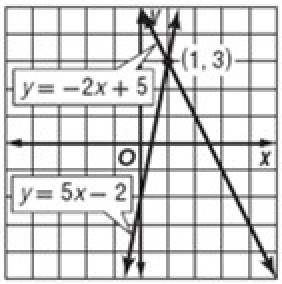
0-5 Systems of Linear Equations and Inequalities

Solve each system of equations by graphing.

1. $y = 5x - 2$
$y = -2x + 5$

SOLUTION:

Graph each equation in the system.



The lines intersect at the point (1, 3). This ordered pair is the solution of the system.

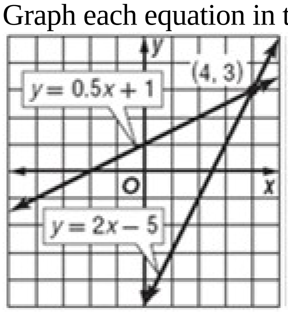
CHECK

$$\begin{aligned} y &= 5x - 2 \\ 3 &= 5(1) - 2 \\ 3 &= 5 - 2 \\ 3 &= 3 \end{aligned} \qquad \begin{aligned} y &= -2x + 5 \\ 3 &= -2(1) + 5 \\ 3 &= -2 + 5 \\ 3 &= 3 \end{aligned}$$

2. $y = 2x - 5$
$y = 0.5x + 1$

SOLUTION:

Graph each equation in the system.



The lines intersect at the point (4, 3). This ordered pair is the solution of the system.

CHECK

$$\begin{aligned} y &= 2x - 5 \\ 3 &= 2(4) - 5 \\ 3 &= 8 - 5 \\ 3 &= 3 \end{aligned} \qquad \begin{aligned} y &= 0.5x + 1 \\ 3 &= 0.5(4) + 1 \\ 3 &= 2 + 1 \\ 3 &= 3 \end{aligned}$$

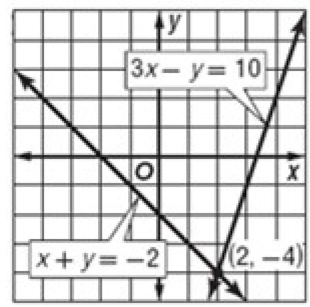
3. $x + y = -2$
$3x - y = 10$

SOLUTION:

Solve each equation for y.

$$\begin{aligned} x + y &= -2 \\ y &= -x - 2 \\ y &= -x - 2 \end{aligned} \qquad \begin{aligned} 3x - y &= 10 \\ -y &= -3x + 10 \\ y &= 3x - 10 \end{aligned}$$

Then graph each equation.



The lines intersect at the point (2, –4). This ordered pair is the solution of the system.

CHECK

$$\begin{aligned} x + y &= -2 \\ 2 + (-4) &= -2 \\ -2 &= -2 \end{aligned} \qquad \begin{aligned} 3x - y &= 10 \\ 3(2) - (-4) &= 10 \\ 6 + 4 &= 10 \\ 10 &= 10 \end{aligned}$$

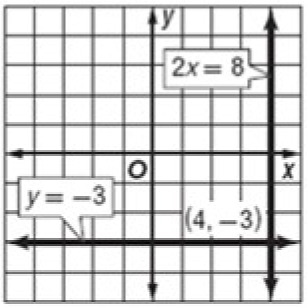
4. $y = -3$
$2x = 8$

SOLUTION:

Solve each equation for y.

$y = -3 \qquad 2x = 8$
$\qquad\qquad\quad x = 4$

Then graph each equation.



The lines intersect at the point (4, –3). This ordered pair is the solution of the system.

CHECK

$y = -3 \qquad 2x = 8$
$-3 = -3 \qquad 2(4) = 8$
$\qquad\qquad 8 = 8$

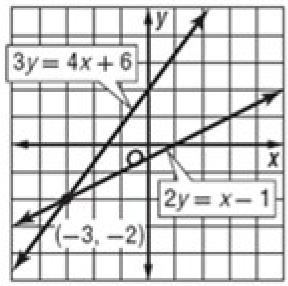
5. $3y = 4x + 6$
$2y = x - 1$

SOLUTION:

Solve each equation for y.

$3y = 4x + 6 \qquad 2y = x - 1$
$y = \frac{4}{3}x + \frac{6}{3} \qquad y = \frac{1}{2}x - \frac{1}{2}$
$y = \frac{4}{3}x + 2$

Then graph each equation.



The lines intersect at the point (–3, –2). This ordered pair is the solution of the system.

CHECK

$3y = 4x + 6 \qquad 2y = x - 1$
$3(-2) = 4(-3) + 6 \qquad 2(-2) = -3 - 1$
$-6 = -12 + 6 \qquad -4 = -3 - 1$
$-6 = -6 \qquad -4 = -4$

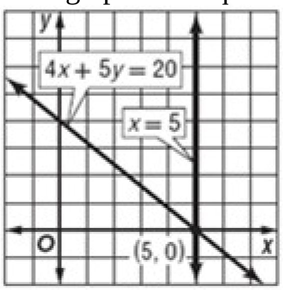
6. $$\begin{aligned} x &= 5 \\ 4x + 5y &= 20 \end{aligned}$$

SOLUTION:

Solve $4x + 5y = 20$ for y.

$$\begin{aligned} 4x + 5y &= 20 \\ 5y &= -4x + 20 \\ y &= -\frac{4}{5}x + \frac{20}{5} \\ y &= -\frac{4}{5}x + 4 \end{aligned}$$

Then graph each equation.



The lines intersect at the point (5, 0). This ordered pair is the solution of the system.

CHECK

$$\begin{aligned} x &= 5 & 4x + 5y &= 20 \\ 5 &= 5 & 4(5) + 5(0) &= 20 \\ & & 20 &= 20 \end{aligned}$$

Use substitution to solve each system of equations.

7. $$\begin{aligned} 5x - y &= 16 \\ 2x + 3y &= 3 \end{aligned}$$

SOLUTION:

To solve the system by substitution, first solve one equation for x or y. In this case, y is easiest to solve for in the first equation.

$$\begin{aligned} 5x - y &= 16 \\ -y &= -5x + 16 \\ y &= 5x - 16 \end{aligned}$$

Then substitute this expression for y into the other equation and solve for x.

$$\begin{aligned} 2x + 3y &= 3 \\ 2x + 3(5x - 16) &= 3 \\ 2x + 15x - 48 &= 3 \\ 17x - 48 &= 3 \\ 17x &= 51 \\ x &= 3 \end{aligned}$$

Substitute this value for x into the equation you solved for y to find the value of y.

$$\begin{aligned} y &= 5x - 16 \\ &= 5(3) - 16 \\ &= -1 \end{aligned}$$

The solution is (3, –1).

8. $$\begin{aligned}3x-5y&=-8\\x+2y&=1\end{aligned}$$

SOLUTION:

To solve the system by substitution, first solve one equation for x or y. In this case, x is easiest to solve for in the second equation.

$$\begin{aligned}x+2y&=1\\x&=-2y+1\end{aligned}$$

Then substitute this expression for x into the other equation and solve for y.

$$\begin{aligned}3x-5y&=-8\\3(-2y+1)-5y&=-8\\-6y+3-5y&=-8\\-11y+3&=-8\\-11y&=-11\\y&=1\end{aligned}$$

Substitute this value for y into the equation you solved for x to find the value of y.

$$\begin{aligned}x&=-2y+1\\&=-2(1)+1\\&=-1\end{aligned}$$

The solution is (–1, 1).

9. $$\begin{aligned}y&=6-x\\x&=4.5+y\end{aligned}$$

SOLUTION:

To solve the system by substitution, substitute the expression given for y into the other equation and solve for x.

$$\begin{aligned}x&=4.5+y\\x&=4.5+(6-x)\\x&=10.5-x\\2x&=10.5\\x&=5.25\end{aligned}$$

Substitute this value for x into the first equation to find the value of y.

$$\begin{aligned}y&=6-x\\&=6-5.25\\&=0.75\end{aligned}$$

The solution is (5.25, 0.75).

10. $$\begin{aligned}x&=2y-8\\2x-y&=-7\end{aligned}$$

SOLUTION:

To solve the system by substitution, substitute the expression given for x into the other equation and solve for y.

$$\begin{aligned}2x-y&=-7\\2(2y-8)-y&=-7\\4y-16-y&=-7\\3y-16&=-7\\3y&=9\\y&=3\end{aligned}$$

Substitute this value for y into the first equation to find the value of x.

$$\begin{aligned}x&=2y-8\\&=2(3)-8\\&=-2\end{aligned}$$

The solution is (−2, 3).

11. $\begin{aligned} 4x - 5y &= 6 \\ x + 3 &= 2y \end{aligned}$

SOLUTION:

To solve the system by substitution, first solve one equation for x or y. In this case, x is easiest to solve for in the second equation.

$$\begin{aligned} x + 3 &= 2y \\ x &= 2y - 3 \end{aligned}$$

Then substitute this expression for x into the other equation and solve for y.

$$\begin{aligned} 4x - 5y &= 6 \\ 4(2y - 3) - 5y &= 6 \\ 8y - 12 - 5y &= 6 \\ 3y - 12 &= 6 \\ 3y &= 18 \\ y &= 6 \end{aligned}$$

Substitute this value for y into the equation you solved for x to find the value of x.

$$\begin{aligned} x &= 2y - 3 \\ &= 2(6) - 3 \\ &= 9 \end{aligned}$$

The solution is (9, 6).

12. $\begin{aligned} x - 3y &= 6 \\ 2x + 4y &= -2 \end{aligned}$

SOLUTION:

To solve the system by substitution, first solve one equation for x or y. In this case, x is easiest to solve for in the first equation.

$$\begin{aligned} x - 3y &= 6 \\ x &= 3y + 6 \end{aligned}$$

Then substitute this expression for x into the other equation and solve for y.

$$\begin{aligned} 2x + 4y &= -2 \\ 2(3y + 6) + 4y &= -2 \\ 6y + 12 + 4y &= -2 \\ 10y + 12 &= -2 \\ 10y &= -14 \\ y &= -1.4 \end{aligned}$$

Substitute this value for y into the equation you solved for x to find the value of x.

$$\begin{aligned} x &= 3y + 6 \\ &= 3(-1.4) + 6 \\ &= -4.2 + 6 \\ &= 1.8 \end{aligned}$$

The solution is (1.8, –1.4).

13. **JOBS** Connor works at a movie rental store earning $8 per hour. He also walks dogs for $10 per hour on the weekends. Connor worked 13 hours this week and made $110. How many hours each did he work at the movie rental store? How many hours did he walk dogs over the weekend?

SOLUTION:

Use the information given to write a system of equations. Let x be the number of hours Connor works at the movie store per week and y be the number of hours he walks dogs per week.

If he worked 13 hours this week then $x + y = 13$. If he earns $8 per hour at renting movies, $10 per hour walking dogs, and he made a total of $110 this week working these two jobs, then $8x + 10y = 110$.

Therefore, a system of equations representing this situation is $\begin{aligned} x + y &= 13 \\ 8x + 10y &= 110 \end{aligned}$.

To solve this system, solve the first equation for y.

$$\begin{aligned} x + y &= 13 \\ y &= -x + 13 \end{aligned}$$

Then substitute this expression for y into the other equation and solve for x.

$$\begin{aligned} 8x + 10y &= 110 \\ 8x + 10(-x + 13) &= 110 \\ 8x - 10x + 130 &= 110 \\ -2x + 130 &= 110 \\ -2x &= -20 \\ x &= 10 \end{aligned}$$

Substitute this value for x into the equation you solved for y to find the value of y.

$$\begin{aligned} y &= -x + 13 \\ &= -10 + 13 \\ &= 3 \end{aligned}$$

The solution is (10, 3), which means that Connor worked 10 hours at the movie rental store and 3 hours walking dogs.

Use elimination to solve each system of equations.

14. $\begin{aligned} 7x + y &= 9 \\ 5x - y &= 15 \end{aligned}$

SOLUTION:

Eliminate the variable y in the system by adding the two equations.

$$\begin{aligned} 7x + y &= 9 \\ (+)\ 5x - y &= 15 \\ \hline 12x \quad &= 24 \\ x &= 2 \end{aligned}$$

Substitute this value for x into one of the original equations and solve for y.

$$\begin{aligned} 7x + y &= 9 \\ 7(2) + y &= 9 \\ 14 + y &= 9 \\ y &= -5 \end{aligned}$$

The solution is (2, −5).

15. $\begin{aligned} 2x - 3y &= 1 \\ 4x - 5y &= 7 \end{aligned}$

SOLUTION:

Eliminate the variable x in the system by multiplying the first equation by –2 and then adding the two equations.

$$\begin{aligned} 2x - 3y &= 1 \quad \Rightarrow & -4x + 6y &= -2 \\ 4x - 5y &= 7 & (+)\ 4x - 5y &= 7 \\ \hline & & y &= 5 \end{aligned}$$

Substitute this value for y into one of the original equations and solve for x.

$$\begin{aligned} 2x - 3y &= 1 \\ 2x - 3(5) &= 1 \\ 2x - 15 &= 1 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

The solution is (8, 5).

16. $-3x + 10y = 5$
$2x + 7y = 24$

SOLUTION:

Eliminate the variable x in the system by multiplying the first equation by 2, multiplying the second equation by 3, and then adding the two equations.

$$\begin{array}{lcr} -3x + 10y = 5 & \Rightarrow & -6x + 20y = 10 \\ 2x + 7y = 24 & & \underline{(+)\quad 6x + 21y = 72} \\ & & 41y = 82 \\ & & y = 2 \end{array}$$

Substitute this value for y into one of the original equations and solve for x.

$$\begin{aligned} 2x + 7y &= 24 \\ 2x + 7(2) &= 24 \\ 2x + 14 &= 24 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

The solution is (5, 2).

17. $2x + 3y = 3$
$12x - 15y = -4$

SOLUTION:

Eliminate the variable y in the system by multiplying the first equation by 5 and then adding the two equations.

$$\begin{array}{lcr} 2x + 3y = 3 & \Rightarrow & 10x + 15y = 15 \\ 12x - 15y = -4 & & \underline{(+)\ 12x - 15y = -4} \\ & & 22x \qquad = 11 \\ & & x = \frac{1}{2} \end{array}$$

Substitute this value for x into one of the original equations and solve for y.

$$\begin{aligned} 2x + 3y &= 3 \\ 2\left(\frac{1}{2}\right) + 3y &= 3 \\ 1 + 3y &= 3 \\ 3y &= 2 \\ y &= \frac{2}{3} \end{aligned}$$

The solution is $\left(\frac{1}{2}, \frac{2}{3}\right)$.

0-5 Systems of Linear Equations and Inequalities

18. $$\begin{aligned} 3x + 4y &= -1 \\ 6x - 2y &= 3 \end{aligned}$$

SOLUTION:

Eliminate the variable y in the system by multiplying the second equation by 2 and then adding the two equations.

$$\begin{array}{lcr} 3x + 4y = -1 & \Rightarrow & 3x + 4y = -1 \\ 6x - 2y = 3 & & \underline{(+)\ 12x - 4y = 6} \\ & & 15x = 5 \\ & & x = \frac{1}{3} \end{array}$$

Substitute this value for x into one of the original equations and solve for y.

$$\begin{aligned} 3x + 4y &= -1 \\ 3\left(\frac{1}{3}\right) + 4y &= -1 \\ 1 + 4y &= -1 \\ 4y &= -2 \\ y &= -\frac{1}{2} \end{aligned}$$

The solution is $\left(\frac{1}{3}, -\frac{1}{2}\right)$.

19. $$\begin{aligned} 5x - 6y &= 10 \\ -2x + 3y &= -7 \end{aligned}$$

SOLUTION:

Eliminate the variable y in the system by multiplying the second equation by 2 and then adding the two equations.

$$\begin{array}{lcr} 5x - 6y = 10 & \Rightarrow & 5x - 6y = 10 \\ -2x + 3y = -7 & & \underline{(+)\ -4x + 6y = -14} \\ & & x \qquad = -4 \end{array}$$

Substitute this value for x into one of the original equations and solve for y.

$$\begin{aligned} 5x - 6y &= 10 \\ 5(-4) - 6y &= 10 \\ -20 - 6y &= 10 \\ -6y &= 30 \\ y &= -5 \end{aligned}$$

The solution is (−4, −5).

Solve each system of equations.

20. $$\begin{aligned} x+2y+3z&=5 \\ 3x+2y-2z&=-13 \\ 5x+3y-z&=-11 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Multiply the third equation by 3 and add it to the first equation to eliminate z.

$$\begin{array}{lcr} x+2y+3z=5 & \Rightarrow & x+2y+3z=5 \\ 5x+3y-z=-11 & & (+)\ 15x+9y-3z=-33 \\ \hline & & 16x+11y \quad =-28 \end{array}$$

Multiply the third equation by –2 and add it to the second to eliminate z.

$$\begin{array}{lcr} 3x+2y-2z=-13 & \Rightarrow & 3x+2y-2z=-13 \\ 5x+3y-z=-11 & & (+)\ -10x-6y+2z=22 \\ \hline & & -7x-4y \quad =9 \end{array}$$

Solve this system of two equations by multiplying the first by 4, the second by 11, and then adding the two equations together to eliminate the variable y.

$$\begin{array}{lcr} 16x+11y=-28 & \Rightarrow & 64x+44y=-112 \\ -7x-4y=9 & & (+)-77x-44y=99 \\ \hline & & -13x \quad =-13 \\ & & x=1 \end{array}$$

$$\begin{aligned} -7x-4y&=9 \\ -7(1)-4y&=9 \\ -7-4y&=9 \\ -4y&=16 \\ y&=-4 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} x+2y+3z&=5 \\ 1+2(-4)+3z&=5 \\ -7+3z&=5 \\ 3z&=12 \\ z&=4 \end{aligned}$$

The solution is (1, −4, 4).

21. $$\begin{aligned} x-y-z&=7 \\ -x+2y-3z&=-12 \\ 3x-2y+7z&=30 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Add the first equation and second equations to eliminate x.

$$\begin{array}{r} x-\ \ y-\ \ z=\ \ 7 \\ (+)\ -x+2y-3z=-12 \\ \hline y-4z=-5 \end{array}$$

Multiply the first equation by –3 and add it to the second to eliminate x.

$$\begin{array}{lcr} x-y-z=7 & \Rightarrow & -3x+3y+3z=-21 \\ 3x-2y+7z=30 & & (+)\ 3x-2y+7z=30 \\ \hline & & y+10z=9 \end{array}$$

Solve this system of two equations by multiplying the first by –1 and then adding the two equations together to eliminate the variable y.

$$\begin{array}{lcr} y-4z=-5 & \Rightarrow & -y+4z=5 \\ y+10z=9 & & (+)\ \ y+10z=9 \\ \hline & & 14z=14 \\ & & z=1 \end{array}$$

$$\begin{aligned} y-4z&=-5 \\ y-4(1)&=-5 \\ y-4&=-5 \\ y&=-1 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} x-y-z&=7 \\ x-(-1)-1&=7 \\ x&=7 \end{aligned}$$

The solution is (7, −1, 1).

22. $$\begin{aligned} 7x+5y+z&=0\\ -x+3y+2z&=16\\ x-6y-z&=-18 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Add the first and third equations to eliminate z.

$$\begin{array}{rl} & 7x+5y+z=0\\ (+) & \underline{x-6y-z=-18}\\ & 8x-y=-18 \end{array}$$

Multiply the third equation by 2 and add it to the second to eliminate z.

$$\begin{array}{lcrl} -x+3y+2z=16 & \Rightarrow & & -x+3y+2z=16\\ x-6y-z=-18 & & (+) & \underline{2x-12y-2z=-36}\\ & & & x-9y=-20 \end{array}$$

Solve this system of two equations by multiplying the first by –9 and then adding the two equations together to eliminate the variable y.

$$\begin{array}{lcrl} 8x-y=-18 & \Rightarrow & & -72x+9y=162\\ x-9y=-20 & & (+) & \underline{x-9y=-20}\\ & & & -71x=142\\ & & & x=-2 \end{array}$$

$$\begin{aligned} x-9y&=-20\\ -2-9y&=-20\\ -9y&=-18\\ y&=2 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} -x+3y+2z&=16\\ -(-2)+3(2)+2z&=16\\ 8+2z&=16\\ 2z&=8\\ z&=4 \end{aligned}$$

The solution is (−2, 2, 4).

23. $$\begin{aligned} 3x-5y+z&=9\\ x-3y-2z&=-8\\ 5x-6y+3z&=15 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Multiply the first equation by 2 and add it to the second equation to eliminate z.

$$\begin{array}{lcrl} 3x-5y+z=9 & \Rightarrow & & 6x-10y+2z=18\\ x-3y-2z=-8 & & (+) & \underline{x-3y-2z=-8}\\ & & & 7x-13y=10 \end{array}$$

Multiply the first equation by –3 and add it to the third to eliminate z.

$$\begin{array}{lcrl} 3x-5y+z=9 & \Rightarrow & & -9x+15y-3z=-27\\ 5x-6y+3z=15 & & (+) & \underline{5x-6y+3z=15}\\ & & & -4x+9y=-12 \end{array}$$

Solve this system of two equations by multiplying the first by 4, the second by 7, and then adding the two equations together to eliminate the variable x.

$$\begin{array}{lcrl} 7x-13y=10 & \Rightarrow & & 28x-52y=40\\ -4x+9y=-12 & & (+) & \underline{-28x+63y=-84}\\ & & & 11y=-44\\ & & & y=-4 \end{array}$$

$$\begin{aligned} -4x+9y&=-12\\ -4x+9(-4)&=-12\\ -4x-36&=-12\\ -4x&=24\\ x&=-6 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} x-3y-2z&=-8\\ (-6)-3(-4)-2z&=-8\\ -6+12-2z&=-8\\ 6-2z&=-8\\ -2z&=-14\\ z&=7 \end{aligned}$$

The solution is (−6, −4, 7).

24. $\begin{aligned} 4x+2y+z&=7 \\ 2x+2y-4z&=-4 \\ x+3y-2z&=-8 \end{aligned}$

SOLUTION:

Eliminate one variable in two pairs of the system.
Multiply the first equation by 4 and add it to the second equation to eliminate z.

$$\begin{array}{lcr} 4x+2y+\ \ z=\ \ 7 & \Rightarrow & 16x+8y+4z=28 \\ 2x+2y-4z=-4 & & (+)\ \ 2x+2y-4z=-4 \\ \hline & & 18x+10y\qquad=24 \end{array}$$

Multiply the first equation by 2 and add it to the third to eliminate z.

$$\begin{array}{lcr} 4x+2y+z=7 & \Rightarrow & 8x+4y+2z=14 \\ x+3y-2z=-8 & & (+)\ \ x+3y-2z=-8 \\ \hline & & 9x+7y\qquad=6 \end{array}$$

Solve this system of two equations by multiplying the second by –2 and then adding the two equations together to eliminate the variable x.

$$\begin{array}{lcr} 18x+10y=24 & \Rightarrow & 18x+10y=\ \ 24 \\ 9x+7y=\ \ 6 & & (+)-18x-14y=-12 \\ \hline & & -4y=12 \\ & & y=-3 \end{array}$$

$$\begin{aligned} 9x+7y&=6 \\ 9x+7(-3)&=6 \\ 9x-21&=6 \\ 9x&=27 \\ x&=3 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} x+3y-2z&=-8 \\ 3+3(-3)-2z&=-8 \\ 3-9-2z&=-8 \\ -6-2z&=-8 \\ -2z&=-2 \\ z&=1 \end{aligned}$$

The solution is (3, −3, 1).

25. $\begin{aligned} x-3z&=7 \\ 2x+y-2z&=11 \\ -x-2y+2z&=6 \end{aligned}$

SOLUTION:

Eliminate the variable y in the system.
Multiply the second equation by 2 and add it to the third equation to eliminate y.

$$\begin{array}{lcr} 2x+y-2z=11 & \Rightarrow & 4x+2y-4z=22 \\ -x-2y+2z=6 & & (+)\ -x-2y+2z=\ \ 6 \\ \hline & & 3x\qquad-2z=28 \end{array}$$

Solve the system of two equations formed by the above equation with the first equation in the original system. To do this, multiply the first equation by –3 and add it to the equation $3x – 2z = 28$ to eliminate the variable x.

$$\begin{array}{lcr} x-3z=\ \ 7 & \Rightarrow & -3x+9z=-21 \\ 3x-2z=28 & & (+)\ \ 3x-2z=\ \ 28 \\ \hline & & 7z=\ \ 7 \\ & & z=1 \end{array}$$

$$\begin{aligned} x-3z&=7 \\ x-3(1)&=7 \\ x-3&=7 \\ x&=10 \end{aligned}$$

Substitute these two values into one of the original equations to find y.

$$\begin{aligned} -x-2y+2z&=6 \\ -(10)-2y+2(1)&=6 \\ -8-2y&=6 \\ -2y&=14 \\ y&=-7 \end{aligned}$$

The solution is (10, −7, 1).

26. $$\begin{aligned} 8x - z &= 4 \\ y + z &= 5 \\ 11x + y &= 15 \end{aligned}$$

SOLUTION:

Eliminate the variable y in the system.
Multiply the second equation by –1 and add it to the third equation to eliminate y.

$$\begin{array}{lcr} y + z = 5 & \Rightarrow & -y - z = -5 \\ 11x + y = 15 & & (+)\ 11x + y \quad = 15 \\ \hline & & 11x \quad - z = 10 \end{array}$$

Solve the system of two equations formed by the above equation with the first equation in the original system. To do this, multiply the first equation by –1 and add it to the equation $11x - z = 10$ to eliminate the variable x.

$$\begin{array}{lcr} 8x - z = 4 & \Rightarrow & -8x + z = -4 \\ 11x - z = 10 & & (+)\ 11x - z = 10 \\ \hline & & 3x \quad = 6 \\ & & x = 2 \end{array}$$

$$\begin{aligned} 8x - z &= 4 \\ 8(2) - z &= 4 \\ 16 - z &= 4 \\ -z &= -12 \\ z &= 12 \end{aligned}$$

Substitute one of these two values into one of the original equations that contains y to find the value of y.

$$\begin{aligned} y + z &= 5 \\ y + 12 &= 5 \\ y &= -7 \end{aligned}$$

The solution is (2, −7, 12).

27. $$\begin{aligned} 4x - 2y + z &= -5 \\ 5x + y + 3z &= 6 \\ -2x + 3y + 2z &= -4 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system.
Multiply the second equation by 2 and add it to the first equation to eliminate y.

$$\begin{array}{lcr} 4x - 2y + z = -5 & \Rightarrow & 4x - 2y + z = -5 \\ 5x + y + 3z = 6 & & (+)\ 10x + 2y + 6z = 12 \\ \hline & & 14x \quad + 7z = 7 \end{array}$$

Multiply the second equation by –3 and add it to the third to eliminate y.

$$\begin{array}{lcr} 5x + y + 3z = 6 & \Rightarrow & -15x - 3y - 9z = -18 \\ -2x + 3y + 2z = -4 & & (+)\ -2x + 3y + 2z = -4 \\ \hline & & -17x \quad - 7z = -22 \end{array}$$

Solve this system of two equations by adding the two equations together to eliminate the variable z.

$$\begin{array}{r} 14x + 7z = 7 \\ (+)\ -17x - 7z = -22 \\ \hline -3x \quad = -15 \\ x = 5 \end{array}$$

$$\begin{aligned} 14x + 7z &= 7 \\ 14(5) + 7z &= 7 \\ 70 + 7z &= 7 \\ 7z &= -63 \\ z &= -9 \end{aligned}$$

Substitute these two values into one of the original equations to find y.

$$\begin{aligned} 5x + y + 3z &= 6 \\ 5(5) + y + 3(-9) &= 6 \\ 25 + y - 27 &= 6 \\ -2 + y &= 6 \\ y &= 8 \end{aligned}$$

The solution is (5, 8, −9).

0-5 Systems of Linear Equations and Inequalities

Solve each system of equations. State whether the system is *consistent and independent*, *consistent and dependent*, or *inconsistent*.

28. $$\begin{aligned} 8x - 5y &= -11 \\ -8x + 9y &= 7 \end{aligned}$$

SOLUTION:

Eliminate the variable *x* in the system by adding the two equations.

$$\begin{array}{rl} & 8x - 5y = -11 \\ (+) & \underline{-8x + 9y = 7} \\ & 4y = -4 \\ & y = -1 \end{array}$$

Substitute this value for y into one of the original equations and solve for *x*.

$$\begin{aligned} 8x - 5y &= -11 \\ 8x - 5(-1) &= -11 \\ 8x + 5 &= -11 \\ 8x &= -16 \\ x &= -2 \end{aligned}$$

The solution is (–2, −1). The system is consistent and independent because it has exactly one solution.

29. $$\begin{aligned} x - y &= 2 \\ 2x &= 2y + 10 \end{aligned}$$

SOLUTION:

Solve the first equation in the system for *x*.

$$\begin{aligned} x - y &= 2 \\ x &= y + 2 \end{aligned}$$

Substitute this value for *x* into the second equation and solve for y.

$$\begin{aligned} 2x &= 2y + 10 \\ 2(y + 2) &= 2y + 10 \\ 2y + 4 &= 2y + 10 \\ 4 &= 10 \end{aligned}$$

Because 4 = 10 is not a true statement, this system has no solutions. Therefore, the system is inconsistent.

30. $$\begin{aligned} 5x + 4y &= 2 \\ 6x + 5y &= 4 \end{aligned}$$

SOLUTION:

Eliminate the variable *x* in the system by multiplying the first equation by –5, the second equation by 4, and then adding the two equations.

$$\begin{array}{lcrl} 5x + 4y = 2 & \Rightarrow & & -25x - 20y = -10 \\ 6x + 5y = 4 & & (+) & \underline{24x + 20y = 16} \\ & & & -x = 6 \\ & & & x = -6 \end{array}$$

Substitute this value for *x* into one of the original equations and solve for y.

$$\begin{aligned} 5x + 4y &= 2 \\ 5(-6) + 4y &= 2 \\ -30 + 4y &= 2 \\ 4y &= 32 \\ y &= 8 \end{aligned}$$

The solution is (–6, 8). The system is consistent and independent because it has exactly one solution.

31. $$\begin{aligned} 12x - 9y &= 3 \\ 4x - 3y &= 1 \end{aligned}$$

SOLUTION:

Eliminate the variable *x* in the system by multiplying the second equation by –3 and then adding the two equations.

$$\begin{array}{lcrl} 12x - 9y = 3 & \Rightarrow & & 12x - 9y = 3 \\ 4x - 3y = 1 & & (+) & \underline{-12x + 9y = -3} \\ & & & 0 = 0 \end{array}$$

Because 0 = 0 is always true, there are an infinite number of solutions. Therefore, the system is consistent and dependent.

32. $$\begin{aligned}1.5x + y &= 3.5\\ 3x + 2y &= 7\end{aligned}$$

SOLUTION:

Eliminate the variable x in the system by multiplying the first equation by –2 and then adding the two equations.

$$\begin{array}{rcrl} 1.5x + y = 3.5 & \Rightarrow & & -3x - 2y = -7 \\ 3x + 2y = 7 & & (+) & 3x + 2y = 7 \\ \hline & & & 0 = 0 \end{array}$$

Because 0 = 0 is always true, there are an infinite number of solutions. Therefore, the system is consistent and dependent.

33. $$\begin{aligned}10x - 3y &= -4\\ -8x + 5y &= 11\end{aligned}$$

SOLUTION:

Eliminate the variable y in the system by multiplying the first equation by 5, the second equation by 3, and then adding the two equations.

$$\begin{array}{rcrl} 10x - 3y = -4 & \Rightarrow & & 50x - 15y = -20 \\ -8x + 5y = 11 & & (+) & -24x + 15y = 33 \\ \hline & & & 26x = 13 \\ & & & x = \frac{1}{2} \end{array}$$

Substitute this value for x into one of the original equations and solve for y.

$$\begin{aligned} 10x - 3y &= -4 \\ 10\left(\frac{1}{2}\right) - 3y &= -4 \\ 5 - 3y &= -4 \\ -3y &= -9 \\ y &= 3 \end{aligned}$$

The solution is $\left(\frac{1}{2}, 3\right)$. The system is consistent and independent because it has exactly one solution.

34. $$\begin{aligned}2x - 2y + 3z &= 2\\ 2x - 3y + 7z &= -1\\ 4x - 3y + 2z &= 0\end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Multiply the first equation by –2 and add it to the third equation to eliminate x.

$$\begin{array}{rcrl} 2x - 2y + 3z = 2 & \Rightarrow & & -4x + 4y - 6z = -4 \\ 4x - 3y + 2z = 0 & & (+) & 4x - 3y + 2z = 0 \\ \hline & & & y - 4z = -4 \end{array}$$

Multiply the second equation by –2 and add it to the third to eliminate x.

$$\begin{array}{rcrl} 2x - 3y + 7z = -1 & \Rightarrow & & -4x + 6y - 14z = 2 \\ 4x - 3y + 2z = 0 & & (+) & 4x - 3y + 2z = 0 \\ \hline & & & 3y - 12z = 2 \end{array}$$

Solve this system of two equations by multiplying the first by –3 and then adding the two equations together.

$$\begin{array}{rcrl} y - 4z = -4 & \Rightarrow & & -3y + 12z = 12 \\ 3y - 12z = 2 & & (+) & 3y - 12z = 2 \\ \hline & & & 0 = 14 \end{array}$$

Because 0 = 14 is not a true statement, this system has no solutions. Therefore, the system is inconsistent.

35. $$\begin{aligned} -3x + 2y + z &= -23 \\ 4x + 2y + z &= 5 \\ 5x + 3y + 3z &= 11 \end{aligned}$$

SOLUTION:

Eliminate one variable in two pairs of the system. Multiply the first equation by –1 and then add it to the second equation.

$$\begin{array}{lcr} -3x + 2y + z = -23 & \Rightarrow & 3x - 2y - z = 23 \\ 4x + 2y + z = 5 & & \underline{(+)\ \ 4x + 2y + z = 5} \\ & & 7x \qquad\quad = 28 \\ & & x = \ 4 \end{array}$$

Multiply the first equation by –3 and then add it to the third equation.

$$\begin{array}{lcr} -3x + 2y + z = -23 & \Rightarrow & 9x - 6y - 3z = 69 \\ 5x + 3y + 3z = 11 & & \underline{(+)\ \ 5x + 3y + 3z = 11} \\ & & 14x - 3y \qquad = 80 \end{array}$$

Substitute the value for x into this equation and solve for y.

$$\begin{aligned} 14x - 3y &= 80 \\ 14(4) - 3y &= 80 \\ 56 - 3y &= 80 \\ -3y &= 24 \\ y &= -8 \end{aligned}$$

Substitute these two values into one of the original equations to find z.

$$\begin{aligned} 4x + 2y + z &= 5 \\ 4(4) + 2(-8) + z &= 5 \\ z &= 5 \end{aligned}$$

The solution is (4, –8, 5). The system is consistent and independent because it has exactly one solution.

36. **CAMPING** The Mountaineers Club held two camping trips during the summer. The club rented 5 tents and 1 cabin for the 30 members who went on the first trip. The club rented 4 tents and 2 cabins for the 36 members who went on the second trip. If the tents and cabins were filled to capacity on both trips, how many people can each tent and each cabin accommodate?

SOLUTION:

Use the information given to write a system of equations. Let t be the number of people each tent can hold and c be the number of people each cabin can hold. If 30 people can fit into 5 tents and 1 cabin then $5t + c = 30$. If 36 people can fit into 4 tents and 2 cabins, then $4t + 2c = 36$.
Therefore, a system of equations representing this situation is $\begin{aligned} 5t + c &= 30 \\ 4t + 2c &= 36 \end{aligned}$.

To solve this system, solve the first equation for c.

$$\begin{aligned} 5t + c &= 30 \\ c &= -5t + 30 \end{aligned}$$

Then substitute this expression for c into the other equation and solve for t.

$$\begin{aligned} 4t + 2c &= 36 \\ 4t + 2(-5t + 30) &= 36 \\ 4t - 10t + 60 &= 36 \\ -6t + 60 &= 36 \\ -6t &= -24 \\ t &= 4 \end{aligned}$$

Substitute this value for t into the equation you solved for c to find the value of c.

$$\begin{aligned} c &= -5t + 30 \\ &= -5(4) + 30 \\ &= 10 \end{aligned}$$

The solution is (4, 10), which means that each tent can hold 4 people and each cabin can hold 10 people.

Solve each system of inequalities. If the system has no solution, state *no solution.*

0-5 Systems of Linear Equations and Inequalities

37. $y \geq x - 3$
$y \leq 2x + 1$

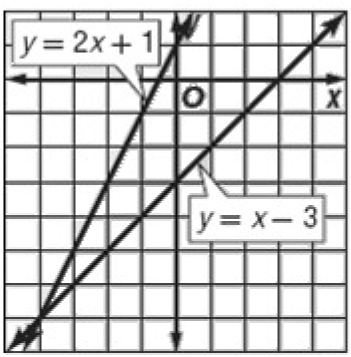
SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

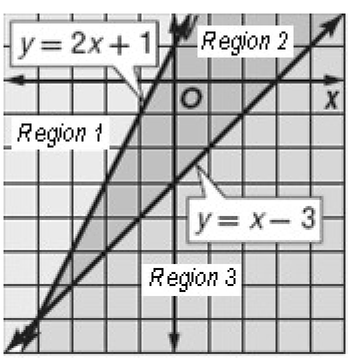
$>$: above the line
$<$: below the line

$y \geq x - 3$
y is greater that or equal to, so graph a solid line along $y = x - 3$ and shade above the line.

$y \leq 2x + 1$
y is less than or equal to, so graph a solid line along $y = 2x + 1$ and shade below the line.



The solution of $y \geq x - 3$ is Regions 1 and 2.
The solution of $y \leq 2x + 1$ is Regions 2 and 3.



Region 2 contains points that are solutions to both inequalities, so this region is the solution to the system.

38. $y + x < 1$
$y > -x - 1$

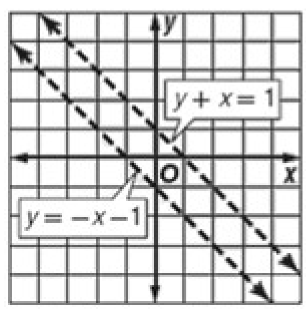
SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

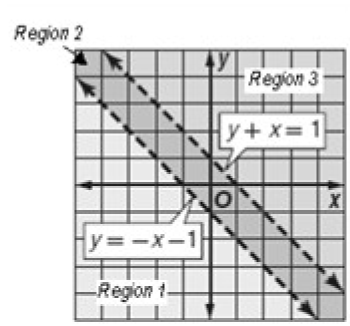
$>$: above the line
$<$: below the line

$y + x < 1$
$y < -x + 1$
y is less than, so graph a dashed line along $y = -x + 1$ and shade below the line.

$y > -x - 1$
y is greater than, so graph a dashed line along $y = -x - 1$ and shade above the line.



The solution of $y + x < 1$ is Regions 1 and 2.
The solution of $y > -x - 1$ is Regions 2 and 3.



Region 2 contains points that are solutions to both inequalities, so this region is the solution to the system.

39. $x + 2y \geq 12$
$x - y \geq 3$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

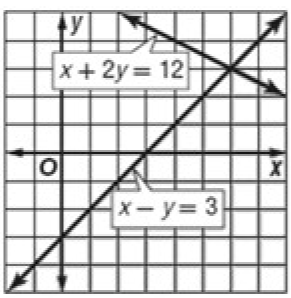
>: above the line
<: below the line

$$x + 2y \geq 12$$
$$2y \geq -x + 12$$
$$y \geq -\tfrac{1}{2}x + 6$$

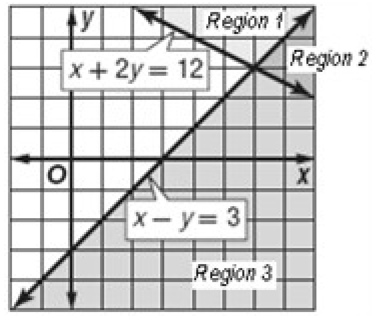
y is greater than or equal to, so graph a solid line along $y = -\frac{1}{2}x + 6$ and shade above the line.

$$x - y \geq 3$$
$$-y \geq -x + 3$$
$$y \leq x - 3$$

y is less than or equal to, so graph a solid line along $y = x - 3$ and shade below the line.



The solution of $x + 2y \geq 12$ is Regions 1 and 2.
The solution of $x - y \geq 3$ is Regions 2 and 3.



Region 2 contains points that are solutions to both inequalities, so this region is the solution to the system.

40. $y \leq \frac{1}{3}x - 7$
$3y \geq x + 6$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

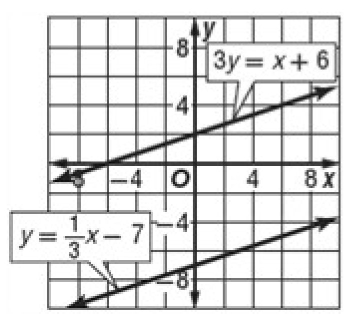
>: above the line
<: below the line

$$y \leq \tfrac{1}{3}x - 7$$

y is less than or equal to, so graph a solid line along $y = \frac{1}{3}x - 7$ and shade below the line.

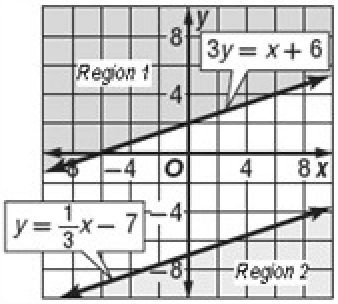
$$3y \geq x + 6$$
$$y \geq \tfrac{1}{3}x + 2$$

y is greater than or equal to, so graph a solid line along $y = \frac{1}{3}x + 2$ and shade above the line.



The solution of $y \le \frac{1}{3}x - 7$ is Region 2.

The solution of $3y \ge x + 6$ is Region 1.



Since these solutions do not contain points that are common to both inequalities, this system of inequalities has no solution.

41. $y + 5 < 4x$
$2y > -2x + 10$

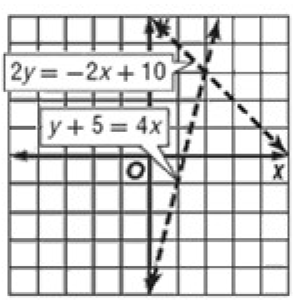
SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

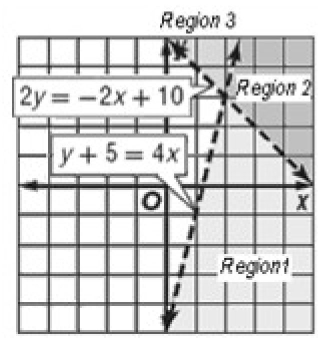
>: above the line
<: below the line

$y + 5 < 4x$
$y < 4x - 5$
y is less than, so graph a dashed line along $y = 4x - 5$ and shade below the line.

$2y > -2x + 10$
$y > -x + 5$
y is greater than, so graph a dashed line along $y = -x + 5$ and shade above the line.



The solution of $y + 5 < 4x$ is Regions 1 and 2.
The solution of $2y > -2x + 10$ is Regions 2 and 3.



Region 2 contains points that are solutions to both inequalities, so this region is the solution to the system.

42. $y \le -x + 8$
$y \ge 0.5x - 4$

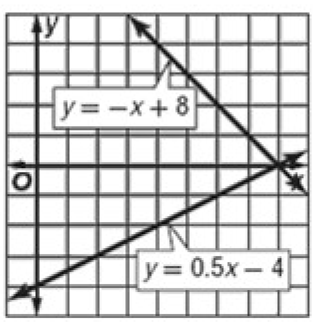
SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

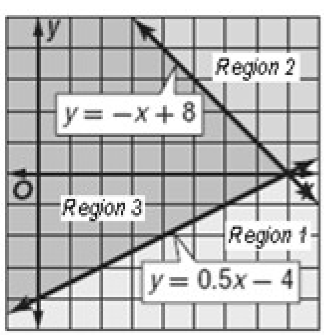
>: above the line
<: below the line

$y \le -x + 8$
y is less than or equal to, so graph a solid line along $y = -x + 8$ and shade below the line.

$y \ge 0.5x - 4$
y is greater than or equal to, so graph a solid line along $y = 0.5x - 4$ and shade above the line.



The solution of $y \le -x + 8$ is Regions 1 and 3.
The solution of $y \ge 0.5x - 4$ is Regions 2 and 3.



Region 3 contains points that are solutions to both inequalities, so this region is the solution to the system.

43. $8y \le -2x - 1$
$4y + x \ge 3$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

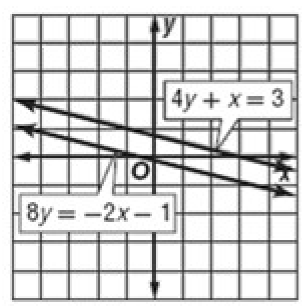
>: above the line
<: below the line

$8y \le -2x - 1$
$y \le -\frac{1}{4}x - \frac{1}{8}$
y is less than or equal to, so graph a solid line along $y = -\frac{1}{4}x - \frac{1}{8}$ and shade below the line.

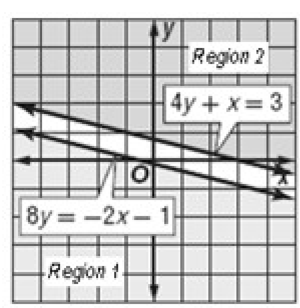
$4y + x \ge 3$
$4y \ge -x + 3$
$y \ge -\frac{1}{4}x + \frac{3}{4}$

y is greater than or equal to, so graph a solid line along $y = -\frac{1}{4}x + \frac{3}{4}$ and shade above the line.

.



The solution of $8y \le -2x - 1$ is Region 1.
The solution of $4y + x \ge 3$ is Region 2.



Since these solutions do not contain points that are common to both inequalities, this system of inequalities has no solution.

44. $y + 7 < 3x$
$2y + 5x > 8$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

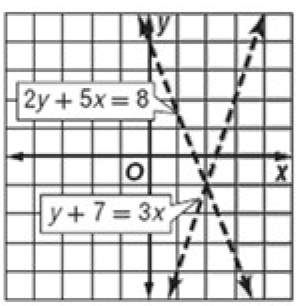
>: above the line
<: below the line

$$y + 7 < 3x$$
$$y < 3x - 7$$

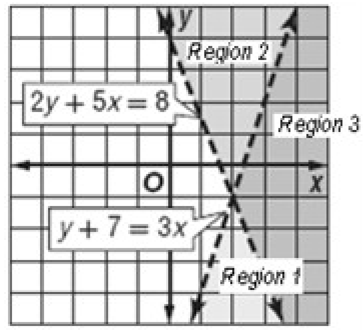
y is less than, so graph a dashed line along $y = 3x - 7$ and shade below the line.

$$2y + 5x > 8$$
$$2y > -5x + 8$$
$$y > -\frac{5}{2}x + 4$$

y is greater than, so graph a dashed line along $y = -\frac{5}{2}x + 4$ and shade above the line.



The solution of $y + 7 < 3x$ is Regions 1 and 3.
The solution of $2y + 5x > 8$ is Regions 2 and 3.



Region 3 contains points that are solutions to both inequalities, so this region is the solution to the system.

45. $-6y \geq -5x + 6$
$y \leq -3x - 1$

SOLUTION:

Rewrite each inequality in slope-intercept form. The determine how to shade each graph.

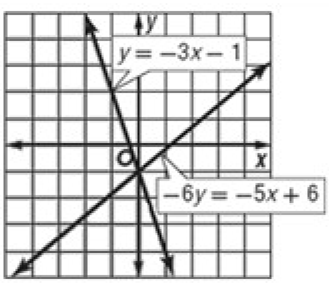
>: above the line
<: below the line

$$-6y \geq -5x + 6$$
$$y \leq \frac{5}{6}x - 1$$

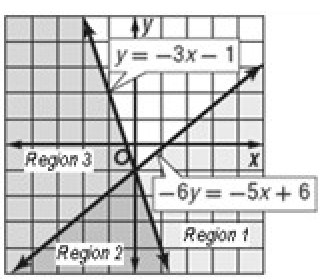
y is less than or equal to, so graph a solid line along $y = \frac{5}{6}x - 1$ and shade below the line.

$$y \leq -3x - 1$$

y is less than or equal to, so graph a solid line along $y = -3x - 1$ and shade below the line.



The solution of $-6y \geq -5x + 6$ is Regions 1 and 2.
The solution of $y \leq -3x - 1$ is Regions 2 and 3.



Region 2 contains points that are solutions to both inequalities, so this region is the solution to the system.

46. $y + 4 \leq \frac{4}{3}x$
$3y \geq 4x + 9$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

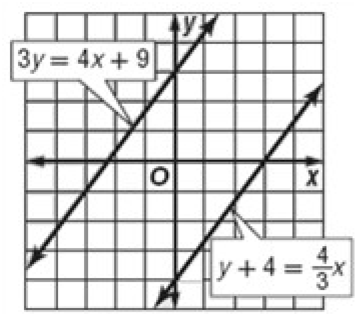
>: above the line
<: below the line

$$y + 4 \leq \frac{4}{3}x$$
$$y \leq \frac{4}{3}x - 4$$

y is less than or equal to, so graph a solid line along $y = \frac{4}{3}x - 4$ and shade below the line.

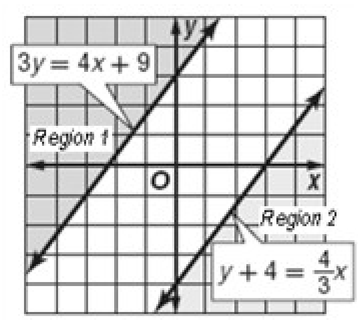
$$3y \geq 4x + 9$$
$$y \geq \frac{4}{3}x + 3$$

y is greater than or equal to, so graph a solid line along $y = \frac{4}{3}x + 3$ and shade above the line.



The solution of $y + 4 \leq \frac{4}{3}x$ is Region 1.

The solution of $3y \geq 4x + 9$ is Region 2.



Since these solutions do not contain points that are common to both inequalities, this system of inequalities has no solution.

47. $y \le 2x + 1$
$y \ge 2x - 2$
$3x + y \le 9$

SOLUTION:

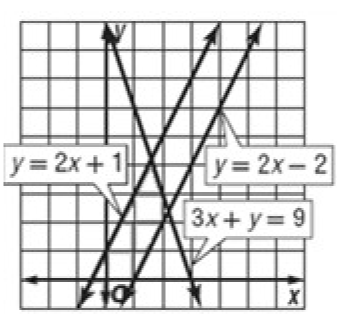
Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

>: above the line
<: below the line

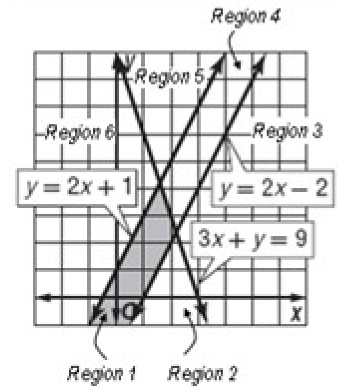
$y \le 2x + 1$
y is less than or equal to, so graph a solid line along $y = 2x + 1$ and shade below the line.

$y \ge 2x - 2$
y is greater than or equal to, so graph a solid line along $y = 2x - 2$ and shade above the line.

$$3x + y \le 9$$
$$y \le -3x + 9$$
y is less than or equal to, so graph a solid line along $y = -3x + 9$ and shade below the line.



The solution of $y \le 2x + 1$ is Regions 1, 2, 3, and 4.
The solution of $y \ge 2x - 2$ is Regions 1, 4, 5, and 6.
The solution of $3x + y \le 9$ is Regions 1, 2, and 6.



Region 1 contains points that are solutions to all three inequalities, so this region is the solution to the system.

48. $x - 3y > 2$
$2x - y < 4$
$2x + 4y \ge -7$

SOLUTION:

Rewrite each inequality in slope-intercept form. Then determine how to shade each graph.

>: above the line
<: below the line

$$x - 3y > 2$$
$$-3y > -x + 2$$
$$y < \frac{1}{3}x - \frac{2}{3}$$

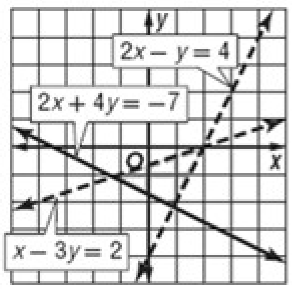
y is less than, so graph a dashed line along $y = \frac{1}{3}x - \frac{2}{3}$ and shade below the line.

$$2x - y < 4$$
$$-y < -2x + 4$$
$$y > 2x - 4$$

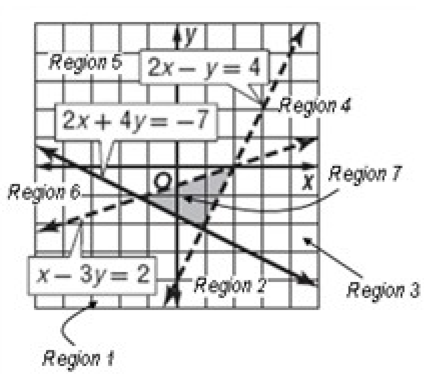
y is greater than, so graph a dashed line along $y = 2x - 4$ and shade above the line.

$$2x + 4y \ge -7$$
$$4y \ge -2x - 7$$
$$y \ge -\frac{1}{2}x - \frac{7}{4}$$

y is greater than or equal to, so graph a solid line along $y = -\frac{1}{2}x - \frac{7}{4}$ and shade above the line.

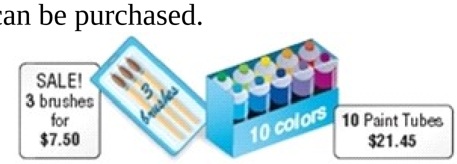


The solution of $x - 3y > 2$ is Regions 1, 2, 3, and 7.
The solution of $2x - y < 4$ is Regions 1, 5, 6, and 7.
The solution of $2x + 4y \geq -7$ is Regions 3, 4, 5, and 7.



Region 7 contains points that are solutions to all three inequalities, so this region is the solution to the system.

49. **ART** Charlie can spend no more than $225 on the art club's supply of brushes and paint. He needs at least 20 brushes and 56 tubes of paint. Graph the region that shows how many packages of each item can be purchased.



SOLUTION:

Use the information given to write a system of equations. Let *x* be the number of brush 3–brush sets and y be the number of 10–paint tubes sets. If Charlie can purchase brush sets for $7.50 each and paint tube sets for $21.45 each, and he can spend no more than $225, then 7.50*x* + 21.45*y* ≤ 225.

If he needs at least 20 brushes, then he must by at least $\frac{20}{3}$ brush sets, so $x \geq \frac{20}{3}$. If he needs at least 56 tubes of paint, then he must buy at least $\frac{56}{10}$ paint tube sets, so $y \geq \frac{56}{10}$.

Therefore, a system of equations representing this situation is:

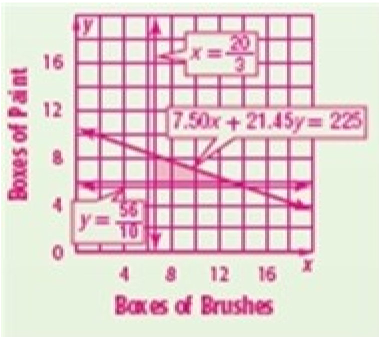
$$7.50x + 21.45y \leq 225$$
$$x \geq \frac{20}{3}.$$
$$y \geq \frac{56}{10}$$

To solve this system of inequalities, graph the related equations $7.50x + 21.45y = 225$, $x = \frac{20}{3}$, $y = \frac{56}{10}$ using a solid line since each inequality contains either ≤ or ≥.

Points on or below the line $7.50x + 21.45y = 225$ make the inequality $7.50x + 21.45y \leq 225$ true, so shade the region below $7.50x + 21.45y = 225$.

Points on or to the right of the line $x = \frac{20}{3}$ make the inequality $x \geq \frac{20}{3}$ true, so shade the region to the right of $x = \frac{20}{3}$. Points on or above the line $y = \frac{56}{10}$ make the inequality $y \geq \frac{56}{10}$ true, so shade the region above $y = \frac{56}{10}$.

The solution to the system is the shade regions that are common to all three inequalities, shown in the triangular regions on the graph below.



$x = \frac{20}{3}$
$7.50x + 21.45y = 225$
$y = \frac{56}{10}$
Boxes of Paint
Boxes of Brushes