NEWTON'S THIRD LAW AND NORMAL FORCE

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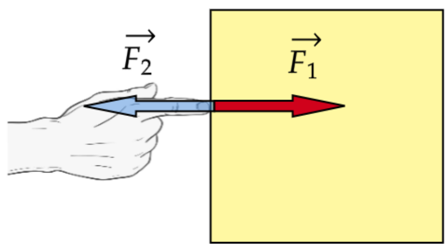
Theory recap

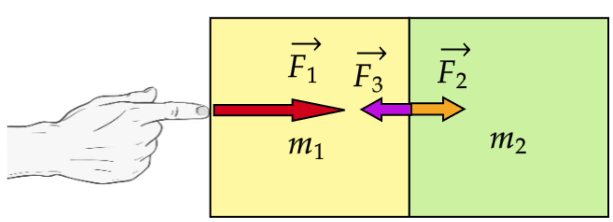
Newton's third law. Last time we learned Newton's first and second laws. We encountered the concept of force which describes strength of interaction between objects. Interaction always occurs between two objects but at our last class we ignored one of them. Today we'll learn what could be said about it.

Assume some two bodies, for example your hand and a block are in contact. You are pushing the block to the right, so you are acting on the block with a force $\vec{F_1}$ directed to the right. But you feel that block is actually pushing you as well. Force $\vec{F_2}$ which the block exerts on your hand is directed to the left. What is the magnitude of this force compared to the one you push the block with? In other words, what is the magnitude of $\vec{F_2}$ compared to $\vec{F_1}$?



The answer to this question is given by **Newton's third law**: basically, $\vec{F_2} = -\vec{F_1}$. Let me remind that if one vector is equal to minus the other it means that they point in the opposite directions but their magnitudes are the same. This means that the block pushes your hand to the left with exactly the same force as your hand pushes the block to the right. Newton's third law is usually stated as follows: for any action there is an equal and opposite reaction. Action here means force with which object 1 acts on object 2 and reaction - force with which object 2 acts on object 1.

Let us consider an example to illustrate an application of Newton's third law. Suppose there are two blocks right next to each other. One of them has mass m_1 and the other one has mass m_2. We push the first block with force $\vec{F_1}$ to the right. With what acceleration will the blocks move?



There are actually two ways to solve this problem and it is instructive to show both of them. Th first option is to say that since blocks move together as a whole, we could consider them together as one big block of mass $m_1 + m_2$. Then the only force acting on this "big block" is $\vec{F_1}$ and applying Newton's second law we could find the acceleration:

$$(m_1 + m_2)a = F_1 \Rightarrow a = \frac{F_1}{m_1 + m_2}$$

This is the answer. But let us try another approach and treat the blocks separately. How does the second block know that it has to move? Our hand does not push it. It is the first block that pushes it - with some force $\vec{F_2}$ (which we don't know yet). If we knew $\vec{F_2}$ we would be able to find the acceleration for the block with mass m_2 since by Newton's second law

$$m_2 a = F_2 \tag{1}$$

But we do not know F_2. Is there some information that we did not use from which we could extract F_2? Actually, there is! We know that the block m_1 also moves with the same acceleration a, so we could write Newton's second law for it. There are two forces acting on this block: $\vec{F_1}$ is the force of our push and $\vec{F_3}$ is the force with which the second block acts on the first block. Now Newton's third law comes in handy: it states that $\vec{F_3} = -\vec{F_2}$. Indeed, $\vec{F_2}$ is the force with which the first block pushes the second block while $\vec{F_3}$ is the force with which the second block pushes the first block. So one of them is action and the other is reaction and by Newton's third law they are equal and opposite.

We see that there are two forces acting on the block m_1, namely $\vec{F_1}$ and $\vec{F_3}$. The net force is $F_1 - F_3 = F_1 - F_2$ and directed to the right, so by Newton's second law for the first block

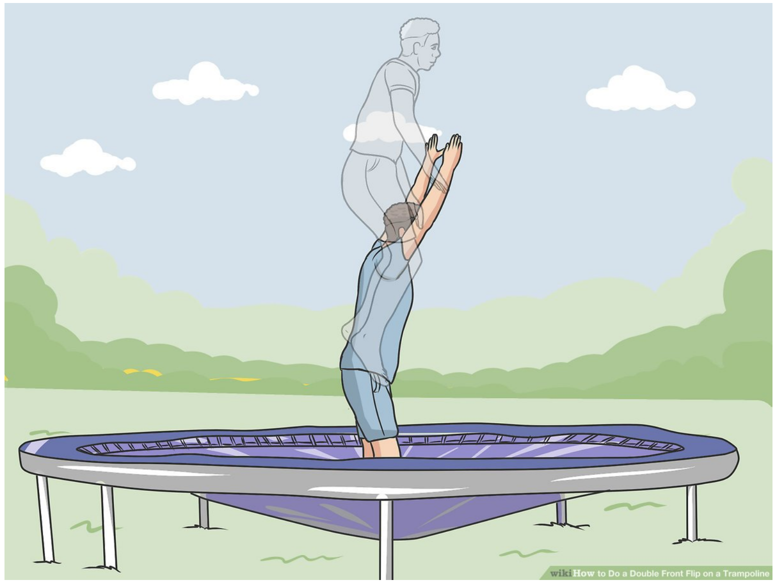
$$m_1 a = F_1 - F_2 \tag{2}$$

Together equations (1) and (2) make a system of two linear equations with two unknowns: a and F_2. Since we are not interested in F_2 we could find a by adding them up, so that F_2 cancels. We then find

$$a = \frac{F_1}{m_1 + m_2}$$

This is the same answer as the one we obtained using the first method (considering two blocks as one). This is of course how it should be and this was made possible by Newton's third law. This example also demonstrated a general procedure of how one could solve problems with various forces : draw all the forces acting on all relevant objects, relate the ones which are action an reaction by Newton's third law. Then use Newton's second law for these objects.

Normal force. Last time we also mentioned that there are different kinds of forces. We have already considered gravitational force. Now let us discuss another force - normal force. When you stand on the floor gravity pulls you down but you don't fall through the floor. What does not let you fall? Of course it is the floor, you can feel that it pushes you feet up supporting the whole weight of your body.



But how does it happen? Where does this force come from? To understand this, let us take a look at a trampoline. When you stand on a trampoline, it bends under your weight in order to support it. The force with which the trampoline acts on you is due to its elastic properties. Now if you imagine trampoline being stiffer, it will bend less. You could think of a trampoline so stiff, that you will not see its deformation under your weight with a naked eye. But you know it is still there - elastic forces are supporting your weight even if you don't see the deformation.

For the floor the situation is the same as for a very stiff trampoline. Floor bends very slightly under your weight and the resulting elastic force supports your weight and don't let you go through the floor. For the same reason an apple doesn't fall through a plate, the plate does not fall through a table, the table does not fall through the floor and a house does not fall through the ground. In all these cases the normal force counteracts against gravity force and as a result an object rests.

The fact that normal force plays against gravity allows us to calculate it in many situations. Assume an apple of mass m rests on a table. There are two forces acting on the apple - gravitational force mg and normal force from the table N. Since the apple is at rest, its' acceleration is zero. So net force must be zero. Gravitational force is directed down while normal force is directed up. So to find the net force down we subtract normal force from gravitational force and obtain

$$mg - N = 0 \;\Rightarrow\; N = mg$$

As we have learned today, Newton's third law tells us that when the table acts with normal force $\vec{N}$ on the apple, the apple acts with force $\vec{P} = -\vec{N}$ on the table. This force $\vec{P}$ is called weight. From the above equation we see that when the apple is at rest, its weight is equal to the gravitational force: $P = mg$. However, it is important not to confuse weight and gravitational force because they act on two different objects. Gravitational force mg acts on the apple. Weight P acts on the table. You need to always keep this distinction in mind in order to avoid confusion. The homework problem about accelerating elevator below further illustrates the distinction between gravity force and weight.

Homework

1. Why can you exert greater force on the pedals of a bicycle if you pull up on the handlebars?

2. You are staying in an elevator on big spring scales. The elevator starts moving with acceleration. Your weight (as the spring scales show) has increased 1.5 times. Find the acceleration of the elevator (both magnitude and direction). It might help you to draw a picture with all the forces.

3. The gravity force on the surface of the Moon is about 6 times less than this on the Earth. What will happen with your weight and mass on the Moon?

***4.** Find the force with which a 1 kg block gravitationally attracts the Earth. What is Earth's acceleration due to this force? Earth mass is approximately $6 \cdot 10^{24}$ kg.