System of Linear Equations with Two Variables

A system of linear equations with two variables is usually *a set of two linear equations*. For example, $\boldsymbol{x + y = 5}$ and $\boldsymbol{2x - y = 1}$ are both linear equations with two variables. When listed together, they form a system of linear equations.

There are three types of systems of linear equations with two variables that can have three types of solutions. An **independent system** has exactly one solution pair (x, y), an **inconsistent system** has no solution, and a **dependent solution** has an infinite number of solutions.

In this handout, you will:

Look at two ways to solve systems of linear equations algebraically: ***substitution and elimination/addition.***

SUBSTITUTION METHOD

How do I solve systems of linear equations using substitution?

When solving a system of equations, our goal is to find the value of each variable within the equations. Since each equation in the system has two variables, one way to reduce the number of variables in an equation is to substitute an expression for a variable.

Steps to solve a system of equations using substitution:

1. ***Isolate*** one of the two variables in one of the equations.
2. ***Substitute the expression*** that is equal to the isolated variable from Step 1 into the other equation. This should result in a linear equation with only one variable.
3. ***Solve*** the linear equation for the remaining variable.
4. ***Use the solution*** of Step 3 to calculate the value of the other variable in the system by using one of the original equations.

Consider the following example:

$x = 2y$
$x + y = 3$

In a system of equations, both equations are simultaneously true. In other words, since the first equation tells us that x is equal to $2y$ the x in the second equation is also equal to $2y$. Therefore, we can plug in $2y$ as a **substitute** for x in the second equation:

$$x + y = 3 \longrightarrow (2y) + y = 3 \longrightarrow 3y = 3$$

From here, we can solve the equation, $3y = 3$, by dividing the coefficient and isolating the y-variable.

$$\frac{3y}{3} = \frac{3}{3} \longrightarrow \boldsymbol{y = 1}$$

Now that we have found the value of y, we can substitute 1 into y to solve for the x-variable in either equation in the system. For this example, we will use $x = 2y$:

$$x = 2y \longrightarrow x = 2\,(1) \longrightarrow \boldsymbol{x = 2}$$

The solution $(\boldsymbol{x}, \boldsymbol{y})$ to the system is $(\mathbf{2}, \mathbf{1})$.

ELIMINATION METHOD

How do I solve systems of linear equations by elimination?

When solving a system of equations, our goal is to find the value of each variable within the equations. Since each equation in the system has two variables, one way to reduce the number of variables is to add or subtract the two equations in the system to cancel out, or eliminate, one of the variables.

To solve a system of equations using elimination:

1. ***Identify a pair of terms*** in the system where the **same variable** has the **same coefficient *but* opposite signs**.
    a. If necessary, rewrite one or both equations so that a pair of terms have both the same variable and coefficients but opposite signs.
2. ***Add (or subtract) the two equations in the system to eliminate the terms*** identified in Step 1. This should result in a linear equation with only one variable.
3. ***Solve the linear equation*** to obtain a value for the variable.
4. ***Use the solution*** of Step 3 to calculate the value of the other variable in the system by using one of the original equations.

Consider the following system of equations:

$3x - y = 7$
$2x + y = 8$

Note that in this example, the pair of terms in the system, $-y$ and y, are the same variable, have the same coefficient, 1, and have opposite signs. Therefore, we can add both equations to cancel the y-variable.

$$\begin{array}{r} 3x - y = 7 \\ +\ 2x + y = 8 \end{array}$$

$\mathbf{5x + 0 = 15} \longrightarrow 5x = 15$

Notice that the y-variables are eliminated as a result of adding the two equations. When solving systems of equations using elimination, our aim is to always look for opportunities to cancel out a pair of terms.

- If two terms have the opposite coefficients like in the system above $(-y \text{ } and \text{ } y)$, we can add the two equations to cancel the terms.
- If two terms have the same coefficients, we can subtract the two equations to cancel the terms.

From here, we can solve the equation $5x = 15$, then use the value of x to find the value of y

First, to solve for x, we must isolate the *x*-variable by dividing the coefficient from both sides:

$$\frac{5x}{5} = \frac{15}{5} \longrightarrow \boldsymbol{x = 3}$$

Now that we have found the value of *x*, we can substitute 3 into *y* to solve for the *x*-variable in either equation in the system. For this example, we will use $2x + y = 8$:

$$2x + y = 8 \longrightarrow 2(3) + y = 8 \longrightarrow 6 + y = 8 \longrightarrow 6 + y - 6 = 8 - 6 \longrightarrow \boldsymbol{y = 2}$$

The solution $(\boldsymbol{x, y})$ to the system is **(3,2)**

References:

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