Lesson Plan: Developing An Understanding Of Standard Deviation And The Effects Of Changing The Mean And Standard Deviation

Aim

To enable students to develop an understanding of standard deviation as a measure of "average distance of all data values from the mean".

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Background

Students are likely to have encountered how to calculate the mean and standard deviation of a small data set and appreciate what they measure.
They need not know the formula for calculating standard deviation; use of statistical functions on a scientific calculator during this lesson is fine (see instruction sheet in this section). The formula for calculation of standard deviation can be introduced after this activity.

Rationale

Considering the standard deviation of a very small data set, and seeing how it changes as items within the data set do, can be an 'easy' way of introducing this measure of spread and getting a feel for it, before introducing formulas and calculations.

In this lesson students spend time investigating what happens to the mean and standard deviation when the data changes and how to change data to change mean & standard deviation.

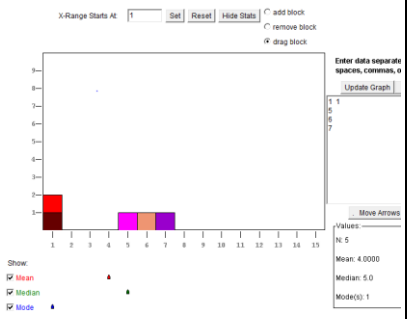
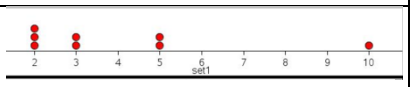
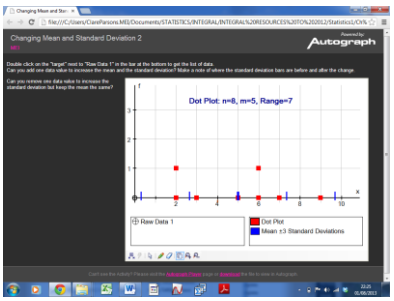
By the end of the lesson

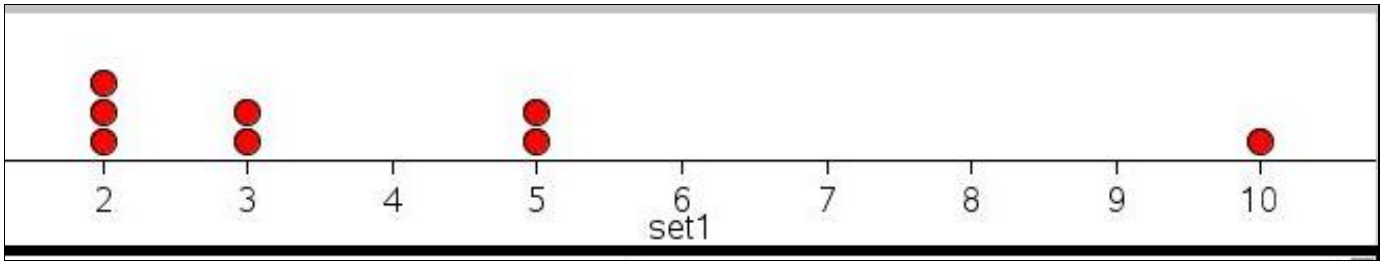
Students should

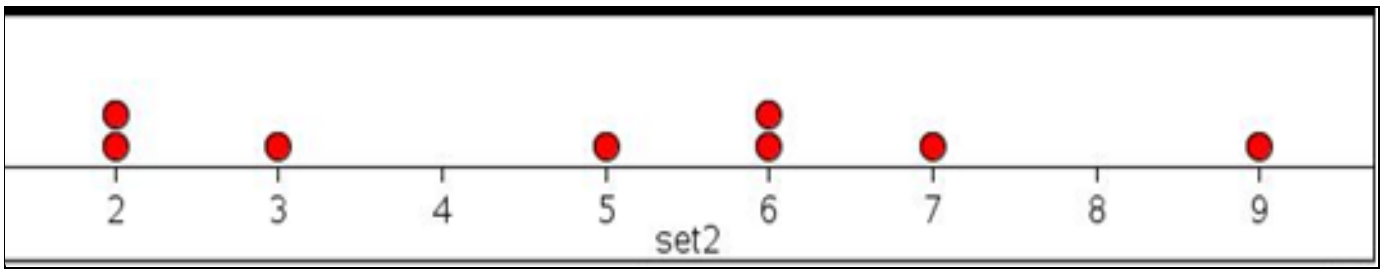
- have a deeper understanding of the difference between measures of central tendency and spread.
- have developed a competence in using their basic scientific calculators to find the mean and standard deviation of a small data set.

The next lesson – students can be introduced to the formula for calculating standard deviation

Resources: display pdf (below), internet access, Mini white boards, pens, wipes, scientific calculators, one worksheet per student (p6),

	CONTENT	ACTIVITY/ QUESTIONS	COMMENTARY
INTRODUCTION (15 mins)	Exploring the mean mode median of small data sets	Give me a set of 5 numbers whose mean is 4 median is 5 mode is 1 Is there a unique answer? Check responses on Plop it	http://www.shodor.org/interactivate/activities/PlopIt/ this animation allows you to drag the blocks and displays the changing averages (click show stats)
		question ideas: a) what will happen if I add another block of size 4? 1? 5? b) where would I add 2 more blocks to keep median as 5, the mean as 4? c) what will happen if I add 3 to all the original blocks values? (move the blocks along 3 places to the right) 4? d) what will happen if I double all the original blocks' values?	
MAIN ACTIVITY (30 mins)	worksheet – investigation (p6)	Display 2 diagrams and data sets (below) and discuss which belongs to which and the important of the values of the summary statistics shown. Discuss what stand dev measures. p3 Run thorugh use of calculators (p4 for basic casio calculators, see instruction manuals for other models) Set the questions, telling the students whether to use s or σ for the standard deviation.p6	
PLENARY(15 mins)	class discussion	Once students have tackled the questions, collect & display their solutions and see whether they agree that all suggested solutions work. Questions to ask include • Will all these ways work to …… (whatever the question says) • Is there another way of doing this? • Why do you think this will work? • Why do you think this won't work?	Two Autograph files are provided which have the data; these allow students to experiment more and are especially useful for showing the consequences of their actions in class discussion 
	plenary question p5	A set of 11 data values has a mean of 8 and a standard deviation of 2. At least one of the data items is the value 8. If this single value is removed, without replacement from the data set, what will happen to the standard deviation of the remaining data? The standard deviation: A) increases B) decreases C) is unchanged D) insufficient information is given	





$\bar{x}$	4
$\sum x$	32
$\sum x^2$	180
s	2.726
σ	2.549
n	8
Min X	2
Q_1	2
median X	3
Q_3	5
Max X	10

$\bar{x}$	5
$\sum x$	40
$\sum x^2$	224
s	2.507
σ	2.345
n	8
Min X	2
Q_1	2.5
median X	5.5
Q_3	6.5
Max X	9

MODE 2 (STAT)

Option 1: 1 - VAR
(for lots of 'x' s given in a list OR in a frequency table)

Put data into the table by inputting number followed by = and use the cursor to move between columns and rows

Don't worry about pressing AC –your data will remain in the table!

	x	FREQ
1		
2		
3		

IGNORE THE FRERQUENCY COLUMN FOR THIS LESSON

PRESS AC, then **STAT** (2nd function above 1)
from menu below choose what you want to know

1:Type	2: Data
3: Sum	4: Var
5: MinMax	

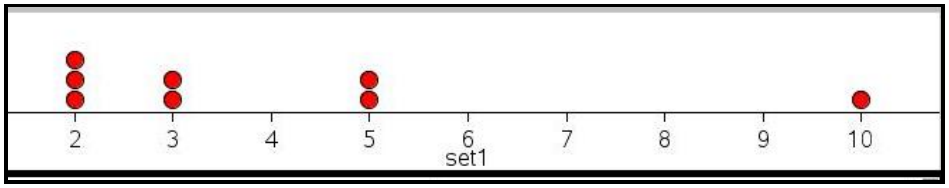
4: Var
gives n, $\bar{x}$, σx , sx

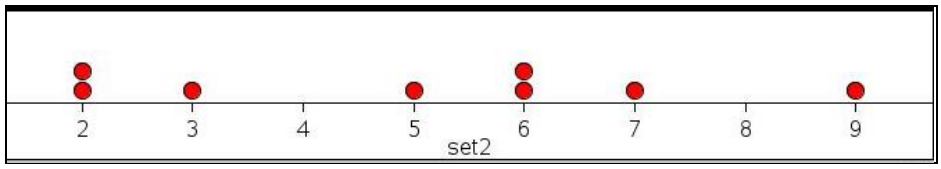
$note: \sigma_x \; uses \; n \; as \; a \; divsor \; in \; finding \; standard \; deviation \; , \; s_x \; uses \; n - 1.$

A set of 11 data values has a mean of 8 and a standard deviation of 2. At least one of the data items is the value 8. If this single value is removed, without replacement from the data set, what will happen to the standard deviation of the remaining data?

The standard deviation:
A) increases
B) decreases
C) is unchanged
D) insufficient information is given

Changing the Mean and Standard Deviation





1. Which set of statistics belongs with which dot plot?

$\bar{x}$	4
$\sum x$	32
$\sum x^2$	180
s	2.726
σ	2.549
n	8
Min X	2
Q_1	2
median X	3
Q_3	5
Max X	10

$\bar{x}$	5
$\sum x$	40
$\sum x^2$	224
s	2.507
σ	2.345
n	8
Min X	2
Q_1	2.5
median X	5.5
Q_3	6.5
Max X	9

2. Can you add one data value to data set 2 to increase the mean and the standard deviation?

3. Can you add one data value to data set 1 to reduce the standard deviation but keep the mean the same?

4. Can you remove one data value from data set 2 to increase the standard deviation but keep the mean the same?

5. Can you remove two data values from data set 1 to increase the standard deviation but keep the mean the same?

6. Can you remove one data value from data set 1 to increase the standard deviation but keep the mean the same?

7. **Challenge question**
Can the standard deviation be more than the range? Try to give a convincing explanation.

Answers

1. The sets of statistics are in the same order as the dot plots. This is an opportunity to ask students how they know which set of statistics goes with which dot plot and so check that they understand the connection between the numerical statistics and the dot plot.

2. Data set 2 has mean 5. Adding any value greater than 5 will increase the mean but not all of these will also increase the standard deviation (8 or 9 will). This is an opportunity to discuss the standard deviation as a measure of "average distance of all the values from the mean".

3. Adding a value at 4 will keep the mean the same but reduce the standard deviation. Why? Is there another way to do this question?

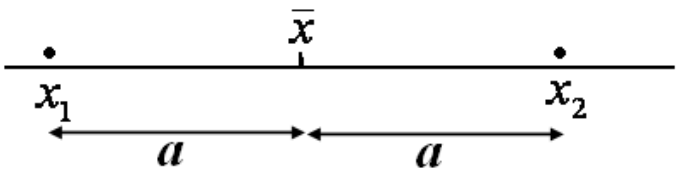
4. Set 2 has mean 5. Taking this value away will keep the mean the same but increase the standard deviation. Why?

5 Set 1 has mean 4. Taking a 3 and a 5 away will keep the mean at 4. Why? It will also increase the standard deviation. Why? Is there another pair that could be removed to keep the mean the same? If so, how would taking them away change the standard deviation?

6. To keep the mean of set 1 the same, you would need to take away a 4 but there isn't one so this cannot be done.

7. Here is a convincing argument rather than a general proof.

For a data set to have a large standard deviation, the points should be as far away from the mean as possible. Suppose there are two values, the mean is halfway between.



Suppose the distance of each value from the mean is *a*.

The range is $2a$

The standard deviation is $\sqrt{\frac{a^2+a^2}{2-1}} = a\sqrt{2}$

The standard deviation is smaller than the range in this case but this is a case where we have deliberately tried to make the standard deviation as big as possible.