

1 Solving Linear Equations

- 1.1 Solving Simple Equations
- 1.2 Solving Multi-Step Equations
- 1.3 Solving Equations with Variables on Both Sides
- 1.4 Solving Absolute Value Equations
- 1.5 Rewriting Equations and Formulas



Density of Pyrite (p. 41)



Cheerleading Competition (p. 29)



Boat (p. 22)



Average Speed (p. 6)



Biking (p. 14)

Maintaining Mathematical Proficiency

Adding and Subtracting Integers

Example 1 Evaluate $4 + (-12)$.

$$4 + (-12) = -8$$

$| -12 | > | 4 |$. So, subtract $| 4 |$ from $| -12 |$.

Use the sign of -12 .

Example 2 Evaluate $-7 - (-16)$.

$$\begin{aligned} -7 - (-16) &= -7 + 16 && \text{Add the opposite of } -16. \\ &= 9 && \text{Add.} \end{aligned}$$

Add or subtract.

1. $-5 + (-2)$

2. $0 + (-13)$

3. $-6 + 14$

4. $19 - (-13)$

5. $-1 - 6$

6. $-5 - (-7)$

7. $17 + 5$

8. $8 + (-3)$

9. $11 - 15$

Multiplying and Dividing Integers

Example 3 Evaluate $-3 \cdot (-5)$.

The integers have the same sign.

$$-3 \cdot (-5) = 15$$

The product is positive.

Example 4 Evaluate $15 \div (-3)$.

The integers have different signs.

$$15 \div (-3) = -5$$

The quotient is negative.

Multiply or divide.

10. $-3(8)$

11. $-7 \cdot (-9)$

12. $4 \cdot (-7)$

13. $-24 \div (-6)$

14. $-16 \div 2$

15. $12 \div (-3)$

16. $6 \cdot 8$

17. $36 \div 6$

18. $-3(-4)$

19. **ABSTRACT REASONING** Summarize the rules for (a) adding integers, (b) subtracting integers, (c) multiplying integers, and (d) dividing integers. Give an example of each.

Specifying Units of Measure

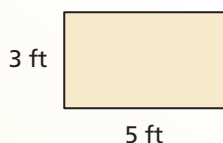
Core Concept

Operations and Unit Analysis

Addition and Subtraction

When you add or subtract quantities, they must have the same units of measure. The sum or difference will have the *same* unit of measure.

Example



Perimeter of rectangle

$$= (3 \text{ ft}) + (5 \text{ ft}) + (3 \text{ ft}) + (5 \text{ ft})$$

$$= 16 \text{ feet}$$

When you add **feet**, you get **feet**.

Multiplication and Division

When you multiply or divide quantities, the product or quotient will have a *different* unit of measure.

Example Area of rectangle $= (3 \text{ ft}) \times (5 \text{ ft})$

$$= 15 \text{ square feet}$$

When you multiply **feet**, you get **feet squared**, or **square feet**.

EXAMPLE 1 Specifying Units of Measure

You work 8 hours and earn \$72. What is your hourly wage?

SOLUTION

$$\begin{array}{lcl} \text{dollars per hour} & & \text{dollars per hour} \\ \text{Hourly wage} & = & \$72 \div 8 \text{ h} \\ (\$ \text{ per h}) & & \\ & = & \$9 \text{ per hour} \end{array}$$

The units on each side of the equation balance. Both are specified in **dollars per hour**.

► Your hourly wage is \$9 per hour.

Monitoring Progress

Solve the problem and specify the units of measure.

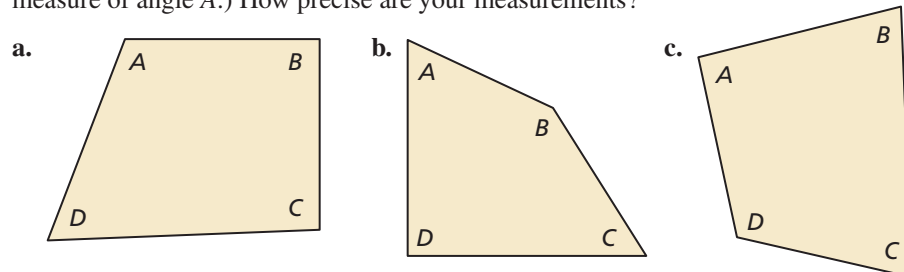
1. The population of the United States was about 280 million in 2000 and about 310 million in 2010. What was the annual rate of change in population from 2000 to 2010?
2. You drive 240 miles and use 8 gallons of gasoline. What was your car's gas mileage (in miles per gallon)?
3. A bathtub is in the shape of a rectangular prism. Its dimensions are 5 feet by 3 feet by 18 inches. The bathtub is three-fourths full of water and drains at a rate of 1 cubic foot per minute. About how long does it take for all the water to drain?

1.1 Solving Simple Equations

Essential Question How can you use simple equations to solve real-life problems?

EXPLORATION 1 Measuring Angles

Work with a partner. Use a protractor to measure the angles of each quadrilateral. Copy and complete the table to organize your results. (The notation $m\angle A$ denotes the measure of angle A.) How precise are your measurements?



UNDERSTANDING MATHEMATICAL TERMS

A **conjecture** is an unproven statement about a general mathematical concept. After the statement is proven, it is called a **rule** or a **theorem**.

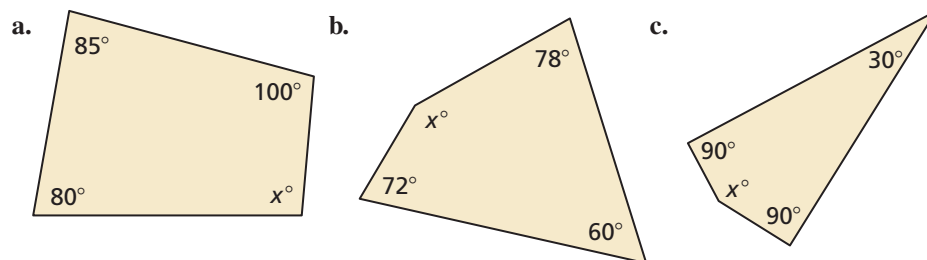
Quadrilateral	$m\angle A$ (degrees)	$m\angle B$ (degrees)	$m\angle C$ (degrees)	$m\angle D$ (degrees)	$m\angle A + m\angle B + m\angle C + m\angle D$
a.					
b.					
c.					

EXPLORATION 2 Making a Conjecture

Work with a partner. Use the completed table in Exploration 1 to write a conjecture about the sum of the angle measures of a quadrilateral. Draw three quadrilaterals that are different from those in Exploration 1 and use them to justify your conjecture.

EXPLORATION 3 Applying Your Conjecture

Work with a partner. Use the conjecture you wrote in Exploration 2 to write an equation for each quadrilateral. Then solve the equation to find the value of x . Use a protractor to check the reasonableness of your answer.



Communicate Your Answer

- How can you use simple equations to solve real-life problems?
- Draw your own quadrilateral and cut it out. Tear off the four corners of the quadrilateral and rearrange them to affirm the conjecture you wrote in Exploration 2. Explain how this affirms the conjecture.

1.1 Lesson

Core Vocabulary

conjecture, p. 3
rule, p. 3
theorem, p. 3
equation, p. 4
linear equation
in one variable, p. 4
solution, p. 4
inverse operations, p. 4
equivalent equations, p. 4

Previous
expression

What You Will Learn

- ▶ Solve linear equations using addition and subtraction.
- ▶ Solve linear equations using multiplication and division.
- ▶ Use linear equations to solve real-life problems.

Solving Linear Equations by Adding or Subtracting

An **equation** is a statement that two expressions are equal. A **linear equation in one variable** is an equation that can be written in the form $ax + b = 0$, where a and b are constants and $a \neq 0$. A **solution** of an equation is a value that makes the equation true.

Inverse operations are two operations that undo each other, such as addition and subtraction. When you perform the same inverse operation on each side of an equation, you produce an equivalent equation. **Equivalent equations** are equations that have the same solution(s).

Core Concept

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a - c = b - c$.

EXAMPLE 1 Solving Equations by Addition or Subtraction

Solve each equation. Justify each step. Check your answer.

a. $x - 3 = -5$

b. $0.9 = y + 2.8$

SOLUTION

a. $x - 3 = -5$

Addition Property of Equality

$$\begin{array}{r} +3 \quad +3 \\ x - 3 = -5 \\ \hline x = -2 \end{array}$$

Write the equation.

Add 3 to each side.

Simplify.

▶ The solution is $x = -2$.

Check

$$\begin{array}{r} x - 3 = -5 \\ -2 - 3 \stackrel{?}{=} -5 \\ -5 = -5 \quad \checkmark \end{array}$$

b. $0.9 = y + 2.8$

Subtraction Property of Equality

$$\begin{array}{r} -2.8 \quad -2.8 \\ 0.9 = y + 2.8 \\ \hline -1.9 = y \end{array}$$

Write the equation.

Subtract 2.8 from each side.

Simplify.

▶ The solution is $y = -1.9$.

Check

$$\begin{array}{r} 0.9 = y + 2.8 \\ 0.9 \stackrel{?}{=} -1.9 + 2.8 \\ 0.9 = 0.9 \quad \checkmark \end{array}$$

Monitoring Progress



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Solve the equation. Justify each step. Check your solution.

1. $n + 3 = -7$

2. $g - \frac{1}{3} = -\frac{2}{3}$

3. $-6.5 = p + 3.9$

Solving Linear Equations by Multiplying or Dividing

Core Concept

REMEMBER

Multiplication and division are inverse operations.

Multiplication Property of Equality

Words Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality

Words Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \div c = b \div c$, $c \neq 0$.

EXAMPLE 2 Solving Equations by Multiplication or Division

Solve each equation. Justify each step. Check your answer.

a. $-\frac{n}{5} = -3$

b. $\pi x = -2\pi$

c. $1.3z = 5.2$

SOLUTION

a. $-\frac{n}{5} = -3$

Write the equation.

Multiplication Property of Equality

$-5 \cdot \left(-\frac{n}{5}\right) = -5 \cdot (-3)$

Multiply each side by -5 .

$n = 15$

Simplify.

▶ The solution is $n = 15$.

Check

$$\begin{aligned} -\frac{n}{5} &= -3 \\ -\frac{15}{5} &\stackrel{?}{=} -3 \\ -3 &= -3 \quad \checkmark \end{aligned}$$

b. $\pi x = -2\pi$

Write the equation.

Division Property of Equality

$\frac{\pi x}{\pi} = \frac{-2\pi}{\pi}$

Divide each side by π .

$x = -2$

Simplify.

▶ The solution is $x = -2$.

Check

$$\begin{aligned} \pi x &= -2\pi \\ \pi(-2) &\stackrel{?}{=} -2\pi \\ -2\pi &= -2\pi \quad \checkmark \end{aligned}$$

c. $1.3z = 5.2$

Write the equation.

Division Property of Equality

$\frac{1.3z}{1.3} = \frac{5.2}{1.3}$

Divide each side by 1.3 .

$z = 4$

Simplify.

▶ The solution is $z = 4$.

Check

$$\begin{aligned} 1.3z &= 5.2 \\ 1.3(4) &\stackrel{?}{=} 5.2 \\ 5.2 &= 5.2 \quad \checkmark \end{aligned}$$

Monitoring Progress



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Solve the equation. Justify each step. Check your solution.

4. $\frac{y}{3} = -6$

5. $9\pi = \pi x$

6. $0.05w = 1.4$

Solving Real-Life Problems

Core Concept

MODELING WITH MATHEMATICS

Mathematically proficient students routinely check that their solutions make sense in the context of a real-life problem.



Four-Step Approach to Problem Solving

1. **Understand the Problem** What is the unknown? What information is being given? What is being asked?
2. **Make a Plan** This plan might involve one or more of the problem-solving strategies shown on the next page.
3. **Solve the Problem** Carry out your plan. Check that each step is correct.
4. **Look Back** Examine your solution. Check that your solution makes sense in the original statement of the problem.

EXAMPLE 3

Modeling with Mathematics

In the 2012 Olympics, Usain Bolt won the 200-meter dash with a time of 19.32 seconds. Write and solve an equation to find his average speed to the nearest hundredth of a meter per second.



REMEMBER

The formula that relates distance d , rate or speed r , and time t is

$$d = rt.$$



SOLUTION

1. **Understand the Problem** You know the winning time and the distance of the race. You are asked to find the average speed to the nearest hundredth of a meter per second.
2. **Make a Plan** Use the Distance Formula to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

$$d = r \cdot t$$

Write the Distance Formula.

$$200 = r \cdot 19.32$$

Substitute 200 for d and 19.32 for t .

$$\frac{200}{19.32} = \frac{19.32r}{19.32}$$

Divide each side by 19.32.

$$10.35 \approx r$$

Simplify.

▶ Bolt's average speed was about 10.35 meters per second.

4. **Look Back** Round Bolt's average speed to 10 meters per second. At this speed, it would take

$$\frac{200 \text{ m}}{10 \text{ m/sec}} = 20 \text{ seconds}$$

to run 200 meters. Because 20 is close to 19.32, your solution is reasonable.

REMEMBER

The symbol \approx means "approximately equal to."



Monitoring Progress

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7. Suppose Usain Bolt ran 400 meters at the same average speed that he ran the 200 meters. How long would it take him to run 400 meters? Round your answer to the nearest hundredth of a second.

Core Concept

Common Problem-Solving Strategies

Use a verbal model.	Guess, check, and revise.
Draw a diagram.	Sketch a graph or number line.
Write an equation.	Make a table.
Look for a pattern.	Make a list.
Work backward.	Break the problem into parts.

EXAMPLE 4 Modeling with Mathematics

On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 54°F at 9:00 A.M. to -4°F at 9:27 A.M. How many degrees did the temperature fall?

SOLUTION

- Understand the Problem** You know the temperature before and after the temperature fell. You are asked to find how many degrees the temperature fell.
- Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
- Solve the Problem**

Words Temperature at 9:27 A.M. = Temperature at 9:00 A.M. - Number of degrees the temperature fell

Variable Let T be the number of degrees the temperature fell.

Equation $-4 = 54 - T$

$$-4 = 54 - T \quad \text{Write the equation.}$$

$$-4 - 54 = 54 - 54 - T \quad \text{Subtract 54 from each side.}$$

$$-58 = -T \quad \text{Simplify.}$$

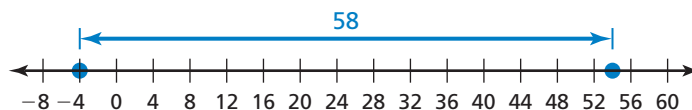
$$58 = T \quad \text{Divide each side by } -1.$$

► The temperature fell 58°F .

REMEMBER

The distance between two points on a number line is always positive.

- Look Back** The temperature fell from 54 degrees *above* 0 to 4 degrees *below* 0. You can use a number line to check that your solution is reasonable.



Monitoring Progress



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- You thought the balance in your checking account was \$68. When your bank statement arrives, you realize that you forgot to record a check. The bank statement lists your balance as \$26. Write and solve an equation to find the amount of the check that you forgot to record.

1.1 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** Which of the operations $+$, $-$, \times , and \div are inverses of each other?
- VOCABULARY** Are the equations $-2x = 10$ and $-5x = 25$ equivalent? Explain.
- WRITING** Which property of equality would you use to solve the equation $14x = 56$? Explain.
- WHICH ONE DOESN'T BELONG?** Which expression does not belong with the other three? Explain your reasoning.

$$8 = \frac{x}{2}$$

$$3 = x \div 4$$

$$x - 6 = 5$$

$$\frac{x}{3} = 9$$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–14, solve the equation. Justify each step. Check your solution. (See Example 1.)

- $x + 5 = 8$
- $m + 9 = 2$
- $y - 4 = 3$
- $s - 2 = 1$
- $w + 3 = -4$
- $n - 6 = -7$
- $-14 = p - 11$
- $0 = 4 + q$
- $r + (-8) = 10$
- $t - (-5) = 9$

- MODELING WITH MATHEMATICS** A discounted amusement park ticket costs \$12.95 less than the original price p . Write and solve an equation to find the original price.

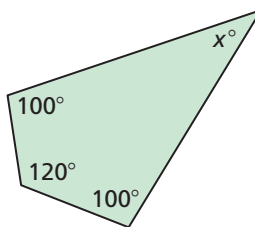


- MODELING WITH MATHEMATICS** You and a friend are playing a board game. Your final score x is 12 points less than your friend's final score. Write and solve an equation to find your final score.

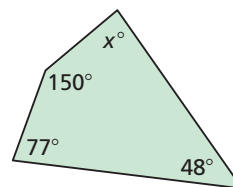
	ROUND 9	ROUND 10	FINAL SCORE
Your Friend	22	12	195
You	9	25	?

USING TOOLS The sum of the angle measures of a quadrilateral is 360° . In Exercises 17–20, write and solve an equation to find the value of x . Use a protractor to check the reasonableness of your answer.

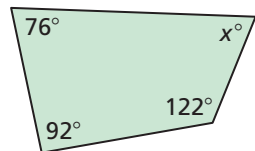
17.



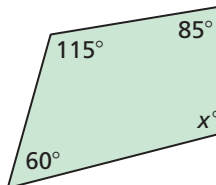
18.



19.



20.




In Exercises 21–30, solve the equation. Justify each step. Check your solution. (See Example 2.)


- $5g = 20$
- $4q = 52$
- $p \div 5 = 3$
- $y \div 7 = 1$
- $-8r = 64$
- $x \div (-2) = 8$
- $\frac{x}{6} = 8$
- $\frac{w}{-3} = 6$
- $-54 = 9s$
- $-7 = \frac{t}{7}$

In Exercises 31–38, solve the equation. Check your solution.

31. $\frac{3}{2} + t = \frac{1}{2}$ 32. $b - \frac{3}{16} = \frac{5}{16}$
 33. $\frac{3}{7}m = 6$ 34. $-\frac{2}{5}y = 4$
 35. $5.2 = a - 0.4$ 36. $f + 3\pi = 7\pi$
 37. $-108\pi = 6\pi j$ 38. $x \div (-2) = 1.4$

ERROR ANALYSIS In Exercises 39 and 40, describe and correct the error in solving the equation.

39. 
$$\begin{aligned} -0.8 + r &= 12.6 \\ r &= 12.6 + (-0.8) \\ r &= 11.8 \end{aligned}$$

40. 
$$\begin{aligned} -\frac{m}{3} &= -4 \\ 3 \cdot \left(-\frac{m}{3}\right) &= 3 \cdot (-4) \\ m &= -12 \end{aligned}$$

41. **ANALYZING RELATIONSHIPS** A baker orders 162 eggs. Each carton contains 18 eggs. Which equation can you use to find the number x of cartons? Explain your reasoning and solve the equation.

- (A) $162x = 18$ (B) $\frac{x}{18} = 162$
 (C) $18x = 162$ (D) $x + 18 = 162$

MODELING WITH MATHEMATICS In Exercises 42–44, write and solve an equation to answer the question. (See Examples 3 and 4.)

42. The temperature at 5 P.M. is 20°F . The temperature at 10 P.M. is -5°F . How many degrees did the temperature fall?

43. The length of an American flag is 1.9 times its width. What is the width of the flag?



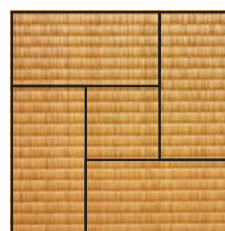
44. The balance of an investment account is \$308 more than the balance 4 years ago. The current balance of the account is \$4708. What was the balance 4 years ago?

45. **REASONING** Identify the property of equality that makes Equation 1 and Equation 2 equivalent.

Equation 1 $x - \frac{1}{2} = \frac{x}{4} + 3$

Equation 2 $4x - 2 = x + 12$

46. **PROBLEM SOLVING** Tatami mats are used as a floor covering in Japan. One possible layout uses four identical rectangular mats and one square mat, as shown. The area of the square mat is half the area of one of the rectangular mats.



Total area = 81 ft^2

- a. Write and solve an equation to find the area of one rectangular mat.
 b. The length of a rectangular mat is twice the width. Use Guess, Check, and Revise to find the dimensions of one rectangular mat.

47. **PROBLEM SOLVING** You spend \$30.40 on 4 CDs. Each CD costs the same amount and is on sale for 80% of the original price.

- a. Write and solve an equation to find how much you spend on each CD.



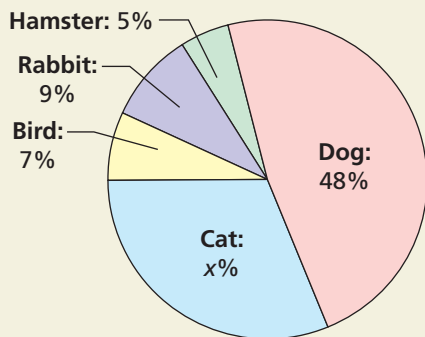
- b. The next day, the CDs are no longer on sale. You have \$25. Will you be able to buy 3 more CDs? Explain your reasoning.

48. **ANALYZING RELATIONSHIPS** As c increases, does the value of x increase, decrease, or stay the same for each equation? Assume c is positive.

Equation	Value of x
$x - c = 0$	
$cx = 1$	
$cx = c$	
$\frac{x}{c} = 1$	

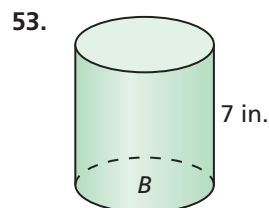
49. **USING STRUCTURE** Use the values -2 , 5 , 9 , and 10 to complete each statement about the equation $ax = b - 5$.
- When $a = \underline{\hspace{1cm}}$ and $b = \underline{\hspace{1cm}}$, x is a positive integer.
 - When $a = \underline{\hspace{1cm}}$ and $b = \underline{\hspace{1cm}}$, x is a negative integer.

50. **HOW DO YOU SEE IT?** The circle graph shows the percents of different animals sold at a local pet store in 1 year.

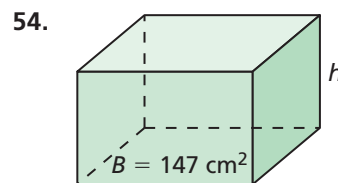


- What percent is represented by the entire circle?
 - How does the equation $7 + 9 + 5 + 48 + x = 100$ relate to the circle graph? How can you use this equation to find the percent of cats sold?
51. **REASONING** One-sixth of the girls and two-sevenths of the boys in a school marching band are in the percussion section. The percussion section has 6 girls and 10 boys. How many students are in the marching band? Explain.
52. **THOUGHT PROVOKING** Write a real-life problem that can be modeled by an equation equivalent to the equation $5x = 30$. Then solve the equation and write the answer in the context of your real-life problem.

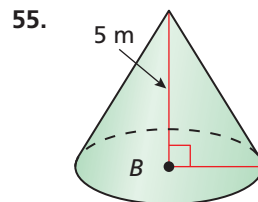
MATHEMATICAL CONNECTIONS In Exercises 53–56, find the height h or the area of the base B of the solid.



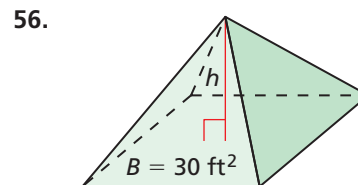
Volume = $84\pi \text{ in.}^3$



Volume = 1323 cm^3



Volume = $15\pi \text{ m}^3$



Volume = 35 ft^3

57. **MAKING AN ARGUMENT** In baseball, a player's batting average is calculated by dividing the number of hits by the number of at-bats. The table shows Player A's batting average and number of at-bats for three regular seasons.

Season	Batting average	At-bats
2010	.312	596
2011	.296	446
2012	.295	599

- How many hits did Player A have in the 2011 regular season? Round your answer to the nearest whole number.
- Player B had 33 fewer hits in the 2011 season than Player A but had a greater batting average. Your friend concludes that Player B had more at-bats in the 2011 season than Player A. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the Distributive Property to simplify the expression. (*Skills Review Handbook*)

58. $8(y + 3)$ 59. $\frac{5}{6}(x + \frac{1}{2} + 4)$ 60. $5(m + 3 + n)$ 61. $4(2p + 4q + 6)$

Copy and complete the statement. Round to the nearest hundredth, if necessary. (*Skills Review Handbook*)

62. $\frac{5 \text{ L}}{\text{min}} = \frac{\boxed{\hspace{1cm}} \text{ L}}{\text{h}}$

63. $\frac{68 \text{ mi}}{\text{h}} \approx \frac{\boxed{\hspace{1cm}} \text{ mi}}{\text{sec}}$

64. $\frac{7 \text{ gal}}{\text{min}} \approx \frac{\boxed{\hspace{1cm}} \text{ qt}}{\text{sec}}$

65. $\frac{8 \text{ km}}{\text{min}} \approx \frac{\boxed{\hspace{1cm}} \text{ mi}}{\text{h}}$

1.2 Solving Multi-Step Equations

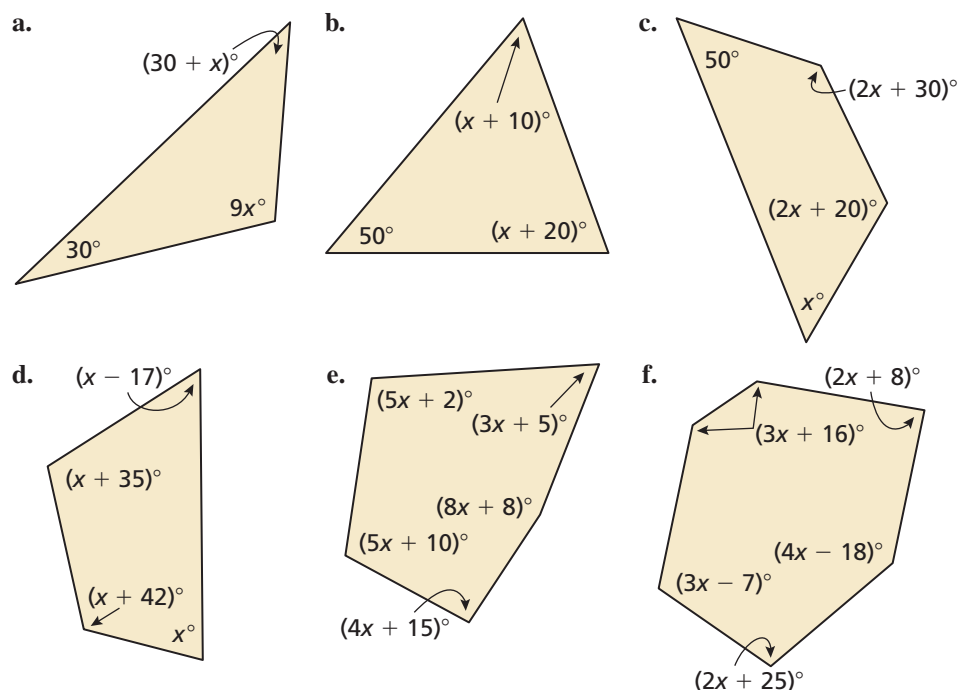
Essential Question How can you use multi-step equations to solve real-life problems?

EXPLORATION 1 Solving for the Angle Measures of a Polygon

Work with a partner. The sum S of the angle measures of a polygon with n sides can be found using the formula $S = 180(n - 2)$. Write and solve an equation to find each value of x . Justify the steps in your solution. Then find the angle measures of each polygon. How can you check the reasonableness of your answers?

JUSTIFYING CONCLUSIONS

To be proficient in math, you need to be sure your answers make sense in the context of the problem. For instance, if you find the angle measures of a triangle, and they have a sum that is not equal to 180° , then you should check your work for mistakes.



EXPLORATION 2 Writing a Multi-Step Equation

Work with a partner.

- Draw an irregular polygon.
- Measure the angles of the polygon. Record the measurements on a separate sheet of paper.
- Choose a value for x . Then, using this value, work backward to assign a variable expression to each angle measure, as in Exploration 1.
- Trade polygons with your partner.
- Solve an equation to find the angle measures of the polygon your partner drew. Do your answers seem reasonable? Explain.

Communicate Your Answer

- How can you use multi-step equations to solve real-life problems?
- In Exploration 1, you were given the formula for the sum S of the angle measures of a polygon with n sides. Explain why this formula works.
- The sum of the angle measures of a polygon is 1080° . How many sides does the polygon have? Explain how you found your answer.

1.2 Lesson

Core Vocabulary

Previous
inverse operations
mean

What You Will Learn

- ▶ Solve multi-step linear equations using inverse operations.
- ▶ Use multi-step linear equations to solve real-life problems.
- ▶ Use unit analysis to model real-life problems.

Solving Multi-Step Linear Equations

Core Concept

Solving Multi-Step Equations

To solve a multi-step equation, simplify each side of the equation, if necessary. Then use inverse operations to isolate the variable.

EXAMPLE 1 Solving a Two-Step Equation

Solve $2.5x - 13 = 2$. Check your solution.

SOLUTION

$$2.5x - 13 = 2$$

Write the equation.

Undo the subtraction.

$$\xrightarrow{+13} \quad +13$$

Add 13 to each side.

$$2.5x = 15$$

Simplify.

Undo the multiplication.

$$\xrightarrow{\div 2.5} \quad \frac{2.5x}{2.5} = \frac{15}{2.5}$$

Divide each side by 2.5.

$$x = 6$$

Simplify.

Check

$$2.5x - 13 = 2$$

$$2.5(6) - 13 \stackrel{?}{=} 2$$

$$2 = 2 \quad \checkmark$$

▶ The solution is $x = 6$.

EXAMPLE 2 Combining Like Terms to Solve an Equation

Solve $-12 = 9x - 6x + 15$. Check your solution.

SOLUTION

$$-12 = 9x - 6x + 15$$

Write the equation.

$$-12 = 3x + 15$$

Combine like terms.

Undo the addition.

$$\xrightarrow{-15} \quad -15$$

Subtract 15 from each side.

$$-27 = 3x$$

Simplify.

Undo the multiplication.

$$\xrightarrow{\div 3} \quad \frac{-27}{3} = \frac{3x}{3}$$

Divide each side by 3.

$$-9 = x$$

Simplify.

Check

$$-12 = 9x - 6x + 15$$

$$-12 \stackrel{?}{=} 9(-9) - 6(-9) + 15$$

$$-12 = -12 \quad \checkmark$$

▶ The solution is $x = -9$.

Monitoring Progress



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Solve the equation. Check your solution.

1. $-2n + 3 = 9$

2. $-21 = \frac{1}{2}c - 11$

3. $-2x - 10x + 12 = 18$

EXAMPLE 3**Using Structure to Solve a Multi-Step Equation**

Solve $2(1 - x) + 3 = -8$. Check your solution.

SOLUTION

Method 1 One way to solve the equation is by using the Distributive Property.

$$2(1 - x) + 3 = -8$$

Write the equation.

$$2(1) - 2(x) + 3 = -8$$

Distributive Property

$$2 - 2x + 3 = -8$$

Multiply.

$$-2x + 5 = -8$$

Combine like terms.

$$\underline{-5} \quad \underline{-5}$$

Subtract 5 from each side.

$$-2x = -13$$

Simplify.

$$\frac{-2x}{-2} = \frac{-13}{-2}$$

Divide each side by -2 .

$$x = 6.5$$

Simplify.

► The solution is $x = 6.5$.

Check

$$2(1 - x) + 3 = -8$$

$$2(1 - 6.5) + 3 \stackrel{?}{=} -8$$

$$-8 = -8 \quad \checkmark$$

Method 2 Another way to solve the equation is by interpreting the expression $1 - x$ as a single quantity.

$$2(1 - x) + 3 = -8$$

Write the equation.

$$\underline{-3} \quad \underline{-3}$$

Subtract 3 from each side.

$$2(1 - x) = -11$$

Simplify.

$$\frac{2(1 - x)}{2} = \frac{-11}{2}$$

Divide each side by 2.

$$1 - x = -5.5$$

Simplify.

$$\underline{-1} \quad \underline{-1}$$

Subtract 1 from each side.

$$-x = -6.5$$

Simplify.

$$\frac{-x}{-1} = \frac{-6.5}{-1}$$

Divide each side by -1 .

$$x = 6.5$$

Simplify.

► The solution is $x = 6.5$, which is the same solution obtained in Method 1.

LOOKING FOR STRUCTURE

First solve for the expression $1 - x$, and then solve for x .

**Monitoring Progress**

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Solve the equation. Check your solution.

4. $3(x + 1) + 6 = -9$

5. $15 = 5 + 4(2d - 3)$

6. $13 = -2(y - 4) + 3y$

7. $2x(5 - 3) - 3x = 5$

8. $-4(2m + 5) - 3m = 35$

9. $5(3 - x) + 2(3 - x) = 14$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics



Use the table to find the number of miles x you need to bike on Friday so that the mean number of miles biked per day is 5.

Day	Miles
Monday	3.5
Tuesday	5.5
Wednesday	0
Thursday	5
Friday	x

SOLUTION

- 1. Understand the Problem** You know how many miles you biked Monday through Thursday. You are asked to find the number of miles you need to bike on Friday so that the mean number of miles biked per day is 5.
- 2. Make a Plan** Use the definition of mean to write an equation that represents the problem. Then solve the equation.
- 3. Solve the Problem** The mean of a data set is the sum of the data divided by the number of data values.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5$$

Write the equation.

$$\frac{14 + x}{5} = 5$$

Combine like terms.

$$5 \cdot \frac{14 + x}{5} = 5 \cdot 5$$

Multiply each side by 5.

$$14 + x = 25$$

Simplify.

$$\underline{-14} \quad \underline{-14}$$

Subtract 14 from each side.

$$x = 11$$

Simplify.

► You need to bike 11 miles on Friday.

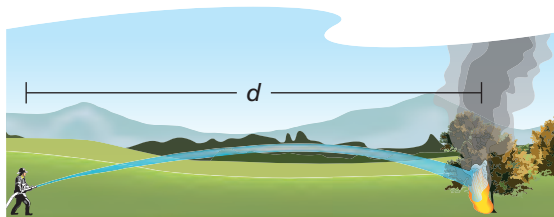
- 4. Look Back** Notice that on the days that you did bike, the values are close to the mean. Because you did not bike on Wednesday, you need to bike about twice the mean on Friday. Eleven miles is about twice the mean. So, your solution is reasonable.

Monitoring Progress



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- The formula $d = \frac{1}{2}n + 26$ relates the nozzle pressure n (in pounds per square inch) of a fire hose and the maximum horizontal distance the water reaches d (in feet). How much pressure is needed to reach a fire 50 feet away?



REMEMBER

When you add **miles** to **miles**, you get **miles**.
But, when you divide **miles** by **days**, you get **miles per day**.

Using Unit Analysis to Model Real-Life Problems

When you write an equation to model a real-life problem, you should check that the units on each side of the equation balance. For instance, in Example 4, notice how the units balance.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5$$

Units: miles, miles per day, per, days

$\frac{\text{mi}}{\text{day}} = \frac{\text{mi}}{\text{day}}$ ✓

EXAMPLE 5 Solving a Real-Life Problem

Your school's drama club charges \$4 per person for admission to a play. The club borrowed \$400 to pay for costumes and props. After paying back the loan, the club has a profit of \$100. How many people attended the play?

SOLUTION

- Understand the Problem** You know how much the club charges for admission. You also know how much the club borrowed and its profit. You are asked to find how many people attended the play.
- Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
- Solve the Problem**

Words Ticket price • Number of people who attended - Amount of loan = Profit

Variable Let x be the number of people who attended.

Equation $\frac{\$4}{\text{person}} \cdot x \text{ people} - \$400 = \$100$ $\$ = \$$ ✓

$$4x - 400 = 100$$

Write the equation.

$$4x - 400 + 400 = 100 + 400$$

Add 400 to each side.

$$4x = 500$$

Simplify.

$$\frac{4x}{4} = \frac{500}{4}$$

Divide each side by 4.

$$x = 125$$

Simplify.

▶ So, 125 people attended the play.

- Look Back** To check that your solution is reasonable, multiply \$4 per person by 125 people. The result is \$500. After paying back the \$400 loan, the club has \$100, which is the profit.

REMEMBER

When you multiply **dollars per person** by **people**, you get **dollars**.



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- You have 96 feet of fencing to enclose a rectangular pen for your dog. To provide sufficient running space for your dog to exercise, the pen should be three times as long as it is wide. Find the dimensions of the pen.

1.2 Exercises

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Vocabulary and Core Concept Check

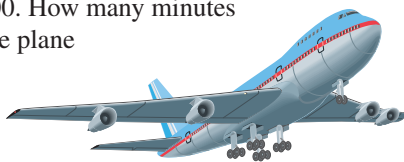
- COMPLETE THE SENTENCE** To solve the equation $2x + 3x = 20$, first combine $2x$ and $3x$ because they are _____.
- WRITING** Describe two ways to solve the equation $2(4x - 11) = 10$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–14, solve the equation. Check your solution. (See Examples 1 and 2.)

- $3w + 7 = 19$
- $2g - 13 = 3$
- $11 = 12 - q$
- $10 = 7 - m$
- $5 = \frac{z}{-4} - 3$
- $\frac{a}{3} + 4 = 6$
- $\frac{h + 6}{5} = 2$
- $\frac{d - 8}{-2} = 12$
- $8y + 3y = 44$
- $36 = 13n - 4n$
- $12v + 10v + 14 = 80$
- $6c - 8 - 2c = -16$

15. **MODELING WITH MATHEMATICS** The altitude a (in feet) of a plane t minutes after liftoff is given by $a = 3400t + 600$. How many minutes after liftoff is the plane at an altitude of 21,000 feet?



16. **MODELING WITH MATHEMATICS** A repair bill for your car is \$553. The parts cost \$265. The labor cost is \$48 per hour. Write and solve an equation to find the number of hours of labor spent repairing the car.

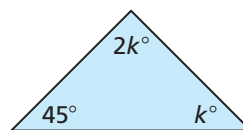
In Exercises 17–24, solve the equation. Check your solution. (See Example 3.)

- $4(z + 5) = 32$
- $-2(4g - 3) = 30$
- $6 + 5(m + 1) = 26$
- $5h + 2(11 - h) = -5$
- $27 = 3c - 3(6 - 2c)$
- $-3 = 12y - 5(2y - 7)$

- $-3(3 + x) + 4(x - 6) = -4$
- $5(r + 9) - 2(1 - r) = 1$

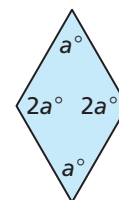
USING TOOLS In Exercises 25–28, find the value of the variable. Then find the angle measures of the polygon. Use a protractor to check the reasonableness of your answer.

25.



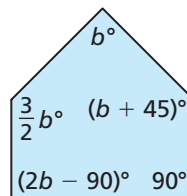
Sum of angle measures: 180°

26.



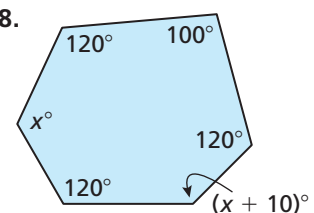
Sum of angle measures: 360°

27.



Sum of angle measures: 540°

28.



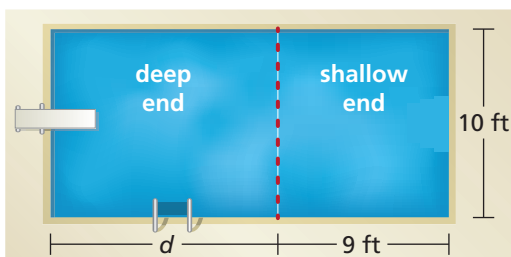
Sum of angle measures: 720°

In Exercises 29–34, write and solve an equation to find the number.

- The sum of twice a number and 13 is 75.
- The difference of three times a number and 4 is -19 .
- Eight plus the quotient of a number and 3 is -2 .
- The sum of twice a number and half the number is 10.
- Six times the sum of a number and 15 is -42 .
- Four times the difference of a number and 7 is 12.

USING EQUATIONS In Exercises 35–37, write and solve an equation to answer the question. Check that the units on each side of the equation balance. (See Examples 4 and 5.)

35. During the summer, you work 30 hours per week at a gas station and earn \$8.75 per hour. You also work as a landscaper for \$11 per hour and can work as many hours as you want. You want to earn a total of \$400 per week. How many hours must you work as a landscaper?
36. The area of the surface of the swimming pool is 210 square feet. What is the length d of the deep end (in feet)?



37. You order two tacos and a salad. The salad costs \$2.50. You pay 8% sales tax and leave a \$3 tip. You pay a total of \$13.80. How much does one taco cost?

JUSTIFYING STEPS In Exercises 38 and 39, justify each step of the solution.

38. $-\frac{1}{2}(5x - 8) - 1 = 6$ Write the equation.

$$-\frac{1}{2}(5x - 8) = 7$$

$$5x - 8 = -14$$

$$5x = -6$$

$$x = -\frac{6}{5}$$

39. $2(x + 3) + x = -9$ Write the equation.

$$2(x) + 2(3) + x = -9$$

$$2x + 6 + x = -9$$

$$3x + 6 = -9$$

$$3x = -15$$

$$x = -5$$

ERROR ANALYSIS In Exercises 40 and 41, describe and correct the error in solving the equation.

40.



$$\begin{aligned} -2(7 - y) + 4 &= -4 \\ -14 - 2y + 4 &= -4 \\ -10 - 2y &= -4 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

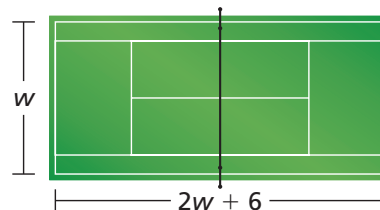
41.



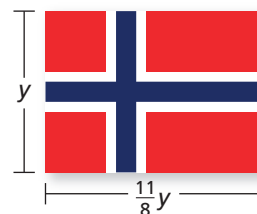
$$\begin{aligned} \frac{1}{4}(x - 2) + 4 &= 12 \\ \frac{1}{4}(x - 2) &= 8 \\ x - 2 &= 2 \\ x &= 4 \end{aligned}$$

MATHEMATICAL CONNECTIONS In Exercises 42–44, write and solve an equation to answer the question.

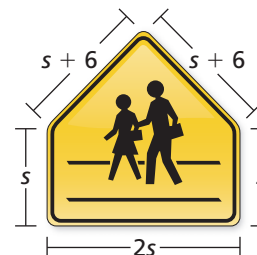
42. The perimeter of the tennis court is 228 feet. What are the dimensions of the court?



43. The perimeter of the Norwegian flag is 190 inches. What are the dimensions of the flag?



44. The perimeter of the school crossing sign is 102 inches. What is the length of each side?



45. **COMPARING METHODS** Solve the equation $2(4 - 8x) + 6 = -1$ using (a) Method 1 from Example 3 and (b) Method 2 from Example 3. Which method do you prefer? Explain.
46. **PROBLEM SOLVING** An online ticket agency charges the amounts shown for basketball tickets. The total cost for an order is \$220.70. How many tickets are purchased?

Charge	Amount
Ticket price	\$32.50 per ticket
Convenience charge	\$3.30 per ticket
Processing charge	\$5.90 per order

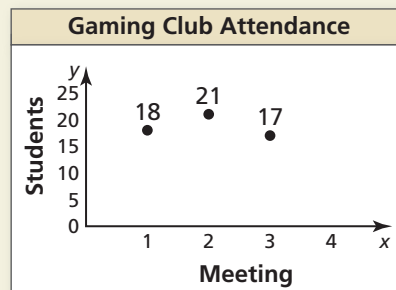
47. **MAKING AN ARGUMENT** You have quarters and dimes that total \$2.80. Your friend says it is possible that the number of quarters is 8 more than the number of dimes. Is your friend correct? Explain.

48. **THOUGHT PROVOKING** You teach a math class and assign a weight to each component of the class. You determine final grades by totaling the products of the weights and the component scores. Choose values for the remaining weights and find the necessary score on the final exam for a student to earn an A (90%) in the class, if possible. Explain your reasoning.

Component	Student's score	Weight	Score \times Weight
Class Participation	92%	0.20	$92\% \times 0.20 = 18.4\%$
Homework	95%		
Midterm Exam	88%		
Final Exam			
Total		1	

49. **REASONING** An even integer can be represented by the expression $2n$, where n is any integer. Find three consecutive even integers that have a sum of 54. Explain your reasoning.

50. **HOW DO YOU SEE IT?** The scatter plot shows the attendance for each meeting of a gaming club.



- The mean attendance for the first four meetings is 20. Is the number of students who attended the fourth meeting greater than or less than 20? Explain.
- Estimate the number of students who attended the fourth meeting.
- Describe a way you can check your estimate in part (b).

REASONING In Exercises 51–56, the letters a , b , and c represent nonzero constants. Solve the equation for x .

- $bx = -7$
- $x + a = \frac{3}{4}$
- $ax - b = 12.5$
- $ax + b = c$
- $2bx - bx = -8$
- $cx - 4b = 5b$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. (*Skills Review Handbook*)

57. $4m + 5 - 3m$

58. $9 - 8b + 6b$

59. $6t + 3(1 - 2t) - 5$

Determine whether (a) $x = -1$ or (b) $x = 2$ is a solution of the equation. (*Skills Review Handbook*)

60. $x - 8 = -9$

61. $x + 1.5 = 3.5$

62. $2x - 1 = 3$

63. $3x + 4 = 1$

64. $x + 4 = 3x$

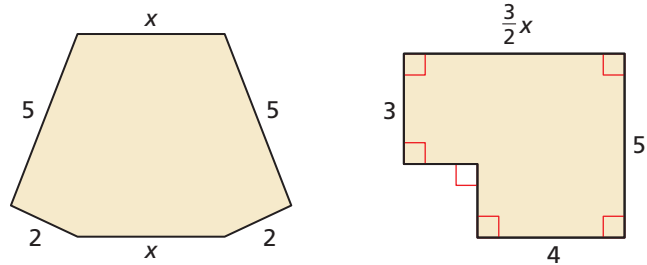
65. $-2(x - 1) = 1 - 3x$

1.3 Solving Equations with Variables on Both Sides

Essential Question How can you solve an equation that has variables on both sides?

EXPLORATION 1 Perimeter

Work with a partner. The two polygons have the same perimeter. Use this information to write and solve an equation involving x . Explain the process you used to find the solution. Then find the perimeter of each polygon.



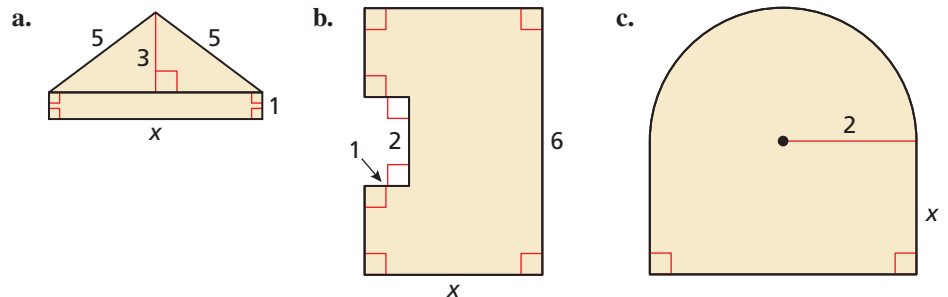
EXPLORATION 2 Perimeter and Area

Work with a partner.

- Each figure has the unusual property that the value of its perimeter (in feet) is equal to the value of its area (in square feet). Use this information to write an equation for each figure.
- Solve each equation for x . Explain the process you used to find the solution.
- Find the perimeter and area of each figure.

LOOKING FOR STRUCTURE

To be proficient in math, you need to visualize complex things, such as composite figures, as being made up of simpler, more manageable parts.



Communicate Your Answer

- How can you solve an equation that has variables on both sides?
- Write three equations that have the variable x on both sides. The equations should be different from those you wrote in Explorations 1 and 2. Have your partner solve the equations.

1.3 Lesson

Core Vocabulary

identity, p. 21

Previous
inverse operations

What You Will Learn

- ▶ Solve linear equations that have variables on both sides.
- ▶ Identify special solutions of linear equations.
- ▶ Use linear equations to solve real-life problems.

Solving Equations with Variables on Both Sides

Core Concept

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, simplify one or both sides of the equation, if necessary. Then use inverse operations to collect the variable terms on one side, collect the constant terms on the other side, and isolate the variable.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $10 - 4x = -9x$. Check your solution.

SOLUTION

$$\begin{array}{ll} 10 - 4x = -9x & \text{Write the equation.} \\ \underline{+ 4x} \quad \underline{+ 4x} & \text{Add } 4x \text{ to each side.} \\ 10 = -5x & \text{Simplify.} \\ \frac{10}{-5} = \frac{-5x}{-5} & \text{Divide each side by } -5. \\ -2 = x & \text{Simplify.} \end{array}$$

Check

$$\begin{array}{l} 10 - 4x = -9x \\ 10 - 4(-2) \stackrel{?}{=} -9(-2) \\ 18 = 18 \quad \checkmark \end{array}$$

- ▶ The solution is $x = -2$.

EXAMPLE 2 Solving an Equation with Grouping Symbols

Solve $3(3x - 4) = \frac{1}{4}(32x + 56)$.

SOLUTION

$$\begin{array}{ll} 3(3x - 4) = \frac{1}{4}(32x + 56) & \text{Write the equation.} \\ 9x - 12 = 8x + 14 & \text{Distributive Property} \\ \underline{+ 12} \quad \underline{+ 12} & \text{Add 12 to each side.} \\ 9x = 8x + 26 & \text{Simplify.} \\ \underline{- 8x} \quad \underline{- 8x} & \text{Subtract } 8x \text{ from each side.} \\ x = 26 & \text{Simplify.} \end{array}$$

- ▶ The solution is $x = 26$.

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Solve the equation. Check your solution.

1. $-2x = 3x + 10$
2. $\frac{1}{2}(6h - 4) = -5h + 1$
3. $-\frac{3}{4}(8n + 12) = 3(n - 3)$

Identifying Special Solutions of Linear Equations

Core Concept

Special Solutions of Linear Equations

Equations do not always have one solution. An equation that is true for all values of the variable is an **identity** and has *infinitely many solutions*. An equation that is not true for any value of the variable has *no solution*.

REASONING

The equation $15x + 6 = 15x$ is not true because the number $15x$ cannot be equal to 6 more than itself.

EXAMPLE 3 Identifying the Number of Solutions

Solve each equation.

a. $3(5x + 2) = 15x$

b. $-2(4y + 1) = -8y - 2$

SOLUTION

a. $3(5x + 2) = 15x$

Write the equation.

$$15x + 6 = 15x$$

Distributive Property

$$\underline{-15x} \quad \underline{-15x}$$

Subtract $15x$ from each side.

$$6 = 0 \quad \times$$

Simplify.

► The statement $6 = 0$ is never true. So, the equation has no solution.

b. $-2(4y + 1) = -8y - 2$

Write the equation.

$$-8y - 2 = -8y - 2$$

Distributive Property

$$\underline{+8y} \quad \underline{+8y}$$

Add $8y$ to each side.

$$-2 = -2$$

Simplify.

► The statement $-2 = -2$ is always true. So, the equation is an identity and has infinitely many solutions.

READING

All real numbers are solutions of an identity.

Monitoring Progress



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Solve the equation.

4. $4(1 - p) = -4p + 4$

5. $6m - m = \frac{5}{6}(6m - 10)$

6. $10k + 7 = -3 - 10k$

7. $3(2a - 2) = 2(3a - 3)$

Concept Summary

Steps for Solving Linear Equations

Here are several steps you can use to solve a linear equation. Depending on the equation, you may not need to use some steps.

Step 1 Use the Distributive Property to remove any grouping symbols.

Step 2 Simplify the expression on each side of the equation.

Step 3 Collect the variable terms on one side of the equation and the constant terms on the other side.

Step 4 Isolate the variable.

Step 5 Check your solution.

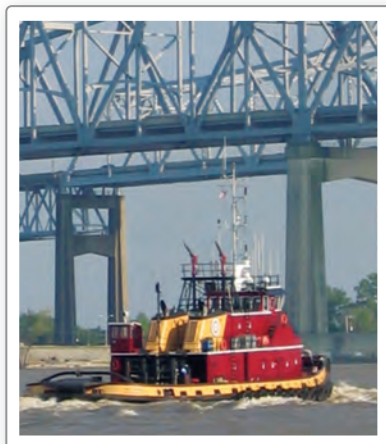
STUDY TIP

To check an identity, you can choose several different values of the variable.

Solving Real-Life Problems

EXAMPLE 4

Modeling with Mathematics



A boat leaves New Orleans and travels upstream on the Mississippi River for 4 hours. The return trip takes only 2.8 hours because the boat travels 3 miles per hour faster downstream due to the current. How far does the boat travel upstream?

SOLUTION

- 1. Understand the Problem** You are given the amounts of time the boat travels and the difference in speeds for each direction. You are asked to find the distance the boat travels upstream.
- 2. Make a Plan** Use the Distance Formula to write expressions that represent the problem. Because the distance the boat travels in both directions is the same, you can use the expressions to write an equation.
- 3. Solve the Problem** Use the formula (distance) = (rate)(time).

Words Distance upstream = Distance downstream

Variable Let x be the speed (in miles per hour) of the boat traveling upstream.

Equation $\frac{x \text{ mi}}{1 \cancel{\text{ h}}} \cdot 4 \cancel{\text{ h}} = \frac{(x + 3) \text{ mi}}{1 \cancel{\text{ h}}} \cdot 2.8 \cancel{\text{ h}}$ (mi = mi) ✓

$$4x = 2.8(x + 3)$$

Write the equation.

$$4x = 2.8x + 8.4$$

Distributive Property

$$\underline{- 2.8x} \quad \underline{- 2.8x}$$

Subtract $2.8x$ from each side.

$$1.2x = 8.4$$

Simplify.

$$\frac{1.2x}{1.2} = \frac{8.4}{1.2}$$

Divide each side by 1.2.

$$x = 7$$

Simplify.

► So, the boat travels 7 miles per hour upstream. To determine how far the boat travels upstream, multiply 7 miles per hour by 4 hours to obtain 28 miles.

- 4. Look Back** To check that your solution is reasonable, use the formula for distance. Because the speed upstream is 7 miles per hour, the speed downstream is $7 + 3 = 10$ miles per hour. When you substitute each speed into the Distance Formula, you get the same distance for upstream and downstream.

Upstream

$$\text{Distance} = \frac{7 \text{ mi}}{1 \cancel{\text{ h}}} \cdot 4 \cancel{\text{ h}} = 28 \text{ mi}$$

Downstream

$$\text{Distance} = \frac{10 \text{ mi}}{1 \cancel{\text{ h}}} \cdot 2.8 \cancel{\text{ h}} = 28 \text{ mi}$$

Monitoring Progress



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8. A boat travels upstream on the Mississippi River for 3.5 hours. The return trip only takes 2.5 hours because the boat travels 2 miles per hour faster downstream due to the current. How far does the boat travel upstream?

1.3 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** Is the equation $-2(4 - x) = 2x + 8$ an identity? Explain your reasoning.
- WRITING** Describe the steps in solving the linear equation $3(3x - 8) = 4x + 6$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–16, solve the equation. Check your solution. (See Examples 1 and 2.)

- $15 - 2x = 3x$
- $26 - 4s = 9s$
- $5p - 9 = 2p + 12$
- $8g + 10 = 35 + 3g$
- $5t + 16 = 6 - 5t$
- $-3r + 10 = 15r - 8$
- $7 + 3x - 12x = 3x + 1$
- $w - 2 + 2w = 6 + 5w$
- $10(g + 5) = 2(g + 9)$
- $-9(t - 2) = 4(t - 15)$
- $\frac{2}{3}(3x + 9) = -2(2x + 6)$
- $2(2t + 4) = \frac{3}{4}(24 - 8t)$
- $10(2y + 2) - y = 2(8y - 8)$
- $2(4x + 2) = 4x - 12(x - 1)$
- MODELING WITH MATHEMATICS** You and your friend drive toward each other. The equation $50h = 190 - 45h$ represents the number h of hours until you and your friend meet. When will you meet?
- MODELING WITH MATHEMATICS** The equation $1.5r + 15 = 2.25r$ represents the number r of movies you must rent to spend the same amount at each movie store. How many movies must you rent to spend the same amount at each movie store?



Membership Fee: \$15





Membership Fee: Free

In Exercises 19–24, solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. (See Example 3.)

- $3t + 4 = 12 + 3t$
- $6d + 8 = 14 + 3d$
- $2(h + 1) = 5h - 7$
- $12y + 6 = 6(2y + 1)$
- $3(4g + 6) = 2(6g + 9)$
- $5(1 + 2m) = \frac{1}{2}(8 + 20m)$

ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in solving the equation.

25. 
$$\begin{aligned} 5c - 6 &= 4 - 3c \\ 2c - 6 &= 4 \\ 2c &= 10 \\ c &= 5 \end{aligned}$$

26. 
$$\begin{aligned} 6(2y + 6) &= 4(9 + 3y) \\ 12y + 36 &= 36 + 12y \\ 12y &= 12y \\ 0 &= 0 \\ \text{The equation has no solution.} \end{aligned}$$

- MODELING WITH MATHEMATICS** Write and solve an equation to find the month when you would pay the same total amount for each Internet service.

	Installation fee	Price per month
Company A	\$60.00	\$42.95
Company B	\$25.00	\$49.95

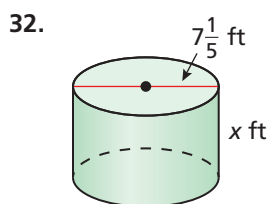
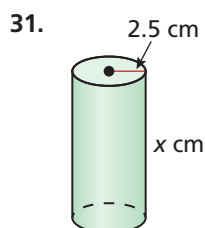
- 28. PROBLEM SOLVING** One serving of granola provides 4% of the protein you need daily. You must get the remaining 48 grams of protein from other sources. How many grams of protein do you need daily?

USING STRUCTURE In Exercises 29 and 30, find the value of r .

29. $8(x + 6) - 10 + r = 3(x + 12) + 5x$

30. $4(x - 3) - r + 2x = 5(3x - 7) - 9x$

MATHEMATICAL CONNECTIONS In Exercises 31 and 32, the value of the surface area of the cylinder is equal to the value of the volume of the cylinder. Find the value of x . Then find the surface area and volume of the cylinder.



33. **MODELING WITH MATHEMATICS** A cheetah that is running 90 feet per second is 120 feet behind an antelope that is running 60 feet per second. How long will it take the cheetah to catch up to the antelope? (See Example 4.)
34. **MAKING AN ARGUMENT** A cheetah can run at top speed for only about 20 seconds. If an antelope is too far away for a cheetah to catch it in 20 seconds, the antelope is probably safe. Your friend claims the antelope in Exercise 33 will not be safe if the cheetah starts running 650 feet behind it. Is your friend correct? Explain.

REASONING In Exercises 35 and 36, for what value of a is the equation an identity? Explain your reasoning.

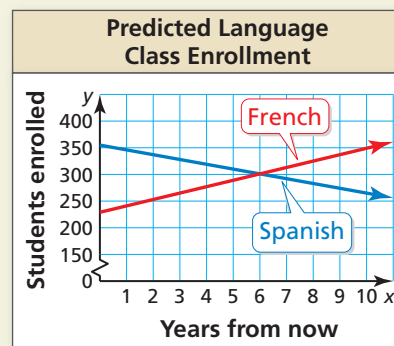
35. $a(2x + 3) = 9x + 15 + x$

36. $8x - 8 + 3ax = 5ax - 2a$

37. **REASONING** Two times the greater of two consecutive integers is 9 less than three times the lesser integer. What are the integers?

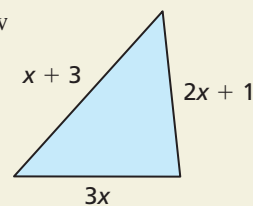
38. **HOW DO YOU SEE IT?** The table and the graph show information about students enrolled in Spanish and French classes at a high school.

	Students enrolled this year	Average rate of change
Spanish	355	9 fewer students each year
French	229	12 more students each year



- a. Use the graph to determine after how many years there will be equal enrollment in Spanish and French classes.
- b. How does the equation $355 - 9x = 229 + 12x$ relate to the table and the graph? How can you use this equation to determine whether your answer in part (a) is reasonable?
39. **WRITING EQUATIONS** Give an example of a linear equation that has (a) no solution and (b) infinitely many solutions. Justify your answers.

40. **THOUGHT PROVOKING** Draw a different figure that has the same perimeter as the triangle shown. Explain why your figure has the same perimeter.



Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Order the values from least to greatest. (Skills Review Handbook)

41. $9, |-4|, -4, 5, |2|$

42. $|-32|, 22, -16, -|21|, |-10|$

43. $-18, |-24|, -19, |-18|, |22|$

44. $-|-3|, |0|, -1, |2|, -2$

1.1–1.3 What Did You Learn?

Core Vocabulary

conjecture, *p. 3*
rule, *p. 3*
theorem, *p. 3*
equation, *p. 4*

linear equation in one variable, *p. 4*
solution, *p. 4*
inverse operations, *p. 4*
equivalent equations, *p. 4*
identity, *p. 21*

Core Concepts

Section 1.1

Addition Property of Equality, *p. 4*
Subtraction Property of Equality, *p. 4*
Multiplication Property of Equality, *p. 5*

Division Property of Equality, *p. 5*
Four-Step Approach to Problem Solving, *p. 6*
Common Problem-Solving Strategies, *p. 7*

Section 1.2

Solving Multi-Step Equations, *p. 12*

Unit Analysis, *p. 15*

Section 1.3

Solving Equations with Variables on Both Sides, *p. 20*

Special Solutions of Linear Equations, *p. 21*

Mathematical Practices

1. How did you make sense of the relationships between the quantities in Exercise 46 on page 9?
2. What is the limitation of the tool you used in Exercises 25–28 on page 16?
3. What definition did you use in your reasoning in Exercises 35 and 36 on page 24?

Study Skills

Completing Homework Efficiently

Before doing homework, review the Core Concepts and examples. Use the tutorials at *BigIdeasMath.com* for additional help.

Complete homework as though you are also preparing for a quiz. Memorize different types of problems, vocabulary, rules, and so on.



1.1–1.3 Quiz

Solve the equation. Justify each step. Check your solution. (Section 1.1)

1. $x + 9 = 7$
2. $8.6 = z - 3.8$
3. $60 = -12r$
4. $\frac{3}{4}p = 18$

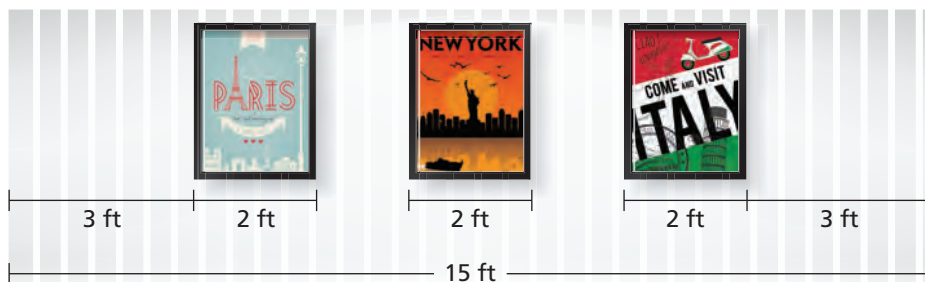
Solve the equation. Check your solution. (Section 1.2)

5. $2m - 3 = 13$
6. $5 = 10 - v$
7. $5 = 7w + 8w + 2$
8. $-21a + 28a - 6 = -10.2$
9. $2k - 3(2k - 3) = 45$
10. $68 = \frac{1}{5}(20x + 50) + 2$

Solve the equation. (Section 1.3)

11. $3c + 1 = c + 1$
12. $-8 - 5n = 64 + 3n$
13. $2(8q - 5) = 4q$
14. $9(y - 4) - 7y = 5(3y - 2)$
15. $4(g + 8) = 7 + 4g$
16. $-4(-5h - 4) = 2(10h + 8)$

17. To estimate how many miles you are from a thunderstorm, count the seconds between when you see lightning and when you hear thunder. Then divide by 5. Write and solve an equation to determine how many seconds you would count for a thunderstorm that is 2 miles away. (Section 1.1)
18. You want to hang three equally-sized travel posters on a wall so that the posters on the ends are each 3 feet from the end of the wall. You want the spacing between posters to be equal. Write and solve an equation to determine how much space you should leave between the posters. (Section 1.2)



19. You want to paint a piece of pottery at an art studio. The total cost is the cost of the piece plus an hourly studio fee. There are two studios to choose from. (Section 1.3)
 - a. After how many hours of painting are the total costs the same at both studios? Justify your answer.
 - b. Studio B increases the hourly studio fee by \$2. How does this affect your answer in part (a)? Explain.



Studio A
Vase: \$10
Studio Fee: \$8 per hour



Studio B
Vase: \$16
Studio Fee: \$6 per hour

1.4 Solving Absolute Value Equations

Essential Question How can you solve an absolute value equation?

EXPLORATION 1 Solving an Absolute Value Equation Algebraically

Work with a partner. Consider the absolute value equation

$$|x + 2| = 3.$$

- Describe the values of $x + 2$ that make the equation true. Use your description to write two linear equations that represent the solutions of the absolute value equation.
- Use the linear equations you wrote in part (a) to find the solutions of the absolute value equation.
- How can you use linear equations to solve an absolute value equation?

MAKING SENSE OF PROBLEMS

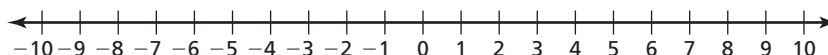
To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Solving an Absolute Value Equation Graphically

Work with a partner. Consider the absolute value equation

$$|x + 2| = 3.$$

- On a real number line, locate the point for which $x + 2 = 0$.



- Locate the points that are 3 units from the point you found in part (a). What do you notice about these points?
- How can you use a number line to solve an absolute value equation?

EXPLORATION 3 Solving an Absolute Value Equation Numerically

Work with a partner. Consider the absolute value equation

$$|x + 2| = 3.$$

- Use a spreadsheet, as shown, to solve the absolute value equation.
- Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?
- How can you use a spreadsheet to solve an absolute value equation?

	A	B
1	x	$ x + 2 $
2	-6	4
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	
10	2	

$\text{abs}(A2 + 2)$

Communicate Your Answer

- How can you solve an absolute value equation?
- What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value equation? Give reasons for your answers.

1.4 Lesson

Core Vocabulary

absolute value equation, p. 28
extraneous solution, p. 31

Previous

absolute value
opposite

What You Will Learn

- ▶ Solve absolute value equations.
- ▶ Solve equations involving two absolute values.
- ▶ Identify special solutions of absolute value equations.

Solving Absolute Value Equations

An **absolute value equation** is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

Core Concept

Properties of Absolute Value

Let a and b be real numbers. Then the following properties are true.

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

Solving Absolute Value Equations

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number that is not negative.

EXAMPLE 1 Solving Absolute Value Equations

Solve each equation. Graph the solutions, if possible.

- a. $|x - 4| = 6$ b. $|3x + 1| = -5$

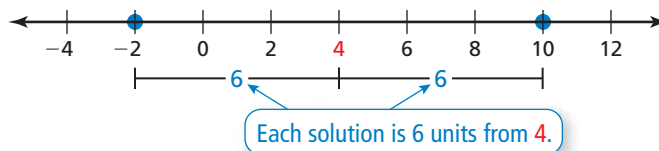
SOLUTION

- a. Write the two related linear equations for $|x - 4| = 6$. Then solve.

$$x - 4 = 6 \quad \text{or} \quad x - 4 = -6 \quad \text{Write related linear equations.}$$

$$x = 10 \quad \quad \quad x = -2 \quad \quad \quad \text{Add 4 to each side.}$$

- ▶ The solutions are $x = 10$ and $x = -2$.



Property of Absolute Value

- b. The absolute value of an expression must be greater than or equal to 0. The expression $|3x + 1|$ cannot equal -5 .

- ▶ So, the equation has no solution.

Monitoring Progress



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Solve the equation. Graph the solutions, if possible.

1. $|x| = 10$ 2. $|x - 1| = 4$ 3. $|3 + x| = -3$

ANOTHER WAY

Using the product property of absolute value, $|ab| = |a| |b|$, you could rewrite the equation as

$$3|x + 3| - 10 = -4$$

and then solve.

EXAMPLE 2 Solving an Absolute Value Equation

Solve $|3x + 9| - 10 = -4$.

SOLUTION

First isolate the absolute value expression on one side of the equation.

$$|3x + 9| - 10 = -4$$

Write the equation.

$$|3x + 9| = 6$$

Add 10 to each side.

Now write two related linear equations for $|3x + 9| = 6$. Then solve.

$$3x + 9 = 6 \quad \text{or} \quad 3x + 9 = -6$$

Write related linear equations.

$$3x = -3$$

$$3x = -15$$

Subtract 9 from each side.

$$x = -1$$

$$x = -5$$

Divide each side by 3.

► The solutions are $x = -1$ and $x = -5$.

EXAMPLE 3 Writing an Absolute Value Equation

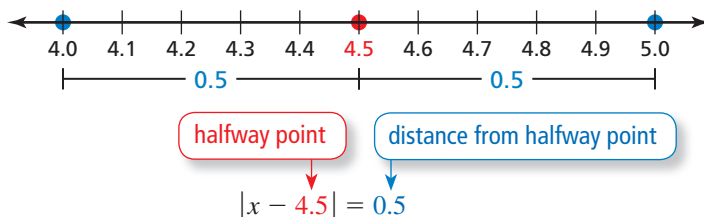
In a cheerleading competition, the minimum length of a routine is 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

SOLUTION

1. Understand the Problem You know the minimum and maximum lengths. You are asked to write an absolute value equation that represents these lengths.

2. Make a Plan Consider the minimum and maximum lengths as solutions to an absolute value equation. Use a number line to find the halfway point between the solutions. Then use the halfway point and the distance to each solution to write an absolute value equation.

3. Solve the Problem



► The equation is $|x - 4.5| = 0.5$.

4. Look Back To check that your equation is reasonable, substitute the minimum and maximum lengths into the equation and simplify.

Minimum

$$|4 - 4.5| = 0.5 \quad \checkmark$$

Maximum

$$|5 - 4.5| = 0.5 \quad \checkmark$$

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Solve the equation. Check your solutions.

4. $|x - 2| + 5 = 9$

5. $4|2x + 7| = 16$

6. $-2|5x - 1| - 3 = -11$

7. For a poetry contest, the minimum length of a poem is 16 lines. The maximum length is 32 lines. Write an absolute value equation that represents the minimum and maximum lengths.



Solving Equations with Two Absolute Values

If the absolute values of two algebraic expressions are equal, then they must either be equal to each other or be opposites of each other.

Core Concept

Solving Equations with Two Absolute Values

To solve $|ax + b| = |cx + d|$, solve the related linear equations

$$ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d).$$

EXAMPLE 4 Solving Equations with Two Absolute Values

Solve (a) $|3x - 4| = |x|$ and (b) $|4x - 10| = 2|3x + 1|$.

SOLUTION

a. Write the two related linear equations for $|3x - 4| = |x|$. Then solve.

$$3x - 4 = x \quad \text{or} \quad 3x - 4 = -x$$

$$\underline{-x} \qquad \underline{-x} \qquad \underline{+x} \qquad \underline{+x}$$

$$2x - 4 = 0 \qquad 4x - 4 = 0$$

$$\underline{+4} \quad \underline{+4} \qquad \underline{+4} \quad \underline{+4}$$

$$2x = 4 \qquad 4x = 4$$

$$\frac{2x}{2} = \frac{4}{2} \qquad \frac{4x}{4} = \frac{4}{4}$$

$$x = 2 \qquad x = 1$$

► The solutions are $x = 2$ and $x = 1$.

b. Write the two related linear equations for $|4x - 10| = 2|3x + 1|$. Then solve.

$$4x - 10 = 2(3x + 1) \quad \text{or} \quad 4x - 10 = 2[-(3x + 1)]$$

$$4x - 10 = 6x + 2 \qquad 4x - 10 = 2(-3x - 1)$$

$$\underline{-6x} \qquad \underline{-6x} \qquad 4x - 10 = -6x - 2$$

$$\underline{-2x} - 10 = 2 \qquad \underline{+6x} \qquad \underline{+6x}$$

$$\underline{+10} \quad \underline{+10} \qquad 10x - 10 = -2$$

$$\underline{-2x} = 12 \qquad \underline{+10} \quad \underline{+10}$$

$$\frac{-2x}{-2} = \frac{12}{-2} \qquad 10x = 8$$

$$x = -6 \qquad \frac{10x}{10} = \frac{8}{10}$$

$$x = 0.8$$

► The solutions are $x = -6$ and $x = 0.8$.

Check

$$|3x - 4| = |x|$$

$$|3(2) - 4| \stackrel{?}{=} |2|$$

$$|2| \stackrel{?}{=} |2|$$

$$2 = 2 \quad \checkmark$$

$$|3x - 4| = |x|$$

$$|3(1) - 4| \stackrel{?}{=} |1|$$

$$|-1| \stackrel{?}{=} |1|$$

$$1 = 1 \quad \checkmark$$

Monitoring Progress



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Solve the equation. Check your solutions.

8. $|x + 8| = |2x + 1|$

9. $3|x - 4| = |2x + 5|$

Identifying Special Solutions

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

EXAMPLE 5 Identifying Extraneous Solutions

Solve $|2x + 12| = 4x$. Check your solutions.

SOLUTION

Write the two related linear equations for $|2x + 12| = 4x$. Then solve.

$2x + 12 = 4x$	or	$2x + 12 = -4x$	Write related linear equations.
$12 = 2x$		$12 = -6x$	Subtract $2x$ from each side.
$6 = x$		$-2 = x$	Solve for x .

Check the apparent solutions to see if either is extraneous.

► The solution is $x = 6$. Reject $x = -2$ because it is extraneous.

When solving equations of the form $|ax + b| = |cx + d|$, it is possible that one of the related linear equations will not have a solution.

EXAMPLE 6 Solving an Equation with Two Absolute Values

Solve $|x + 5| = |x + 11|$.

SOLUTION

By equating the expression $x + 5$ and the opposite of $x + 11$, you obtain

$x + 5 = -(x + 11)$	Write related linear equation.
$x + 5 = -x - 11$	Distributive Property
$2x + 5 = -11$	Add x to each side.
$2x = -16$	Subtract 5 from each side.
$x = -8$	Divide each side by 2.

However, by equating the expressions $x + 5$ and $x + 11$, you obtain

$x + 5 = x + 11$	Write related linear equation.
$x = x + 6$	Subtract 5 from each side.
$0 = 6$	Subtract x from each side.

which is a false statement. So, the original equation has only one solution.

► The solution is $x = -8$.

Check

$$|2x + 12| = 4x$$

$$|2(6) + 12| \stackrel{?}{=} 4(6)$$

$$|24| \stackrel{?}{=} 24$$

$$24 = 24 \quad \checkmark$$

$$|2x + 12| = 4x$$

$$|2(-2) + 12| \stackrel{?}{=} 4(-2)$$

$$|8| \stackrel{?}{=} -8$$

$$8 \neq -8 \quad \times$$

REMEMBER

Always check your solutions in the original equation to make sure they are not extraneous.

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Solve the equation. Check your solutions.

- | | |
|-------------------------|---------------------------|
| 10. $ x + 6 = 2x$ | 11. $ 3x - 2 = x$ |
| 12. $ 2 + x = x - 8 $ | 13. $ 5x - 2 = 5x + 4 $ |

1.4 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** What is an extraneous solution?
- WRITING** Without calculating, how do you know that the equation $|4x - 7| = -1$ has no solution?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression.

- $|-9|$
- $-|15|$
- $|14| - |-14|$
- $|-3| + |3|$
- $-|-5 \cdot (-7)|$
- $|-0.8 \cdot 10|$
- $\left|\frac{27}{-3}\right|$
- $\left|-\frac{-12}{4}\right|$

In Exercises 11–24, solve the equation. Graph the solution(s), if possible. (See Examples 1 and 2.)

- $|w| = 6$
- $|r| = -2$
- $|y| = -18$
- $|x| = 13$
- $|m + 3| = 7$
- $|q - 8| = 14$
- $|-3d| = 15$
- $\left|\frac{t}{2}\right| = 6$
- $|4b - 5| = 19$
- $|x - 1| + 5 = 2$
- $-4|8 - 5n| = 13$
- $-3\left|1 - \frac{2}{3}v\right| = -9$
- $3 = -2\left|\frac{1}{4}s - 5\right| + 3$
- $9|4p + 2| + 8 = 35$

- WRITING EQUATIONS** The minimum distance from Earth to the Sun is 91.4 million miles. The maximum distance is 94.5 million miles. (See Example 3.)

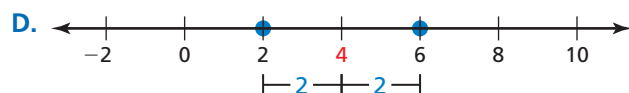
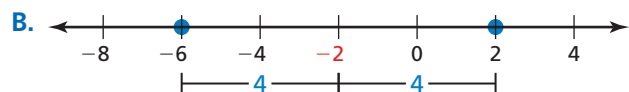
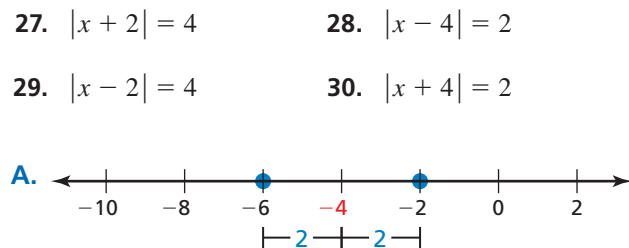
- Represent these two distances on a number line.
- Write an absolute value equation that represents the minimum and maximum distances.

- WRITING EQUATIONS** The shoulder heights of the shortest and tallest miniature poodles are shown.



- Represent these two heights on a number line.
- Write an absolute value equation that represents these heights.

USING STRUCTURE In Exercises 27–30, match the absolute value equation with its graph without solving the equation.



In Exercises 31–34, write an absolute value equation that has the given solutions.

31. $x = 8$ and $x = 18$ 32. $x = -6$ and $x = 10$
 33. $x = 2$ and $x = 9$ 34. $x = -10$ and $x = -5$

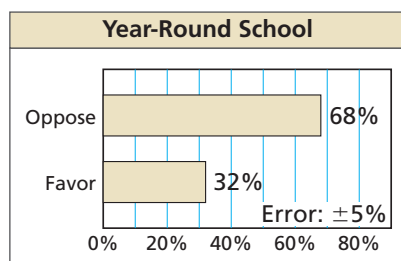
In Exercises 35–44, solve the equation. Check your solutions. (See Examples 4, 5, and 6.)

35. $|4n - 15| = |n|$ 36. $|2c + 8| = |10c|$
 37. $|2b - 9| = |b - 6|$ 38. $|3k - 2| = 2|k + 2|$
 39. $4|p - 3| = |2p + 8|$ 40. $2|4w - 1| = 3|4w + 2|$
 41. $|3h + 1| = 7h$ 42. $|6a - 5| = 4a$
 43. $|f - 6| = |f + 8|$ 44. $|3x - 4| = |3x - 5|$

45. **MODELING WITH MATHEMATICS** Starting from 300 feet away, a car drives toward you. It then passes by you at a speed of 48 feet per second. The distance d (in feet) of the car from you after t seconds is given by the equation $d = |300 - 48t|$. At what times is the car 60 feet from you?

46. **MAKING AN ARGUMENT** Your friend says that the absolute value equation $|3x + 8| - 9 = -5$ has no solution because the constant on the right side of the equation is negative. Is your friend correct? Explain.

47. **MODELING WITH MATHEMATICS** You randomly survey students about year-round school. The results are shown in the graph.



The error given in the graph means that the actual percent could be 5% more or 5% less than the percent reported by the survey.

- a. Write and solve an absolute value equation to find the least and greatest percents of students who could be in favor of year-round school.
 b. A classmate claims that $\frac{1}{3}$ of the student body is actually in favor of year-round school. Does this conflict with the survey data? Explain.

48. **MODELING WITH MATHEMATICS** The recommended weight of a soccer ball is 430 grams. The actual weight is allowed to vary by up to 20 grams.



- a. Write and solve an absolute value equation to find the minimum and maximum acceptable soccer ball weights.
 b. A soccer ball weighs 423 grams. Due to wear and tear, the weight of the ball decreases by 16 grams. Is the weight acceptable? Explain.

ERROR ANALYSIS In Exercises 49 and 50, describe and correct the error in solving the equation.

49. $|2x - 1| = -9$
 $2x - 1 = -9$ or $2x - 1 = -(-9)$
 $2x = -8$ $2x = 10$
 $x = -4$ $x = 5$
 The solutions are $x = -4$ and $x = 5$.

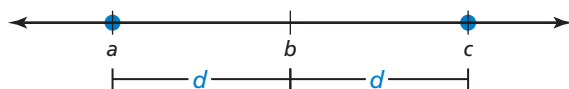
50. $|5x + 8| = x$
 $5x + 8 = x$ or $5x + 8 = -x$
 $4x + 8 = 0$ $6x + 8 = 0$
 $4x = -8$ $6x = -8$
 $x = -2$ $x = -\frac{4}{3}$
 The solutions are $x = -2$ and $x = -\frac{4}{3}$.

51. **ANALYZING EQUATIONS** Without solving completely, place each equation into one of the three categories.

No solution	One solution	Two solutions
$ x - 2 + 6 = 0$	$ x + 3 - 1 = 0$	
$ x + 8 + 2 = 7$	$ x - 1 + 4 = 4$	
$ x - 6 - 5 = -9$	$ x + 5 - 8 = -8$	

52. **USING STRUCTURE** Fill in the equation

$|x - \boxed{}| = \boxed{}$ with a, b, c , or d so that the equation is graphed correctly.



ABSTRACT REASONING In Exercises 53–56, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

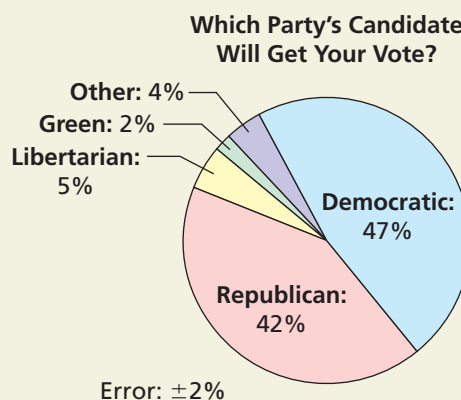
53. If $x^2 = a^2$, then $|x|$ is _____ equal to $|a|$.
54. If a and b are real numbers, then $|a - b|$ is _____ equal to $|b - a|$.
55. For any real number p , the equation $|x - 4| = p$ will _____ have two solutions.
56. For any real number p , the equation $|x - p| = 4$ will _____ have two solutions.
57. **WRITING** Explain why absolute value equations can have no solution, one solution, or two solutions. Give an example of each case.

58. **THOUGHT PROVOKING** Describe a real-life situation that can be modeled by an absolute value equation with the solutions $x = 62$ and $x = 72$.

59. **CRITICAL THINKING** Solve the equation shown. Explain how you found your solution(s).

$$8|x + 2| - 6 = 5|x + 2| + 3$$

60. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey of registered voters the day of an election.



The error given in the graph means that the actual percent could be 2% more or 2% less than the percent reported by the survey.

- What are the minimum and maximum percents of voters who could vote Republican? Green?
 - How can you use absolute value equations to represent your answers in part (a)?
 - One candidate receives 44% of the vote. Which party does the candidate belong to? Explain.
61. **ABSTRACT REASONING** How many solutions does the equation $a|x + b| + c = d$ have when $a > 0$ and $c = d$? when $a < 0$ and $c > d$? Explain your reasoning.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Identify the property of equality that makes Equation 1 and Equation 2 equivalent. (*Section 1.1*)

62.

Equation 1	$3x + 8 = x - 1$
Equation 2	$3x + 9 = x$

63.

Equation 1	$4y = 28$
Equation 2	$y = 7$

Use a geometric formula to solve the problem. (*Skills Review Handbook*)

64. A square has an area of 81 square meters. Find the side length.
65. A circle has an area of 36π square inches. Find the radius.
66. A triangle has a height of 8 feet and an area of 48 square feet. Find the base.
67. A rectangle has a width of 4 centimeters and a perimeter of 26 centimeters. Find the length.

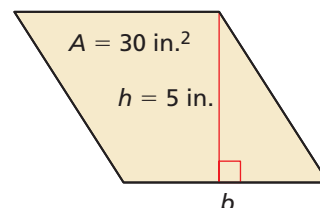
1.5 Rewriting Equations and Formulas

Essential Question How can you use a formula for one measurement to write a formula for a different measurement?

EXPLORATION 1 Using an Area Formula

Work with a partner.

- Write a formula for the area A of a parallelogram.
- Substitute the given values into the formula. Then solve the equation for b . Justify each step.
- Solve the formula in part (a) for b without first substituting values into the formula. Justify each step.
- Compare how you solved the equations in parts (b) and (c). How are the processes similar? How are they different?



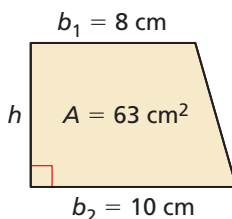
REASONING QUANTITATIVELY

To be proficient in math, you need to consider the given units. For instance, in Exploration 1, the area A is given in square inches and the height h is given in inches. A unit analysis shows that the units for the base b are also inches, which makes sense.

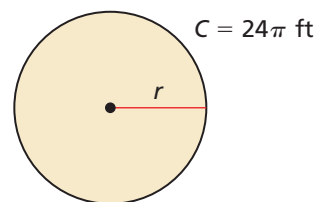
EXPLORATION 2 Using Area, Circumference, and Volume Formulas

Work with a partner. Write the indicated formula for each figure. Then write a new formula by solving for the variable whose value is not given. Use the new formula to find the value of the variable.

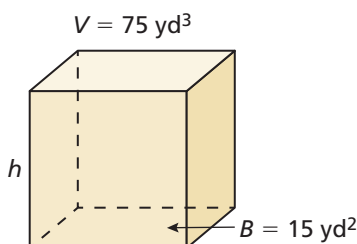
- a. Area A of a trapezoid



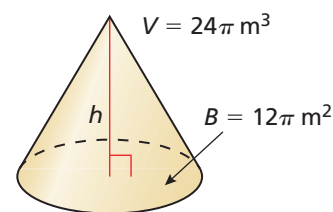
- b. Circumference C of a circle



- c. Volume V of a rectangular prism



- d. Volume V of a cone



Communicate Your Answer

- How can you use a formula for one measurement to write a formula for a different measurement? Give an example that is different from those given in Explorations 1 and 2.

1.5 Lesson

Core Vocabulary

literal equation, p. 36
formula, p. 37

Previous
surface area

What You Will Learn

- ▶ Rewrite literal equations.
- ▶ Rewrite and use formulas for area.
- ▶ Rewrite and use other common formulas.

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

EXAMPLE 1 Rewriting a Literal Equation

Solve the literal equation $3y + 4x = 9$ for y .

SOLUTION

$$3y + 4x = 9$$

Write the equation.

$$3y + 4x - 4x = 9 - 4x$$

Subtract $4x$ from each side.

$$3y = 9 - 4x$$

Simplify.

$$\frac{3y}{3} = \frac{9 - 4x}{3}$$

Divide each side by 3.

$$y = 3 - \frac{4}{3}x$$

Simplify.

- ▶ The rewritten literal equation is $y = 3 - \frac{4}{3}x$.

EXAMPLE 2 Rewriting a Literal Equation

Solve the literal equation $y = 3x + 5xz$ for x .

SOLUTION

$$y = 3x + 5xz$$

Write the equation.

$$y = x(3 + 5z)$$

Distributive Property

$$\frac{y}{3 + 5z} = \frac{x(3 + 5z)}{3 + 5z}$$

Divide each side by $3 + 5z$.

$$\frac{y}{3 + 5z} = x$$

Simplify.

- ▶ The rewritten literal equation is $x = \frac{y}{3 + 5z}$.

REMEMBER

Division by 0 is undefined.

In Example 2, you must assume that $z \neq -\frac{3}{5}$ in order to divide by $3 + 5z$. In general, if you have to divide by a variable or variable expression when solving a literal equation, you should assume that the variable or variable expression does not equal 0.

Monitoring Progress



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Solve the literal equation for y .

1. $3y - x = 9$

2. $2x - 2y = 5$

3. $20 = 8x + 4y$

Solve the literal equation for x .

4. $y = 5x - 4x$

5. $2x + kx = m$

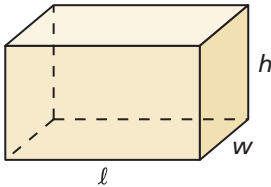
6. $3 + 5x - kx = y$

Rewriting and Using Formulas for Area

A **formula** shows how one variable is related to one or more other variables. A formula is a type of literal equation.

EXAMPLE 3 Rewriting a Formula for Surface Area

The formula for the surface area S of a rectangular prism is $S = 2\ell w + 2\ell h + 2wh$. Solve the formula for the length ℓ .



SOLUTION

$$S = 2\ell w + 2\ell h + 2wh$$

Write the equation.

$$S - 2wh = 2\ell w + 2\ell h + 2wh - 2wh$$

Subtract $2wh$ from each side.

$$S - 2wh = 2\ell w + 2\ell h$$

Simplify.

$$S - 2wh = \ell(2w + 2h)$$

Distributive Property

$$\frac{S - 2wh}{2w + 2h} = \frac{\ell(2w + 2h)}{2w + 2h}$$

Divide each side by $2w + 2h$.

$$\frac{S - 2wh}{2w + 2h} = \ell$$

Simplify.

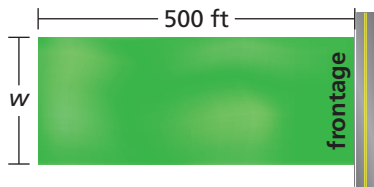
► When you solve the formula for ℓ , you obtain $\ell = \frac{S - 2wh}{2w + 2h}$.

EXAMPLE 4 Using a Formula for Area

You own a rectangular lot that is 500 feet deep. It has an area of 100,000 square feet. To pay for a new water system, you are assessed \$5.50 per foot of lot frontage.

- Find the frontage of your lot.
- How much are you assessed for the new water system?

SOLUTION



- In the formula for the area of a rectangle, let the width w represent the lot frontage.

$$A = \ell w$$

Write the formula for area of a rectangle.

$$\frac{A}{\ell} = w$$

Divide each side by ℓ to solve for w .

$$\frac{100,000}{500} = w$$

Substitute 100,000 for A and 500 for ℓ .

$$200 = w$$

Simplify.

► The frontage of your lot is 200 feet.

- Each foot of frontage costs \$5.50, and $\frac{\$5.50}{1 \cancel{\text{ft}}} \cdot 200 \cancel{\text{ft}} = \1100 .

► So, your total assessment is \$1100.

Monitoring Progress



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Solve the formula for the indicated variable.

7. Area of a triangle: $A = \frac{1}{2}bh$; Solve for h .

8. Surface area of a cone: $S = \pi r^2 + \pi r\ell$; Solve for ℓ .

Rewriting and Using Other Common Formulas

Core Concept

Common Formulas

Temperature F = degrees Fahrenheit, C = degrees Celsius

$$C = \frac{5}{9}(F - 32)$$

Simple Interest I = interest, P = principal,
 r = annual interest rate (decimal form),
 t = time (years)

$$I = Prt$$

Distance d = distance traveled, r = rate, t = time

$$d = rt$$

EXAMPLE 5 Rewriting the Formula for Temperature

Solve the temperature formula for F .

SOLUTION

$$C = \frac{5}{9}(F - 32) \quad \text{Write the temperature formula.}$$

$$\frac{9}{5}C = F - 32 \quad \text{Multiply each side by } \frac{9}{5}.$$

$$\frac{9}{5}C + 32 = F - 32 + 32 \quad \text{Add 32 to each side.}$$

$$\frac{9}{5}C + 32 = F \quad \text{Simplify.}$$

► The rewritten formula is $F = \frac{9}{5}C + 32$.

EXAMPLE 6 Using the Formula for Temperature

Which has the greater surface temperature: Mercury or Venus?

SOLUTION

Convert the Celsius temperature of Mercury to degrees Fahrenheit.

$$F = \frac{9}{5}C + 32 \quad \text{Write the rewritten formula from Example 5.}$$

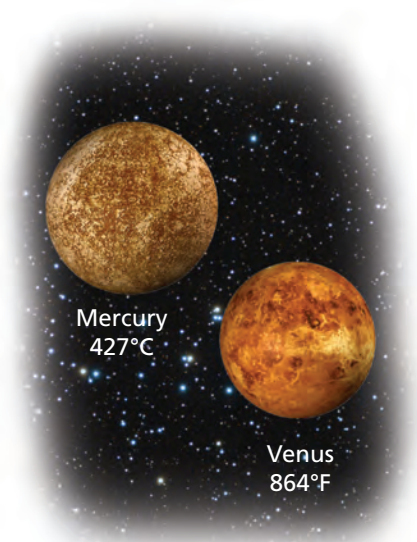
$$= \frac{9}{5}(427) + 32 \quad \text{Substitute 427 for } C.$$

$$= 800.6 \quad \text{Simplify.}$$

► Because 864°F is greater than 800.6°F , Venus has the greater surface temperature.

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9. A fever is generally considered to be a body temperature greater than 100°F . Your friend has a temperature of 37°C . Does your friend have a fever?



EXAMPLE 7**Using the Formula for Simple Interest**

You deposit \$5000 in an account that earns simple interest. After 6 months, the account earns \$162.50 in interest. What is the annual interest rate?

COMMON ERROR

The unit of t is years. Be sure to convert months to years.

$$\frac{1 \text{ yr}}{12 \text{ mo}} \cdot 6 \text{ mo} = 0.5 \text{ yr}$$

SOLUTION

To find the annual interest rate, solve the simple interest formula for r .

$$I = Prt$$

Write the simple interest formula.

$$\frac{I}{Pt} = r$$

Divide each side by Pt to solve for r .

$$\frac{162.50}{(5000)(0.5)} = r$$

Substitute 162.50 for I , 5000 for P , and 0.5 for t .

$$0.065 = r$$

Simplify.

► The annual interest rate is 0.065, or 6.5%.

EXAMPLE 8**Solving a Real-Life Problem**

A truck driver averages 60 miles per hour while delivering freight to a customer. On the return trip, the driver averages 50 miles per hour due to construction. The total driving time is 6.6 hours. How long does each trip take?

SOLUTION

Step 1 Rewrite the Distance Formula to write expressions that represent the two trip times. Solving the formula $d = rt$ for t , you obtain $t = \frac{d}{r}$. So, $\frac{d}{60}$ represents the delivery time, and $\frac{d}{50}$ represents the return trip time.

Step 2 Use these expressions and the total driving time to write and solve an equation to find the distance one way.

$$\frac{d}{60} + \frac{d}{50} = 6.6$$

The sum of the two trip times is 6.6 hours.

$$\frac{11d}{300} = 6.6$$

Add the left side using the LCD.

$$11d = 1980$$

Multiply each side by 300 and simplify.

$$d = 180$$

Divide each side by 11 and simplify.

The distance one way is 180 miles.

Step 3 Use the expressions from Step 1 to find the two trip times.

► So, the delivery takes $180 \text{ mi} \div \frac{60 \text{ mi}}{1 \text{ h}} = 3$ hours, and the return trip takes

$$180 \text{ mi} \div \frac{50 \text{ mi}}{1 \text{ h}} = 3.6 \text{ hours.}$$

Monitoring Progress

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10. How much money must you deposit in a simple interest account to earn \$500 in interest in 5 years at 4% annual interest?
11. A truck driver averages 60 miles per hour while delivering freight and 45 miles per hour on the return trip. The total driving time is 7 hours. How long does each trip take?

1.5 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** Is $9r + 16 = \frac{\pi}{5}$ a literal equation? Explain.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Solve $3x + 6y = 24$ for x .

Solve $24 - 3x = 6y$ for x .

Solve $6y = 24 - 3x$ for y in terms of x .

Solve $24 - 6y = 3x$ for x in terms of y .

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the literal equation for y .
(See Example 1.)

- $y - 3x = 13$
- $2x + y = 7$
- $2y - 18x = -26$
- $20x + 5y = 15$
- $9x - y = 45$
- $6x - 3y = -6$
- $4x - 5 = 7 + 4y$
- $16x + 9 = 9y - 2x$
- $2 + \frac{1}{6}y = 3x + 4$
- $11 - \frac{1}{2}y = 3 + 6x$

In Exercises 13–22, solve the literal equation for x .
(See Example 2.)

- $y = 4x + 8x$
- $m = 10x - x$
- $a = 2x + 6xz$
- $y = 3bx - 7x$
- $y = 4x + rx + 6$
- $z = 8 + 6x - px$
- $sx + tx = r$
- $a = bx + cx + d$
- $12 - 5x - 4kx = y$
- $x - 9 + 2wx = y$

- MODELING WITH MATHEMATICS** The total cost C (in dollars) to participate in a ski club is given by the literal equation $C = 85x + 60$, where x is the number of ski trips you take.

- Solve the equation for x .
- How many ski trips do you take if you spend a total of \$315? \$485?



- MODELING WITH MATHEMATICS** The penny size of a nail indicates the length of the nail. The penny size d is given by the literal equation $d = 4n - 2$, where n is the length (in inches) of the nail.

- Solve the equation for n .
- Use the equation from part (a) to find the lengths of nails with the following penny sizes: 3, 6, and 10.



ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in solving the equation for x .

25.
$$\begin{aligned} 12 - 2x &= -2(y - x) \\ -2x &= -2(y - x) - 12 \\ x &= (y - x) + 6 \end{aligned}$$

26.
$$\begin{aligned} 10 &= ax - 3b \\ 10 &= x(a - 3b) \\ \frac{10}{a - 3b} &= x \end{aligned}$$

In Exercises 27–30, solve the formula for the indicated variable. (See Examples 3 and 5.)

- Profit: $P = R - C$; Solve for C .
- Surface area of a cylinder: $S = 2\pi r^2 + 2\pi rh$; Solve for h .
- Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$; Solve for b_2 .
- Average acceleration of an object: $a = \frac{v_1 - v_0}{t}$; Solve for v_1 .

31. **REWRITING A FORMULA** A common statistic used in professional football is the quarterback rating. This rating is made up of four major factors. One factor is the completion rating given by the formula

$$R = 5\left(\frac{C}{A} - 0.3\right)$$

where C is the number of completed passes and A is the number of attempted passes. Solve the formula for C .

32. **REWRITING A FORMULA** Newton's law of gravitation is given by the formula

$$F = G\left(\frac{m_1 m_2}{d^2}\right)$$

where F is the force between two objects of masses m_1 and m_2 , G is the gravitational constant, and d is the distance between the two objects. Solve the formula for m_1 .

33. **MODELING WITH MATHEMATICS** The sale price S (in dollars) of an item is given by the formula $S = L - rL$, where L is the list price (in dollars) and r is the discount rate (in decimal form). (See Examples 4 and 6.)

- Solve the formula for r .
- The list price of the shirt is \$30. What is the discount rate?



34. **MODELING WITH MATHEMATICS** The density d of a substance is given by the formula $d = \frac{m}{V}$, where m is its mass and V is its volume.

Pyrite
Density: 5.01g/cm^3 Volume: 1.2 cm^3



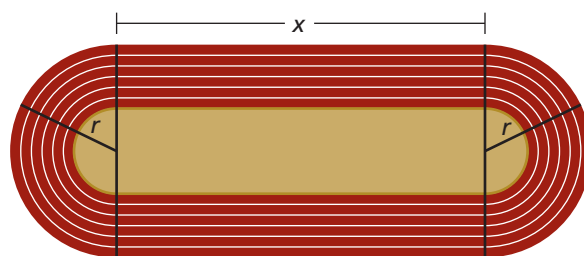
- Solve the formula for m .
- Find the mass of the pyrite sample.

35. **PROBLEM SOLVING** You deposit \$2000 in an account that earns simple interest at an annual rate of 4%. How long must you leave the money in the account to earn \$500 in interest? (See Example 7.)

36. **PROBLEM SOLVING** A flight averages 460 miles per hour. The return flight averages 500 miles per hour due to a tailwind. The total flying time is 4.8 hours. How long is each flight? Explain. (See Example 8.)

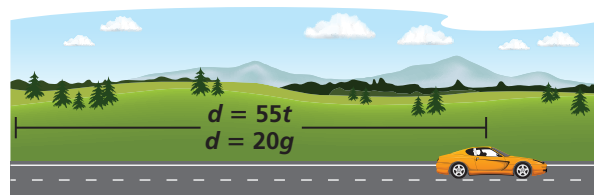


37. **USING STRUCTURE** An athletic facility is building an indoor track. The track is composed of a rectangle and two semicircles, as shown.



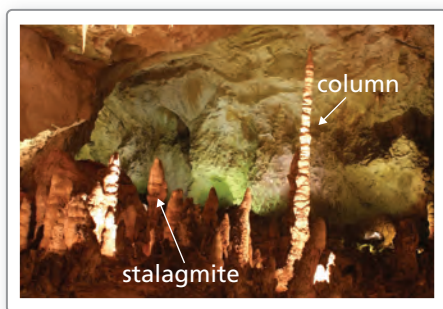
- Write a formula for the perimeter of the indoor track.
- Solve the formula for x .
- The perimeter of the track is 660 feet, and r is 50 feet. Find x . Round your answer to the nearest foot.

38. **MODELING WITH MATHEMATICS** The distance d (in miles) you travel in a car is given by the two equations shown, where t is the time (in hours) and g is the number of gallons of gasoline the car uses.



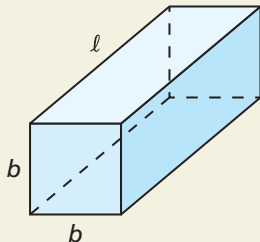
- Write an equation that relates g and t .
- Solve the equation for g .
- You travel for 6 hours. How many gallons of gasoline does the car use? How far do you travel? Explain.

39. **MODELING WITH MATHEMATICS** One type of stone formation found in Carlsbad Caverns in New Mexico is called a column. This cylindrical stone formation connects to the ceiling and the floor of a cave.



- Rewrite the formula for the circumference of a circle, so that you can easily calculate the radius of a column given its circumference.
- What is the radius (to the nearest tenth of a foot) of a column that has a circumference of 7 feet? 8 feet? 9 feet?
- Explain how you can find the area of a cross section of a column when you know its circumference.

40. **HOW DO YOU SEE IT?** The rectangular prism shown has bases with equal side lengths.

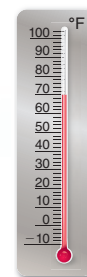


- Use the figure to write a formula for the surface area S of the rectangular prism.
- Your teacher asks you to rewrite the formula by solving for one of the side lengths, b or l . Which side length would you choose? Explain your reasoning.

41. **MAKING AN ARGUMENT** Your friend claims that Thermometer A displays a greater temperature than Thermometer B. Is your friend correct? Explain your reasoning.

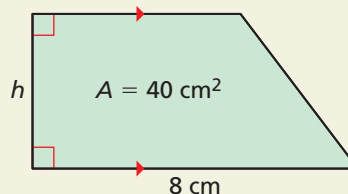


Thermometer A



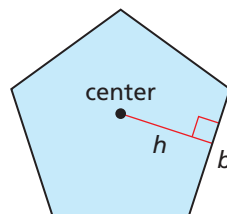
Thermometer B

42. **THOUGHT PROVOKING** Give a possible value for h . Justify your answer. Draw and label the figure using your chosen value of h .

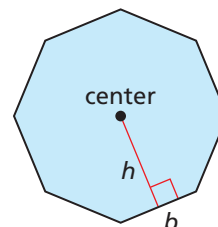


MATHEMATICAL CONNECTIONS In Exercises 43 and 44, write a formula for the area of the regular polygon. Solve the formula for the height h .

43.



44.



REASONING In Exercises 45 and 46, solve the literal equation for a .

45. $x = \frac{a + b + c}{ab}$

46. $y = x\left(\frac{ab}{a - b}\right)$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. (*Skills Review Handbook*)

47. $15 - 5 + 5^2$

48. $18 \cdot 2 - 4^2 \div 8$

49. $3^3 + 12 \div 3 \cdot 5$

50. $2^5(5 - 6) + 9 \div 3$

Solve the equation. Graph the solutions, if possible. (*Section 1.4*)

51. $|x - 3| + 4 = 9$

52. $|3y - 12| - 7 = 2$

53. $2|2r + 4| = -16$

54. $-4|s + 9| = -24$

1.4–1.5 What Did You Learn?

Core Vocabulary

absolute value equation, *p.* 28
extraneous solution, *p.* 31

literal equation, *p.* 36
formula, *p.* 37

Core Concepts

Section 1.4

Properties of Absolute Value, *p.* 28
Solving Absolute Value Equations, *p.* 28
Solving Equations with Two Absolute Values, *p.* 30
Special Solutions of Absolute Value Equations, *p.* 31

Section 1.5

Rewriting Literal Equations, *p.* 36
Common Formulas, *p.* 38

Mathematical Practices

1. How did you decide whether your friend's argument in Exercise 46 on page 33 made sense?
2. How did you use the structure of the equation in Exercise 59 on page 34 to rewrite the equation?
3. What entry points did you use to answer Exercises 43 and 44 on page 42?

Performance Task

Magic of Mathematics

Have you ever watched a magician perform a number trick?
You can use algebra to explain how these types of tricks work.

To explore the answers to these questions and more, go to
BigIdeasMath.com.



1.1 Solving Simple Equations (pp. 3–10)a. Solve $x - 5 = -9$. Justify each step.

$$x - 5 = -9$$

Write the equation.

Addition Property of Equality

$$\xrightarrow{+5} \quad +5 \quad +5$$

Add 5 to each side.

$$x = -4$$

Simplify.

▶ The solution is $x = -4$.b. Solve $4x = 12$. Justify each step.

$$4x = 12$$

Write the equation.

Division Property of Equality

$$\xrightarrow{\div 4} \quad \frac{4x}{4} = \frac{12}{4}$$

Divide each side by 4.

$$x = 3$$

Simplify.

▶ The solution is $x = 3$.

Solve the equation. Justify each step. Check your solution.

1. $z + 3 = -6$

2. $2.6 = -0.2t$

3. $-\frac{n}{5} = -2$

1.2 Solving Multi-Step Equations (pp. 11–18)Solve $-6x + 23 + 2x = 15$.

$$-6x + 23 + 2x = 15$$

Write the equation.

$$-4x + 23 = 15$$

Combine like terms.

$$-4x = -8$$

Subtract 23 from each side.

$$x = 2$$

Divide each side by -4 .▶ The solution is $x = 2$.

Solve the equation. Check your solution.

4. $3y + 11 = -16$

5. $6 = 1 - b$

6. $n + 5n + 7 = 43$

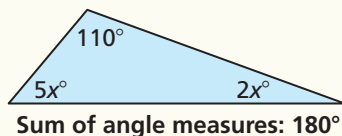
7. $-4(2z + 6) - 12 = 4$

8. $\frac{3}{2}(x - 2) - 5 = 19$

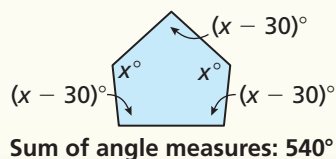
9. $6 = \frac{1}{5}w + \frac{7}{5}w - 4$

Find the value of x . Then find the angle measures of the polygon.

10.



11.



1.3 Solving Equations with Variables on Both Sides (pp. 19–24)

Solve $2(y - 4) = -4(y + 8)$.

$$2(y - 4) = -4(y + 8)$$

Write the equation.

$$2y - 8 = -4y - 32$$

Distributive Property

$$6y - 8 = -32$$

Add 4y to each side.

$$6y = -24$$

Add 8 to each side.

$$y = -4$$

Divide each side by 6.

▶ The solution is $y = -4$.

Solve the equation.

12. $3n - 3 = 4n + 1$

13. $5(1 + x) = 5x + 5$

14. $3(n + 4) = \frac{1}{2}(6n + 4)$

1.4 Solving Absolute Value Equations (pp. 27–34)

a. Solve $|x - 5| = 3$.

$$x - 5 = 3 \quad \text{or} \quad x - 5 = -3$$

Write related linear equations.

$$\begin{array}{r} +5 \\ x - 5 = 3 \\ \hline x = 8 \end{array}$$

$$\begin{array}{r} +5 \\ x - 5 = -3 \\ \hline x = 2 \end{array}$$

Add 5 to each side.

Simplify.

▶ The solutions are $x = 8$ and $x = 2$.

b. Solve $|2x + 6| = 4x$. Check your solutions.

$$2x + 6 = 4x \quad \text{or} \quad 2x + 6 = -4x$$

Write related linear equations.

$$\begin{array}{r} -2x \\ 2x + 6 = 4x \\ \hline 6 = 2x \end{array}$$

$$\begin{array}{r} -2x \\ 2x + 6 = -4x \\ \hline 6 = -6x \end{array}$$

$$\begin{array}{r} -2x \\ 2x + 6 = 4x \\ \hline 6 = 2x \end{array}$$

$$\begin{array}{r} -2x \\ 2x + 6 = -4x \\ \hline 6 = -6x \end{array}$$

Subtract 2x from each side.

$$6 = 2x$$

$$6 = -6x$$

Simplify.

$$\frac{6}{2} = \frac{2x}{2}$$

$$\frac{6}{-6} = \frac{-6x}{-6}$$

Solve for x.

$$3 = x$$

$$-1 = x$$

Simplify.

Check the apparent solutions to see if either is extraneous.

▶ The solution is $x = 3$. Reject $x = -1$ because it is extraneous.

Check

$$|2x + 6| = 4x$$

$$|2(3) + 6| \stackrel{?}{=} 4(3)$$

$$|12| \stackrel{?}{=} 12$$

$$12 = 12 \quad \checkmark$$

$$|2x + 6| = 4x$$

$$|2(-1) + 6| \stackrel{?}{=} 4(-1)$$

$$|4| \stackrel{?}{=} -4$$

$$4 \neq -4 \quad \times$$

Solve the equation. Check your solutions.

15. $|y + 3| = 17$

16. $-2|5w - 7| + 9 = -7$

17. $|x - 2| = |4 + x|$

18. The minimum sustained wind speed of a Category 1 hurricane is 74 miles per hour. The maximum sustained wind speed is 95 miles per hour. Write an absolute value equation that represents the minimum and maximum speeds.

1.5 Rewriting Equations and Formulas (pp. 35–42)

- a. The slope-intercept form of a linear equation is $y = mx + b$. Solve the equation for m .

$$y = mx + b$$

Write the equation.

$$y - b = mx + b - b$$

Subtract b from each side.

$$y - b = mx$$

Simplify.

$$\frac{y - b}{x} = \frac{mx}{x}$$

Divide each side by x .

$$\frac{y - b}{x} = m$$

Simplify.

▶ When you solve the equation for m , you obtain $m = \frac{y - b}{x}$.

- b. The formula for the surface area S of a cylinder is $S = 2\pi r^2 + 2\pi rh$. Solve the formula for the height h .

$$S = 2\pi r^2 + 2\pi rh$$

Write the equation.

$$S - 2\pi r^2 = 2\pi rh$$

Subtract $2\pi r^2$ from each side.

$$S - 2\pi r^2 = 2\pi rh$$

Simplify.

$$\frac{S - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

Divide each side by $2\pi r$.

$$\frac{S - 2\pi r^2}{2\pi r} = h$$

Simplify.

▶ When you solve the formula for h , you obtain $h = \frac{S - 2\pi r^2}{2\pi r}$.

Solve the literal equation for y .

19. $2x - 4y = 20$

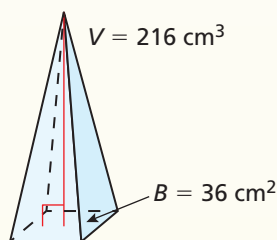
20. $8x - 3 = 5 + 4y$

21. $a = 9y + 3yx$

22. The volume V of a pyramid is given by the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.

a. Solve the formula for h .

b. Find the height h of the pyramid.



23. The formula $F = \frac{9}{5}(K - 273.15) + 32$ converts a temperature from kelvin K to degrees Fahrenheit F .

a. Solve the formula for K .

b. Convert 180°F to kelvin K . Round your answer to the nearest hundredth.

Solve the equation. Justify each step. Check your solution.

1. $x - 7 = 15$

2. $\frac{2}{3}x + 5 = 3$

3. $11x + 1 = -1 + x$

Solve the equation.

4. $2|x - 3| - 5 = 7$

5. $|2x - 19| = 4x + 1$

6. $-2 + 5x - 7 = 3x - 9 + 2x$

7. $3(x + 4) - 1 = -7$

8. $|20 + 2x| = |4x + 4|$

9. $\frac{1}{3}(6x + 12) - 2(x - 7) = 19$

Describe the values of c for which the equation has no solution. Explain your reasoning.

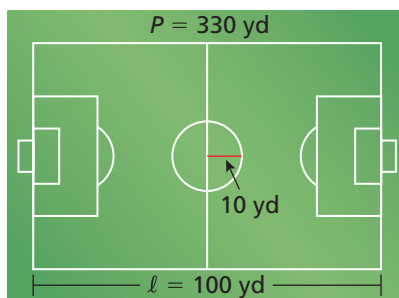
10. $3x - 5 = 3x - c$

11. $|x - 7| = c$

12. A safety regulation states that the minimum height of a handrail is 30 inches. The maximum height is 38 inches. Write an absolute value equation that represents the minimum and maximum heights.

13. The perimeter P (in yards) of a soccer field is represented by the formula $P = 2\ell + 2w$, where ℓ is the length (in yards) and w is the width (in yards).

- Solve the formula for w .
- Find the width of the field.
- About what percent of the field is inside the circle?



14. Your car needs new brakes. You call a dealership and a local mechanic for prices.

	Cost of parts	Labor cost per hour
Dealership	\$24	\$99
Local Mechanic	\$45	\$89

- After how many hours are the total costs the same at both places? Justify your answer.
 - When do the repairs cost less at the dealership? at the local mechanic? Explain.
15. Consider the equation $|4x + 20| = 6x$. Without calculating, how do you know that $x = -2$ is an extraneous solution?
16. Your friend was solving the equation shown and was confused by the result “ $-8 = -8$.” Explain what this result means.

$$\begin{aligned}
 4(y - 2) - 2y &= 6y - 8 - 4y \\
 4y - 8 - 2y &= 6y - 8 - 4y \\
 2y - 8 &= 2y - 8 \\
 -8 &= -8
 \end{aligned}$$

1

Cumulative Assessment

1. A mountain biking park has 48 trails, 37.5% of which are beginner trails. The rest are divided evenly between intermediate and expert trails. How many of each kind of trail are there?

- (A) 12 beginner, 18 intermediate, 18 expert
- (B) 18 beginner, 15 intermediate, 15 expert
- (C) 18 beginner, 12 intermediate, 18 expert
- (D) 30 beginner, 9 intermediate, 9 expert

2. Which of the equations are equivalent to $cx - a = b$?

$$cx - a + b = 2b$$

$$0 = cx - a + b$$

$$2cx - 2a = \frac{b}{2}$$

$$x - a = \frac{b}{c}$$

$$x = \frac{a + b}{c}$$

$$b + a = cx$$

3. Let N represent the number of solutions of the equation $3(x - a) = 3x - 6$. Complete each statement with the symbol $<$, $>$, or $=$.

- a. When $a = 3$, N ____ 1.
- b. When $a = -3$, N ____ 1.
- c. When $a = 2$, N ____ 1.
- d. When $a = -2$, N ____ 1.
- e. When $a = x$, N ____ 1.
- f. When $a = -x$, N ____ 1.

4. You are painting your dining room white and your living room blue. You spend \$132 on 5 cans of paint. The white paint costs \$24 per can, and the blue paint costs \$28 per can.

- a. Use the numbers and symbols to write an equation that represents how many cans of each color you bought.

x	132	5	24	28	=
()	+	-	\times	\div

- b. How much would you have saved by switching the colors of the dining room and living room? Explain.

5. Which of the equations are equivalent?

$$6x + 6 = -14$$

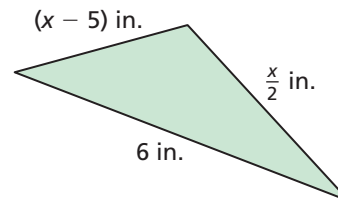
$$8x + 6 = -2x - 14$$

$$5x + 3 = -7$$

$$7x + 3 = 2x - 13$$

6. The perimeter of the triangle is 13 inches. What is the length of the shortest side?

- (A) 2 in.
(B) 3 in.
(C) 4 in.
(D) 8 in.



7. You pay \$45 per month for cable TV. Your friend buys a satellite TV receiver for \$99 and pays \$36 per month for satellite TV. Your friend claims that the expenses for a year of satellite TV are less than the expenses for a year of cable.
- Write and solve an equation to determine when you and your friend will have paid the same amount for TV services.
 - Is your friend correct? Explain.
8. Place each equation into one of the four categories.

No solution	One solution	Two solutions	Infinitely many solutions
$ 8x + 3 = 0$ $-2x + 4 = 2x + 4$ $12x - 2x = 10x - 8$	$-6 = 5x - 9$ $0 = x + 13 + 2$ $9 = 3 2x - 11 $		$3x - 12 = 3(x - 4) + 1$ $-4(x + 4) = -4x - 16$ $7 - 2x = 3 - 2(x - 2)$

9. A car travels 1000 feet in 12.5 seconds. Which of the expressions do *not* represent the average speed of the car?

$$80 \frac{\text{second}}{\text{feet}}$$

$$80 \frac{\text{feet}}{\text{second}}$$

$$\frac{80 \text{ feet}}{\text{second}}$$

$$\frac{\text{second}}{80 \text{ feet}}$$