PUNNETT SQUARES AND BINOMIALS

Teacher: Paul McNally
School: Carusi Middle School
Class: Enriched Algebra I
Lesson: Squaring a binomial (1 day)

Defining the Problem: To improve students' understanding of squaring a binomial. Students normally have misconceptions/problems when squaring a binomial because they think that they can distribute the power to each term in the binomial. This misconception stems from the unit on monomials, where a power *is* distributed to a monomial.

Learning Goals: The overarching goals of this lesson will be if the students can understand the pattern of a squared binomial by visualizing it geometrically with a Punnett square.

Objectives: The students will understand the pattern of a squared binomial using geometric representations (Punnett squares) and extend the concept by exploring the similarities between the binomial pattern and the pattern in Pascal's triangle.

Prior Knowledge: Students have learned how to multiply binomials using the FOIL pattern and multiply higher degree polynomials using the distributive property. Students have also learned about mitosis, meiosis, dominant genes, recessive genes, and incomplete dominance.

New Jersey Core Curriculum Standard(s) and explanation.
4.1.8 A *Numerical Operations*
　　　* *Understand and apply laws of exponents to simplify expressions involving numbers raised to powers*. (How) Students will perform manipulations involving the laws of exponents to simplify the polynomial expressions.
4.2.8 A *Geometric Properties*
　　　* *Use geometric models to represent real-world situations and objects to solve problems using those models. (*How) Students will use geometric models to solve/represent the special product of a binomial
4.3.8 A *Patterns and Algebra*
　　　* *Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers*. (How) Students will recognize the pattern of a squared binomial and extend it to Pascal's triangle.
4.3.8.D *Patterns and Algebra*
　　　**Create, evaluate, and simplify algebraic expressions involving variables.* (How) Students will simplify squared binomial expressions.
4.4.8 B *Data Analysis and Probability*
　　　**Interpret probabilities as ratios, percents, and decimals.* (How) Students will observe Punnett squares to determine probability and use algebra to verify the probability.
4.5. ABCD *Mathematical Processes*

*A, *Learn mathematics through problem solving, inquiry, and discovery; B,*
Communicate their mathematical thinking coherently and clearly to peers,
teachers, and others, both orally and in writing; C, Recognize recurring themes
across mathematical domains (e.g., patterns in number, algebra, and geometry);
D, *Recognize that mathematical facts, procedures, and claims must be justified.*

Anticipation of Questions: I expect students to have difficulty remembering the pattern of
a squared binomial. Some of the questions I anticipate the students asking are as follows:
      1. Why can't you just distribute the power like we did when we did monomials?
      2. Why are there two middle terms when you square a binomial?
      3. How is a Punnett square similar to a squared binomial?
*More questions will arise as the lesson proceeds.

Vocabulary:
      1. Binomial
      2. Polynomial
      3. Punnett square
      4. Homozygous
      5. Heterozygous

Motivational questions:
      1. Why do we resemble one parent more than they other?
      2. Why is there sometimes a mix of looking like both parents?

Phase of Inquiry:
      Punnett squares are diagrams that are used to show the possible ways that genes
can combine at fertilization. In a Punnett square, **dominant** genes are shown in capital
letters. **Recessive** genes are shown with lowercase letters. Letters representing parents'
genes are placed on the two of the outer sides of the Punnett square. Letters inside the
squares of the Punnett square show the possible gene combinations of their offspring.
      The Punnett square below represents an area model of a cross between two
heterozygous pea plants. The Punnett square shows all the possible outcomes of there
offspring and the probability of each scenario. Let *T* represent the dominant gene for
tallness, and let *t* represent the recessive gene for shortness. Notice that there are three
dark green squares inside the Punnett square. The reason for this is because *T* is the
dominant gene.

	T	t
T	TT	Tt
t	Tt	tt

Q: By observing the Punnett square, what is the probability of having a tall pea plant
from this cross, and the probability of having a short pea plant?

A: Since each square represents $\left(\dfrac{1}{4}\right)$ (25%) of the total area, the probability of getting a tall pea plant is $3 \times \left(\dfrac{1}{4}\right) = 75\%$. Therefore, the probability of short pea plant is $\left(\dfrac{1}{4}\right)$ (25%).

- After an introduction into Punnett squares, I would explain to them that Punnett squares are an excellent geometric representation of "squaring a binomial". Learning the special pattern of "squaring a binomial" can be very beneficial because it will save time and make "factoring" and "completing the square" much easier to understand in the future. I anticipate that most students will have the following misconception because they are familiar with distributing a power in a monomial: $(a+b)^2 \neq a^2 + b^2$

Introduction of Special Products of Polynomials
$$(a+b)^2 = a^2 + 2ab + b^2$$

- The area model shown below gives a geometric representation of a squared binomial: $(a+b)^2 = a^2 + 2ab + b^2$

- The Punnett square below is an example of **incomplete dominance**. It is an area model that shows the possible results of crossing two *pink* snapdragons, each with one red gene R and one with one white gene W. Each parent snapdragon passes along only one gene for color to its offspring.

Q: How can the square of a binomial can be used to model this Punnett square?

A: Each parent snapdragon has half red genes and half white genes. You can model the genetic makeup of each parent as $0.5R + 0.5W$. The genetic makeup of the offspring can be modeled by $(0.5R + 0.5W)^2$

$$(0.5R + 0.5W)^2 = (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2$$

$$= 0.25R^2 + 0.5RW + 0.25W^2$$
$$\text{(red)} \quad \text{(pink)} \quad \text{(white)}$$

Q: What does this model tell use about the probability of each event, and how is this model consistent with the Punnett square?

A: This model implies that the probability of a red snapdragon offspring will be $\left(\dfrac{1}{4}\right)$ (25%), a pink offspring will be $\left(\dfrac{1}{2}\right)$ (50%), and a white offspring will be $\left(\dfrac{1}{4}\right)$ (25%). This is consistent with the area model because 25% of the area is red (RR), 50% of the area is pink (RW), and 25% of the area is white (WW).

Closure/Extension:

      Q: This is an amazing that pattern because it can be shown algebraically and geometrically. Now that you have learned the pattern of a squared binomial, can anyone think of this pattern of coefficients 1 : 2 : 1 in another part of mathematics?

      A: Pascal's triangle. Notice the coefficients when you perform various powers to the same binomial. The pattern of coefficients represents Pascal's triangle. Pascal's triangle is a pattern formed by adding two adjacent numbers and writing the solution below the two numbers.

$(a+b)^0$	$=$	1
$(a+b)^1$	$=$	1 1
$(a+b)^2$	$=$	**1 2 1**
$(a+b)^3$	$=$	1 3 3 1
$(a+b)^4$	$=$	1 4 6 4 1
• • •	$=$	• • • • • •

      Q: Using Pascal's triangle, determine the coefficients of a binomial that is raised to the 5th power? 6th power?

A: 5th power = 1 5 10 10 5 1

      6th power = 1 6 15 20 15 6 1

Enriched Algebra I   Name: _____
Punnett Square Activity   Date: _____

1. In tigers, the normal color gene C is dominant and the white color gene c is recessive. This means that a tiger whose color genes are CC or Cc will have normal coloring, and a tiger whose color genes are cc will be white.

a. Create a Punnett square that represents the cross between two heterozygous $(Cc \times Cc)$ tigers.

b. Find a model that can be used to represent the Punnett square and write the model as a polynomial.

c. Use the polynomial to determine what percent of the offspring will have normal coloring? What percent will be white?

2. In chickens, there is an incomplete dominance between the normal-feathered gene F and the frizzle-feathered gene f.

a. Create a Punnet square that represents the cross between two heterozygous $(Ff \times Ff)$ chickens.

b. Find a model that can be used to represent the Punnett square and write the model as a polynomial.

c. Use the polynomial to determine the probability of all possible outcomes.

3. One cat carries heterozygous, long-haired traits (Ss), and its mate carries homozygous short-haired traits (ss). Use a Punnett square to determine the probability of one of their offspring having long hair.

Probability: _____

4. One cat carries homozygous, long-haired traits (SS), and its mate carries homozygous short-haired traits (ss). Use a Punnett square to determine the probability of one of their offspring having short hair.

Probability: _____

5. One flower is heterozygous red (Rr) and it is crossed with a homozygous white (rr) plant. Use a Punnett square to determine the probability of one of their offspring having a red color.

Probability: _____

6. One flower is homozygous red (RR) and it is crossed with a homozygous white (rr) plant. Use a Punnett square to determine the probability of one of their offspring having a white color.

Probability: _____