

## Solving Compound Interest Problems

### What is Compound Interest?

If you walk into a bank and open up a savings account you will earn interest on the money you deposit in the bank. If the interest is calculated once a year then the interest is called “simple interest”. If the interest is calculated more than once per year, then it is called “compound interest”.

### Compound Interest Formula

The mathematical formula for calculating compound interest depends on several factors. These factors include the amount of money deposited called the principal, the annual interest rate (in decimal form), the number of times the money is compounded per year, and the number of years the money is left in the bank. These factors lead to the formula

$$FV = P \left( 1 + \frac{r}{n} \right)^{nt}$$

FV = future value of the deposit  
P = principal or amount of money deposited  
r = annual interest rate (in decimal form)  
n = number of times compounded per year  
t = time in years.

### Solving Compound Interest Problems

To solve compound interest problems, we need to take the given information and plug the information into the compound interest formula and solve for the missing variable. The method used to solve the problem will depend on what we are trying to find. If we are solving for the time, t, then we will need to use logarithms because the compound interest formula is an exponential equation and solving exponential equations with different bases requires the use of logarithms.

**Examples** – Now let’s solve a few compound interest problems.

**Example 1:** If you deposit \$4000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?

$$FV = 4000 \left( 1 + \frac{0.06}{4} \right)^{4(5)}$$

Plug in the given information,  $P = 4000$ ,  $r = 0.06$ ,  $n = 4$ , and  $t = 5$ .

$$FV = 4000(1.015)^{20}$$

Use the order of operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 4000(1.346855007)$$

$$FV = 5387.42$$

Round your final answer to two decimal places.

After 5 years there will be \$5387.42 in the account.

**Example 2:** If you deposit \$6500 into an account paying 8% annual interest compounded monthly, how much money will be in the account after 7 years?

$$FV = 6500 \left( 1 + \frac{0.08}{12} \right)^{12(7)}$$

Plug in the giving information,  $P = 6500$ ,  $r = 0.08$ ,  $n = 12$ , and  $t = 7$ .

$$FV = 6500(1.006666666)^{84}$$

Use the order of operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 6500(1.747422051)$$

$$FV = 11358.24$$

Round your final answer to two decimals places.

After 7 years there will be \$11358.24 in the account.

**Example 3:** How much money would you need to deposit today at 9% annual interest compounded monthly to have \$12000 in the account after 6 years?

$$12000 = P \left( 1 + \frac{0.09}{12} \right)^{12(6)}$$

Plug in the giving information,  $FV = 12000$ ,  $r = 0.09$ ,  $n = 12$ , and  $t = 6$ .

$$12000 = P(1.0075)^{72}$$

Use the order of operations to simplify the problem. If the problem has decimals, keep as many decimals as possible until the final step.

$$12000 = P(1.712552707)$$

$$P = 7007.08$$

Divide and round your final answer to two decimals places.

You would need to deposit \$7007.08 to have \$12000 in 6 years.

In the last 3 examples we solved for either FV or P and when solving for FV or P is mostly a calculator exercise. Be careful not to try and type too much into the calculator in one step and let the calculator store as many decimals as possible. Do not round off too soon because your answer may be slightly off and when dealing with money people want every cent they deserve.

In the next 3 examples we will be solving for time,  $t$ . When solving for time, we will need to solve exponential equations with different bases. Remember that to solve exponential equations with different bases we will need to take the common logarithm or natural logarithm of each side. Taking the logarithm of each side will allow us to use Property 5 and rewrite the problem as a multiplication problem. Once the problem is rewritten as a multiplication problem we should be able to solve the problem.

**Example 4:** If you deposit \$5000 into an account paying 6% annual interest compounded monthly, how long until there is \$8000 in the account?

$$8000 = 5000 \left( 1 + \frac{0.06}{12} \right)^{12t}$$

Plug in the giving information,  $FV = 8000$ ,  $P = 5000$ ,  $r = 0.06$ , and  $n = 12$ .

$$8000 = 5000(1.005)^{12t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$1.6 = 1.005^{12t}$$

Divide each side by 5000.

$$\log(1.6) = \log(1.005^{12t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 1.6 = (12t)(\log 1.005)$$

$$\frac{\log 1.6}{\log 1.005} = 12t$$

Divide each side by  $\log 1.005$ .

$$94.23553232 \approx 12t$$

Use a calculator to find  $\log 1.6$  divided by  $\log 1.005$ .

$$t \approx 7.9$$

Finish solving the problem by dividing each side by 12 and round your final answer.

It will take approximately 7.9 years for the account to go from \$5000 to \$8000.

**Example 5:** If you deposit \$8000 into an account paying 7% annual interest compounded quarterly, how long until there is \$12400 in the account?

$$12400 = 8000 \left( 1 + \frac{0.07}{4} \right)^{4t}$$

Plug in the giving information,  $FV = 12400$ ,  $P = 8000$ ,  $r = 0.07$ , and  $n = 4$ .

$$12400 = 8000(1.0175)^{4t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$1.55 = 1.0175^{4t}$$

Divide each side by 8000.

$$\log(1.55) = \log(1.0175^{4t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 1.55 = (4t)(\log 1.0175)$$

$$\frac{\log 1.55}{\log 1.0175} = 4t$$

Divide each side by  $\log 1.0175$ .

$$25.26163279 \approx 4t$$

Use a calculator to find  $\log 1.55$  divided by  $\log 1.0175$ .

$$t \approx 6.3$$

Finish solving the problem by dividing each side by 4 and round your final answer.

It will take approximately 6.3 years for the account to go from \$8000 to \$12400.

**Example 6:** At 3% annual interest compounded monthly, how long will it take to double your money?

At first glance it might seem that this problem cannot be solved because we do not have enough information. It can be solved as long as you double whatever amount you start with. If we start with \$100, then  $P = \$100$  and  $FV = \$200$ .

$$200 = 100 \left( 1 + \frac{0.03}{12} \right)^{12t}$$

Plug in the giving information,  $FV = 200$ ,  $P = 100$ ,  $r = 0.03$ , and  $n = 12$ .

$$200 = 100(1.0025)^{12t}$$

Use the order of operations to simplify the problem. Keep as many decimals as possible until the final step.

$$2 = 1.0025^{12t}$$

Divide each side by 100.

$$\log(2) = \log(1.0025^{12t})$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\log 2 = (12t)(\log 1.0025)$$

$$\frac{\log 2}{\log 1.0025} = 12t$$

Divide each side by  $\log 1.0025$ .

$$277.6053016 \approx 12t$$

Use a calculator to find  $\log 2$  divided by  $\log 1.0025$ .

$$t \approx 23.1$$

Finish solving the problem by dividing each side by 12 and round your final answer.

At 3% annual interest it will take approximately 23.1 years to double your money.

### **Addition Examples**

If you would like to see more examples of solving compound interest problems, just click on the link below.

[Additional Examples](#)

## Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

**Problem 1:** If you deposit \$4500 at 5% annual interest compounded quarterly, how much money will be in the account after 10 years?

**Problem 2:** If you deposit \$4000 into an account paying 9% annual interest compounded monthly, how long until there is \$10000 in the account?

**Problem 3:** If you deposit \$2500 into an account paying 11% annual interest compounded quarterly, how long until there is \$4500 in the account?

**Problem 4:** How much money would you need to deposit today at 5% annual interest compounded monthly to have \$20000 in the account after 9 years?

**Problem 5:** If you deposit \$6000 into an account paying 6.5% annual interest compounded quarterly, how long until there is \$12600 in the account?

**Problem 6:** If you deposit \$5000 into an account paying 8.25% annual interest compounded semiannually, how long until there is \$9350 in the account?

[Solutions to Practice Problems](#)