Relating the Standard and Factored Forms of Quadratic Equations

- Quadratic functions can be written in standard form, i.e., $f(x) = ax^2 + bx + c$, or in factored form, i.e., $f(x) = a(x - r)(x - s)$. These forms are equivalent.
- Factoring functions is helpful in finding the zeros of the function. The zeros often help to answer certain questions about a function.
- The vertex of the parabola is the maximum if the parabola opens downward and the minimum if it opens upward.
- In factored form, the zeros are *r* and *s*. The vertex of the parabola is halfway between the two zeros. To find the *x* coordinate of the vertex, add the two zeros together and divide by two.
- In standard form, *c* is the *y*-intercept of the function.
- In both forms, *a* determines if the parabola opens upward or downward.

Converting Between Standard and Factored Forms

- An equation in standard form has a trinomial on the right side, which can *sometimes* be factored using the usual techniques for factoring trinomials. It is important to look for the greatest common factor (GCF) before trying to factor the trinomial.
- An equation in factored form can be converted to standard form by expanding, i.e. by multiplying the binomial factors together (using the distributive property) and then multiplying each term of the result by the value of *a*. The terms in the resulting expression must then be rearranged (if necessary) so that they appear in descending order of powers of *x*.
- In order to determine whether two quadratic functions are equivalent, they must be in the same form, so you typically have to convert one of them to the form of the other, and then compare.

Sketching the Graph of a Quadratic Function, Given its Equation

- The graph of a quadratic function in either standard or vertex form is always a parabola.
- Use the *a* value in the equation to determine whether the parabola opens upward or downward.
- The following points are useful in sketching the graph:
  - *x*-intercepts
  - vertex
  - *y*-intercept
- If the given equation is in standard form, you can:
  - determine the *y*-intercept directly from the value of *c* in the given equation
  - change the equation to factored form to get the *x*-intercepts (zeros)
  - determine the *x*-coordinate of the vertex by finding the average of the zeros (call this value *h*); determine the *y*-coordinate of the vertex by evaluating *f*(*h*)
- If the given equation is in factored form, you can:
  - determine the *x*-intercepts (zeros) directly from the equation
  - determine the *x*-coordinate of the vertex by finding the average of the zeros (call this value *h*); determine the *y*-coordinate of the vertex by evaluating *f*(*h*)
  - change the equation to standard form (or just substitute *x*=0) to get the *y*-intercept

To find the equation of a function, given its graph, see example 4 on page 138.