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A modified Lotka–Volterra model for the evolution of coordinate symbiosis in energy enterprise

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Abstract. Recent developments in energy markets make the operating industries more dynamic and complex, and energy enterprises cooperate more closely in the industrial chain and symbiosis. In order to further discuss the evolution of coordinate symbiosis in energy enterprises, a modified Lotka-Volterra equation is introduced to develop a symbiosis analysis model of energy groups. According to the equilibrium and stability analysis, a conclusion is obtained that if the upstream energy group and the downstream energy group are in symbiotic state, the growth of their utility will be greater than their independent value. Energy enterprises can get mutual benefits and positive promotions in industrial chain by their cooperation.

1. Introduction
Recent developments in energy markets make the operating environment of process industries more dynamic and complex (Merkert L et al. 2015). Under the new situation and stiff global competition, energy enterprises cooperate in industrial chain and symbiosis. Sokka et al. (2011) point out that the relevance of industrial symbiosis is important in sustainable development. Yu et al. (2014) assume that industrial symbiosis study is evolved from practice-oriented research toward coherent theory building with systematic and expanded topics. Yazan (2016) states that joint production chains can increase the efficiency in the energy area. Fraccascia et al. (2017) propose that by taking advantage of industrial symbiosis, two different firms can obtain mutual environmental and economic benefits. Accordingly, this paper introduces Lotka-Volterra model to further discuss the evolution of coordinate symbiosis in energy enterprises, helping improve the utility and efficiency in energy area.

2. Methodology
The Lotka-Volterra model is a form of Logistic Equation developed according to the theory from Alfred Lotka (A.J.Lotka, 1925) and Victoria Wall Taylor (V.Volterra 1926). It is proposed to analyze the diffusion process with mathematical description. The main idea of Lotka-Volterra model is based

on the independence and interaction of different species. The resolution of the growth and relationship between species comes from the benefit analysis of positive and negative values among groups, transforming multiple populations of logistic equation.

In this research, energy enterprises can be described as a set of communities. Then, the evolutionary path of relationships in those energy communities can be represented by mathematical models. Assuming that there are two kinds of enterprise group, *a* and *b*, in a certain community, and in this situation, *N* stands for the number of the group population, and *t* means growth rate (or growth variables) which denotes the growth over time, respectively, r_a and r_b to group *a* and *b*. Moreover, we assume that the limitation size of the group is H, which means the energy community will stop growing when the population is H. Accordingly, by population theory with evolution equation, Lotka-Volterra model, an application equation set can be obtained as follow:

$$\begin{cases} \dfrac{dN_a}{dt} = r_a N_a (1 - \dfrac{N_a}{H_a}) \\ \dfrac{dN_b}{dt} = r_b N_b (1 - \dfrac{N_b}{H_b}) \end{cases} \tag{1}$$

The general form of the equations is $\frac{dN}{dt} = rN(1 - \frac{N}{H})$, which describes the growth of one unit is subject to the constraints of ecology. When the population grows in a single unit, it will cause pressure on the population and decrease the unit growth rate of the community by $\frac{r}{H}$. Besides, a correction term, $(1 - \frac{N}{H})$, is introduced into the model, representing the growth potential of the population. When the number of energy enterprise group increases to the maximum value *H*, $(1 - \frac{N}{H})$ will decrease to *0*, and the growth potential of the group will become 0, ceasing the growth of energy enterprise.

According to the nature of function, when $N = \frac{H}{2}$, $\frac{dN}{dt}$ will get maximum. Meanwhile, when $0 \le N \le \frac{H}{2}$, energy group is in accelerated growing period, and $\frac{dN}{dt}$ will increase with the growth of group; However, when $\frac{H}{2} \le N \le H$, it will be a decelerating growing period for energy enterprise group and $\frac{dN}{dt}$ will decrease with the growing of the population. After that, when $N = H$, the community get its saturation. In the energy enterprise community Lotka Wohl Taylor Lotka-Volterra model, if group *a* and group *b* are independent of each other, each enterprise group is not affected by other effects, then the part of growth will be $\frac{dN}{dt} = rN$; if group *a* and group *b* affects each other mutually, then a complement correction for interaction, $g\frac{N}{H}$, is required. As a result, the function of group *a* will expand to $\frac{dN_a}{dt} = r_a N_a (1 - \frac{N_a}{H_a} + g_a \frac{N_b}{H_b})$, while the function of group *b* will expand to $\frac{dN_b}{dt} = r_b N_b (1 - \frac{N_b}{H_b} + g_b \frac{N_a}{H_a})$.

3. Analysis

According to the literature review of energy enterprise and previous empirical research, energy enterprises in this area become interdependent and symbiotic around their value chain. In the case of dependence and symbiosis, energy groups are directly related to the business cooperation or industry chain, and the development of energy enterprise size is closely related. Generally, the strong relationship between the upstream and downstream is based on the industry chain that without the raw materials and basic products from upstream units, downstream energy enterprise cannot make any improvement in its market as one cannot make bricks without straw; meanwhile, if there is no downstream units offer products to develop the market, the energy enterprises in upstream will lose the market demand, as a competent man has no opportunity to show his talents. Therefore, the upstream and downstream energy groups in the industry chain need to coexist in the market offering mutual help, developing market together and benefiting each other. Assuming that *a* is the upstream energy unit, and *b* is the downstream energy unit, according to the Lotka-Volterra model in the previous section and the analysis in former sentences, a model of energy dependency symbiosis can be developed and shown as follows:

$$\begin{cases} \frac{dN_a}{dt} = r_a N_a (-1 - \frac{N_a}{H_a} + g_a \frac{N_b}{H_b}) \\ \frac{dN_b}{dt} = r_b N_b (-1 - \frac{N_b}{H_b} + g_b \frac{N_a}{H_a}) \end{cases} \tag{2}$$

Since *a* and *b* are not independent, the exponential growth rate is $\frac{dN}{dt} = -rN$ （$r \geq 0$）. Among the parameters, g_a represents contribution rate of growth for upstream energy group *a* from the downstream energy group *b* by the value creation of $\frac{N_b}{H_b}$; if $g_a > 1$, the utility promotion from energy group *b* is greater than the utility contribution in energy group *a*. Similarly, the parameter g_b represents contribution rate of growth for downstream energy group *b* from the upstream energy group *a* by the value creation of $\frac{N_a}{H_a}$; if $g_b > 1$, the utility promotion from energy group *a* is greater than its own utility contribution. In order to analyze the symbiotic evolution of energy group *a* and *b*, let $t \to \infty$, then we get energy symbiosis Lotka-Volterra equations:

$$\begin{cases} r_a N_a (-1 - \frac{N_a}{H_a} + g_a \frac{N_b}{H_b}) = 0 \\ r_b N_b (-1 - \frac{N_b}{H_b} + g_b \frac{N_a}{H_a}) = 0 \end{cases} \tag{3}$$

After calculation, there are four equilibrium points $E_i(N_a^*, N_b^*)$, and they were: $(0,0)$, $(-H_a,0)$, $(0,-H_b)$, $(H_a\frac{g_a+1}{g_a g_b-1}, H_b\frac{g_b+1}{g_a g_b-1})$. Only when $N \geq 0$, the equilibrium solution is of practical significance in reality. For that reason, we delete $(-H_a,0)$ and $(0,-H_b)$, only preserve the $(0,0)$ and $(H_a\frac{g_a+1}{g_a g_b-1}, H_b\frac{g_b+1}{g_a g_b-1})$ in further analysis. Besides, in the following equations, $\frac{g_a+1}{g_a g_b-1}$ and $\frac{g_b+1}{g_a g_b-1}$ are not less than *0*.

The stability of the points can be calculated after the equilibrium is analyzed. According to the stability theory of differential equations, if the equilibrium point $E_i(N_a^*, N_b^*)$ satisfies $\lim_{t\to\infty} N_a(t) = N_a^*, \lim_{t\to\infty} N_b(t) = N_b^*)$ from any initial condition, the equilibrium point $E_i(N_a^*, N_b^*)$ is a stable equilibrium point, otherwise it is unstable.

For general nonlinear equations, the stability of equilibrium can be determined by the approximate line of Taylor expansion. Accordingly, the Taylor series expansion of the original equation, $E_i(N_a^*, N_b^*)$, is shown as follows:

$$\begin{cases} \dot{N}_a(t) = F_{N_a}(N_a^*, N_b^*)(N_a - N_a^*) + F_{N_b}(N_a^*, N_b^*)(N_b - N_b^*) \\ \dot{N}_b(t) = G_{N_a}(N_a^*, N_b^*)(N_a - N_a^*) + G_{N_b}(N_a^*, N_b^*)(N_b - N_b^*) \end{cases} \tag{4}$$

Inputting parameters to equation (4), then we can get:

$$\begin{cases} \frac{dN_a}{dt} = r_a(-1 - \frac{2N_a}{H_a} + g_a\frac{N_b}{H_b})(N_a - N_a^*) + r_a g_a\frac{N_a}{H_b}(N_b - N_b^*) \\ \frac{dN_b}{dt} = r_b g_b\frac{N_b}{H_a}(N_a - N_a^*) + r_b(-1 - \frac{2N_b}{H_b} + g_b\frac{N_a}{H_a})(N_b - N_b^*) \end{cases} \tag{5}$$

Coefficient matrix of equations can be obtained:

$$A = \begin{bmatrix} F_{N_a} & F_{N_b} \\ G_{N_a} & G_{N_a} \end{bmatrix}\Bigg|_{E_i(N_a^*, N_b^*)} = \begin{bmatrix} r_a(-1 - \frac{2N_a}{H_a} + g_a\frac{N_b}{H_b}) & r_a g_a\frac{N_a}{H_b} \\ r_b g_b\frac{N_b}{H_a} & r_b(-1 - \frac{2N_b}{H_b} + g_b\frac{N_a}{H_a}) \end{bmatrix}$$

The coefficient of characteristic equation can be obtained: $p = -(F_{N_a} + G_{N_b})$, $q = -\det A$, take $(0,0)$, $(H_a\frac{g_a+1}{g_a g_b-1}, H_b\frac{g_b+1}{g_a g_b-1})$ into the characteristic equation, we can get the stability conditions that are shown in Table1.

Table 1. Symbiosis stability of energy enterprise group

Equilibrium Point	Feature Coefficient p	Feature Coefficient q	Conditional Stability
$(0,0)$	$r_a + r_b$	$r_a r_b$	Stable

$(H_a \frac{g_a+1}{g_a g_b -1}, H_b \frac{g_b+1}{g_a g_b -1})$	$r_a \frac{g_a+1}{g_a g_b -1} + r_b \frac{g_b+1}{g_a g_b -1}$	$r_a r_b \frac{(g_a+1)(g_b+1)}{1-g_a g_b}$	Unstable in reality

Owing to the stability condition at the point, $(H_a \frac{g_a+1}{g_a g_b -1}, H_b \frac{g_b+1}{g_a g_b -1})$, is $g_a < -1, g_b < -1$, and $g_a g_b < 1$, which is contradictory to the previous assumption that a and b are mutually beneficial relations and both of g_a and g_b are greater than 0, therefore the point $(H_a \frac{g_a+1}{g_a g_b -1}, H_b \frac{g_b+1}{g_a g_b -1})$ is unstable. The stable equilibrium of energy enterprise point is (0, 0).

4. Conclusions

It is proved that energy enterprises in the upstream and downstream of the industrial chain can positively promote the value creation to each other by modified Lotka-Volterra model analysis. And there is no limit to resource support between the upstream energy group and the downstream energy group. As long as one of the symbiotic energy groups realizes the value creation, the other side will increase utility production as well. According to equilibrium and stability analysis, when the upstream energy group and the downstream energy group are out of symbiosis, the utility is negative. In other words, when the upstream energy group and the downstream energy group are symbiotic, the growth of their utility is certainly greater than their independent value. As a result, the cooperation between energy enterprises should be enhanced to improve the efficiency and benefits.

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References

[1] Merkert L, Harjunkoski I, Isaksson A, et al. Scheduling and energy–Industrial challenges and opportunities[J]. Computers & Chemical Engineering, 2015, 72: 183-198.

[2] Sokka L, Pakarinen S, Melanen M. Industrial symbiosis contributing to more sustainable energy use–an example from the forest industry in Kymenlaakso, Finland[J]. Journal of Cleaner Production, 2011, 19(4): 285-293.

[3] Yu C, Davis C, Dijkema G P J. Understanding the evolution of industrial symbiosis research[J]. Journal of Industrial Ecology, 2014, 18(2): 280-293.

[4] Yazan D M. Constructing joint production chains: An enterprise input-output approach for alternative energy use[J]. Resources, conservation and recycling, 2016, 107: 38-52.

[5] Fraccascia L, Albino V, Garavelli C A. Technical efficiency measures of industrial symbiosis networks using enterprise input-output analysis[J]. International Journal of Production Economics, 2017, 183: 273-286.