Top Ten Summation Formulas

	Name	Summation formula	Constraints
1.	Binomial theorem	$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$	integer $n \geq 0$
	Binomial series	$\sum_k \binom{\alpha}{k} x^k = (1+x)^\alpha$	$\lvert x\rvert < 1$ if $\alpha \neq$ integer $n \geq 0$.
2.	Geometric sum	$\sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$	$r \neq 1$
	Geometric series	$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$	$\lvert r\rvert < 1$
3.	Telescoping sum "Fundamental Theorem"	$\sum_{a \leq k < b} \Delta F(k) = F(b) - F(a)$ of summation calculus	integers $a \leq b$
4.	Sum of powers See related formulas.	$\sum_{a \leq k < b} k^{\underline{m}} = \frac{k^{\underline{m+1}}}{m+1}\Big\rvert_a^b$	integers $a \leq b$ integer $m \neq -1$
5.	Vandermonde convolution	$\sum_k \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$	integer n
6.	Exponential series	$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$	complex x
7.	Taylor series $a = 0$: Maclaurin series	$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(x)$	$\lvert x-a\rvert < R =$ radius of convergence
8.	Newton's advancing difference formula	$\sum_k \frac{\Delta^k f(a)}{k!} x^{\underline{k}} = \sum_k \binom{x}{k} \Delta^k f(a) = f(a+x)$	real a, x $f =$ polynomial
9.	Euler's summation formula	$\sum_{a \leq k < b} f(k) = \int_a^b f(x)\,dx + \sum_{k=1}^{m} \frac{B_k}{k!} f^{(k-1)}(x)\Big\rvert_a^b$ $+(-1)^{m+1} \int_a^b \frac{B_m(x - \lfloor x \rfloor)}{m!} f^{(m)}(x)dx$	integers $a \leq b$ integer $m \geq 1$
	$m = 1$: trapezoidal rule:	$\sum_{a \leq k < b} f(k) \approx \int_a^b f(x)\,dx - (1/2)[f(b) - f(a)]$	
10.	Inclusion-exclusion Also true if "P" = "#"	$P(\bigcup_{j=1}^{n} A_j) = \sum_{k=1}^{n} (-1)^{k-1} S_k$ where $S_k = \sum_{1 \leq j_1 < \cdots < j_k \leq n} P(\bigcap_{i=1}^{k} A_{j_i})$	events $A_1, \ldots, A_n$

Other Contenders and Related Formulas

Name	Summation formula	Constraints
Hypergeometric series	$F\left(\begin{matrix} a_1,\dots,a_m \\ b_1,\dots,b_n \end{matrix} \middle\vert z\right) = \sum_{k\ge 0} \frac{a_1^{\overline{k}}\cdots a_m^{\overline{k}}}{b_1^{\overline{k}}\cdots b_n^{\overline{k}}}\frac{z^k}{k!}$	
Sum of powers	$\sum_{k=0}^{n-1} k^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k n^{m+1-k}$	integer $n \ge 1$
Thus	$\sum_{k=0}^{n-1} k^m = \frac{n^{m+1}}{m+1} +$ lower order terms	
Formulas relating factorial powers and ordinary powers		
Stirling numbers of the second kind	$x^n = \sum_k \left\{ \begin{matrix} n \\ k \end{matrix} \right\} x^{\underline{k}}$	integer $n \ge 0$
Stirling numbers of the first kind	$x^{\overline{n}} = \sum_k \left[\begin{matrix} n \\ k \end{matrix} \right] x^k$	integer $n \ge 0$
Stirling numbers of the first kind	$x^{\underline{n}} = \sum_k \left[\begin{matrix} n \\ k \end{matrix} \right] (-1)^{n-k} x^k$	integer $n \ge 0$
Bernoulli numbers	$\begin{array}{c\|cccccc} n & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline B_n & 1 & -1/2 & 1/6 & 0 & -1/30 & 0 \end{array}$ $\begin{array}{c\|cccccc} n & 6 & 7 & 8 & 9 & 10 & 11 \\ \hline B_n & 1/42 & 0 & -1/30 & 0 & 5/66 & 0 \end{array}$	$B_{12} = -691/2730$
Implicit recursion	$\sum_{j=0}^{m}\binom{m+1}{j} B_m = [m=0]$	
Generating function	$\frac{z}{e^z-1} = \sum_{n\ge 0} B_n \frac{z^n}{n!}$	
Bernoulli polynomial	$B_m(x) = \sum_k \binom{m}{k} B_k x^{m-k}$	