn \$100 for the day, Marcos solves an equation and justifies his steps. Is Marcos' conclusion right? If not, what mistake did he make, and how many shirts would he need to sell to earn \$100?

Marcos writes:	**Marcos says:**
$100 = 3\boldsymbol{s} + 50 - 1\boldsymbol{s}$	To find how many shirts I have to sell to earn \$100, I substitute 100 for $\boldsymbol{e}$ in the function equation.
$100 = 3\boldsymbol{s} - 1\boldsymbol{s} + 50$	I used the Commutative Property to rearrange terms and collect the $\boldsymbol{s}$ terms together.
$100 = (3 - 1)\boldsymbol{s} + 50$	I applied the Distributive Property in reverse to factor out $\boldsymbol{s}$ from $3\boldsymbol{s} - 1\boldsymbol{s}$.
$100 = 2\boldsymbol{s} + 50$	I simplified by subtracting 1 from 3 to get 2.
$\frac{100}{2} = \frac{2\boldsymbol{s}}{2} + 50$	I used the Division Property of Equality, dividing expressions on both sides of the equal sign by 2.
$50 = s + 50$	I divided to simplify the expressions.
$50 - 50 = \boldsymbol{s} + 50 - 50$	I used the Subtraction Property of Equality, subtracting 50 from both sides of the equal sign.
$0 = \boldsymbol{s}$	I subtracted to get a solution of zero shirts. So even if I don't sell any shirts, I still make \$100!

When Marcos divided the expressions on both sides of the equation by 2, he did not divide the entire expression on the right side of the equation by 2. Each term must be divided by 2, like this:

$$100 = 2\boldsymbol{s} + 50$$
$$\frac{100}{2} = \frac{2\boldsymbol{s} + 50}{2}$$
$$50 = \frac{2\boldsymbol{s}}{2} + \frac{50}{2}$$
$$50 = \boldsymbol{s} + 25$$
$$50 - 25 = \boldsymbol{s} + 25 - 25$$
$$25 = \boldsymbol{s}$$

Marcos would need to sell 25 shirts to earn \$100 for the day.

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2. Ava and Bernardo each tried to solve the equation $-3(x - 5) = 20$. They each got different results.

Ava	Bernardo
$-3(x - 5) = 20$	$-3(x - 5) = 20$
$-3x - 15 = 20$	$-3x + 15 = 20$
$-3x - 15 + 15 = 20 + 15$	$-3x + 15 - 15 = 20 - 15$
$-3x = 35$	$-3x = 5$
$-3x \cdot \left(-\frac{1}{3}\right) = 35 \cdot \left(-\frac{1}{3}\right)$	$\frac{-3x}{-3} = \frac{5}{-3}$
$x = -\frac{35}{3}$	$x = -\frac{5}{3}$

a. Whose solution is correct? What mistake did the other student make?

Bernardo's solution is correct.

Ava did not apply the Distributive Property correctly in her first step. Multiplying the "– 5" term in the parentheses by –3 would result in a value of +15, not –15.

b. What is Bernardo's justification for rewriting the original equation as $-3x + 15 = 20$ in the first step of his solution?

Bernardo is applying the Distributive Property of Multiplication over Subtraction for the expression on the left side of the equal sign.

3. Chandra, David, and Elise each tried to solve the equation $5x + 8 = 12 - 2(x + 3)$. They got three different results.

Chandra	David	Elise
$5x + 8 = 12 - 2(x + 3)$	$5x + 8 = 12 - 2(x + 3)$	$5x + 8 = 12 - 2(x + 3)$
$5x + 8 = 12 - 2x - 6$	$5x + 8 = 12 - 2x + 3$	$5x + 8 + (-12) = 12 + (-12) - 2(x + 3)$
$5x + 8 = -2x + 6$	$5x + 8 = -2x + 15$	$5x - 4 = -2(x + 3)$
$5x + 2x + 8 = -2x + 2x + 6$	$\frac{1}{5} \cdot 5x + 8 = -2x + 15 \cdot \frac{1}{5}$	$5x - 4 - 3 = -2(x + 3 - 3)$
$7x + 8 = 6$	$x + 8 = -2x + 3$	$5x - 7 = -2x$
$7x + 8 - 8 = 6 - 8$	$x - x + 8 = -2x - x + 3$	$5x - 5x - 7 = -2x - 5x$
$7x = -2$	$8 = -3x + 3$	$-7 = -7x$
$x = -\frac{2}{7}$	$8 - 3 = -3x + 3 - 3$	$1 = x$
	$5 = -3x$	
	$-\frac{5}{3} = x$	

Whose solution is correct? What mistakes did each of the others make?

Chandra's solution is correct.

David did not use the Distributive Property correctly. In his first step, he did not multiply both terms in the parentheses by –2. In the third step, he did not multiply all of the terms in each expression by 1/5 when applying the Multiplication Property of Equality.

Elise did not apply the Subtraction Property of Equality correctly. When subtracting 3 from expressions on each side of the equal sign, she did so inside of the parentheses on the right side. This would actually result in adding 6 to the right side of the equal sign if the –3 inside of the parentheses is multiplied by the –2 outside of the parentheses.

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4. Solve these equations using algebraic properties. Compare your solutions with your partner.

   a. $5x - 14 = 8x + 4$

   $5x - 8x - 14 = 8x - 8x + 4$

   $-3x - 14 = 4$

   $-3x - 14 + 14 = 4 + 14$

   $-3x/-3 = 18/-3$

   $x = -6$

   b. $-\frac{1}{2}(-4x + 3) - 2 = 1 + \frac{1}{2}x$

	Justifications for question 5:
$2x - 3/2 - 2 = 1 + \frac{1}{2}x$	Distributive Property
$2x - 7/2 = 1 + \frac{1}{2}x$	Subtract to simplify the expression on the left
$2x - \frac{1}{2}x - 7/2 = 1 + \frac{1}{2}x - \frac{1}{2}x$	Use Subtraction Property of Equality to isolate x-term
$3x/2 - 7/2 = 1$	Subtract to simplify expressions on both sides
$3x/2 - 7/2 + 7/2 = 1 + 7/2$	Use Subtraction Property of Equality to isolate constant term
$3x/2 = 9/2 \Rightarrow x = 3$	Add and multiply to simplify expressions

5. Write a justification next to each step of your solution to the equation in question 4b. Below are some properties and phrases you can use to help. You may want to use the example in question 1 as a guide for justifying steps.

- Addition Property of Equality
- Subtraction Property of Equality
- Multiplication Property of Equality
- Division Property of Equality
- Commutative Property
- Distributive Property
- Simplifying an expression by adding, subtracting, multiplying, or dividing terms
- Associative Property

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LESSON 6: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Justify each step in the solution of the equation 5(***x*** +2) = 24 – 2***x***.

Distributive Property of Multiplication over Addition	**Associative Property of Addition**	**Commutative Property of Addition**

Addition Property of Equality	**Additive Inverse Property**	**Additive Identity Property**	**Addition of integers**

5x + 10 = 24 – 2x	Distributive Property of Multiplication over Addition
5x + 10 + (2x) = 24 – 2x + (2x)	Addition Property of Equality
5x + (2x) + 10 = 24 – 2x + (2x)	Commutative Property of Addition
[5x + (2x)] + 10 = 24 + [-2x + (2x)]	Associative Property of Addition
7x + 10 = 24 + 0	Additive Identity Property
7x + 10 = 24	Addition of integers

2. Finish solving the equation in problem 1.
   7x + 10 - 10 = 24 - 10
   7x = 14
   7x ÷ 7 = 14 ÷ 7 ⇒ x = 2

3. Use a combination of inverse operations to solve the following equations.

a. 3**x** – 4 = 11

3x - 4 + 4 = 11 + 4

3x/3 = 15/3

x = 5

b. 2(x – 1) = -6

x - 1 = -3

x - 1 + 1 = -3 + 1

x = -2

c 4(**x** – 5) = –3(**x** – 5)

4x - 20 = -3x + 15

6x - 11 = 12x + 7

-18 = 6x

-3 = x

d. 4**x** – 5 + 2(**x** – 3) = 12**x** + 7

4x - 5 + 2x - 6 = 12x + 7

7x = 35

x = 5

e. $\frac{1}{3}x - 4 = \frac{5}{3}x + 8$

$-12 = \frac{4}{3}x$

$-12\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)\frac{4}{3}x$

$-9 = x$

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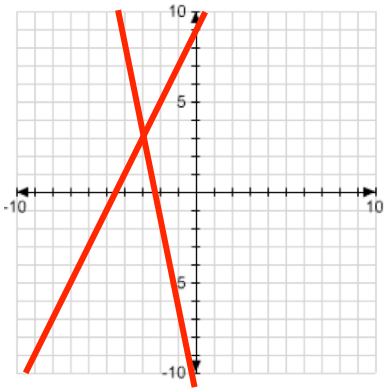
4. Use algebraic operations to solve the following equations. Then use a graph to verify each solution.

a. $4x + 9 - 2x = -5x - 12$

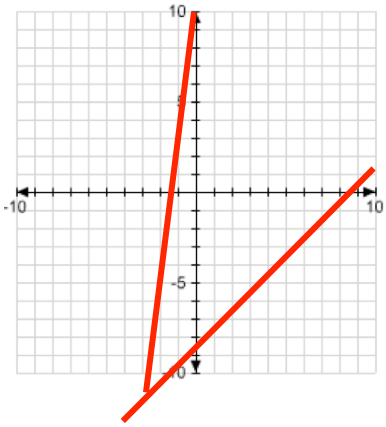


$2x + 9 = -5x - 12$

$7x = -21$

$x = -3$

b. $7(x + 2) - 4 = 8 + x - 14$



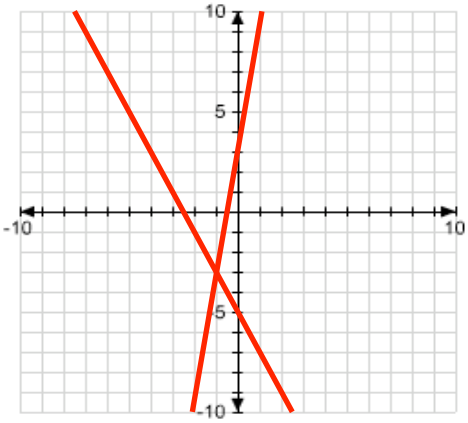
$7x + 14 - 4 = -6 + x$

$7x + 10 = -6 + x$

$6x = -16$

$x = -8/3$

c. $3(2\mathbf{x} + 1) = -5\mathbf{x} - 5 + 3\mathbf{x}$

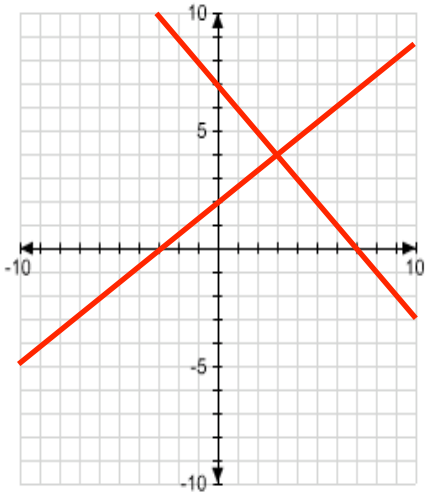


$6\mathbf{x} + 3 = -2\mathbf{x} - 5$

$8\mathbf{x} = -8$

$\mathbf{x} = -1$

d. $2(\frac{1}{3}\boldsymbol{x}+4)-6=-\frac{1}{3}(3\boldsymbol{x}-21)$



$$2(\frac{1}{3}x+4)-6=-\frac{1}{3}(3x-21)$$

$$\frac{2}{3}x+8-6=-x+7$$

$$\frac{2}{3}x+2=-x+7$$

$$\frac{5}{3}x=5$$

$$x=3$$

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5. Louis created the following table to solve a linear equation. Use the information in the table to determine the equation he was trying to solve. Then find the solution to his equation.

x	?	?
-3	9	12
–2	7	7
–1	5	2
0	3	–3
1	1	–8

Louis was trying to solve the equation $-2x + 3 = -5x - 3$. The solution to this equation is $x = -2$.

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LESSON 6: STAYING SHARP

Practicing skills & concepts	**1.** A balance scale like the one shown here is in balance when it has 11 equally sized pieces of gold on one side and 3 1-pound bricks on the other. How much does each piece of gold weigh? Answer with supporting work: 3 pounds ÷ 11 = 3/11 pounds	**2.** A line is described by the equation $6x + 10y = 30$. Identify the slope and y-intercept of the line. Answer with supporting work: $6x + 10y = 30$ $10y = -6x + 30$ $y = -3/5\ x + 3$ slope = -3/5 y-intercept = 3
Preparing for upcoming lessons	**3.** Write the coordinates of one point that makes the inequality $5x - 2y > 11$ true. Show that the point you selected makes the inequality true. Answer with supporting work: (10,10) $5(10) - 2(10) = 30 \Rightarrow 30 > 11$ Student responses will vary	**4.** Write the coordinates of one point that does NOT make the inequality $5x - 2y > 11$ true. Show that the point you selected does NOT make the inequality true. Answer with supporting work: (0,0) $5(0) - 2(0) = 0$ $0 < 11$ Student responses will vary
Reviewing ideas from earlier grades	**5.** The circumference of a circle is given by the formula $C = 2\pi r$, where r represents the length of its radius. If a circle has a circumference of 24π inches, what is the length of its radius? $24\pi = 2\pi r$ r = 12 inches	**6.** Find a common denominator for the following fractions: $\frac{3}{4}, \frac{1}{9}$, and $\frac{13}{3}$ Answer: $4 \bullet 9 = 36$ $9 \bullet 4 = 36$ $3 \bullet 12 = 36$ Common denominator = 36

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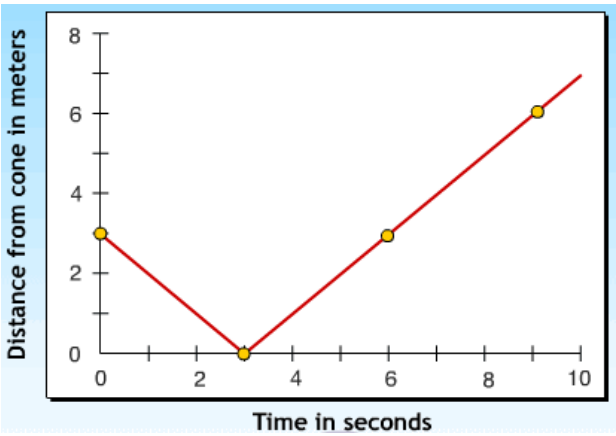
Lesson 7: Solving absolute value equations

LESSON 7: OPENER



The graph of Terrence's skate is shown. The function that models his skate is the absolute value function $y = |x - 3|$.

1. Use the function rule to create an equation that will tell you when Terrence was exactly 2 meters from the safety cone.

   $|x - 3| = 2$

2. Can you tell from the graph when Terrence was 2 meters from the cone for the first time? How many times was Terrence 2 meters from the cone?

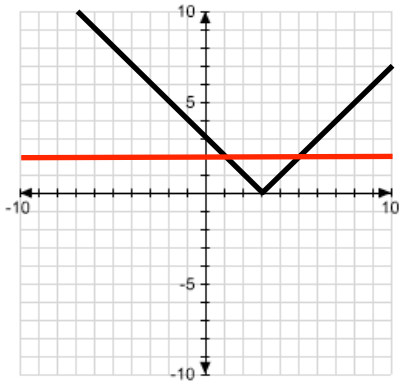
   By looking at points on the graph and analyzing the y-coordinate, you can see that Terrence's distance from the cone was 2 meters around 1 second. He was also 2 meters from the cone at around 5 seconds.

LESSON 7: CORE ACTIVITY

1. Consider the equation $|x - 3| = 2$.

   a. What line could you add to the graph of $y = |x - 3|$ to represent this equation? Add this line to the graph of $y = |x - 3|$.

   You could add the line $y = 2$ to the graph of $|x - 3| = 2$



   b. How does the graph you created help you solve the equation $|x - 3| = 2$? What are the solutions to this equation?

   The solution to the equation can be found by identifying the x-values where the two lines intersect. The solution to $|x - 3| = 2$ is $x = 1$ or $x = 5$.

   c. What do these solutions mean in context of Terrence's skate?

   These solutions mean that Terrence was exactly 2 meters away from the safety cone at two different times: once at 1 second, and again at 5 seconds.

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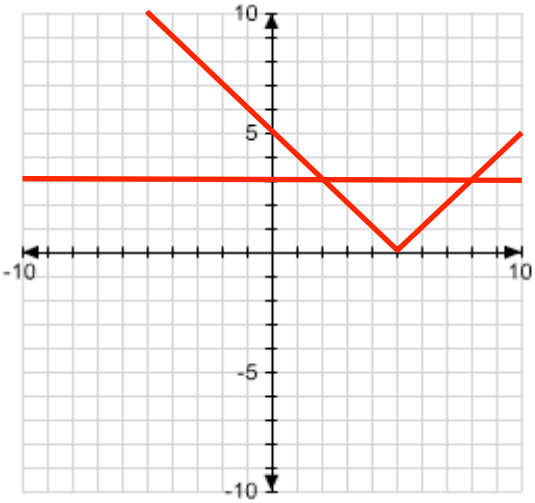
2. Now consider a different skate by Terrence. This time, Terrence starts 5 meters away from the cone and skates toward it at 1 meter per second. The function that represents this skate is $y=|x-5|$.

   a. Write an equation you could solve to find the times when Terrence is 3 meters away from the cone.

   $|x - 5| = 3$

   b. Solve your equation by graphing.



   The solutions to the equation $|x - 5| = 3$ are $x = 2$ and $x = 8$.

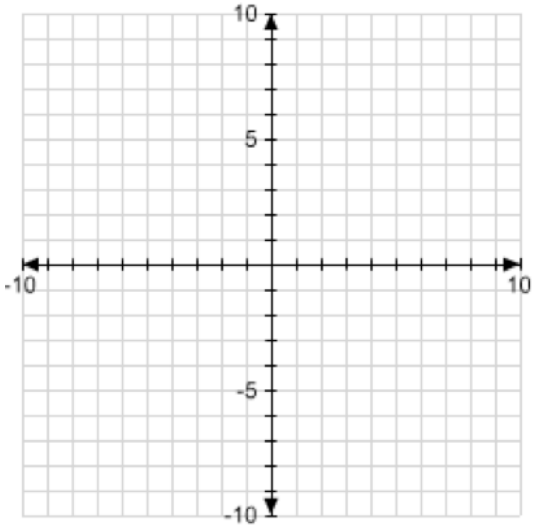
   c. What do these solutions mean in the context of Terrence's new skate?

   These solutions mean that Terrence was 3 meters from the safety cone at 2 seconds and again at 8 seconds.

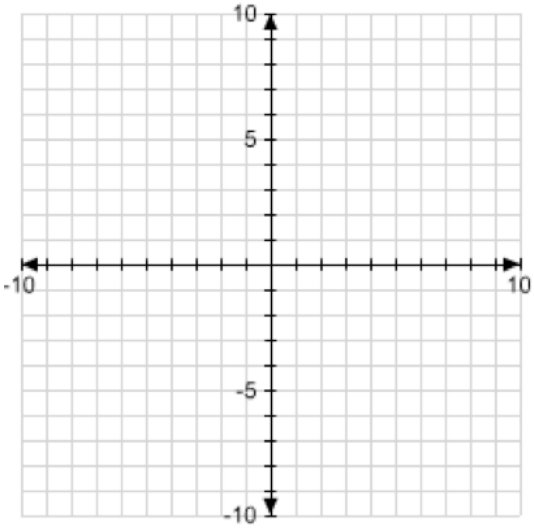
3. The two equations you just solved both had two solutions. Think about whether it is possible for an absolute value equation to have one solution or no solution.

   Create a graph that represents an equation that has one solution. Then create a graph that represents an equation that has no solution. Explain why your graphs represent one solution and no solution.

One solution



No solution



Graphs will vary, but for the graph that represents an equation with one solution, the horizontal line should intersect the graph of the absolute value function at its vertex. For the graph that represents an equation with no solution, the graph of the horizontal line should lie below the graph of the absolute value function.

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4. Explain the "two-for-one trade" using your own words. Provide an example.

   Answers will vary.

5. Use the "two-for-one trade" to solve the equation $|x - 3| = 2$.

1	$-x + 3$	5	-1
$-x - 3$	-5	$x - 3$	$\lvert x - 3 \rvert$

	$x - 3 < 0$	$x - 3 \geq 0$
Step 1	$\lvert x - 3 \rvert = -x + 3 = 2$	$\lvert x - 3 \rvert = x - 3 = 2$
Step 2	$x = 1$	$x = 5$

6. Consider the equation $|x + 4| = 5$.

   a. Use the two-for-one trade to solve the equation analytically.

   When $x + 4 < 0$, $|x + 4| = -x - 4$.

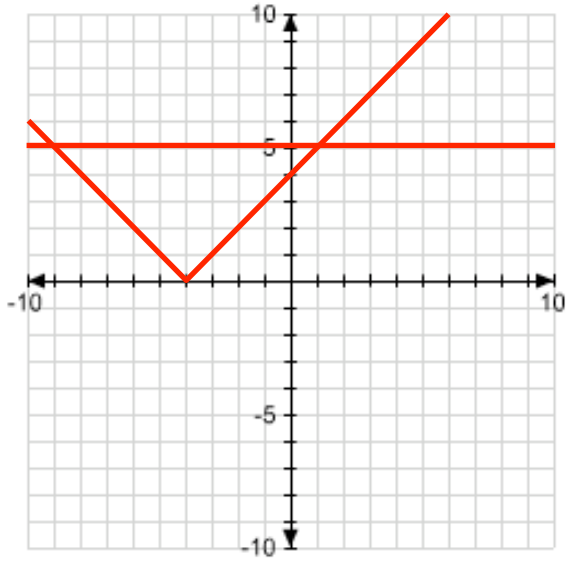
   $-x - 4 = 5$
   $-x = 9$
   $x = -9$

   When $x + 4 \geq 0$, $|x + 4| = x + 4$.

   $x + 4 = 5$
   $x = 1$

   The solutions to the equation are $x = -9$ and $x = 1$

   b. Verify your solution using a graph.



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7. Consider the equation $|2x - 5| = 7$

a. Use the two-for-one trade to solve the equation analytically.

When $2x - 5 < 0$, $|2x - 5| = -2x + 5$.

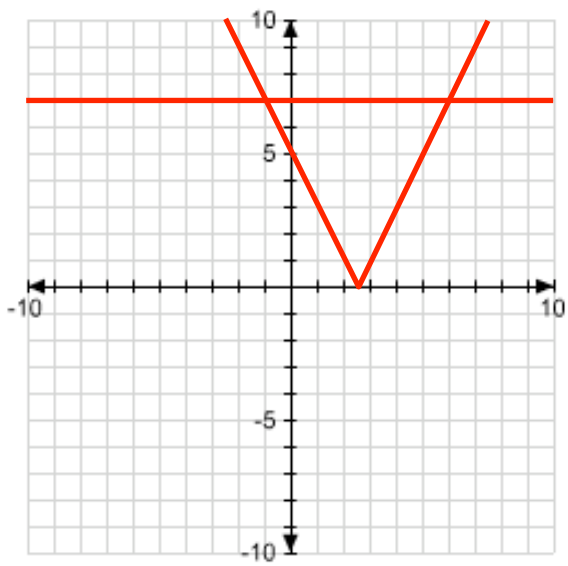
$-2x + 5 = 7$
$-2x = 2$
$x = -1$

When $2x - 5 \geq 0$, $|2x - 5| = 2x - 5$.

$2x - 5 = 7$
$2x = 12$
$x = 6$.

The solutions to the equation are $x = -1$ and $x = 6$.

b. Verify your solution using a graph.



LESSON 7: CONSOLIDATION ACTIVITY

1. Consider the functions $y = |3x + 3|$ and $y = 3$.

a. Values for x are given in the table. Complete the table to show values for each function for each given x value.

x	$\|3x + 3\|$	3
–4	9	3
–3	6	3
–2	3	3
–1	0	3
0	3	3
1	6	3
2	9	3

b. Use the table to solve the absolute value equation $|3x + 3| = 3$. Explain how the table supports the solution to this equation.

The values of $|3x + 3|$ are equal to 3 when x is equal to -2 or x is equal to 0. This is written as $x = -2$ or $x = 0$.

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c. Now solve the equation analytically. Don't forget to use the two-for-one trade.

When $3x + 3 < 0$, $|3x+3| = -3x-3$.

$-3x - 3 = 3$

$-3x = 6$

$x = -2$

When $3x + 3 \geq 0$, $|3x+3| = 3x+3$..

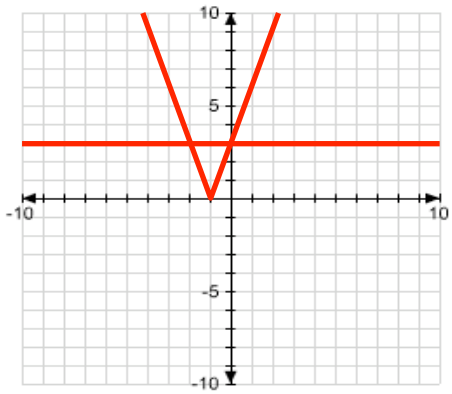
$3x + 3 = 3$

$3x = 0$

$x = 0$

The solution to the equation $|3x+3| = 3$ is $x = -2$ or $x = 0$.

d. Use the table to create a graph of the two functions. Use your graph to justify the solution you found.



The graph of the absolute value function intersects the line $y =$ at $x = -2$ and $x = 0$.

e. How do the two-for-one trade equations appear in the graph you drew in part d? Which equation is associated with each part of the graph? Label each part of the graph in part d with the correct equation.

To the left of the vertex, the function $y = -3x - 3$ is graphed. The left intersection point is found by solving the equation $-3x - 3 = 3$. To the right of the vertex, the function $y = 3x + 3$ is graphed. The right intersection point is found by solving the equation $3x + 3 = 3$.

2. Consider the absolute value equation $\left|\frac{1}{2}x - 1\right| = 1$.

a. Values for x are given in the table. Complete the table to show the values of the expressions on each side of the equation for each given x value.

x	$\left\|\frac{1}{2}x - 1\right\|$	1
–1	1.5	1
0	1	1
1	0.5	1
2	0	1
3	0.5	1
4	1	1
5	1.5	1

b. What is the solution to the equation?

The solution to the equation is $x = 0$ or $x = 4$.

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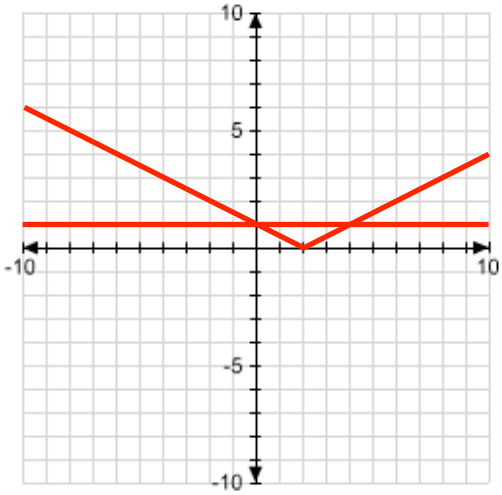
c. Verify your solution using either analytical techniques or a graph. Why did you choose the method you did?

Explanations regarding the preferred solution method will vary.

Using a graph:



Solving analytically:

When $\frac{1}{2}x - 1 < 0$, $\left|\frac{1}{2}x - 1\right| = -\frac{1}{2}x + 1$.

$$-\frac{1}{2}x + 1 = 1$$
$$-\frac{1}{2}x = 0$$
$$x = 0$$

When $\frac{1}{2}x - 1 \geq 0$, $\left|\frac{1}{2}x - 1\right| = \frac{1}{2}x - 1$.

$$\frac{1}{2}x - 1 = 1$$
$$\frac{1}{2}x = 2$$
$$x = 4$$

The solutions to the equation are $x = 0$ and $x = 4$.

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LESSON 7: HOMEWORK

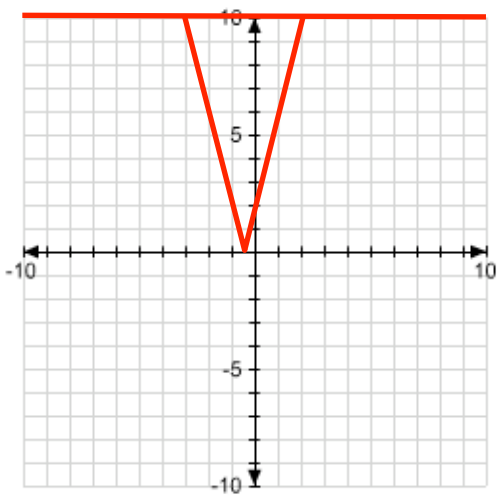
Notes or additional instructions based on whole-class discussion of homework assignment:

1. Solve the equation $|4x+2|=10$. Confirm your solution using a graph or table.

The two equations to solve are $4x + 2 = 10$ and $-4x - 2 = 10$.

$4x + 2 = 10$
$4x = 8$
$x = 2$

$-4x - 2 = 10$
$-4x = 12$
$x = -3$



x	$\lvert 4x+2 \rvert$	10
-4	14	10
-3	10	10
-2	6	10
-1	2	10
0	2	10
1	6	10
2	10	10
3	14	10

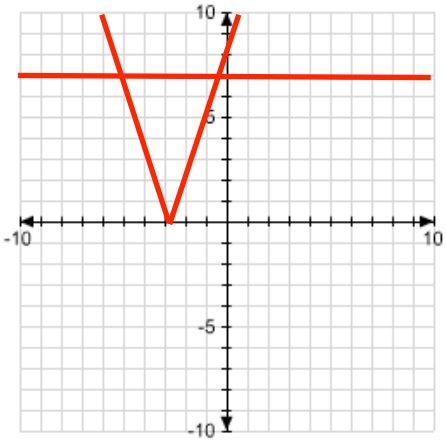
2. Solve $|3x+8|=7$ analytically. Verify your solution using a graph or a table.

First, solve the equation when $3x + 8 \geq 0$. If $3x + 8 \geq 0$, then $|3x+8| = 3x + 8$. So the equation becomes:

$3x + 8 = 7$
$3x = -1$
$x = -\frac{1}{3}$

Next, solve the equation when $3x + 8 < 0$. If $3x + 8 < 0$, then $|3x+8| = -3x - 8$. So the equation becomes:

$-3x - 8 = 7$
$-3x = 15$
$x = -5$



x	$\lvert 3x+8 \rvert$	7
-6	10	7
-5	7	7
-4	4	7
-3	1	7
-2	2	7
-1	5	7
$-\frac{1}{3}$	7	7

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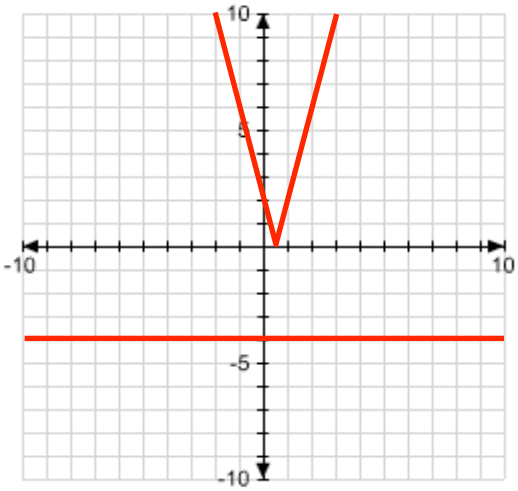
3. Solve the equation $|4x-2|=-4$ using either a graph or table.

This equation has no solution. The graph of the absolute value function will never fall below the x-axis, so it will never intersect the line $y = -4$.



x	$\lvert 4x-2 \rvert$	-4
-2	10	-4
-1	6	-4
0	2	-4
1	2	-4
2	6	-4
3	10	-4

4. How would you change the equation in question 3 so that there is only one solution to the equation? How would you change it so that the equation has two solutions?

The equation $|4x+2|=0$ has exactly one solution, at $x = 0.5$.

To have two solutions, change the equation so that the absolute value function equals a positive number.

5. You have solved absolute value equations analytically, by graphing and by using tables. Which method do you prefer and why?

Answers will vary.

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LESSON 7: STAYING SHARP

Practicing skills & concepts

1. Solve for the value of the star in the equation.

$-2\,(\star + 2) = -12$

Answer with supporting work:

$\star = 4$. $-2(6) = -12$, so $\star + 2 = 6$, which means $\star = 4$.

2. Consider a line that represents the equation $y - 5 = \frac{1}{3}(x + 6)$. What are the slope and y-intercept of the line?

Answer with supporting work:

The slope of the line is $\frac{1}{3}$. To find the y-intercept, convert to slope-intercept form.

$$y - 5 = \frac{1}{3}(x + 6)$$

$$y - 5 = \frac{1}{3}x + 2$$

$$y = \frac{1}{3}x + 7$$

The y-intercept is 7.

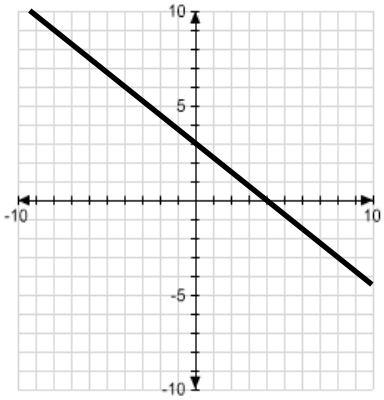
Preparing for upcoming lessons

3. Find two points that make the inequality $3x + 4y > 12$ true. Then find two points that make the inequality false.

True points: **False points:**

Answers will vary.

4. The graph of $3x + 4y = 12$ is shown. Where do the points that satisfy the inequality from question 3 lie? Where do the points that do not satisfy the inequality lie?



The points that satisfy the inequality lie above the line, while points that do not satisfy the inequality lie below the line.

Reviewing ideas from earlier grades

5. A cube has a volume of 64 cm^3. What is the length of a side of the cube?

4 cm

6. What is the least common denominator of the fractions $\frac{5}{8}$ and $\frac{3}{10}$? Explain.

The least common denominator is 40. The smallest number that 8 can be multiplied by to reach a factor of 10 is 5. 8(5) = 40 and 10(4) = 40.

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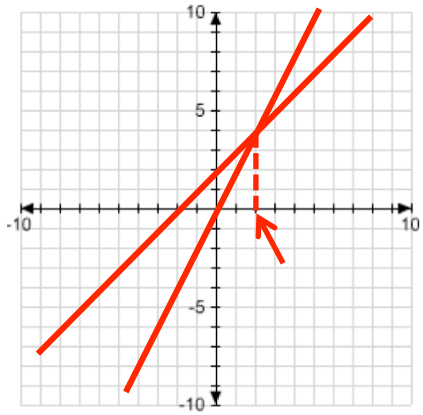
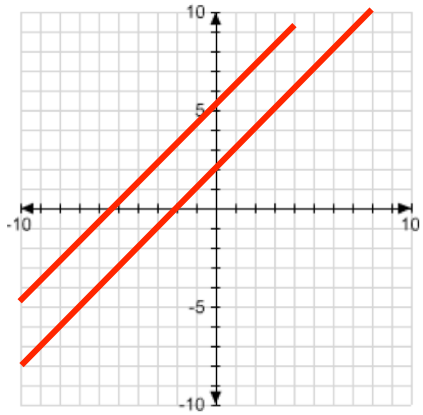
Lesson 8: How many solutions?

LESSON 8: OPENER

1. Use your number sense to explain why it is impossible to find a number that makes this equation true:
   $x + 2 = x + 5$
   Since x will always be the same on both sides of the equation, if you add two different numbers (for instance, 2 and 5) to the same starting value, the two sums will never be the same.
2. Use your number sense to explain why any number you choose will make this equation true:
   $x + 2 = x + 2$
   Since x will always be the same on both sides of the equation, you will always be adding identical expressions on each side of the equation.

LESSON 8: CORE ACTIVITY

1. Solve each equation by using algebraic operations and by using a graph. Then explain the meaning of the solution in the third column of the table.

Solve the equation using algebraic operations.	Graph the functions on either side of the equation to find the solution.	Explain what the solution means.
a. $2x = x + 2$ $2x - x = x - x + 2$ $x = 2$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is true when $x = 2$. This is the value of x for which both sides of the equation are equal. It is the x-coordinate of the point where the graphs of the functions on either side of the equal sign intersect.
b. $x + 2 = x + 5$ $x - x + 2 = x - x + 5$ $2 = 5$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is never true for any value of x.

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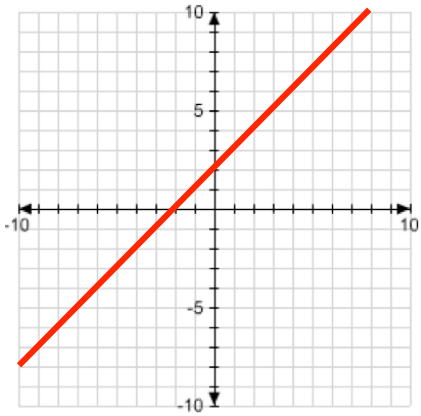
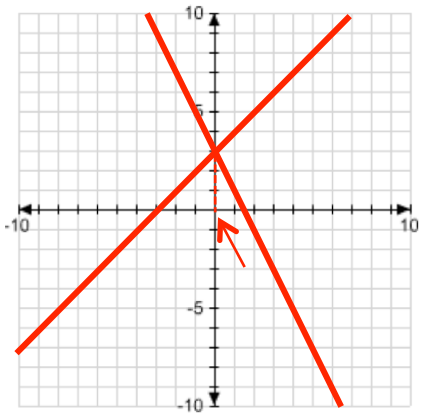
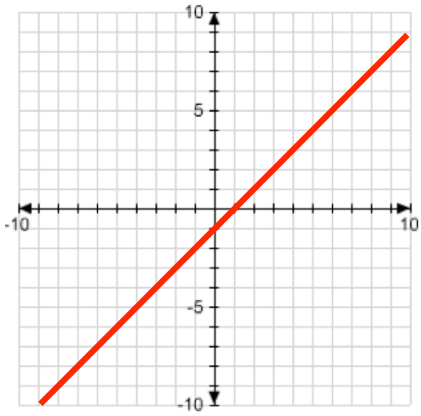
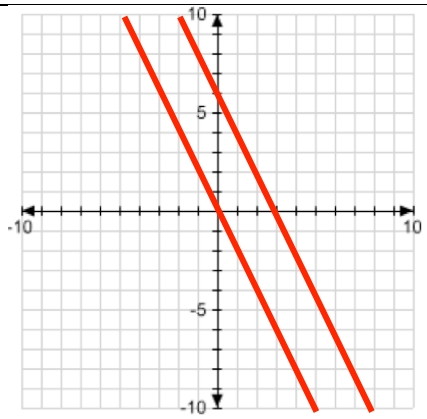
Solve the equation using algebraic operations.	Graph the functions on either side of the equation to find the solution.	Explain what the solution means.
c. $x + 2 = x + 2$ $x + 2 - 2 = x + 2 - 2$ $x = x$ $x - x = x - x$ $0 = 0$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is true for all values of x.
d. $x + 3 = -2x + 3$ $x + 3 - 3 = -2x + 3 - 3$ $x = -2x$ $x - x = -2x - x$ $0 = -3x$ $\frac{0}{-3} = \frac{-3x}{-3}$ $0 = x$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is true when $x = 0$.
e. $x - 1 = \frac{1}{2}(2x - 2)$ $2(x - 1) = \frac{1}{2} \cdot 2(2x - 2)$ $2(x - 1) = 2x - 2$ $2x - 2 = 2x - 2$ $2x - 2 + 2 = 2x - 2 + 2$ $2x = 2x$ $2x - 2x = 2x - 2x$ $0 = 0$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is true for all values of x.
f. $2(3 - x) = -2x$ $6 - 2x = -2x$ $6 - 2x + 2x = -2x + 2x$ $6 = 0$		Show on the graph where the solution to the equation is. What is the solution? (For what value of x is the equation true?) The equation is never true for any value of x.

LESSON 8: ONLINE ASSESSMENT

Today you will take an online assessment.

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LESSON 8: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

1. Consider the following equation: $4 + 6x = 6x + 4$. Explain why the equation has many solutions.
   The two linear equations y = 4 + 6x and y = 6x + 4 are the same line so they share all points.

2. Consider the following equation: $2 + 2x = 3 + 2x$. Explain why the equation does not have a solution.

   The two linear equations y = 2 + 2x and y = 3 + 2x have the same slope but different y-intercepts. These lines never intersect; they are parallel.

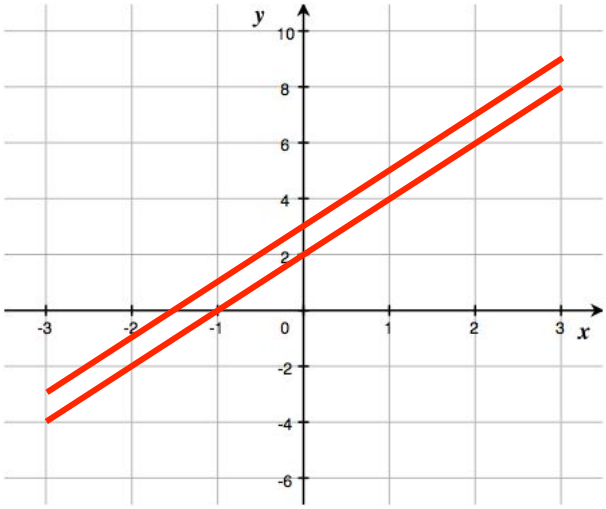
3. Solving $2 + 2x = 3 + 2x$ can provide justification that the equation does not have a solution. Investigate this equation using tables, a graph, and inverse operations.
   a. **Tables:** Complete the tables and look for a solution. How can you tell from the tables that the equation does not have a solution?

x	$y = 2 + 2x$
-2	-2
-1	0
0	2
1	4
2	6
3	8

x	$y = 3 + 2x$
-2	-1
-1	1
0	3
1	5
2	7
3	9

The y-values for the second table are always 1 greater than the corresponding y-value in the first table (at the same x-value). They will never have the same y-value at the same x-value.

   b. **Graph:** Graph both equations on the coordinate plane provided. How can you tell from their graphs that the equation does not have a solution?



The two lines are parallel; they will never intersect.

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c. **Inverse operations:** Solve 2 + 2***x*** = 3 + 2***x*** using inverse operations. How does the algebraic result show you that there is no solution?

2 + 2x = 3 + 2x

2 + 2x – 2x = 3 + 2x – 2x

2 = 3

The algebraic result is an equation that is untrue, meaning there is no solution.

4. How many solutions do you think the equation -10***x*** + 3 = 5***x*** – 27 has? Explain. Then, find the solution(s).

-10x + 3 = 5x - 27
3 = 15x – 27
30 = 15x ⇒ x = 2

y = -10x + 3 and y = 5x – 27 have different slopes so there will be one solution.

5. Indicate whether each statement about the equation 5**x** = 7**x** – 2(**x**–8) is true or false.

Statement	True or False
The solution is *x* = -2.	False
Any number may be substituted for ***x*** and a true equation will result.	False
There is no number you can substitute for ***x*** to make the equation true.	True
Graphing each side of the equation will result in 2 parallel lines.	True
There is no solution; the solution set is { } or Ø.	True

6. A common formula used to find the area of a triangle is $A = \frac{bh}{2}$, where ***b*** is the measurement of the length of the base and ***h*** is the measurement of the height. The area of a triangle is 15 cm^2, and the base is 3 cm. Use algebraic operations to solve an equation to find the measurement of the height of the triangle.

2(15) = (3h/2)•(2)

30 = 3h

30/3 = 3h/3

10 = h

The height is 10 cm

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LESSON 8: STAYING SHARP

Practicing skills & concepts

1. Solve for the value of the star in the equation.

$$4 \cdot \star = 9 \cdot \star + 30$$

Answer with supporting work:

$4\star - 9\star = 30$

$-5\star = 30$

$\star = -6$

2. Determine the coordinates of the ***x***- and ***y***-intercepts of a line with the equation $y = \frac{3}{4}x - 20$ in slope-intercept form.

Answer with supporting work:

x-intercept:

$0 = \frac{3}{4}x - 20$

$20 = \frac{3}{4}x$

$^4/_3(20) = x \Rightarrow x = {}^{80}/_3$

y-intercept:

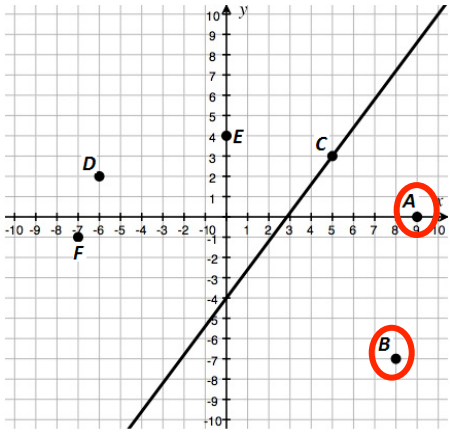
$y = \frac{3}{4}(0) - 20 \Rightarrow y = -20$

Preparing for upcoming lessons

3. Identify whether each point listed below satisfies the inequality $7x - 5y > 20$.

Point	Coordinates	Does point satisfy inequality?
A	(9,0)	YES / NO
B	(8,-7)	YES / NO
C	(5,3)	YES / NO
D	(-6,2)	YES / NO
E	(0,4)	YES / NO
F	(-7,-1)	YES / NO

4. The graph of the equation $7x - 5y = 20$ is shown here, and the points from question 3 are plotted. Circle the points that satisfy the inequality. What do you notice?

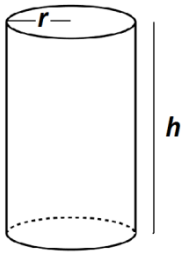


They are on the same side of the line.

Reviewing ideas from earlier grades

5. The volume of a cylinder is given by the formula $V = \pi r^2 h$ where ***r*** represents the length of the radius of the base of the cylinder and ***h*** represents the height of the cylinder. A cylinder has a volume of 54 m^3 and a radius of 3 m. What is its height, ***h***?

Answer with supporting work:



$54 = \pi(3)^2 h$

$54 = 9\pi h$

$\frac{54}{9\pi} = h$

$\frac{6}{\pi} = h$

$h \approx 1.91$ meters

6. By what number would you multiply $\frac{9}{5}$ to get a product of –1?

$9/5 \bullet -5/9 = -1$

Multiply by -5/9

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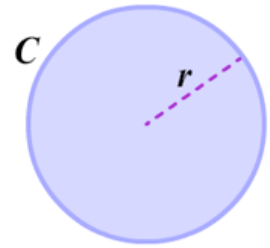
Lesson 9: Rearranging formulas

LESSON 9: OPENER

1. Find the circumference of a circle with a radius of 7 inches. Explain the process you used to find the answer.
   $C = 2\pi(7) = 14\pi$ inches

2. Find the radius of a circle with a circumference of 32π. Explain the process you used to find the answer.
   $32\pi = 2\pi r$
   $r = 16$ inches

$C = 2\pi r$



LESSON 9: CORE ACTIVITY

1. Solve the equation $\boldsymbol{C} = 2\pi\boldsymbol{r}$ for $\boldsymbol{r}$. Explain the process you used to find the answer.
   $C = 2\pi r$
   $r = C/(2\pi)$
   Divide each side by 2π

2. Explain another process you could have used to solve the equation $\boldsymbol{C} = 2\pi\boldsymbol{r}$ for $\boldsymbol{r}$.
   Multiply each side by $1/(2\pi)$

3. Solve the following equations for the specified variable.

a. $\boldsymbol{V} = \boldsymbol{lwh}$ for $\boldsymbol{w}$

$V/(lh) = (lwh)/(lh)$

$V/(lh) = w$

b. $\boldsymbol{ab} - \boldsymbol{d} = \boldsymbol{c}$ for $\boldsymbol{b}$

$ab - d + d = c + d$

$ab/a = (c+d)/a$

$b = (c+d)/a$

c. $\boldsymbol{C} = \frac{5}{9}(\boldsymbol{F} - 32)$ for $\boldsymbol{F}$

$^9/_5\, C = {}^9/_5\,({}^5/_9)\,(F - 32)$

$32 + {}^9/_5\, C = F - 32 + 32$

$^9/_5\, C + 32 = F$

d. $\boldsymbol{V} = \boldsymbol{\pi r^2 h}$ for $\boldsymbol{h}$

$V \div \pi r^2 = \pi r^2 h \div \pi r^2$

$V/(\pi r^2) = h$

e. $\boldsymbol{P} = 2\boldsymbol{l} + 2\boldsymbol{w}$ for $\boldsymbol{l}$

$P = 2l + 2w$

$P - 2w = 2t + 2w - 2w$

$(P-2w)/2 = (2l)/2 \Rightarrow (P - 2w)/2 = l$

f. $\boldsymbol{D} = \boldsymbol{rt}$ for $\boldsymbol{t}$

$\frac{D}{r} = t$

g. $\boldsymbol{V} = \frac{1}{3}\pi \boldsymbol{r}^2 \boldsymbol{h}$ for $\boldsymbol{h}$

$3V = \pi r^2 h$

$\frac{3V}{\pi r^2} = h$

h. $\boldsymbol{A} = \frac{1}{2}\boldsymbol{h}(\boldsymbol{b}_1 + \boldsymbol{b}_2)$ for $\boldsymbol{h}$

$2A = h(b_1 + b_2)$

$\frac{2A}{b_1 + b_2} = h$

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Another equation that you need to be able to work with in this course and future courses is the equation of a line in standard form, ***Ax*** + ***By*** = ***C***. You need to know how to transform an equation in this form to slope-intercept form.

4. Transform the following equations from standard form to slope-intercept form by solving for ***y***.

a. $25x - 5y = 30$

$25x - 25x - 5y = 30 - 25x$

$-5y = 30 - 25x$

$y = -6 + 5x$

b. $2x + 5y = 7$

$2x - 2x + 5y = 7 - 2x$

$5y = 7 - 2x$

$y = {}^{7}/_{5} - {}^{2}/_{5}\,x$

LESSON 9: REVIEW ONLINE ASSESSMENT

You will work with your class to review the online assessment questions.

Problems we did well on:	**Skills and/or concepts that are addressed in these problems:**
Problems we did not do well on:	**Skills and/or concepts that are addressed in these problems:**

Addressing areas of incomplete understanding

Use this page and notebook paper to take notes and re-work particular online assessment problems that your class identifies.

Problem #_____	**Work for problem:**
Problem #_____	**Work for problem:**
Problem #_____	**Work for problem:**

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LESSON 9: HOMEWORK

Notes or additional instructions based on whole-class discussion of homework assignment:

Next class period, you will take a mid-unit assessment. One good way to prepare for tests is to review the important skills and ideas you have learned. Use this list to help you review these skills and concepts, especially by reviewing related course materials.

Important skills and ideas you have learned so far in the unit **Linear equations and inequalities**:

- Analyze situations involving linear functions and formulate linear equations to solve problems
- Solve linear equations by inspection, tables, graphs, and use of algebraic operations in connection with the properties of equality
- For given contexts, interpret and determine the reasonableness of solutions to linear equations
- Apply techniques for solving equations in one variable to solve literal equations

Homework Assignment

Part I: Study for the mid-unit assessment by reviewing the key ideas listed above.

Part II: Complete the online *More practice* in the topic **Solving linear equations**. Note the skills and ideas for which you need more review, and refer back to related activities and animations from this topic to help you study.

Part III: Complete Lesson 9: Staying Sharp.

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LESSON 9: STAYING SHARP

Practicing skills & concepts

1. Solve for the value of the star in the equation.

$$17 = -2 \cdot \star + 3$$

Answer with supporting work:

$17 = -2\star + 3$

$14 = -2\star$

$-7 = \star$

2. A line has the equation $8\boldsymbol{x} - 2\boldsymbol{y} = 24$ in standard form. Rewrite the equation of the line in slope-intercept form. Then report the slope and ***y***-intercept.

Answer with supporting work:

$8x - 2y = 24$

$-2y = -8x + 24$

$y = 4x - 12$

Slope = 4, y-intercept = -12

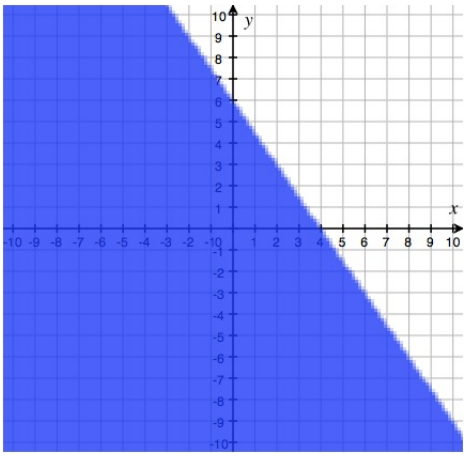
Preparing for upcoming lessons

3. The point (−5,3) is in the shaded region of the graph. Show whether (−5,3) satisfies the inequality $3\boldsymbol{x} + 2\boldsymbol{y} < 12$.

$3(-5) + 2(3)$

$-15 + 6 = -9$

$-9 < 12 \Rightarrow$ the point satisfies the inequality



4. Find another point satisfying the inequality $3\boldsymbol{x} + 2\boldsymbol{y} < 12$. What do your point and (−5,3) have in common on the graph?

(0,0)

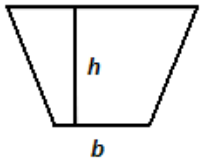
$3(0) + 2(0) = 0$

$0 < 12$

Points (0,0) and (-5,3) are on the same side of the line.

Reviewing ideas from earlier grades

5. The height of a trapezoid (***h***) measures 4 cm. Its shorter base (***b***) measures 3 cm. A similar trapezoid has a shorter base measuring 15 cm. Use proportional reasoning to find the height of that similar trapezoid.



Answer with supporting work:

$3/15 = 4/h$

$3h = 60$

$h = 20$ cm

6. Find a common denominator for these fractions:

$$\frac{5}{24} \text{ and } \frac{7}{18}$$

Is the common denominator you found the **least** common denominator? Explain how you know.

Answer with supporting work:

$$\frac{5}{24} \bullet \frac{3}{3} + \frac{7}{18} \bullet \frac{4}{4} = \frac{15}{72} + \frac{28}{72}$$

72 is the least common denominator as it is the first number that both 24 and 18 go into evenly.

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