Indices and Standard Form/ Scientific Notation

Laws of Indices	***Example***
$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^8$
$a^m \div a^n = a^{m-n}$	$5^7 \div 5^3 = 5^4$
$(a^m)^n = a^{m \times n}$	$(10^3)^7 = 10^{21}$
$a^1 = a$	$115^1 = 115$
$a^0 = 1$	$34^0 = 1$
$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$(\frac{5}{6})^3 = \frac{125}{216}$
$a^{-m} = \frac{1}{a^m}$	$9^{-2} = \frac{1}{81}$
$a^{\frac{x}{y}} = \sqrt[y]{a^x}$	$32^{\frac{2}{5}} = \sqrt[5]{32^2}$ 5th root of 32^2 $45^{\frac{1}{2}} = \sqrt{45}$ square root of 45

Standard Form / Scientific Notation:

Rule: There should be one non-zero digit before the decimal.

Ordinary Notation	*conversion*	*Standard Form / Scientific Notation*
384,400,000	→ decimal moves 8 places left → ←decimal moves 8 places right←	3.844×10^8
0.000000005	→ decimal moves 9 places right → ← decimal moves 9 places left ←	5×10^{-9}

Q1. Evaluate the following ***without using calculator***:

(a) $6.4\times10^6+8\times10^8$ (Ans: 8.064×10^8)
(b) $1.45\times10^{-6}+5.7\times10^9$ (Ans: 5.7×10^9)
(c) $6\times10^{-11}+3.887\times10^{-6}$ (Ans: 3.887×10^{-6})
(d) $3.3\times10^7-7.811\times10^9$ (Ans: -7.778×10^9)
(e) $7\times10^{-8}-3.7\times10^{12}$ (Ans: -3.7×10^{12})
(f) $1.088\times10^{-13}-2.4\times10^{-8}$ (Ans: -2.4×10^{-8})
(g) $6\times10^6\times 8.75\times10^9$ (Ans: 5.25×10^{16})
(h) $4.6\times10^{-15}\times 8\times10^8$ (Ans: 3.68×10^{-6})
(i) $3.9\times10^{-8}\times 1.1\times10^{-6}$ (Ans: 4.29×10^{-14})
(j) $2.76\times10^7\div 6.9\times10^{16}$ (Ans: 4×10^{-10})
(k) $2.7\times10^{-14}\div 2.16\times10^7$ (Ans: 1.25×10^{-21})
(l) $5\times10^{-13}\div 7.5\times10^{-8}$ (Ans: 6.667×10^{-6})

Steps: For addition and subtraction, take the smaller index as common. For multiplication and division, apply laws of indices.

Q1: W2018 P1 Q21

(a) Evaluate $9^1 + 9^0$ (Ans: 10) [1]
(b) Evaluate *n* where $4^n = 2^{n-1}$ (Ans: n= -1) [2]

Q2, 3: W2017 P1

Q12. Given that $a^x = 5$. Find: (a) a^{2x} (Ans: 25) [1]
(b) a^{-x} (Ans: 0.2) [1]

Q16. (a) Given that $p = 5 \times 10^9$ and $q = 9 \times 10^{-16}$, evaluate and express your answer in standard form:
(i) $p \times q$ (Ans: 4.5×10^{-6}) [1]
(ii) $\sqrt{q}$ (Ans: 3×10^{-8}) [1]

Step: Square root means index $\frac{1}{2}$

(b)Write 360 million in standard form. (Ans: 3.6×10^8)[1]

Explore the following with reference to the given question:

kilo (K)	10^3	a thousand
mega (M)	10^6	a million
giga (G)	10^9	a billion
tera (T)	10^{12}	a trillion
milli (m)	10^{-3}	a thousandth
micro (μ)	10^{-6}	a millionth
nano (n)	10^{-9}	a billionth
pico (p)	10^{-12}	a trillionth

Q4, 5: W2016 P1

Q8. (a) Write 513,000 in standard form. (Ans: 5.13×10^5) [1]

(b) Evaluate and express your answer in standard form:
$(4\times10^{-5}) \times (6\times10^{-4})$ (Ans: 2.4×10^{-8}) [2]

Q16. (a) Evaluate $3^2 + 3^1 + 3^0$ (Ans: 13) [1]

(b) Evaluate $\left(\frac{4}{3}\right)^{-2}$ (Ans: $\frac{9}{16}$) [1]

(c) $(16y^6)^{\frac{1}{2}}$ (Ans: $4y^3$) [1]

Q6: W2016 P2 Q2(a) Simplify $\frac{3a^2}{10bc} \div \frac{9a}{5b^2c}$ (Ans: $\frac{ab}{6}$) [2]

Q7, 8: S2016 P1

Q11. Simplify: (a) $\frac{5x^7y}{15\,x^3y^4}$ (Ans: $\frac{x^4}{3y^3}$) [1]

(b) $\left(\frac{4t^2}{v^4}\right)^{\frac{1}{2}}$ (Ans: $\frac{2t}{v^2}$) [1]

Q22. The table shows the populations of some countries in 2014:

Country	*Population*
Nigeria	
Sudan	3.6×10^7
Chad	1.1×10^7
Namibia	2.2×10^6

(a) In 2014, the population of Nigeria was 177 156 000. Complete the table with the population of Nigeria using standard form, correct to 2 significant figures. (Ans: 1.8×10^8) [2]

(b) Complete the following: The population of Chad was ..... times the population of Namibia. (Ans: five) [1]

(c) The population density of a country is measured as the number of people per square kilometer. It can be found by dividing the population of the country by its area in km^2. The area of Sudan is $1.86\times10^6 km^2$. Estimate the population density of Sudan. Give your answer correct to 1 significant figure. (Ans: 20) [2]

Q9, 10: W2015 P1

Q10 (a) Express 0.0000045 in standard form. (Ans: 4.5×10^{-6}) [1]

(b) Given that $p = 6\times10^8$ and $q = 4\times10^7$, expressing each answer in standard form, find:

(i) $p \times q$ (Ans: 2.4×10^{16}) [1]

(ii) $p - q$. (Ans: 5.6×10^8) [1]

Q11 (a) Evaluate $\left(\frac{3}{2}\right)^0$ (Ans: 1) [1]

(b) Evaluate $\left(\frac{3}{2}\right)^{-1}$ (Ans: $\frac{2}{3}$) [1]

(c) Simplify $(9x^3)^2$ (Ans: $81x^6$) [1]

Q11: S2015 P1 Q16

(a) Evaluate (i) $2^0 + 2^3$ (Ans: 9) [1]

(ii) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$ (Ans: $\frac{1}{3}$) [1]

(b) Simplify $(4x^2)^{-2}$ (Ans: $\frac{1}{16x^4}$) [1]

Q12: W2014 P1 Q17(a) Simplify $p^2\,(p^3 - 3p^{-2})$ (Ans: $p^5 - 3$) [2]

Q13: S2013 P1 Q18

The annual coffee production of some countries in 2010 is as follows:

Country	*Number of bags per year*
Brazil	
Vietnam	1.85×10^7
Colombia	9.2×10^6
Indonesia	8.5×10^6

(a) In 2010, Brazil produced 48 million bags of coffee. Complete the table using standard form. (Ans: 4.8×10^7) [1]

(b) How many more bags of coffee were produced in Vietnam than in Colombia? (Ans: 9.3×10^6) [2]

(c) the mass of a bag of coffee is 60 kb. Work out the number of kilograms of coffee produced in Indonesia. Give your answer in standard form. (Ans: 5.1×10^8) [1]

Q14. W2012 P1 Q11

(a) Find the value of a when $3^a \div 3^4 = 3^2$ (Ans: 6) [1]

(b) Find the value of b when $8^b = 2$ (Ans: $\frac{1}{3}$) [1]