Chapter 5 – The Standard Deviation as a Ruler and the Normal Model

1. **Stats test.**

   Nicole scored 65 points on the test.

   That is one standard deviation above the mean, so her z-score is 1.

   $$z = \frac{y-\mu}{\sigma}$$
   $$z = \frac{80-75}{5}$$
   $$z = 1$$

2. **Horsepower.**

   This car has 195 horsepower.

   That is one and a half standard deviations above the mean, so its z-score is 1.5.

   $$z = \frac{y-\mu}{\sigma}$$
   $$z = \frac{195-135}{40}$$
   $$z = 1.5$$

3. **Homeruns.**

   Roger Maris hit 61 homeruns when the average was 18.8 home runs with a standard deviation of 13.37 home runs. That's a z-score of about 3.16.

   $$z = \frac{y-\mu}{\sigma}$$
   $$z = \frac{61-18.8}{13.37}$$
   $$z \approx 3.16$$

   Mark McGwire hit 70 homeruns when the average was 20.7 home runs, with a standard deviation of 12.74 homeruns. That's a z-score of about 3.87.

   $$z = \frac{y-\mu}{\sigma}$$
   $$z = \frac{70-20.7}{12.74}$$
   $$z \approx 3.87$$

   Mark McGwire's season home run total was more impressive than Roger Maris's season home run total, since he had the higher z-score. You'll have to decide for yourself if the steroid use made it less impressive anyway.

4. **Teenage mothers.**

   The teenage birth rate for Hispanic mothers in Maine was 31.3 births per 1000. The US mean was 96.7 births per 1000 with a standard deviation of 31.60 births per 1000, resulting in a z-score of approximately –2.07.

   $$z = \frac{y-\mu}{\sigma}$$
   $$z = \frac{31.3-96.7}{31.60}$$
   $$z \approx -2.07$$

The teenage birth rate for Non-Hispanic black mothers in Hawaii was 17.4 births per 1000. The US mean was 64.0 births per 1000 with a standard deviation of 16.12 births per 1000, resulting in a *z*-score of approximately –2.89.

$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{17.4 - 64.0}{16.12}$$
$$z \approx -2.89$$

The teenage birth rate for non-Hispanic white mothers in the District of Columbia was 4.3 births per 1000. The US mean was 29.0 births per 1000 with a standard deviation of 11.55 births per 1000, resulting in a *z*-score of approximately –2.14.

$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{4.3 - 29.0}{11.55}$$
$$z \approx -2.14$$

The teenage birth rate for Non-Hispanic black mothers in Hawaii, at 17.4 births per 1000, was the most extreme low rate, since it had the most extreme *z*-score.

5. SAT or ACT?

Measures of center and position (lowest score, top 25% above, mean, and median) will be multiplied by 40 and increased by 150 in the conversion from ACT to SAT by the rule of thumb. Measures of spread (standard deviation and IQR) will only be affected by the multiplication.

Lowest score = 910	Mean = 1230	Standard deviation = 120
Top 25% above = 1350	Median = 1270	IQR = 240

6. Cold U?

Measures of center and position (maximum, median, and mean) will be multiplied by $\frac{9}{5}$ and increased by 32 in the conversion from Fahrenheit to Celsius. Measures of spread (range, standard deviation, IQR) will only be affected by the multiplication.

Maximum temperature = 51.8°F	Range = 59.4°F
Mean = 33.8°F	Standard deviation = 12.6°F
Median = 35.6°F	IQR = 28.8°F

7. Stats test, part II.

A *z*-score of 2.20 means that your score was 2.20 standard deviations above the mean.

8. Checkup.

A *z*-score of –1.88 means that the boy's height was 1.88 standard deviations below the mean.

9. Stats test, part III.

At two standard deviations below the mean, Gregor scored 65 points.

$$z = \frac{y - \mu}{\sigma}$$
$$-2 = \frac{y - 75}{5}$$
$$y = 65$$

10. Mensa.

In order to be considered a genius, your IQ must be 140 or above.

$$z = \frac{y - \mu}{\sigma}$$
$$2.5 = \frac{y - 100}{16}$$
$$y = 140$$

11. Temperatures.

In January, with mean temperature 36° and standard deviation in temperature 10°, a high temperature of 55° is almost 2 standard deviations above the mean. In July, with mean temperature 74° and standard deviation 8°, a high temperature of 55° is more than two standard deviations below the mean. A high temperature of 55° is less likely to happen in July, when 55° is farther away from the mean.

12. Placement Exams.

On the French exam, the mean was 72 and the standard deviation was 8. The student's score of 82 was 10 points, or 1.25 standard deviations, above the mean. On the math exam, the mean was 68 and the standard deviation was 12. The student's score of 86 was 18 points or 1.5 standard deviations above the mean. The student did better on the math exam.

13. Combining test scores.

The *z*-scores, which account for the difference in the distributions of the two tests, are 1.5 and 0 for Derrick and 0.5 and 2 for Julie. Derrick's total is 1.5 which is less than Julie's 2.5.

14. Combining scores again.

The *z*-scores, which account for the difference in the distributions of the two tests, are 0 and 1 for Reginald, for a total of 1.0. For Sara, they are 2.0 and –0.33 for a total of 1.67. While her raw score is lower, her *z*-score is higher.

15. Final Exams.

a) Anna's average is $\frac{83+83}{2} = 83$. Megan's average is $\frac{77+95}{2} = 86$.

Only Megan qualifies for language honors, with an average higher than 85.

b) On the French exam, the mean was 81 and the standard deviation was 5. Anna's score of 83 was 2 points, or 0.4 standard deviations, above the mean. Megan's score of 77 was 4 points, or 0.8 standard deviations below the mean.

On the Spanish exam, the mean was 74 and the standard deviation was 15. Anna's score of 83 was 9 points, or 0.6 standard deviations, above the mean. Megan's score of 95 was 21 points, or 1.4 standard deviations, above the mean.

Measuring their performance in standard deviations is the only fair way in which to compare the performance of the two women on the test.

Anna scored 0.4 standard deviations above the mean in French and 0.6 standard deviations above the mean in Spanish, for a total of 1.0 standard deviation above the mean.

Megan scored 0.8 standard deviations below the mean in French and 1.4 standard deviations above the mean in Spanish, for a total of only 0.6 standard deviations above the mean.

Anna did better overall, but Megan had the higher average. This is because Megan did very well on the test with the higher standard deviation, where it was comparatively easy to do well.

16. MP3s.

a) Standard deviation measures variability, which translates to consistency in everyday use. A type of batteries with a small standard deviation would be more likely to have lifespans close to their mean lifespan than a type of batteries with a larger standard deviation.

b) RockReady batteries have a higher mean lifespan and smaller standard deviation, so they are the better battery. 8 hours is $2\frac{2}{3}$ standard deviations below the mean lifespan of RockReady and $1\frac{1}{2}$ standard deviations below the mean lifespan of DuraTunes. DuraTunes batteries are more likely to *fail* before the 8 hours have passed.

c) 16 hours is $2\frac{1}{2}$ standard deviations higher than the mean lifespan of DuraTunes, and $2\frac{2}{3}$ standard deviations higher than the mean lifespan of RockReady. Neither battery has a good chance of lasting 16 hours, but DuraTunes batteries have a greater chance than RockReady batteries.

17. Cattle.

a) A steer weighing 1000 pounds would be about 1.81 standard deviations below the mean weight.

$$z = \frac{y-\mu}{\sigma} = \frac{1000-1152}{84} \approx -1.81$$

b) A steer weighing 1000 pounds is more unusual. Its z-score of –1.81 is further from 0 than the 1250 pound steer's z-score of 1.17.

18. Car speeds.

a) A car going the speed limit of 20 mph would be about 1.08 standard deviations below the mean speed.

$$z = \frac{y-\mu}{\sigma} = \frac{20-23.84}{3.56} \approx -1.08$$

b) A car going 10 mph would be more unusual. Its z-score of –3.89 is further from 0 than the 34 mph car's z-score of 2.85.

19. More cattle.

a) The new mean would be 1152 – 1000 = 152 pounds. The standard deviation would not be affected by subtracting 1000 pounds from each weight. It would still be 84 pounds.

b) The mean selling price of the cattle would be 0.40(1152) = $460.80. The standard deviation of the selling prices would be 0.40(84) = $33.60.

20. Car speeds again.

a) The new mean would be 23.84 – 20 = 3.84 mph over the speed limit. The standard deviation would not be affected by subtracting 20 mph from each speed. It would still be 3.56 miles per hour.

b) The mean speed would be 1.609(23.84) = 38.359 kph. The speed limit would convert to 1.609(20) = 32.18 kph. The standard deviation would be 1.609(3.56) = 5.728 kph.

21. Cattle, part III.

Generally, the minimum and the median would be affected by the multiplication and subtraction. The standard deviation and the IQR would only be affected by the multiplication.

Minimum = 0.40(980) – 20 = $372.00
Median = 0.40(1140) – 20 = $436
Standard deviation = 0.40(84) = $33.60
IQR = 0.40(102) = $40.80

22. Caught speeding.

Generally, the mean and the maximum would be affected by the multiplication and addition. The standard deviation and the IQR would only be affected by the multiplication.

Mean = 100 + 10(28 – 20) = $180
Maximum = 100 +10(33 – 20) = $230
Standard deviation = 10(2.4) = $24
IQR = 10(3.2) = $32

23. Professors.

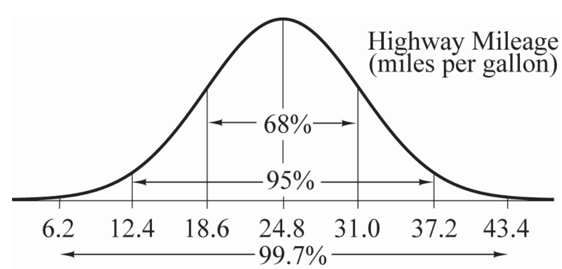
The standard deviation of the distribution of years of teaching experience for college professors must be 6 years. College professors can have between 0 and 40 (or possibly 50) years of experience. A workable standard deviation would cover most of that range of values with ± 3 standard deviations around the mean. If the standard deviation were 6 months ($\frac{1}{2}$ year), some professors would have years of experience 10 or 20 standard deviations away from the mean, whatever it is. That isn't possible. If the standard deviation were 16 years, ± 2 standard deviations would be a range of 64 years. That's way too high. The only reasonable choice is a standard deviation of 6 years in the distribution of years of experience.

24. Rock concerts.

The standard deviation of the distribution of the number of fans at the rock concerts would most likely be 2000. A standard deviation of 200 fans seems much too consistent. With this standard deviation, the band would be very unlikely to draw more than a 1000 fans (5 standard deviations!) above or below the mean of 21,359 fans. It seems like rock concert attendance could vary by much more than that. If a standard deviation of 200 fans is too small, then so is a standard deviation of 20 fans. 20,000 fans is too large for a likely standard deviation in attendance, unless they played several huge venues. Zero attendance is only a bit more than 1 standard deviation below the mean, although it seems very unlikely. 2000 fans is the most reasonable standard deviation in the distribution of number of fans at the concerts.

25. Guzzlers?

a) The Normal model for auto fuel economy is at the right.



b) Approximately 68% of the cars are expected to have highway fuel economy between 18.6 mpg and 31.0 mpg.

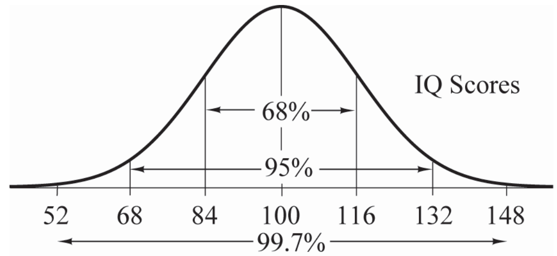
c) Approximately 16% of the cars are expected to have highway fuel economy above 31 mpg.

d) Approximately 13.5% of the cars are expected to have highway fuel economy between 31 mpg and 37 mpg.

e) The worst 2.5% of cars are expected to have fuel economy below approximately 12.4 mpg.

26. IQ.

a) The Normal model for IQ scores is at the right.



b) Approximately 95% of the IQ scores are expected to be within the interval 68 to 132 IQ points.

c) Approximately 16% of IQ scores are expected to be above 116 IQ points.

d) Approximately 13.5% of IQ scores are expected to be between 68 and 84 IQ points.

e) Approximately 2.5% of the IQ scores are expected to be above 132.

27. Small steer.

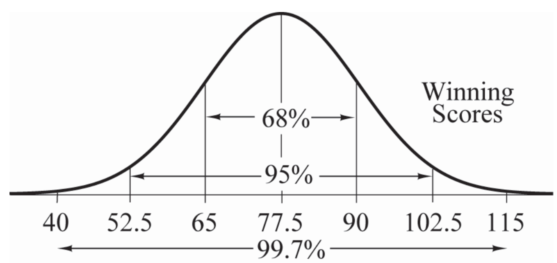
Any weight more than 2 standard deviations below the mean, or less than 1152 – 2(84) = 984 pounds might be considered unusually low. We would expect to see a steer below 1152 – 3(84) = 900 very rarely.

28. High IQ.

Any IQ more than 2 standard deviations above the mean, or more than 100 + 2 (16) = 132 might be considered unusually high. We would expect to find someone with an IQ over 100 + 3(16) = 148 very rarely.

29. College hoops.

a) The Normal model for the distribution of the winning scores of 2011-2012 college men's basketball games is at the right.



b) According to the Normal model, we expect 95% of winning scores to be between 52.5 and 102.5 points.

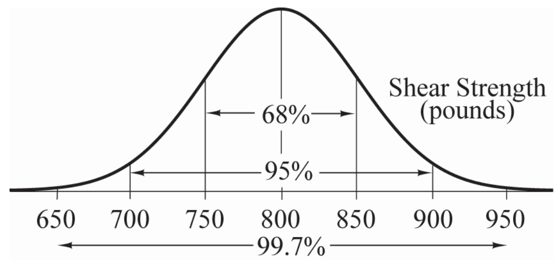
c) 65 points is 1 standard deviation below the mean. The Normal model predicts that approximately 16% of winning scores are expected to be less than 65 points.

d) According to the Normal model, we expect about 81.5% of the winning scores to be between 65 and 102 points. (95 – 68 = 27, so each of the "slices" between 1 and 2 standard deviations away from the mean is 27/2 = 13.5, and 68 + 13.5 = 81.5)

e) According to the Normal model, we only expect about 2.5% of the winning scores to be above 102 points. (100 – 95 = 5, and 5/2 = 2.5% for each tail.)

30. Rivets.

a) The Normal model for the distribution of shear strength of rivets is at the right.



b) 750 pounds is 1 standard deviation below the mean, meaning that the Normal model predicts that approximately 16% of the rivets are expected to have a shear strength of less than 750 pounds. These rivets are a poor choice for a situation that requires a shear strength of 750 pounds, because 16% of the rivets would be expected to fail. That's too high a percentage.

c) Approximately 97.5% of the rivets are expected to have shear strengths below 900 pounds.

d) In order to make the probability of failure very small, these rivets should only be used for applications that require shear strength several standard deviations below the mean, probably farther than 3 standard deviations. (The chance of failure for a required shear strength 3 standard deviations below the mean is still approximately 3 in 2000.) For example, if the required shear strength is 550 pounds (5 standard deviations below the mean), the chance of one of these bolts failing is approximately 1 in 1,000,000.

31. Trees.

The use of the Normal model requires a distribution that is unimodal and symmetric. The distribution of tree diameters is neither unimodal nor symmetric, so use of the Normal model is not appropriate.

32. Car speeds, the picture.

The distribution of cars speeds shown in the histogram is unimodal and roughly symmetric, and the normal probability plot looks quite straight, so a normal model is appropriate.

33. Wisconsin ACT math.

a) The distribution of mean ACT math scores bimodal, so it is not approximately Normal, and 78.8% of scores are within one standard deviation of the mean. If a Normal model is useful, we would need approximately 68% of the scores within one standard deviation of the mean.

b) With Milwaukee area schools removed, the distribution of mean ACT math scores is slightly skewed, but the Normal probability plot is reasonably straight, so the Normal model is appropriate.

34. Wisconsin ACT math II.

a) The distribution of ACT scores for Wisconsin students with the Milwaukee schools removed is unimodal and roughly symmetric.

b) The Normal model seems appropriate, since this distribution of ACT scores is unimodal and symmetric.

c) This supports our assertion that the Normal model can be used to approximate this distribution. According to the Normal model, we expect 68% of the ACT scores to be within one standard deviation of the mean, and 68.9% is fairly close to that.

d) The model is not really correct, because in some locations the bars extend above the model and in other places they are below it. The model is a smooth, continuous curve, but the bars are discrete. However, the model will give reasonable estimates of the proportion of ACT scores in particular ranges.

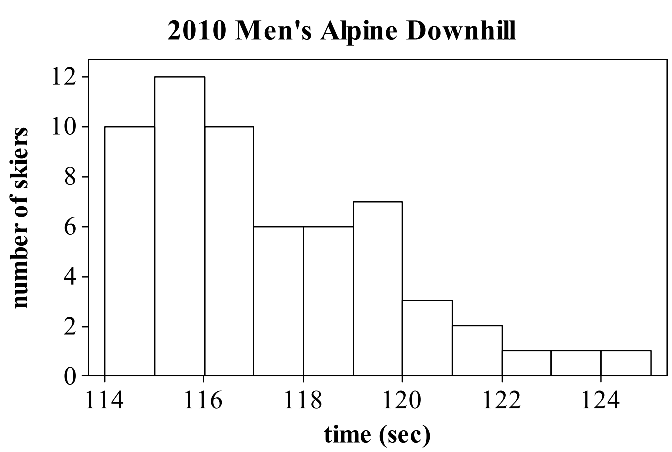
35. Winter Olympics 2010 downhill.

a) The 2010 Winter Olympics downhill times have mean of 117.34 seconds and standard deviation 2.456 seconds. 114.875 seconds is 1 standard deviation below the mean. If the Normal model is appropriate, 16% of the times should be below 99.7 seconds.

b) 8 out of 59 times (13.56%) are below 114.875 seconds.

c) The percentages in parts a and b do not agree because the Normal model is not appropriate in this situation.

d) The histogram of 2010 Winter Olympic Downhill times is skewed to the right. The Normal model is not appropriate for the distribution of times, because the distribution is not symmetric.



36. Check the model.

a) We know that 95% of the observations from a Normal model fall within 2 standard deviations of the mean. That corresponds to 23.84 – 2(3.56) = 16.72 mph and 23.84 + 2(3.56) = 30.96 mph. These are the 2.5 percentile and 97.5 percentile, respectively. According to the Normal model, we expect only 2.5% of the speeds to be below 16.72 mph, and 97.5% of the speeds to be below 30.96 mph.

b) The actual 2.5 percentile and 97.5 percentile are 16.638 and 30.976 mph, respectively. These are very close to the predicted values from the Normal model. The histogram from Exercise 24 is unimodal and roughly symmetric. It is very slightly skewed to the right and there is one outlier, but the Normal probability plot is quite straight. We should not be surprised that the approximation from the Normal model is a good one.

37. Receivers 2010.

a) Approximately 2.5% of the receivers are expected to gain more yards than 2 standard deviations above the mean number of yards gained.

b) The distribution of the number of yards gained has mean 397.15 yards and standard deviation 362.4 yards. According to the Normal model, we expect 2.5% of the receivers, or 4.8 of them, to gain more than 2 standard deviations above the mean number of yards. This means more than 397.15 + 2(362.4) = 1122 yards. In 2010, 8 receivers ran for more than 1122 yards.

c) The distribution of the number of yards run by wide receivers is skewed heavily to the right. Use of the Normal model is not appropriate for this distribution, since it is not symmetric.

38. Customer database.

a) The median of 93% is the better measure of center for the distribution of the percentage of white residents in the neighborhoods, since the distribution is skewed to the left. Median is a better summary for skewed distributions since the median is resistant to effects of the skewness, while the mean is pulled toward the tail.

b) The IQR of 17% is the better measure of spread for the distribution of the percentage of white residents in the neighborhoods, since the distribution is skewed to the left. IQR is a better summary for skewed distributions since the IQR is resistant to effects of the skewness, and the standard deviation is not.

c) According to the Normal model, approximately 68% of neighborhoods are expected to have a percentage of whites within 1 standard deviation of the mean.

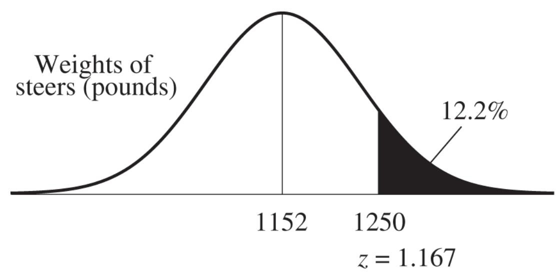
d) The mean percentage of whites in a neighborhood is 83.59%, and the standard deviation is 22.26%. 83.59% ± 22.26% = 61.33% to 105.85%. Estimating from the graph, more than 80% of the neighborhoods have a percentage of whites greater than 61.33%.

e) The distribution of the percentage of whites in the neighborhoods is strongly skewed to the left. The Normal model is not appropriate for this distribution. There is a discrepancy between c) and d) because c) is wrong!

39. Normal cattle.

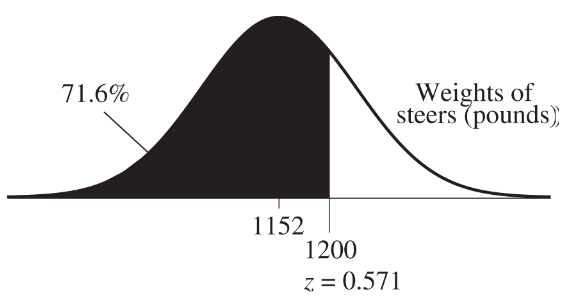
a)

$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{1250 - 1152}{84}$$
$$z \approx 1.167$$



According to the Normal model, 12.2% of steers are expected to weigh over 1250 pounds.

b)

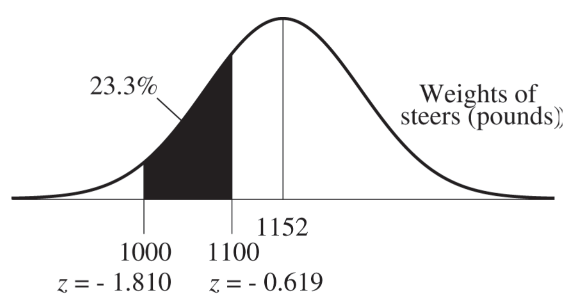


$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{1200 - 1152}{84}$$
$$z \approx 0.571$$

According to the Normal model, 71.6% of steers are expected to weigh less than 1200 pounds.

c)

$$z = \frac{y - \mu}{\sigma} \qquad z = \frac{y - \mu}{\sigma}$$
$$z = \frac{1000 - 1152}{84} \qquad z = \frac{1100 - 1152}{84}$$
$$z \approx -1.810 \qquad z \approx -0.619$$

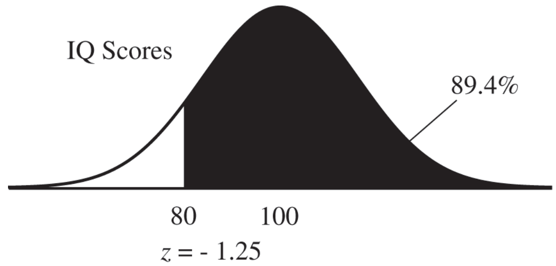


According to the Normal model, 23.3% of steers are expected to weigh between 1000 and 1100 pounds.

40. IQs revisited.

a)

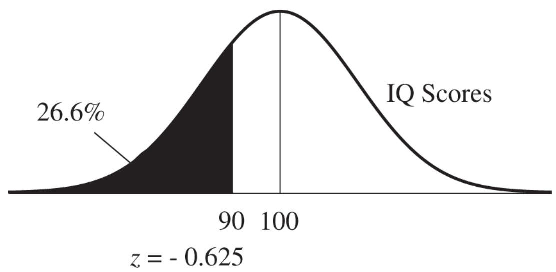
$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{80 - 100}{16}$$
$$z = -1.25$$



According to the Normal model, 89.4% of IQ scores are expected to be over 80.

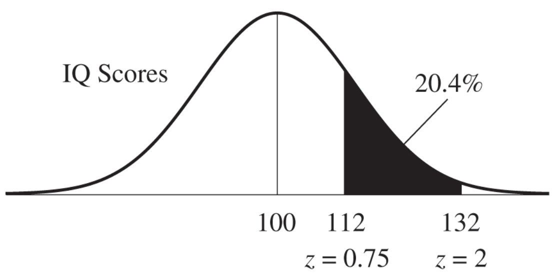
b)

$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{90 - 100}{16}$$
$$z = -0.625$$



According to the Normal model, 26.6% of IQ scores are expected to be under 90.

c)

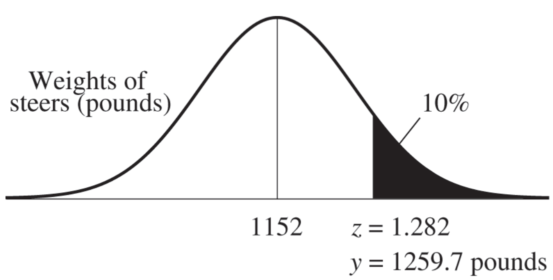


$$z = \frac{y - \mu}{\sigma} \qquad z = \frac{y - \mu}{\sigma}$$
$$z = \frac{112 - 100}{16} \qquad z = \frac{132 - 100}{16}$$
$$z = 0.75 \qquad z = 2$$

According to the Normal model, about 20.4% of IQ scores are between 112 and 132.

41. More cattle.

a)

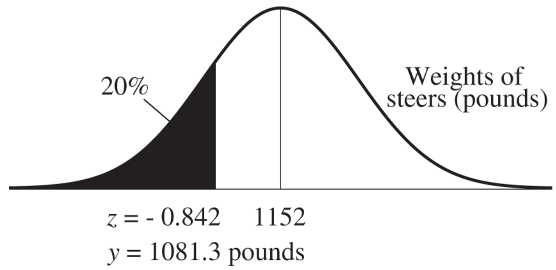


$$z = \frac{y - \mu}{\sigma}$$
$$1.282 = \frac{y - 1152}{84}$$
$$y \approx 1259.7$$

According to the Normal model, the highest 10% of steer weights are expected to be above approximately 1259.7 pounds.

b)

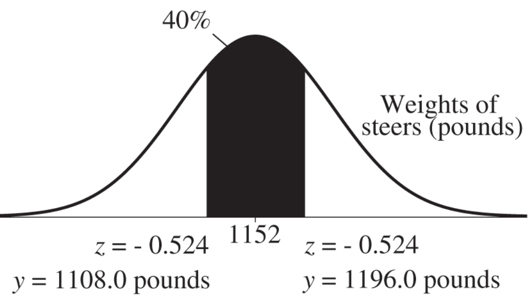
$$z = \frac{y - \mu}{\sigma}$$
$$-0.842 = \frac{y - 1152}{84}$$
$$y \approx 1081.3$$



According to the Normal model, the lowest 20% of weights of steers are expected to be below approximately 1081.3 pounds.

c)

$$z = \frac{y - \mu}{\sigma}$$
$$-0.524 = \frac{y - 1152}{84}$$
$$y \approx 1108.0$$

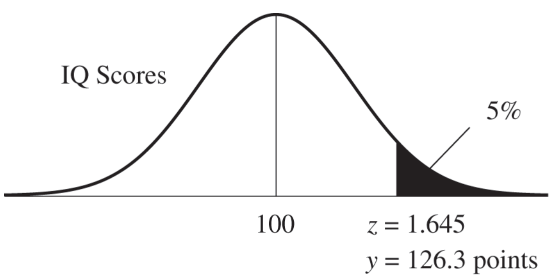


$$z = \frac{y - \mu}{\sigma}$$
$$0.524 = \frac{y - 1152}{84}$$
$$y \approx 1196.0$$

According to the Normal model, the middle 40% of steer weights is expected to be between about 1108.0 pounds and 1196.0 pounds.

42. More IQs.

a)

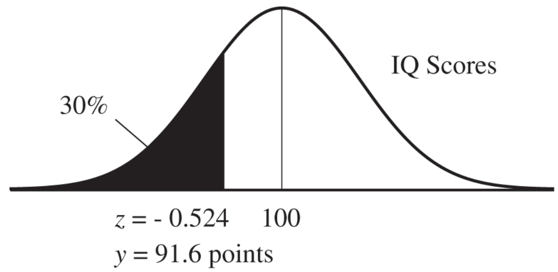


$$z = \frac{y - \mu}{\sigma}$$
$$1.645 = \frac{y - 100}{16}$$
$$y \approx 126.3$$

According to the Normal model, the highest 5% of IQ scores are above about 126.3 points.

b)

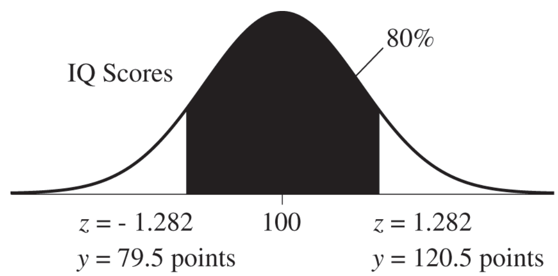
$$z = \frac{y - \mu}{\sigma}$$
$$-0.524 = \frac{y - 100}{16}$$
$$y \approx 91.6$$



According to the Normal model, the lowest 30% of IQ scores are expected to be below about 91.6 points.

c)

$$z = \frac{y - \mu}{\sigma}$$
$$-1.282 = \frac{y - 100}{16}$$
$$y \approx 79.5$$



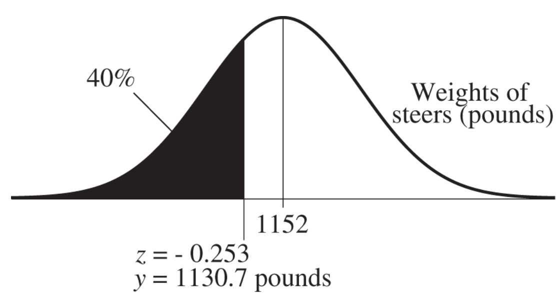
$$z = \frac{y - \mu}{\sigma}$$
$$1.282 = \frac{y - 100}{16}$$
$$y \approx 120.5$$

According to the Normal model, the middle 80% of IQ scores is expected to be between 79.5 points and 120.5 points.

43. Cattle, finis.

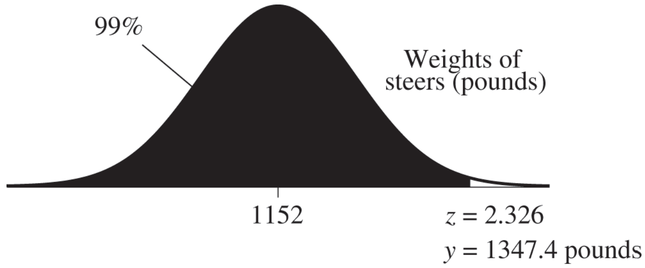
a)

$$z = \frac{y - \mu}{\sigma}$$
$$-0.253 = \frac{y - 1152}{84}$$
$$y \approx 1130.7$$



According to the Normal model, the weight at the 40th percentile is 1130.7 pounds. This means that 40% of steers are expected to weigh less than 1130.7 pounds.

b)

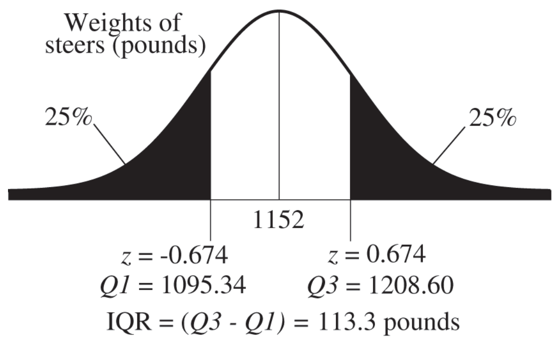


$$z = \frac{y-\mu}{\sigma}$$
$$2.326 = \frac{y-1152}{84}$$
$$y \approx 1347.4$$

According to the Normal model, the weight at the 99th percentile is 1347.4 pounds. This means that 99% of steers are expected to weigh less than 1347.4 pounds.

c)

$$z = \frac{y-\mu}{\sigma}$$
$$-0.674 = \frac{Q1-1152}{84}$$
$$Q1 \approx 1095.34$$



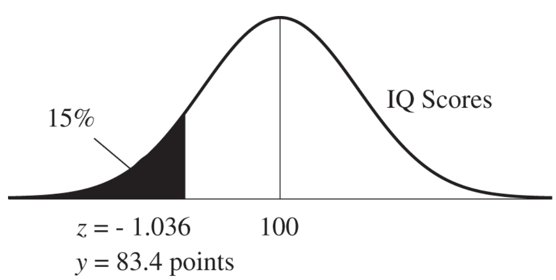
$$z = \frac{y-\mu}{\sigma}$$
$$0.674 = \frac{Q3-1152}{84}$$
$$Q3 \approx 1208.60$$

According to the Normal model, the IQR of the distribution of weights of Angus steers is about 113.3 pounds.

44. IQ, finis.

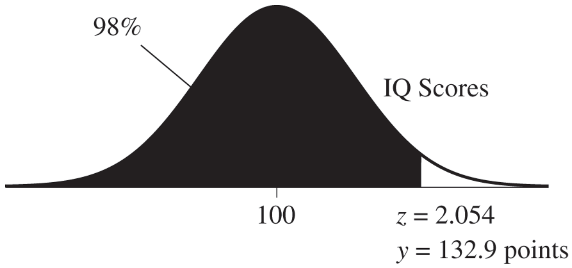
a)

$$z = \frac{y-\mu}{\sigma}$$
$$-1.036 = \frac{y-100}{16}$$
$$y \approx 83.4$$



According to the Normal model, the 15th percentile of IQ scores is about 83.4 points. This means that we expect 15% of IQ scores to be lower than 83.4 points.

b)

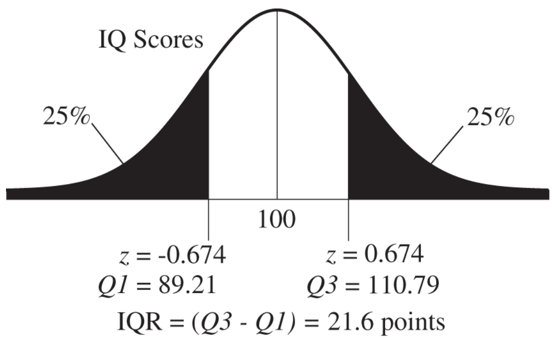


$$z = \frac{y-\mu}{\sigma}$$
$$2.054 = \frac{y-100}{16}$$
$$y \approx 132.9$$

According to the Normal model, the 98th percentile of IQ scores is about 132.9 points. This means that we expect 98% of IQ scores to be lower than 132.9 points.

c)

$$z = \frac{y-\mu}{\sigma}$$
$$-0.674 = \frac{Q1-100}{16}$$
$$Q1 \approx 89.21$$

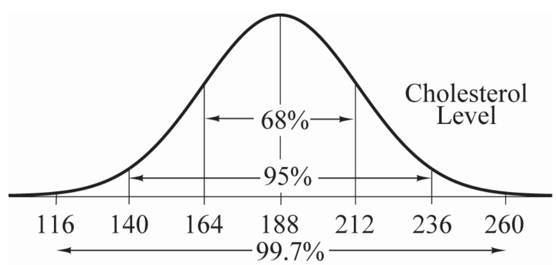


$$z = \frac{y-\mu}{\sigma}$$
$$0.674 = \frac{Q3-100}{16}$$
$$Q3 \approx 110.79$$

According to the Normal model, the IQR of the distribution of IQ scores is 21.6 points.

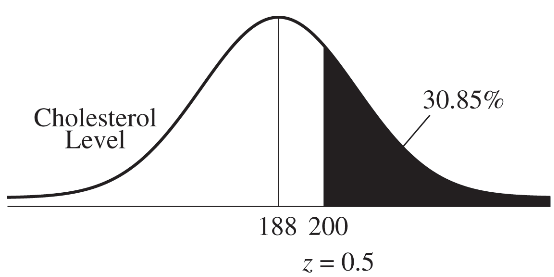
45. Cholesterol.

a) The Normal model for cholesterol levels of adult American women is at the right.



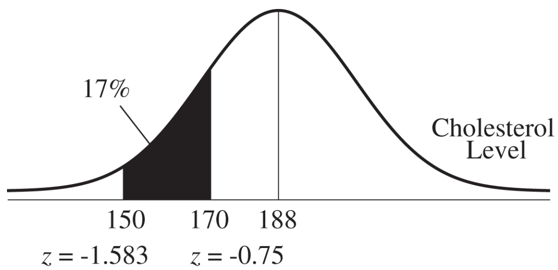
b)

$$z = \frac{y-\mu}{\sigma}$$
$$z = \frac{200-188}{24}$$
$$z = 0.5$$



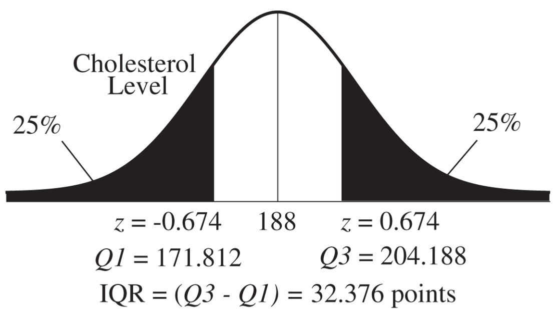
According to the Normal model, 30.85% of American women are expected to have cholesterol levels over 200.

c)



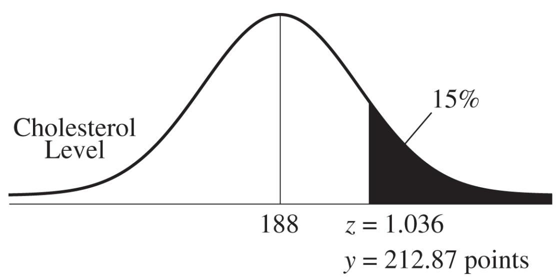
According to the Normal model, 17.00% of American women are expected to have cholesterol levels between 150 and 170.

d)



According to the Normal model, the interquartile range of the distribution of cholesterol levels of American women is approximately 32.38 points.

e)



$$z = \frac{y - \mu}{\sigma}$$

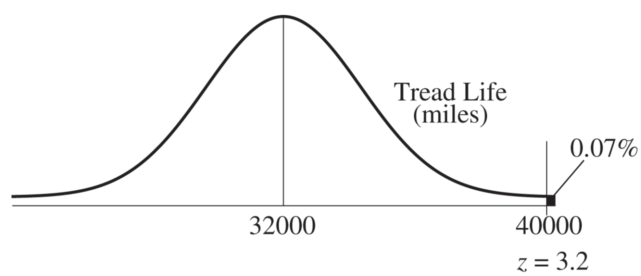
$$1.036 = \frac{y - 188}{24}$$

$$y = 212.87$$

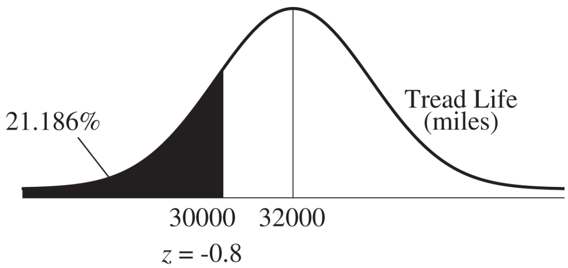
According to the Normal model, the highest 15% of women's cholesterol levels are above approximately 212.87 points.

46. Tires.

a) A tread life of 40,000 miles is 3.2 standard deviations above the mean tread life of 32,000. According to the Normal model, only approximately 0.07% of tires are expected to have a tread life greater than 40,000 miles. It would not be reasonable to hope that your tires lasted this long.



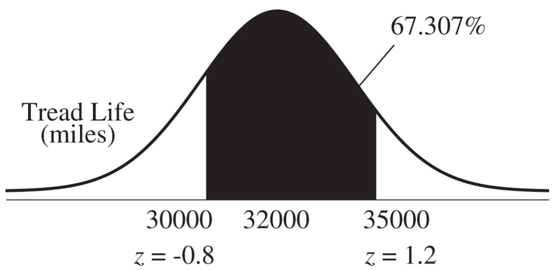
b)



$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{30000 - 32000}{2500}$$
$$z = -0.8$$

According to the Normal model, approximately 21.19% of tires are expected to have a tread life less than 30,000 miles.

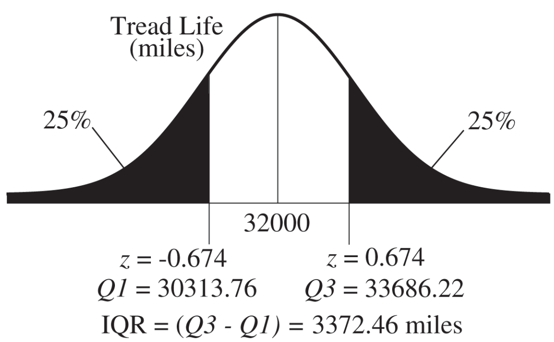
c)



$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{35000 - 32000}{2500}$$
$$z = 1.2$$

According to the Normal model, approximately 67.31% of tires are expected to last between 30,000 and 35,000 miles.

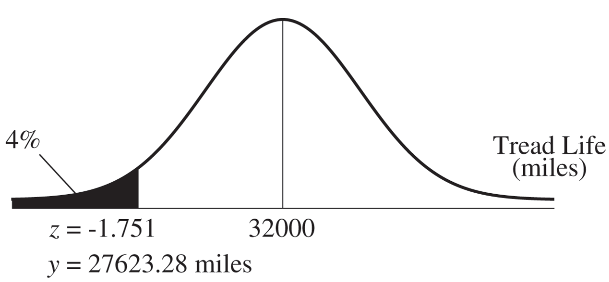
d)



$$z = \frac{y - \mu}{\sigma}$$
$$0.674 = \frac{Q3 - 32000}{2500}$$
$$Q3 = 33686.22$$

According to the Normal model, the interquartile range of the distribution of tire tread life is expected to be 3372.46 miles.

e)



$$z = \frac{y - \mu}{\sigma}$$
$$-1.751 = \frac{y - 32000}{2500}$$
$$y = 27623.28$$

According to the Normal model, 1 of every 25 tires is expected to last less than 27,623.28 miles. If the dealer is looking for a round number for the guarantee, 27,000 miles would be a good tread life to choose.

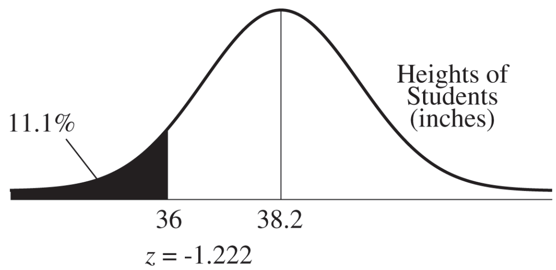
47. Kindergarten.

a)

$$z = \frac{y - \mu}{\sigma}$$

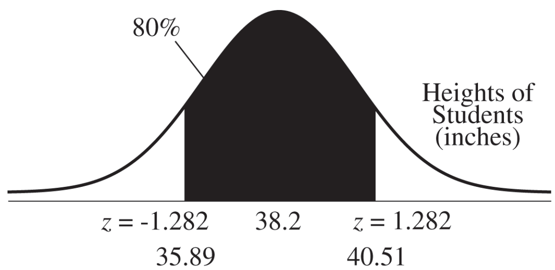
$$z = \frac{36 - 38.2}{1.8}$$

$$z = -1.222$$



According to the Normal model, approximately 11.1% of kindergarten kids are expected to be less than three feet (36 inches) tall.

b)



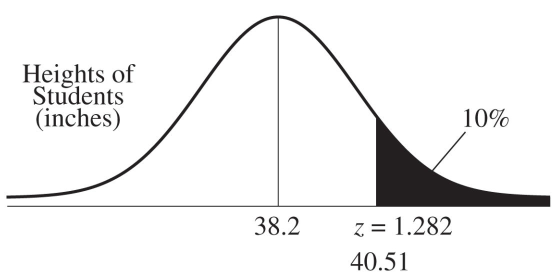
$$z = \frac{y - \mu}{\sigma} \qquad z = \frac{y - \mu}{\sigma}$$

$$-1.282 = \frac{y_1 - 38.2}{1.8} \qquad 1.282 = \frac{y_2 - 38.2}{1.8}$$

$$y_1 = 35.89 \qquad y_2 = 40.51$$

According to the Normal model, the middle 80% of kindergarten kids are expected to be between 35.89 and 40.51 inches tall. (The appropriate values of $z = \pm 1.282$ are found by using right and left tail percentages of 10% of the Normal model.)

c)



$$z = \frac{y - \mu}{\sigma}$$

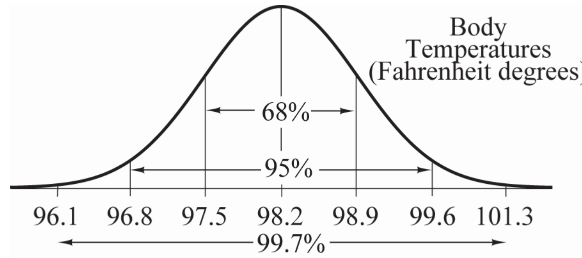
$$1.282 = \frac{y - 38.2}{1.8}$$

$$y = 40.51$$

According to the Normal model, the tallest 10% of kindergarteners are expected to be at least 40.51 inches tall.

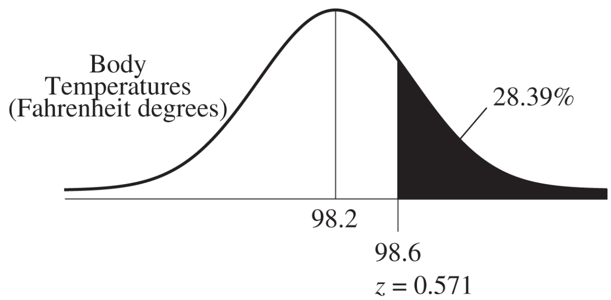
48. Body temperatures.

a) According to the Normal model (and based upon the 68-95-99.7 rule), 95% of people's body temperatures are expected to be between 96.8° and 99.6°. Virtually all people (99.7%) are expected to have body temperatures between 96.1° and 101.3°.



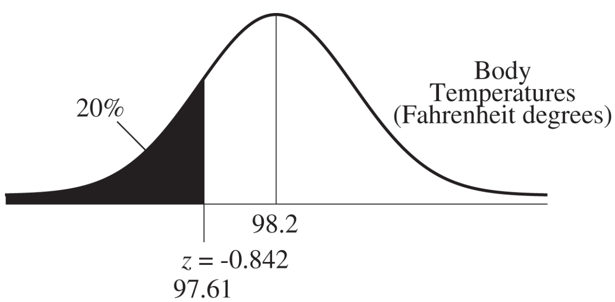
b)

$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{98.6 - 98.2}{0.7}$$
$$z = 0.571$$



According to the Normal model, approximately 28.39% of people are expected to have body temperatures above 98.6°.

c)

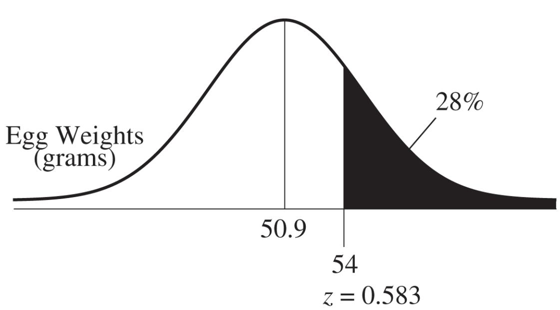


$$z = \frac{y - \mu}{\sigma}$$
$$-0.842 = \frac{y - 98.2}{0.7}$$
$$y = 97.61$$

According to the Normal model, the coolest 20% of all people are expected to have body temperatures below 97.6°.

49. Eggs.

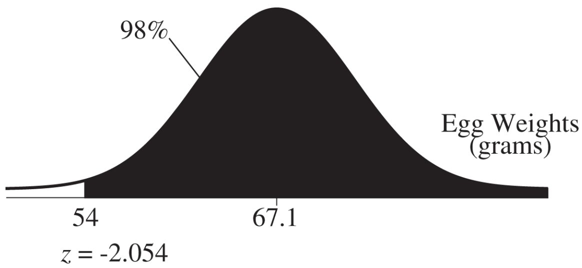
a)



$$z = \frac{y - \mu}{\sigma}$$
$$0.583 = \frac{54 - 50.9}{\sigma}$$
$$0.583\sigma = 3.1$$
$$\sigma = 5.317$$

According to the Normal model, the standard deviation of the egg weights for young hens is expected to be 5.3 grams.

b)



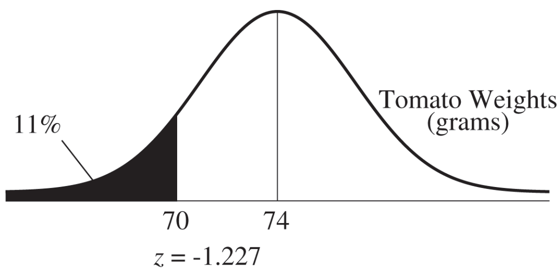
$$z=\frac{y-\mu}{\sigma}$$
$$-2.054=\frac{54-67.1}{\sigma}$$
$$-2.054\sigma=-13.1$$
$$\sigma=6.377$$

According to the Normal model, the standard deviation of the egg weights for older hens is expected to be 6.4 grams.

c) The younger hens lay eggs that have more consistent weights than the eggs laid by the older hens. The standard deviation of the weights of eggs laid by the younger hens is lower than the standard deviation of the weights of eggs laid by the older hens.

48. Tomatoes.

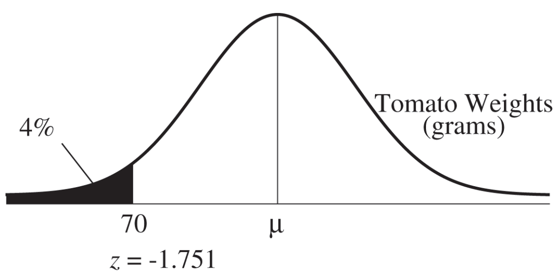
a)



$$z=\frac{y-\mu}{\sigma}$$
$$-1.227=\frac{70-74}{\sigma}$$
$$-1.227\sigma=-4$$
$$\sigma=3.260$$

According to the Normal model, the standard deviation of the weights of Roma tomatoes now being grown is 3.26 grams.

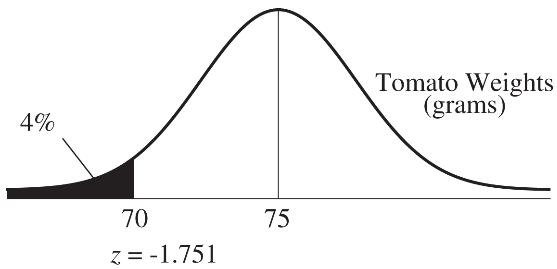
b)



$$z=\frac{y-\mu}{\sigma}$$
$$-1.751=\frac{70-\mu}{3.260}$$
$$-5.708=70-\mu$$
$$\mu=75.71$$

According to the Normal model, the target mean weight for the tomatoes should be 75.71 grams.

c)



$$z = \frac{y - \mu}{\sigma}$$

$$-1.751 = \frac{70 - 75}{\sigma}$$

$$\sigma = 2.856$$

According to the Normal model, the standard deviation of these new Roma tomatoes is expected to be 2.86 grams.

d) The weights of the new tomatoes have a lower standard deviation than the weights of the current variety. The new tomatoes have more consistent weights.