

Permutations with and without Repetition

1. Permutations with Repetition

These are the easiest to calculate.

When we have n things to choose from ... we have n choices each time!

When choosing r of them, the permutations are:

$$n \times n \times \dots (r \text{ times})$$

(In other words, there are n possibilities for the first choice, THEN there are n possibilities for the second choice, and so on, multiplying each time.)

Which is easier to write down using an exponent of r :

$$n \times n \times \dots (r \text{ times}) = n^r$$



Figure 1 So, we should really call this a "Permutation Lock"!

Example: in the lock above, there are 10 numbers to choose from (0,1,...9) and we choose 3 of them:

$$10 \times 10 \times \dots (3 \text{ times}) = 10^3 = 1,000 \text{ permutations}$$

Remark:

$$n^r$$

where n is the number of things to choose from, and we choose r of

them
(Repetition allowed, order matters)

Ex: how many 3 letter words can be created, if Repetition is allowed?
 $26^3=17576$

2. Permutations without Repetition

In this case, we have to **reduce** the number of available choices each time.



For example, what order could 16 pool balls be in? After choosing, say, number "14" we can't choose it again.

So, our first choice has 16 possibilities, and our next choice has 15 possibilities, then 14, 13, etc. And the total permutations are:

$$16 \times 15 \times 14 \times 13 \times \dots = 20,922,789,888,000$$

But maybe we don't want to choose them all, just 3 of them, so that is only:

$$16 \times 15 \times 14 = 3,360$$

In other words, there are 3,360 different ways that 3 pool balls could be arranged out of 16 balls.

Without repetition our choices get reduced each time.

$$\frac{n!}{(n-r)!}$$

where n is the number of things to choose from, and we choose r of them

(No repetition, order matters)

Examples:

Our "order of 3 out of 16 pool balls example" is:

$$\frac{16!}{(16-3)!} = \frac{16!}{13!} = \frac{20,922,789,888,000}{6,227,020,800} = 3,360$$

(which is just the same as: $16 \times 15 \times 14 = 3,360$)

Example:- How many ways can first and second place be awarded to 10 people?

$$\frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{3,628,800}{40,320} = 90$$

(which is just the same as: $10 \times 9 = 90$)

Ex:- How many permutations of 3 **different** digits are there, chosen from the ten digits 0 to 9 inclusive?

Sol:- The number of permutations of 3 digits chosen from 10 is ${}^{10}P_3 = 10 \times 9 \times 8 = 720$.

Ex:- How many permutations of 4 **different** letters are there, chosen from the twenty six letters of the alphabet?

Sol:- The number of permutations of 4 letters chosen from 26 is ${}^{26}P_4 = 26 \times 25 \times 24 \times 23 = 358,800$

Ex:- How many different committees of 5 people can be chosen from 10 people?

sol:- In choosing a committee, order doesn't matter; so we need the number of combinations of 5 people chosen from 10

$$= {}^{10}C_5$$

$$= 10!/(5!)(5!)$$

$$= (10 \times 9 \times 8 \times 7 \times 6)/(5 \times 4 \times 3 \times 2 \times 1) = 30,240/120 = 252.$$

Example: Jones is the Chairman of a committee. In how many ways can a committee of 5 be chosen from 10 people given that Jones must be one of them?

Sol: Jones is already chosen, so we need to choose another 4 from 9.

In choosing a committee, order doesn't matter; so we need the number of combinations of 4 people chosen from 9

$$\begin{aligned}
 &= {}^9C_4 \\
 &= 9!/(4!)(5!) \\
 &= (9 \times 8 \times 7 \times 6)/(4 \times 3 \times 2 \times 1) \\
 &= 3,024/24 \\
 &= 126
 \end{aligned}$$

Example:- A special type of password consists of four **different** letters of the alphabet, where each letter is used only once. How many different possible passwords are there?

Sol:- The number of permutations of 4 letters chosen from 26 is ${}^{26}P_4 = 26 \times 25 \times 24 \times 23 = 358,800$

Example:- What is the total number of possible 4-letter arrangements of the letters **m, a, t, h** if each letter is used only once in each arrangement?

$${}_4P_4 = 4 \cdot 3 \cdot 2 \cdot 1 = 24 \quad \text{or} \quad {}_4P_4 = \frac{4!}{(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$$

Or $n! = 4!$

Example:

There are 12 boys and 14 girls in Mrs. Ali math class. Find the number of ways Mrs. Ali can select a team of 3 students from the class to work on a group project. The team is to consist of 1 girl and 2 boys.

Order, or position, is not important. Using the multiplication counting principle,

$$\underbrace{{}_{12}C_2}_{\text{boys}} \cdot \underbrace{{}_{14}C_1}_{\text{girls}} = \frac{12 \cdot 11}{2 \cdot 1} \cdot \frac{14}{1} = 66 \cdot 14 = 924$$

Example: A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?

Sol;

A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?