# Analytical Skills-I

## DEPEA515

Edited by
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LOVELY PROFESSIONAL UNIVERSITY

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# Unit 01: Number System

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand what are different types of numbers
- Understand Learn Multiplication short cuts.
- Get Distributive Laws.
- Understand Division laws.
- Understand Tests of Divisibility
- Understand Methods of Finding L.C.M.
- Understand Methods of Finding H.C.F.

## Introduction

Numbers are an integral part of our everyday lives, right from the number of hours we sleep at night to the number of rounds we run around the racing track and much more. In math, numbers can be even and odd numbers, prime and composite numbers, decimals, fractions, rational and irrational numbers, natural numbers, integers, real numbers, rational numbers, irrational numbers, and whole numbers. In this chapter, we'll get an introduction to the different types of numbers and to all the concepts related to it. Numbers form the basis of mathematics. We should befriend numbers in order to understand math. Numbers are of various kinds. We have a long list that includes ordinal numbers, consecutive numbers, odd numbers, even numbers, natural numbers, whole numbers, integers, real numbers, rational numbers, irrational numbers, and complex numbers.

Along with numbers, we come across the interesting world of factors and multiples. This world includes prime numbers, composite numbers, co-prime numbers, perfect numbers (yes, numbers could be perfect!) HCF, LCM, and prime factorization.

## 1.1 Types of Numbers

1. Natural Numbers: Counting numbers 1, 2, 3, 4, 5, ..... are called natural numbers.
2. Whole Numbers: All counting numbers together with zero form the set of whole numbers. Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3. Integers: All natural numbers, 0 and negatives of counting numbers i.e.,

{…, - 3, - 2, - 1, 0, 1, 2, 3, …..} together form the set of integers.

(i) Positive Integers: {1, 2, 3, 4, …..} is the set of all positive integers.

(ii) Negative Integers: {- 1, - 2, - 3, …..} is the set of all negative integers.

(iii) Non-Positive and Non-Negative Integers: 0 is neither positive nor

negative. So, {0, 1, 2, 3, ….} represents the set of non-negative integers, while

{0, - 1, - 2, - 3, …..} represents the set of non-positive integers.

4. Even Numbers: A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8,

10, etc.

5. Odd Numbers: A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7,

9, 11, etc.

6. Prime Numbers: A number greater than 1 is called a prime number, if it has exactly

two factors, namely 1 and the number itself.

Prime numbers upto 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43,

47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

7. Composite Numbers: Numbers greater than 1 which are not prime, are known as

composite numbers, e.g., 4, 6, 8, 9, 10, 12.

Note: (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 and 100.

8. Co-primes: Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g.,

(2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

8. Rational Numbers

The numbers of the form p

q , where p and q are integers

and q ≠ 0, are known as rational numbers,

The set of all rational numbers is denoted by Q.

That is, Q = {x : x = $\frac{p}{q}$; p, q ∈ I, q ≠ 0}

Since every natural number 'a' can be written as $\frac{a}{1}$,every natural number is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$ every integer is a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or in non-terminating repeating decimals.

For example, $\frac{1}{5}$ = 0.2, ,$\frac{1}{3}$ = 0.333..., , $\frac{22}{7}$ =3.1428714287, , $\frac{8}{44}$ = 0.181818 ..., etc.

The recurring decimals have been given a short notation as

0.333... = $0\underline{.3}$

4.1555... = $4.0\underline{5}$

0.323232... = $0.\underline{32}$

9. Irrational numbers**:** Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g., $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, etc.

Note that the exact value of π is not $\frac{22}{7}$ . $\frac{22}{7}$ is rational while π is irrational number. $\frac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

10. Real numbers: The rational and irrational numbers combined together to form real numbers, e.g., $\frac{13}{21}, \frac{2}{5}, -\frac{3}{7}, \sqrt{3}$ , $4 + \sqrt{2}$ etc. are real numbers. The set of all real numbers is denoted by R

Note that the sum, difference or product of a rational and irrational number is irrational, e.g., 3 + $\sqrt{2}$ , 4 -$\sqrt{3}$ , $\frac{2}{5}$ - $\sqrt{5}$ , $\sqrt[4]{3}$ , - $\sqrt[7]{5}$ are all irrational.

11. Even numbers**:** All those numbers which are exactly divisible by 2 are called even numbers, e.g., 2, 6, 8, 10, etc., are even numbers.

12. Odd numbers**:** All those numbers which are not exactly divisible by 2 are called odd numbers, e.g., 1, 3, 5, 7, etc., are odd numbers.

13. Composite numbers**:** Natural numbers greater than 1 which are not prime, are known as composite numbers. For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g., 16 and 17 are relatively prime, although 16 is not a prime number.

## 1.2 Multiplication

**1.** Multiplication By Distributive Law :

(i) a x (b + c) = a x b + a x c (ii) ax(b-c) = a x b-a x c.

Example 1.(i) 567958 x 99999 = 567958 x (100000 - 1)

= 567958 x 100000 - 567958 x 1 = (56795800000 - 567958) = 56795232042. (ii) 978 x 184 + 978 x 816 = 978 x (184 + 816) = 978 x 1000 = 978000.

2. Multiplication of a Number By $5^n$: Put n zeros to the right of the multiplicand and divide the number so formed by $2^n$.

Ex. 975436 x 625 = 975436 x $5^4$= 9754360000 = 609647600.

Multiplication of a given number by 9, 99, 999, etc., that is by $10^n$-1Method: Put as many zeros to the right of the multiplicant as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

Example 2: Multiply:

(a) 3893 by 99

3893 × 99 = 389300 – 3893 = 385407

(b) 4327 by 999

4327 × 999 = 4327000 – 4327 = 4322673

(c) 5863 by 99

5863 × 9999 = 58630000 – 5863 = 58624137

Multiplication of a given number by 11, 101, 1001, etc., that is, by $10^n + 1$.

Method: Place n zeros to the right of the multiplicant and then add the multiplicant to the number so obtained.

Example 3: Multiply

4782 x 11

Solution: (a) 4782 × 11 = 47820 + 4782 = 52602

b) 9836 x 101

Solution :- 9836 × 101 = 983600 + 9836 = 993436

c) 6538 x 1001

Solution : - 6538 × 1001 = 6538000 + 6538 = 6544538

Multiplication of a given number by 15, 25, 35, etc. Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2

Example 4: Multiply

7054 x 15

$= \frac{1}{2}(7054 * 30)$

$= \frac{1}{2}(2116203)$

= 105810

b) 3897 x 25

$= \frac{1}{2}(3897\ X\ 50)$

$= \frac{1}{2}(194850)$

= 97425

c) 4536 x 35

$= \frac{1}{2}(4563\ x\ 30)$

$= \frac{1}{2}(319410)$

= 159705

Example 5: Multiply

3982 x 2

$= \frac{39820}{2}$

= 19910

b) 4739 x 25

$= \frac{473900}{2^3}$

= 118475

c) 7894 X 125

$= \frac{7894000}{2^3}$

$= \frac{7894000}{8}$

d) 4863 x 625

$= \frac{48630000}{2^4}$

$= \frac{48630000}{16}$

= 3039375

## 1.3 Distributive Laws

For any three numbers a , b, c we have

a) a x b + a x c = a x(b + c)

b) A x b –a x c = a x (b-c)

Example 6: 438 x 637 + 438 x 367 =?

Solution: - 438 x 637 + 438 x 367 = 438 x ( 637 +367) = 430 x 1000= 438000

Example 7: 674 x 832 – 674 x 632 -?

Solution:- : 674 × 832 – 674 × 632

= 674 × (832 – 632)

= 674 × 200

= 134800

## 1.4 Division

Division is repeated subtraction. For example, when we divide 63289 by 43, it means 43 can be repeatedly subtracted 1471 times from 63289 and they remainder 36 is left.

```
               1471 ← Quotient
Divisor → 43) 63289  ← Dividend
              43
               202
               172
                308
                301
                 79
                 43
                 36  ← Remainder
```

Dividend = (Divisor x quotient ) + remainder

Divisor = $\frac{dividend - remainder}{Quotient}$

Example 8: On dividing 7865321 by a certain number, the quotient is 33612 and the remainder is 113. Find the divisor.

Solution: Divisor = Dividend - Remainder / Quotient

$= \frac{7865321 - 113}{33612}$

$= \frac{7865208}{33612}$

= 234

Example 9: A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.

Solution: Number = (Divisor × Quotient) + Remainder = (315 × 7) + 46 = 2205 + 46 = 2251

Example 10: Find the least number of 5 digits which is exactly divisible by 632.

Solution: The least number of 5 digits is 10000. Dividing this number by 632, the remainder is 520. So, the required number = 10000 + (632 + 520) = 10112

$$\begin{array}{r} 15 \\ 632\overline{)\,10000} \\ \underline{632} \\ 3680 \\ \underline{3160} \\ 520 \end{array}$$

Example 11: Find the greatest number of 5 digits which is exactly divisible by 463.

Solution: The greatest number of 5 digits is 99999. Dividing this number by 463, the remainder is 454. So, the required number = 99999 – 454 = 99545

$$\begin{array}{r} 215 \\ 463\overline{)\,99999} \\ \underline{926} \\ 739 \\ \underline{463} \\ 2769 \\ \underline{2315} \\ 454 \end{array}$$

Example 12: Find the number nearest to 13700 which is exactly divisible by 235.

Solution: On dividing the number 13700 by 235, the remainder is 70. Therefore, the nearest number to 13700, which is exactly divisible by 235 = 13700 – 70 = 13630.

$$\begin{array}{r} 58 \\ 235\overline{)\,13700} \\ \underline{1175} \\ 1950 \\ \underline{1880} \\ 70 \end{array}$$

## 1.5 Tests of Divisibility

1. Divisibility by 2:- A number is divisible by 2 if the unit's digit is zero or divisible by For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. Divisibility by 3 :- A number is divisible by 3 if the sum of digits in the number is divisible by 3.

For example, the number 3792 is divisible by 3 since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

3. Divisibility by 4:- A number is divisible by 4 if the number formed by the last two digits (ten's digit

and unit's digit) is divisible by 4 or are both zero. For example, the number 2616 is divisible by 4 since 16 is divisible by 4. 4.

4. Divisibility by 5:- A number is divisible by 5 if the unit's digit in the number is 0 or 5. For example, 13520, 7805, 640, 745, etc., are all divisible by 5. 5. 5. Divisibility by 6 A number is divisible by 6 if the number is even and sum of its digits is divisible by 3. For example, the number 4518 is divisible by 6 since it is even and sum of its digits 4 + 5 + 1 + 8 = 18 is divisible by 3.

6. Divisibility by 7:- The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7. For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16. Then, 44 – 16 = 28.

Since 28 is divisible by 7, 448 is divisible by 7.

7. Divisibility by 8:- A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e., 784 is divisible by 8.

8. Divisibility by 9:- A number is divisible by 9 if the sum of its digits is divisible by 9. For example, the number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9. 9.

9. Divisibility by 10:- A number is divisible by 10, if it ends in zero.

For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. Divisibility by 11:- A number is divisible by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

For example, in the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17 = 0, so the number is divisible by 11.

11. Divisibility by 12 :- A number is divisible by 12 if it is divisible by 3 and 4.

12. Divisibility by 18:- An even number satisfying the divisibility test of 9 is divisible by 18.

13. Divisibility by 25 A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero. For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25

14. Divisibility by 88 A number is divisible by 88 if it is divisible by 11 and 8.

15. Divisibility by 125 A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero. For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

## 1.6 Squares

To square any number ending with 5.

Method: $(A5)^2 = A(A + 1)/25$

**Example 13:**

(a) $(25)^2 = 2(2 + 1)/25 = 6/25 = 625$

(b) $(45)^2 = 4(4 + 1)/25 = 20/25 = 2025$

(c) $(85)^2 = 8(8 + 1)/25 = 72/25 = 7225$

To square a number in which every digit is one. Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

**Example 14:**

(a) $11^2 = 121$

(b) $111^2 = 12321$

(c) $1111^2 = 1234321$

(d) $222^2 = 2^2 (111)^2 = 4(12321) = 49284$

(e) $33332 = 3^2 (1111)^2 = 9(1234321) = 11108889$

To square a number which is nearer to 10x. Method: Use the formula: $x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$

**Example 15:**

(a) $(97)^2 = (97 + 3) (97 - 3) + 32 = 9400 + 9 = 9409$

(b) $(102)^2 = (102 - 2) (102 + 2) + 2^2 = 10400 + 4 = 10404$

(c) $(994)^2 = (994 + 6) (994 - 6) + 6^2 = 988000 + 36 = 988036$

(d) $(1005)^2 = (1005 - 5) (1005 + 5) + 5^2 = 1010000 + 25 = 1010025$

**Common Factor**

If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a. A common factor of two or more numbers is a number which divides each of them exactly. For example, 4 is a common factor of 8 and 12.

**Highest Common Factor**

Highest common factor of two or more numbers is the greatest number that divides each one of them

Exactly For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called Greatest Common Divisor or Greatest Common Measure. Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

## 1.7 Methods of Finding H.C.F.

I. Method of Prime Factors

Step1 Express each one of the given numbers as the product of prime factors. [A number is said to be a prime number if it is exactly divisible by 1 and itself, but not by any other number, e.g., 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose common factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Example 16: Find the H.C.F. of 70 and 90.

Solution: $70 = 2 \times 5 \times 7$

$90 = 2 \times 5 \times 9$

Common factors are 2 and 5.

∴ H.C.F. $= 2 \times 5 = 10$.

Example 17: Find the H.C.F. of 3332, 3724 and 4508.

Solution: $3332 = 2 \times 2 \times 7 \times 7 \times 17$

$3724 = 2 \times 2 \times 7 \times 7 \times 19$

$4508 = 2 \times 2 \times 7 \times 7 \times 23$

∴ H.C.F. $= 2 \times 2 \times 7 \times 7 = 196$.

Example 18: Find the H.C.F. of 360 and 132.

Solution: $360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

∴ H.C.F. $= 2^2 \times 3^1 = 12$.

Example 19: If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution: The factors common to both x and y are $2^3 3^5$ and $5^9$.

∴ H.C.F. $= 2^3 \times 3^5 \times 5^9$.

II. Method of Division

A. For two numbers:

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder. This is continued until no remainder is left.

H.C.F. is the divisor of last step.

Example 20: Find the H.C.F. of 3556 and 3444.

```
3444)3556( 1
     3444
     ----
      112)3444( 30
          3360
          ----
            84)112( 1
               84
               --
               28)84( 3
                  84
                  --
                  ×
```

$\therefore$ H.C.F. = 28.

A. For more than two numbers:

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Example 21: Find the H.C.F. of 13915, 9499 and 2553 by division method.

**Solution:**

```
9499) 13915 (1
      9499
      ----
      4416) 9499 (2
            8832
            ----
             667) 4416 (6
                  4002
                  ----
                   414) 667 (1
                        414
                        ---
                        253) 414 (1
                             253
                             ---
                             161) 253 (1
                                  161
                                  ---
                                   92) 161 (1
                                       92
                                       --
                                       69) 92 (1
                                           23
                                           --
                                           69) 69 (3
                                               69
                                               --
                                               ×
```

Now, in the next step, we will find the H.C.F. of 23 and 2553.

23) 2553 (111
23
25
23
23
23
×

Thus, H.C.F. of 13915, 9499 and 2553 = 23.

Example 22: Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution: Required length

= (H.C.F. of 700, 385, 1295) cm = 35 cm.

**Common multiple**

A common multiple of two or more numbers is a number which is exactly divisible by each one of them. For example, 32 is a common multiple of 8 and 16.

$8 \times 4 = 32$

$16 \times 2 = 32$.

## 1.8 Least Common Multiple

The least common multiple of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 60, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

∴L east common multiple, i.e., L.C.M. of 12 and 18 is 36.

**Methods of Finding L.C.M.**

**Method of Prime Factors**

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.

Example 23: Find the L.C.M. of 32, 48, 60 and 320.

Solution: $32 = 2^5 \times 1$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$320 = 2^6 \times 6$

∴ L.C.M. = $2^6 \times 3 \times 5 = 960$.

II. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers 2, 3, 5, 7, 11, ... which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line, which is the required L.C.M.

Example 24: Find the L.C.M. of 12, 15, 20 and 54.

**Solution:**

| | | | | |
|---|---|---|---|---|
| 2 | 12, | 15, | 20, | 54 |
| 2 | 6, | 15, | 10, | 27 |
| 3 | 3, | 15, | 5, | 27 |
| 5 | 1, | 5, | 5, | 9 |
| | 1, | 1, | 1, | 9 |

L.C.M. = 2 × 2 × 3 × 5 × 1 × 1 × 1 × 9 = 540.

Notes

Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit

**Short-cut methods**

01 H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of Step 2) leaving as many digits on its right as there are in each of the numbers.

Illustration 10: Find the L.C.M. of 1.2, 0.24 and 6. Solution: The given numbers can be written as 1.20, 0.24 and 6.00.

Now, ignoring the decimal we find the L.C.M. of 120, 24 and 600.

| | | | |
|---|---|---|---|
| 2 | 120, | 24, | 600 |
| 2 | 60, | 12, | 300 |
| 2 | 30, | 6, | 150 |
| 3 | 15, | 3, | 75 |
| 5 | 5, | 1, | 25 |
| | 1, | 1, | 5 |

∴ L.C.M. = 2 × 2 × 2 × 3 × 5 × 1 × 5 = 600

Thus, the required L.C.M. = 6.00, i.e., 6.

Example 25: Find the H.C.F. of 6.16 and 13. Solution: The given numbers can be written as 6.16 and 13.00.

N ow, ignoring the decimals we find the H.C.F. of 616 and 1300.

$$
\begin{array}{l}
616\,)\,1300\,(\,2 \\
\quad\;\;\; 1232 \\
\quad\;\;\; \overline{\;\;\;68\,)\,616\,(\,9} \\
\qquad\qquad\;\; 612 \\
\qquad\qquad\;\; \overline{\;\;\;4\,)\,68\,(\,17} \\
\qquad\qquad\qquad\quad 68 \\
\qquad\qquad\qquad\quad \overline{\times}
\end{array}
$$

∴ H.C.F. of 616 and 1300 is 4. Thus, the required H.C.F. = 0.04

02 L.C.M. and H.C.F. of Fractions

L.C.M = $\frac{L.C.M. of\ the\ numbers\ in\ numerators}{H.C.F. of\ the\ numbers\ inn\ denominators}$

H.C.F. = $\frac{H.C.F. of\ the\ numbers\ in\ numerators}{L.C.M. of\ the\ numbers\ inn\ denominators}$

Example 26: Find the L.C.M. of $\frac{2}{5}, \frac{3}{10}, and\ \frac{6}{25}$

Solution: L.C.M. of numerators 2, 3 and 6 is 6.

H.C.F. of denominators 5, 10 and 25 is 5.

∴ Required L.C.M. $=\frac{L.C.M. of\ Numerators}{H.C.F. of\ Denominators}=\frac{6}{5}$

Illustration 13: Find the H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

Solution: H.C.F. of numerators 4, 10 and 20 is 2. L.C.M. of denominators 9, 21 and 63 is 63.

∴ Required H.C.F $=\frac{H.C.F\ of\ numerators}{L.C.M\ of\ denominators}=\frac{2}{63}$

Notes

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.

2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

03 Product of two numbers = L.C.M. of the numbers × H.C.F. of the numbers

Example 27: The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Solution: The required number

$$=\frac{L.C.M \times H.C.F}{First\ number}=\frac{1260 \times 63}{315}=252$$

04 To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z.

Example 28: Find the greatest number that will exactly divide 200 and 320. Solution: The required greatest number = H.C.F. of 200 and 320 = 40.

05 To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively. Required number = H.C.F. of (x – a), (y – b) and (z – c).

Illustration 16: Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution: The required greatest number = H.C.F. of (148 – 4), (246 – 6) and (623 – 11), i.e., H.C.F. of 144, 240 and 612 = 12.

06 To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z.

Example 29: What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution: The required smallest number = L.C.M. of 36, 45, 63 and 80 = 5040.

07 To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that (x – a) = (y – b) = (z – c) = k (say)

∴ Required number = (L.C.M. of x, y and z) – k.

Example 30: Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Solution: Since, (36 – 25) = (48 – 37) = (64 – 53) = 11, therefore, the required smallest number = (L.C.M. of 36, 48 and 64) – 11 = 576 – 11 = 565.

08 To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) + r.

Example 31: Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution: The required smallest number = (L.C.M. of 12, 16 and 18) + 5 = 144 + 5 = 149.

09 To find the greatest number that will divide x, y and z leaving the same remainder in each case. (a) When the value of remainder r is given: Required number = H.C.F. of (x – r), (y – r) and (z – r). (b) When the value of remainder is not given: Required number = H.C.F. of | (x – y) |, | (y – z) | and | (z – x) |

Example 32: Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution: The required greatest number = H.C.F. of (772 – 5) and (2778 – 5) = H.C.F. of 767 and 2773 = 59.

Example 33: Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder. Solution: The required greatest number = H.C.F. of |(x – y)|, |(y – z)| and |(z – x)| = H.C.F. of |(152 – 277)|, |(277 – 427)| and |(427 – 152)| = H.C.F. of 125, 150 and 275 = 25.

10 To find the n-digit greatest number which, when divided by x, y and z,

leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and z = L

Step 2 $\frac{{}^{l}\sqrt{n\ digit\ greatest\ number}}{Remainder=R}$

Step 3 Required number = n-digit greatest number – R

(b) leaves remainder K in each case Required number = (n-digit greatest number – R) + K.

Example 34: Find the greatest number of 4-digit number which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252

```
252)9999 ( 39
    9828
     171
```

∴ The required number = (9999 – 171) + 3 = 9931.

Example 35: Find the greatest number of four digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution: L.C.M. of 12, 15, 20 and 35 = 420.

```
420) 9999( 23
     9660
      339
```

∴ The required number = 9999 – 339 = 9663.

11 To find the n-digit smallest number which when divided by x, y and z (a) leaves no remainder (i.e., exactly divisible)

Step 1 L.C.M. of x, y and z = L

Step 2 $\frac{l)\ n\ digit\ smallest\ number(}{Remainder=R}$

Step 3 Required number = n-digit smallest number + (L – R).

(b) leaves remainder K in each case. Required number = n-digit smallest number + (L – R) + K.

Example 36: Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution: L.C.M. of 4, 6, 8 and 10 = 120.

$$\begin{array}{r} 120\overline{)1000}(8 \\ \underline{960} \\ 40 \end{array}$$

∴ The required number = 1000 + (120 – 40) = 1080.

Example 37: Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution: L.C.M. of 12, 18, 21 and 28 = 252.

$$\begin{array}{r} 252\overline{)1000}(3 \\ \underline{756} \\ 244 \end{array}$$

∴ The required number = 1000 + (252 – 244) + 3 = 1011.

Cyclicity of unit digit

Numbers are classified into three categories to find unit digit.
1. Digits 0,1,5,6
2. Digits 4,9
3. Digits 2,3,7,8

**Digits 0,1,5,6**

When we have these numbers (0,1,5,6) in the unit place, we get the same digit itself at the unit place when raised to any power, i.e. $0^n=0$, $1^n=1$, $5^n=5$, $6^n=6$. Let us apply this concept to the following questions.

Example 38**:** Find the Unit place digit of the following numbers:

1. $360^{244}$
Answer= 0
2. $2974281^{307}$
Answer=1
3. $4575^{400000666}$
Answer=5
4. $5687686^{265749375}$
Answer=6

Digits 4 & 9
Both these numbers have a cyclicity of only two different digits as their unit's digit.
In the case of 4 & 9
• If the Power of 4 is Even, the result will be 6
• If the Power of 4 is Odd, the result will be 4
• If the Power of 9 is Even, the result will be 1
• If the Power of 9 is Odd, the result will be 9**.**

**Example:** Find the Unit place digit of the following numbers:
1. $4568474^{26734258}$
Answer= 6.

3456445767843
Answer= 4
3. $54857465789^{5768454}$
Answer= 1
4. $4576348567895627369^{765787}$
Answer= 9

**Digits 2,3,7,8**

For Digit 2

When we have number 2 in the unit place then follow the given steps to find the unit digit.
Step 1- Divide the last two digits of the power of a given number with 4
Step 2- You get the remainder n
Step 3- Since you have got n as a remainder, put it as the power of 2, i.e $(2)^n$
Step 4- Have a look at the table below and mark your answer.

Example 39: Find the Unit place digit of the following numbers:
**1.** $\mathbf{46572^{33}}$
Here, the unit place is 2 and power is 33. To solve follow the given steps
Step 1- Divide 33 by 4.
Step 2- You get remainder 1.
Step 3- Since you have got remainder 1, put it as a power of 2, i.e $(2)^1$
Step 4- Have a look at the table above, $(2)^1=2$.
Answer= 2.

Example 40: Find the Unit place digit of the following numbers:
**1.** $\mathbf{46573^{33}}$
Here, the unit place is 3 and power is 33. To solve follow the given steps
Step 1- Divide 33 by 4.
Step 2- You get remainder 1.
Step 3- Since you have got remainder 1, put it as a power of 3, i.e $(3)^1$
Step 4- Have a look at the table above, $(3)^1=3$.
Answer= 3.

For Digit 7

Example 41: Find the Unit place digit of the following numbers:
**1.** $\mathbf{46577^{18}}$
Here, the unit place is 7 and power is 18. To solve follow the given steps
Step 1- Divide 18 by 4.
Step 2- You get remainder 2.
Step 3- Since you have got remainder 2, put it as a power of 7, i.e $(7)^2$
Step 4- Have a look at the table above, $(7)^2=9$.
Answer= 9.

For Digit 8

Example 42: Find the Unit place digit of the following numbers:
**1.** $\mathbf{46578^{59}}$
Here unit place is 8 and power is 59. To solve follow the given steps
Step 1- Divide 59 by 4.
Step 2- You get remainder 3.
Step 3- Since you have got remainder 3, put it as a power of 8, i.e (8)3
Step 4- Have a look at the table above, (8)3=2.
Answer= 2

## Summary

The key concepts learned from this unit are: -

- We have learnt what are different types of numbers
- We have learntMultiplication short cuts.
- We have learnt Distributive Laws.
- We have learnt Division laws.
- We have learnt Tests of Divisibility
- We have learnt Methods of Finding L.C.M.
- We have learnt Methods of Finding H.C.F.

## Keywords

- Multiplication.
- Distributive Laws.
- Division laws.
- Tests of Divisibility
- L.C.M.
- H.C.F.

## Self Assessment

1. the unit's digit in the product (2467)163 x (341)72

A. 1
B. 0
C. 7
D. 9

2. 387 x 387 + 114 x 114 + 2 x 387 x 114=

A. 251001
B. 251002
C. 251003
D. 251006

3. Which of the following statements is true?

A. 541326 is divisible by 3, 5967013 is not divisible by 3
B. 541326 is not divisible by 3, 5967013 is not divisible by 3
C. 541326 is divisible by 3, 5967013 is divisible by 3
D. 541326 is not divisible by 3, 5967013 is divisible by 3

4. Which of the following statements is true?

A. 67920594 is divisible by 4, 618703572 is not divisible by 4
B. 67920594 is not divisible by 4, 618703572 is not divisible by 4
C. 67920594 is divisible by 4, 618703572 is divisible by 4
D. 67920594 is not divisible by 4, 618703572 is divisible by 4

5. 4832718 is divisible by ?

A. 11
B. 5
C. 10
D. 25

6. Least number must be added to 3000 to obtain a number exactly divisible by 19 is?

A. 1
B. 2
C. 3
D. 4

7. The H.C.F. of 108, 288 and 360 is?

A. 24
B. 28
C. 36

D. 48

8. The H.C.F. of 513, 1134 and 1215 is?
A. 13
B. 27
C. 81
D. 124

9. The L.C.M. of 72, 108 and 2100 is?
A. 38700
B. 32500
C. 37800
D. 32

10. The L.C.M. of 16, 24, 36 and 54 is?
A. 432
B. 411
C. 32
D. 24

11. The H.C.F. and L.C.M. of 0.63, 1.05 and 2.1 is?
A. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.33, 1.05 and 2.1 is 5
B. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.93, 1.05 and 2.1 is 6.30
C. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.63, 1.05 and 2.1 is 6.30
D. H.C.F. of 0.63, 1.05 and 2.1 is 0.21, L.C.M. of 0.63, 1.05 and 2.1 is 9.30

12. Two numbers are in the ratio of 15:11. If their H.C.F. is 13, then the numbers are?
A. 195 and 200.
B. 200 and 143.
C. 195 and 145.
D. 195 and 143.

13. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the
14. numbers is 77, then the other is?
A. 95
B. 99
C. 120
D. 200

15. $(994)^2$ =
A. 1988036
B. 998036
C. 108036
D. 988036

16. The Unit place digit of the number $46572^{33}$is
A. 2
B. 3
C. 4
D. 5

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | C | 2. | A | 3. | A | 4. | D | 5. | A |
| 6. | B | 7. | C | 8. | B | 9. | C | 10. | A |
| 11. | D | 12. | D | 13. | B | 14. | D | 15. | A |

## Review Questions

1. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.
2. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.
3. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case
4. Find the least number exactly divisible by 12,15,20,27.
5. Find the least number which when divided by 6,7,8,9, and 12 leave the same remainder 1 each case.
6. The traffic lights at three different road crossings change after every 48 sec., 72 sec and 108 sec. respectively .If they all change simultaneously at 8:20:00hours,then at what time they again change simultaneously.
7. A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.
8. Find the least number of 5 digits which is exactly divisible by 632.
9. Find the greatest number of 5 digits which is exactly divisible by 463.
10. Find the number nearest to 13700which is exactly divisible by 235.

## Further Readings


1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 02: Average

CONTENTS



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand the average of all numbers.
- Understand what Weighted Average are

## Introduction

Find the average of all prime numbers between 30 and 50? Sol: There are five prime numbers between 30 and 50.

They are 31,37,41,43 and 47.

Therefore the required average=(31+37+41+43+47)/5 ⇔199/5 ⇔ 39.8.

Whenever we are asked the marks we scored in any examination, we usually tell the marks in terms of percentage that is, taking the percentage of total marks of all subjects. This percentage is called average percentage. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about their average age. Introduction Whenever we are asked the marks we scored in any examination, we usually tell the marks in terms of percentage, that is, taking the percentage of total marks of all subjects. This percentage is called average percentage. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about their average age.$\frac{3+9+11+15+18+19+23}{7} = \frac{98}{7} = 14$

## 2.1 Average

01 Average=$\frac{Sum\ of\ qualities}{Number\ of\ qualities}$

02 Sum of quantities = Average × Number of quantities

03 Number of quantities =$\frac{sum\ of\ qualities}{Average}$

Find the average of first 40 natural numbers?
Sol: sum of first n natural numbers=n(n+1)/2;So,sum of 40 natural numbers=(40*41)/2 ⇔820.
Therefore the required average=(820/40) ⇔20.5.

Example 1: A man purchased 5 toys at rs.200 each, 6 toys at rs250 each and 9 toys at `300 each. Calculate the average cost of 1 toy.

Solution: Price of 5 toys = 200 × 5 = rs.1000

Price of 6 toys = 250 × 6 = rs.1500

Price of 9 toys = 300 × 9 = rs.2700

Total number of toys = 5 + 6 + 9 = 20

Average price of 1 toy =$\frac{1000+1500+2700}{20}$

=$\frac{5200}{20} = rs. 260.$

Find the average of first 20 multiples of 7?

Sol: Required average =7(1+2+3+…….+20)/20 ⇔(7*20*21)/(20*2) ⇔(147/2)=73.5.

Example 2: The average marks obtained by 200 students in a certain examination is 45. Find the total marks.

Solution: Total marks

= Average marks × Number of students

= 200 × 45 = 900.

The average of four consecutive even numbers is 27. find the largest of these numbers?

Sol: let the numbers be x,x+2,x+4 andx+6. then,

(x+(x+2)+(x+4)+(x+6))/4=27⇔(4x+12)/4=27⇔x+3=27⇔x=24.

Therefore the largest number=(x+6)= 24+6=30.

Example 3: Total temperature for the month of September is 840°C. If the average temperature of that month is 28°C, find out the number of days is the month of September.

Solution: Number of days in the month of September

=$\frac{Total\ temperature}{Average\ temperature} = \frac{840}{28} = 30\ days.$

Short cut methods

01 Average of two or more groups taken together. (a) If the number of quantities in two groups are $n_1$ and $n_2$ and their average is x and y, respectively, the combined average (average of all of them put together) is

$$\frac{n_1 x + n_2 y}{n_1 + n_2}$$

Explanation

Number of quantities in the first group = $n_1$

Their average = x

∴ Sum = $n_1$× x Number of quantities in the second group = $n_2$ Their average = y

∴ Sum = $n_2$× y

Number of quantities in the combined group = $n_1$ + $n_2$. Total sum (sum of quantities of the first group and the second group) = $n_1$x + $n_2$y.

∴ Average of the two groups

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

(b) I f the average of $n_1$ quantities is x, and the average of $n_2$ quantities out of them is y, the average of the remaining group (rest of the quantities) is

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

Explanation

Number of quantities = $n_1$

Their average = x

∴ Sum = $n_1$ x Number of quantities taken out = $n_2$

Their average = y

∴ Sum = $n_2$y

Sum of remaining quantities = $n_1$x − $n_2$y Number of remaining quantities = $n_1 - n_2$

∴ Average of the remaining group=$\frac{n_1x+n_2y}{n_1-n_2}$.

There are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40kg and that of section B is 35kg, find the average weight of the whole class?

Sol: total weight of(36+44) students=(36*40+44*35)kg =2980kg.

Therefore weight of the total class=(2980/80)kg =37.25kg.

Example 4: The average weight of 24 students of section A of a class is 58 Kg, whereas the average weight of 26 students of section B of the same class is 60.5 Kg. Find out average weight of all the 50 students of the class.

Solution: Here, $n_1$ = 24, $n_2$ = 26, x = 58, and y = 60.5.

∴ Average weight of all the 50 students

$$\frac{n_1x + n_2y}{n_1 + n_2}$$

$=\frac{24\times58+26\times60.5}{24+26}$

$=\frac{1392+1573}{50} = \frac{2965}{50} = 59.3\ kg$

Nine persons went to a hotel for taking their meals 8 of them spent Rs.12 each on their meals and the ninth spent Rs.8 more than the average expenditure of all the nine. What was the total money spent by them?

Sol: Let the average expenditure of all nine be Rs. x

Then 12*8+(x+8)=9x or 8x=104 or x=13.

Total money spent = 9x=Rs.(9*13)=Rs.117

Example 5: Average salary of all the 50 employees including 5 officers of a company is rs.850. If the average salary of the officers is rs.2500, find the average salary of the remaining staff of the company. Solution: Here, $n_1$ = 50, $n_2$ = 5, x = 850 and y = 2500.

∴ Average salary of the remaining staff

$\frac{n_1x+n_2y}{n_1-n_2} = \frac{50\times850-5\times2500}{50-5}$

$=\frac{42500-12500}{45}=\frac{30000}{45}$

=rs 667(approx)

02 If x is the average of $x_1$, $x_2$,..., $x_n$, then

(b) T he averages of $x_1$ − a, $x_2$ − a, $x_n$ − a is x − a.

(c) The average of a$x_1$, a$x_2$, ..., axn is ax, provided a ≠ 0.

(d) The average of $\frac{x_1}{a}\frac{x_2}{a}$, ..., $\frac{x_s}{a}$is $\frac{\bar{x}}{a}$, provided a ≠ 0.

Example 6: The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Solution: The new average = $\bar{x}$ +a.

The average of 25 result is 18.The average of 1st 12 of them is 14 & that of last 12 is 17.Find the 13th result.

Sol: Clearly 13th result = (sum of 25 results)-(sum of 24 results)=(18*25)-(14*12)+(17*12)=450-(168+204)=450-372=78

Example 7: The average of x numbers is 5x. If x − 2 is subtracted from each given number, what will be the new average?

Solution: The new average = = $\bar{x}$ − a

= 5x − (x − 2) = 4x + 2.

Example 8: The average of 8 numbers is 21. If each of the numbers is multiplied by 8, find the average of a new set of numbers. Solution: The average of a new set of numbers = a$\bar{x}$ = 8 × 21 = 168

03 T he average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then q = p + n(y − x).

The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.

Sol. Total weight increased = (1.8 x 10) kg =18 kg.

:. Weight of the new man = (53 + 18) kg =71 kg

Example 9: The average weight of 25 persons is increased by 2 Kg when one of them whose weight is 60 Kg, is replaced by a new person. What is the weight of the new person?

Solution: The weight of the new person

= p + n(y − x)

= 60 + 25(2) = 110 Kg.

04 (a) The average of n quantities is equal to x. When a quantity is removed, the average becomes y. The value of the removed quantity is n (x − y) + y.

(b) The average of n quantities is equal to y. When a quantity is added, the average becomes y. The value of the new quantity is n (y − x) + y.

Example 10: The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year. What is the age of the class teacher?

Solution: The age of class teacher

= n(x − y) + y

= 25(16 − 15) + 15 = 40 years.

Example 11: The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacher's age

. Solution: The teacher's age

= n(y − x) + y

= 30(10 − 9) + 10 = 40 years.

A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by

3. Find his average after 17th inning.

Sol. Let the average after 17th inning = x. Then, average after 16th inning = (x - 3).

:. 16 (x - 3) + 87 = 17x or x = (87 - 48) = 39

05 (a) T he average of first n natural numbers is $\frac{n+1}{2}$

(b) The average of square of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$

(c) The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$

(d) The average of odd numbers from 1 to n is $\frac{last\ odd\ number\ +1}{2}$

(e) The average of even numbers from 1 to n is $\frac{last\ even\ number\ +2}{2}$

Example 12: Find the average of first 81 natural numbers.

Solution: The required average

$$= \frac{n+1}{2} = \frac{81+1}{2} = 41$$

Example 13: What is the average of squares of the natural numbers from 1 to 41?

Solution: The required average

$$= \frac{(n+1)(2n+1)}{6} = \frac{(41+1)(2\times 41+1)}{6}$$

$=\frac{42\times 83}{6} = \frac{3486}{6} = 581$

Example 14: Find the average of cubes of natural numbers from 1 to 27.

Solution: The required average

$=\frac{n(n+1)^2}{4} = \frac{27\times(27+1)^2}{4}$

$=\frac{27\times 28\times 28}{4} = \frac{21168}{4} = 5292$

Example 15: What is the average of odd numbers from 1 to 40?

Solution: The required average

$=\frac{last\ odd\ number+1}{2} = \frac{39+1}{2} = 20.$

Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km per hour. Find the average speed of the train during the whole journey. Sol. Required average speed = ((2xy)/(x+y)) km / hr =(2 x 84 x 56)/(84+56)km/hr = (2*84*56)/140 km/hr=67.2 km/hr.

Example 16: What is the average of even numbers from 1 to 81? Solution: The required average

$$\frac{last\ even\ number + 1}{2} = \frac{80+2}{2} = 41$$

06 (a) If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.

(b) I f n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.

(c) T he averages of first n consecutive even numbers is (n + 1).

(d) The average of first n consecutive odd numbers is n.

(e) T he averages of squares of first n consecutive even numbers is $\frac{2(n+1)(2n+1)}{3}$

(f) The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$

(g) T he average of squares of consecutive odd numbers till n is $\frac{n(n+2)}{3}$

(h) If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is 2(n − 1).

Example 17: Find the average of 7 consecutive numbers 3, 4, 5, 6, 7, 8, 9.

Solution: The required average = middle number = 6.

Example 18: Find the average of consecutive odd numbers 21, 23, 25, 27, 29, 31, 33, 35.

Solution: The required average = average of middle two numbers

= average of 27 and 29

$$= \frac{27+29}{2} = 28$$

Example 19: Find the average of first 31 consecutive even numbers.

Solution: The required average = (n + 1) = 31 + 1 = 32.

Example 20: Find the average of first 50 consecutive odd numbers.

Solution: The required average = n = 50.

Example 21: Find the average of squares of fi rst 19 consecutive even numbers.

Solution: The required average= $\frac{2(n+1)(2n+1)}{3}$

$= \frac{2\times20\times39}{3} = \frac{1560}{3} = 520$

Example 22: Find the average of squares of consecutive even numbers from 1 to 25.

Solution: The required average

$= \frac{(n+1)(n+2)}{3} = \frac{(25+1)(25+2)}{3}$

$= \frac{26\times27}{3} = \frac{702}{3} = 234$

Example 23: Find the average of squares of consecutive odd numbers from 1 to 31.

Solution: The required average

$=\frac{n(n+2)}{3}=\frac{31\times(31+2)}{3}$

$=\frac{31\times 33}{3}=341$

Example 24: If the average of 6 consecutive numbers is 48, what is the difference between the smallest and the largest number?

Solution: The required difference

= 2(n − 1) = 2(6 − 1) = 10.

07 Geometric Mean or Geometric Average.

Geometric mean of $x_1, x_2..., x_n$ *is denoted by*

G.M=$\sqrt[n]{x_1 \times x_2 \times .. \times x_n}$

Geometric mean is useful in calculating averages of ratios such as average population growth rate, average percentage increase and, so on.

Example 25: The production of a company for three successive years has increased by 10%, 20% and 40%, respectively. What is the average annual increase of production?

Solution: Geometric mean of x, y and z =$(x \times y \times z)^{1/3}$

∴ Average increase = (10 × 20 × 40)$^{1/3\%}=20\%$

Example 26: The population of a city in two successive years increases at the rates of 16% and 4%, respectively. Find out the average increase in two years.

Solution: In case of population increase, the geometric mean is required.

∴ Geometric mean of 16% and 4% is

= (16 × 4)$^{1/2}$ %, i.e., 8%

08 Harmonic Mean or Harmonic Average.

Harmonic mean of$x_1, x_2..., x_n$ is denoted by

$$H.M=\frac{1}{\frac{1}{n}\left(\frac{1}{x_1}+\frac{1}{x_2}+..+\frac{1}{x_n}\right)}$$

Harmonic mean is useful in finding out average speed of a vehicle, average production per day and, so on.

Example 27: A man runs 1 Km at a speed of 15 Km/h and another 1 Km he walks at a speed of 5 Km/h. Find out his average speed in covering 2 Km.

Solution: Harmonic mean is used when distance remains constant and speed varies. Harmonic mean of x and y is$\frac{2}{\frac{1}{x}+\frac{1}{y}}$ $or, \frac{2xy}{x+y}$.

∴ Average speed for the whole distance

$=\frac{2\times 15\times 5}{15+5}=7.5\ km/h$

09 I f a certain distance is covered at a speed of x Km/h and the same distance is covered at a speed of y Km/h, the average speed during the entire journey is

$$\left(\frac{2xy}{x+y}\right)km/h$$

Example 28: If half of the journey is travelled at a speed of 15 Km/h and the remaining half at a speed of 12 Km/h, find out average speed during the entire journey

Solution: The average speed

$$\left(\frac{2xy}{x+y}\right) = \frac{2 \times 15 \times 12}{15 + 12} = \frac{360}{27} = 13\frac{1}{3} km/h$$

Example 29: A man goes to a certain place at a speed of 30 Km/h and returns to the original place at a speed of 20 Km/h, find out his average speed during this up and down journey.

Solution: The average speed

$=\left(\frac{2xy}{x+y}\right) = \left(\frac{2\times30\times20}{30+20}\right) = \frac{1200}{50} = 24km/h$

10 If a person or a motor car covers three equal distances at the speed of x Km/h, y Km/h and z Km/h, respectively, then for the entire journey average speed of the person or motor car is

$$\left(\frac{3xyz}{xy + yz + zx}\right)$$

Example 30: A train covers the first 160 Km at a speed of 120 Km/h, another 160 Km at 140 Km/h and the last 160 Kms at 80 Km/h. Find out average speed of the train for the entire journey.

Solution: Average speed $=\frac{3xyz}{xy+yz+zx}$

$=\frac{3\times120\times140\times80}{120\times140+140\times80+80\times120}$

$=\frac{360140\times80}{16800+11200+9600} = \frac{4032000}{37600}$

$=107\frac{11}{47} km/h$

11 If a person covers A Km at a speed of x Km/h, B Km at a speed of y Km/h and C Km at a speed of z Km/h, the average speed during the entire journey is

$$\left(\frac{A + B + C}{\frac{A}{x} + \frac{B}{y} + \frac{C}{z}}\right) km/h$$

Example 31: A person covers 9 Km at a speed of 3 Km/h, 25 Km at a speed of 5 Km/h and 30 Km at a speed of 10 Km/h. Find out average speed for the entire journey.

Solution: The average speed$=\left(\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) km/h$

$$\left(\frac{9 + 25 + 30}{\frac{9}{3} + \frac{25}{5} + \frac{30}{10}}\right)$$

$$= \frac{64}{11} = 5\frac{9}{11} km/h$$

12 If a person covers A th part of the distance at x Km/h, Bth part of the distance at y Km/h and the remaining C th part at z Km/h, then the average speed during the entire journey is

$$\left(\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)km/h$$

Example 32: A person covers the first $\frac{1}{4}$ of the distance at 8 Km/h, the next $\frac{3}{5}$at 6 Km/h and the remaining distance at 15 Km/h. Find the average speed during the entire journey. Solution: The average speed

$$=\frac{1}{\left(\frac{A}{x}+\frac{B}{y}+\frac{C}{z}\right)}=\left(\frac{1}{\frac{1/4}{8}+\frac{3/5}{6}+\frac{3/20}{15}}\right)$$

$$\left[Here, A=\frac{1}{4}, B=\frac{3}{5}\ and\ C=1-\left(\frac{1}{4}+\frac{3}{5}\right)=\frac{3}{20}\right]$$

$$=\frac{1}{\frac{1}{31}+\frac{1}{10}+\frac{1}{100}}=\frac{3200}{452}=7\frac{9}{113}km/h$$

Example 33: A train covers 50% of the journey at 30 Km/h, 25% of the journey at 25 Km/h and the remaining at 20 Km/h. Find the average speed of the train during the entire journey.

Solution: Let the total journey be = 100m. The average speed: = 100 km

$$\left(\frac{100}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)=\left(\frac{100}{\frac{50}{30}+\frac{25}{25}+\frac{25}{20}}\right)$$

$$[Here, A=50, B=25\ and\ C=25]$$

$$=\frac{100}{47/12}=\frac{1200}{47}=25\frac{25}{47}km/h$$

## 2.2 Weighted Average

To find the weighted average (or weighted mean) of a data set, multiply

each value by its weight and add the products together, then divide by the

sum of the weights.

For a set of data x

1, $x_2, \ldots, x_n$, with nonnegative weights $w_1, w_2, \ldots, w_n$,

the weighted average is

$[x_1w_1 + x_2w_2 + \ldots + x_nw_n]/w_1 + w_2 + \ldots + w_n$ .

Finding a Weighted Average

A grade point average is a weighted average that gives greater weight to courses

that earn more credits. Hailey's grade points are 4.0 in Chemistry, which is worth

4 credits, 3.5 in English, which is worth 3 credits, and 3.7 in Physics, which is

worth 2 credits. What is Hailey's grade point average?

SOLUTION

Multiply each grade by its weight. Add the products and then divide by the sum

of the weights.

$4.0(4) + 3.5(3) + 3.7(2)/4 + 3 + 2 = 33.9/9 = 3.7\overline{6}$.

Hailey's grade point average is approximately 3.77.

Sometimes the weights associated with data values are numbers between 0 and the sum of the weights is 1. In this case, you can use the following equivalent formula for the weighted average.

## Summary

The key concepts learned from this unit are: -

- We have learnt the average of all numbers.
- We have learnt what are Weighted Average.

## Keywords

- average
- Weighted Average.

## Self Assessment

1. The average of 10 numbers is 7. What will be the new average if each of the numbers is multiplied by 8?

A. 45
B. 52
C. 56
D. 55

2. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by Rs42 per day while the average expenditure per head decreased by Rs 1. Find out the actual expenditure of the mess.

A. Rs 480
B. Rs 440
C. Rs 520
D. Rs 420

3. An aero plane travels 2500 Km, 1200 Km and 500 Km at 500 Km/h, 400 Km, and 250 Km/h, respectively. The average speed is:

A. 420 Km/h
B. 410 Km/h
C. 405 Km/h
D. 575 Km/h

4. The average age of 5 members is 21 years. If the age of the youngest member be 5 years, find out the average age of the family at the birth of the youngest member.

A. 24 years
B. 25 years
C. 20 years
D. 28 years

5. The average weight of 10 students is increased by half a Kg when one of the students weighing 50 Kg is replaced by a new student. Find out the weight of the new student.

A. 55 Kg

B. 60 Kg

C. 45 Kg

D. 40 Kg

6. The mean marks of 10 boys in a class is 70%, whereas the mean marks of 15 girls is 60%. The mean marks of all the 25 students is:

A. 64%

B. 60%

C. 55%

D. 52%

7. The average of five consecutive even numbers starting with 4, is:

A. 6

B. 7

C. 8

D. 7.5

8. The average of 17 numbers is 10.9. If the average of first nine numbers is 10.5 and that of the last 9 numbers is 11.4, the middle number is:

A. 11.8

B. 11.4

C. 10.9

D. 11.7

9. The sum of three numbers is 98. If the ratio between first and second be 2:3 and between second and third be 5:8, then the second number is:

A. 30

B. 20

C. 58

D. 48

10. The average age of A, B, C, D five years ago was 45 years. By including x, the present average age of all the five is 49 years. The present age of x is:

A. 64 years

B. 48 years

C. 45 years

D. 40 years

11. The average age of 8 men is increased by 2 years. When 2 of them, whose ages are 20 years and 24 years respectively, are replaced by 2 women. What is the average age of these two women?

A. 36 years

B. 30 years

C. 40 years

D. 42 years

12.At a language school, each student is given a score that measures his or her fluency. The fluency score is a weighted average that is determined by rating the student on a scale of 0 to 10 in three categories: grammar, vocabulary, and pronunciation. Grammar counts for 40% of the score, vocabulary counts for 25%, and pronunciation counts for 35%. Thomas gets ratings of 8 for grammar, 6 for vocabulary, and 5 for pronunciation. What is his fluency score?

A. 7.45

B. 6.45

C. 7.50

D. 7.70

13. A grade point average is a weighted average that gives greater weight to courses that earn more credits. Hailey's grade points are 4.0 in Chemistry, which is worth 4 credits, 3.5 in English, which is worth 3 credits, and 3.7 in Physics, which is worth 2 credits. What is Hailey's grade point average?

A. 8.65

B. 3.76

C. 9.88

D. 7.70

14. The 30 students in Ms. Chen's class had an average grade of 85 on a standardized test. The 20 students in Mr. Jackson's class had an average grade of 90 on the same test. What is the average test grade of all 50 students?

A. 80.0

B. 60

C. 19.5

D. 67.20

15. A batsman makes a score of 87 runs in the 17th inning and thus increases his avg by. Find his average after 17th inning.

A. 39

B. 40

C. 42

D. 69

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | C | 2. | D | 3. | A | 4. | C | 5. | A |
| 6. | A | 7. | C | 8. | B | 9. | A | 10. | C |
| 11. | B | 12. | B | 13. | B | 14. | D | 15. | A |

## Review Questions

1. There were 35 students in a hostel. Due to the admission of 7 new students, he expenses of the mess were increased by Rs. 42 per day while the average expenditureper head diminished by Rs 1. What was the original expenditure of the mess?
2. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the newman.
3. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by3 months. Find the age of the teacher.
4. The average weight of A,B,C is 45 Kg. The avg wgt of A & B be 40Kg & that of B,C be 43Kg. Find the wgt of B.
5. Find the average of first 40 natural numbers?
6. A tourist covers half of this journey by train at60 km/h, half of the remainder by bus at 30 km/hand the rest by cycle at 10 km/h. What is the average speed of the tourist in km/h during his entire journey.
7. What is the average of all multiples of 10 from 2 to198?
8. The average weight of 16 boys in a class is 50.25 kgand that of the remaining 8 boys is 45.15 kg.Find the average weights of all the boys in theclass.
9. In what ratio must water be mixed with milk togain 16 2 /3% on selling the mixture at cost price?
10. In what ratio must a grocer mix two varieties ofpulses costing H15 and H20 per kg respectively soas to get a mixture worth H16.50 kg?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing.
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 03: Mathematical Operations

**CONTENTS**



## Objectives

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand what is BODMAS Rule.
- Understand conversion of symbols into signs.

## Introduction

What is BODMAS Rule?

BODMAS is an acronym used for Brackets, Order, Division, Multiplication, Addition and Subtraction. In some regions, people/students use PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition and Subtraction) which is the synonym or equivalent of the BODMAS rule and can be used interchangeably.

It explains the mathematical operations to be performed while solving a mathematical expression. According to this rule, if multiple brackets (vinculum,+,×,÷) are there in the expression, start solving inside the vinculum or bar or line bracket first, round bracket then followed by curly bracket, then square bracket and then solve the order (means power and roots etc), then division, multiplication, addition and then subtraction.

So, the BODMAS rule is used to evaluate mathematical expressions and to deal with complex calculations in a much easier way and correctly.

## 3.1 Definition of BODMAS Rule

According to the rule, to solve any mathematical expression, first, solve the terms written inside the brackets, and then simplify exponential terms and move ahead to division and multiplication operations, then, at last, addition and subtraction.

Here, multiplication and division can be considered as level one operations as they must be solved first, addition and subtraction can be considered as level two operations. Simplification of terms inside the brackets can be done directly. This means we can perform the operations inside the bracket in order of division, multiplication, addition, and subtraction.

Sticking to this order of operations in the BODMAS rule, always gives the correct answer. If there are multiple brackets in an expression, all the same types of brackets can be solved simultaneously.

Example:- (31+2)÷(13−2)=33÷11=3

Look at the below diagram to understand the terms and operations denoted by the BODMAS acronym in the proper order.

There are few conditions and rules for general simplification as given below:

| Condition | Rule |
|---|---|
| p+(q+r)⇒p+q+rp+(q+r)⇒p+q+r | Open the bracket and add the terms. |
| p−(q+r)⇒p−q−rp−(q+r)⇒p−q−r | Open the bracket and multiply the negative sign with each term inside the bracket.<br>(All positive terms will be negative and negative terms will be positive) |
| p(q+r)⇒pq+prp(q+r)⇒pq+pr | Multiply the outside term with each term inside the round bracket |

BODMAS is an acronym used to remember the order of operations to be followed during solving mathematical expressions. According to this rule, first solve the expression inside the brackets (vinculum,(),{},[]) then solve order or of (power or roots), then division or multiplication (as division and multiplication is having the same precedence, perform whatever comes first from left to right), then solve the addition or subtraction (as addition and subtraction are having the same precedence, perform whatever comes first from left to right). It makes simplification easy and error-free.

BODMAS (Bracket's order or of Division Multiplication Addition Subtraction) rule is correct. But in some regions, people also use PEMDAS (Parentheses Exponent Multiplication Division Addition Subtraction) or BIDMAS (Brackets Indices Division Multiplication Addition Subtraction). By the way, all three acronyms are correct.

## 3.2 Simple Arithmetic Operation

It is a common need to simplify the expressions

formulated according to the statements of the problems related to practical life. To do this, it is essential to follow in sequence the mathematical operations given

by the term, 'BODMAS'

## 3.3 BODMAS

Each letter of the word BODMAS stands as follows:

B for Bracket : [{(-)}]

There are four brackets, namely, - i.e., bar, ( ), { } and

[ ]. They are removed, strictly in the order -, ( ), { } and [ ].

O for Of : of

D for Division : ÷

M for Multiplication : ×

A for Addition : +

S for Subtraction : -

The order of various operations in exercises involving brackets and fractions must be strictly performed according to the order of the letters of the word BODMAS.

**Illustration 1: simplify**

$8\frac{1}{2}$ - $\left[3\frac{1}{5}+4\frac{1}{2}\, of\ 5\frac{1}{3}\left\{11-\left(3-1\overline{\frac{1}{4}-\frac{5}{8}}\right)\right\}\ \right]$

Solution : - given expression

$= \frac{17}{2} - \left[\frac{16}{5}\ \div \frac{9}{2}\ of\ \frac{16}{3} + \left\{11-\left(3-1\overline{\frac{5}{4}-\frac{5}{8}}\right)\right\}\ \right]$

$= \frac{17}{2}$ - $\left[\frac{16}{5} \div \frac{9}{2}\ of\ \frac{16}{3} + \frac{69}{8}\right]$

$= \frac{17}{2} - \ \left[\frac{16}{5} \div \frac{9}{2}\ X \frac{16}{3} + \frac{69}{8}\right]$

$=\frac{17}{2} - \left[\frac{16}{5} \div \frac{24}{1} + \frac{69}{8}\right]$

$= \frac{17}{2}$ - $\left[\frac{16}{5}\ X\ \frac{1}{24} + \ \frac{69}{8}\right] = \frac{17}{2}$-$\left[\frac{16}{120} + \frac{69}{8}\right]$

$= \frac{17}{2} - \left[\frac{16+1035}{120}\right]$

$= \frac{17}{2}$ - $\frac{1051}{120}$

$= \frac{1020\ -1051}{120}$

$=$ -$\frac{31}{120}$

**Illustration 2: simplify**

$5\frac{1}{3}$ - $\left\{4\frac{1}{3}\ -\ \left(3\frac{1}{3} - 2\overline{\frac{1}{3} - \frac{1}{3}}\right)\right\}$

Solution : given expression

$= \ \frac{16}{3} - \left\{\frac{13}{3} - \left(\frac{10}{3} - \overline{\frac{7}{3} - \frac{1}{3}}\right)\right\}$

$= \frac{16}{3}\ -\ \left\{\frac{13}{3} - \left(\frac{10}{3} - \frac{6}{3}\right)\right\}$

$= \frac{16}{3}\ -\ \left\{\frac{13}{3} - \frac{4}{3}\right\}$

$=\frac{16}{3} - \left\{\frac{9}{3}\right\} = \frac{16}{3} - \frac{9}{3}$

$= \frac{7}{3}$

$= 2\frac{1}{3}$

## 3.4 Use of Algebraic Formulae

The following formulae are sometimes found useful in dealing with the simplification :

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (1-b)^2 = 2(a^2 + b^2)$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $(a^2 - b^2) = (a+b)(a-b)$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

7. $(a-b)^3 = a^3 - 3a^2b + 3ab2 - b^3$

$= a^3 - b^3 - 3ab(a-b)$

8. $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

9 $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

10 . $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$

11. $a^4 - a^4 = (a^2 + b^2)(a+b)(a-b)$

**Illustration 3: simplify the following**

1) 0.32 x .032 +0.64 x 0.68 x 0.68 x 0.68

**Solution:-** Given expression = 0.32 × 0.32 + 2 × 0.32 × 0.68 + 0.68 × 0.68

$= (0.32)^2 + 2 \times 0.32 \times 0.68 + (0.68)^2$

$= (0.32 + 0.68)^2$ [ $a^2 + 2ab + b^2 = (a + b)^2$ ]

$= 1^2$

= 1

ii)2.45 × 2.45 - 0.9 × 2.45 + 0.45 × 0.45

**Solution:** Given expression = 2.45 × 2.45 - 2 × 2.45 × 0.45 + 0.45 × 0.45

= (2.45)2 - 2 × 2.45 × 0.45 + (0.45)2

= (2.45 - 0.45)2 [ $a^2 - 2ab + b^2 = (a - b)^2$ ]

$= (2)^2$

= 4

iii) $\frac{7x\{(146+92)^2(146-92)^2\}}{(146)^2+(92)^2}$

**Solution:** given expression

$= \frac{7\,X\,2\{(146)^2+(92)^2\}}{(146)^2+(92)^2}$ [$\because (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

= 14

iv) $\frac{(0.345+0.255)^2-(0345-0.2555)^2}{0.345\,X\,1.02}$

**Solution:** given expression

$= \frac{(0.345+0.255)^2-(0345-0.2555)^2}{0.345\,X\,1.02}$

$= \frac{4\,x\,0.345\,x\,0.255}{4\,X\,0.345\,x\,0.255}$ [$\because (a+b)^2 - (a-b)^2 = 4ab$ ]

=1

(v) $\frac{0.682\,x\,0.682-0.318x\,0.318}{0.682-0.318}$

**Solution :**Given expression

$= \frac{(0.682)^2-(0.318)^2}{0.682-0.318}$

= ( 0.682 + 0.318) $\left[\because \frac{a^2-b^2}{a-b}\right] = a + b$

= 1

(vi) $\frac{(3.29)^2-(0.81)^2}{4}$

**Solution :** given expression

$= \frac{(3.29)^2-(0.81)^2}{3.29+0.81}$

= (3.29 – 0.81) $\left[\because \frac{a^2-b^2}{a-b}\right] = a - b$

= 2.48

(vii) $(2.35)^3 + 1.95 \times (2.35)^2 + 7.05 \times (0.65)^2 + (0.65)^3$

**Solution:** Given expression

$= (2.35)^3 + 3 \times 0.65 \times (2.35)^2$

$+ 3 \times 2.35 \times (0.65)^2 + (0.65)^3$

$= (2.35 + 0.65)^3$ $[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3]$

$= (3)^3$

= 27.

(viii) $\frac{(4.23)3-3x\ 0.32\ x\ (4.32)2+3x4.32x(0.x2)2-(0.32)2}{4\ X\ 4\ X\ 4}$

**Solution:** given expression

$= \frac{(4.23)3-3x\ 0.32\ x\ (4.32)2+3x4.32x(0.32)2-(0.32)2}{4\ X\ 4\ X\ 4}$

$\frac{(4.32-0.32)^3}{4^3}$ $[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3]$

$=\left(\frac{4}{4}\right)^3$

= 1

(ix) $\frac{885\ x\ 855X\ 885+115\ X115\ X\ 115}{885X\ 885+115\ X\ 115\ -885\ X\ 115}$

**Solution:** - $\frac{(885)^3+(115)^3}{(885)^2+(115)^2-885\ X\ 115}$

(885 + 115) $[\because \frac{a^3+b^3}{a^2-ab+b^2} = a+b\ ]$

= 1000

(x) $\frac{0.62\ X\ 0.62\ X\ 0.62\ -0.41\ X\ 0.41\ X\ 0.41}{0.62\ X\ 0.62+0.62\ X\ 0.41+0.41+0.41}$

**Solution:** given expression

$= \frac{(0.62)^3-(0.41)^3}{(0.62)^2+0.62X\ 0.41+(0.41)^2}$

= (0.62 - 0.41) ) $[\because \frac{a^3-b^3}{a^2+ab+b^2} = a-b\ ]$

= 0.21

(xi) $\frac{(2.3)^3+(1.5)^3+(1.2)^3-3X\ 2.3\ X\ 1.5\ X\ 1.2}{2.3X\ 2.3+1.5X1.5+1.2\ X\ 1.2-2.3\ X\ 1.5X-2.3X1.2-1.5X1.2}$

**Solution:** Given expression

$$\frac{(2.3)^3 + (1.5)^3 + (1.2)^3 - 3X\ 2.3\ X\ 1.5\ X\ 1.2}{(2.3)^2 + (1.5)^2 + (1.2)^2 - 2.3\ X\ 1.5X - 2.3X1.2 - 1.5X1.2}$$

= (2.3 + 1.5 + 1.2) $[\because \frac{a^3+b^3+c^2-3abc}{a^2+b^2+c^2-ab-ac-bc} = a+ b + c\ ]$

=5

surds and indices an is called, ‘a surd’ if n is a fraction and an is called, ‘an index’ if n is an integer. a is called, ‘the base’.

Common Errors While Using BODMAS Rule

Someone can make some common mistakes while applying the BODMAS rule to solve an expression and those mistakes are given below,

1. The presence of multiple brackets may cause confusion and we may get the wrong answer.

2. Error occurs due to lack of proper understanding of addition and subtraction of integers.

For example: $2-5+6=-3+6=3$.

But if we simplify $2-5+6=2-11=-9$, we will get a wrong answer.

3. By assuming that the division has higher precedence over multiplication and addition has higher precedence than the subtraction.

Multiplication and division are same level operations and must be performed from left to right sequence (whichever comes first in the expression) and same with addition and subtraction which are same levels operations to be performed after multiplication and division. If one solves division first before multiplication (which is on the left side of division operation) as D comes before M in BODMAS, they may get the wrong answer.

## Summary

The key concepts learned from this unit are: -

- Understand what BODMAS Rule is.
- Understand conversion of symbols into signs.

## Keywords

- BODMAS Rule.
- Precedence
- Symbols into signs.

## Self Assessment

1. 3+(6+7)=

A. 16

B. 17

C. 18

D. 12

2. 15–(3+2)=

A. 16

B. 17

C. 18

D. 10

3. 2×(3×8)=

A. 16

B. 22

C. 18

D. 10

4. [{50–(2+3)}+15]÷12=

A. 5

B. 1

C. 10

D. 20

5. 8+9÷9+5×2−7=

A. 12

B. 15

C. 20

D. 25

6. [25–3(6+1)]÷4+9=

A. 10

B. 12

C. 15

D. 19

7. 180÷15{(12–6)–(14–12)}=

A. (a)12

B. (b)6

C. (c)48

D. (d)20

8. $3+2^4\times(15\div3)$ using BODMAS rule is

A. 24

B. 20

C. 25

D. 83

9. If '+' means 'divided by', '_' means 'multiplied by', 'x' means 'minus' and ' ' means'plus', which of the following will be the value of the expression 16÷8-4+ 2*4?

A. 16

B. 28

C. 32

D. 44

10. If A means 'plus', B means 'minus', C means 'divided by' and D means 'multiplied by',then 18 A 12 C 6 D 2 B 5 =?

A. 15

B. 25

C. 27

D. 17

11. If a means 'plus', b means 'minus', c means 'multiplied by', d means 'divided by',then 18 c 14 a 6 b 16 d 4 =?

A. 63
B. 254
C. 288
D. 1208

12. If x stands for 'add', y stands for 'subtract', z stands for 'divide' and p stands for'multiply', then (7 p 3) y 6 x 5?

A. 5
B. 10
C. 15
D. 20

13. If A stands for +, B stands for -, C stands for x , then (10 C 4) A (4 C 4) B 6?

A. 60
B. 56
C. 50
D. 46

14. Which of the following two signs need to be interchanged to make the givenequation correct? 10+10÷10-10x10=10

A. + and –
B. + and ÷
C. + and x
D. ÷ and +

15. If A means -, B means ÷ , C means + and D meansx , then 15 B 3 C 24 A 12 D 2 =?

A. 34
B. 2
C. (c)9/5
D. none of these

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | D | 3. | B | 4. | A | 5. | A |
| 6. | A | 7. | C | 8. | D | 9. | B | 10. | D |
| 11. | B | 12. | D | 13. | C | 14. | C | 15. | A |

## Review Questions

1. 15+(6+7)=
2. 25–(3+2)=
3. 2×(3×8)=

4. [{10–(2+3)}+15]÷12=
5. 18+19÷19+15×12−17=
6. 180÷15{(12–6)–(14–12)}=
7. Using BODMAS rule solve 30+25×(25÷30)
8. If '-' means 'divided by', '+' means 'multiplied by', 'x' means 'minus' and ' ÷ ' means 'plus', which of the following will be the value of the expression 16÷8-4+ 2*4?
9. If A means 'plus', B means 'divided by', C means 'minus' and D means 'multiplied by', then 18 A 12 C 6 D 2 B 5 =?
10. If a means 'plus', b means 'minus', c means 'multiplied by', d means 'divided by', then 18 c 14 a 6 b 16 d 4 =?
11. If x stands for 'divide', y stands for 'subtract', z stands for 'add' and p stands for 'multiply', then (7 p 3) y 6 x 5?

## Further Readings


1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 04: Percentage

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand percentage commodity price increase/decrease
- Understand successive percent changes and budget based problems

## Introduction

Concept of Percentage: By a certain percent, we mean that many hundredths. Thus x percent means x hundredths, written as x%. To express x% as a fraction: We have, x% = x/100.

Thus, 20% =20/100 =1/5; 48% =48/100 =12/25, etc. To express a/b as a percent: We have, a/b =((a/b)*100)%. Thus, ¼ =[(1/4)*100] = 25%; 0.6 =6/10 =3/5 =[(3/5)*100]% =60%

The term percent means per hundred or for every hundred. It is the abbreviation of the Latin phrase percentum. Scoring 60 per cent marks means out of every 100 marks the candidate scored 60 marks. The term percent is sometimes abbreviated as p.c. The symbol % is often used for the term percent. Thus, 40 percent will be written as 40%. A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called rate percent, e.g., $\frac{5}{100}$ and 5 percent means the same thing, i.e., 5 parts out of every hundred parts.

## 4.1 Basic Formulae

01 To convert a fraction into a percent: To convert any fraction $\frac{1}{m}$ to rate percent, multiply it by 100 and put % sign, i.e., $\frac{1}{m} \times 100\%$

Example 1: What percentage is equivalent to $\frac{3}{5}$?

Solution: $\frac{3}{5} \times 100 = 60\%$.

02 To convert a per cent into a fraction: To convert a percent into a fraction, drop the percent sign and divide the number by 100.

Example 2: What fraction is $16\frac{2}{3}\%$?

Solution: $16\frac{2}{3}\%=\frac{\left(\frac{50}{3}\right)}{100} = \left(\frac{50}{3} \times \frac{1}{100}\right) = \frac{1}{6}$

03 To find a percentage of a given number: x% of given number (N) = $\frac{x}{100} \times N$

Example 3: 75% of 400 =?

Solution: 75% of 400 = $\frac{75}{100}$ × 400 = 300.

Example 4: Find a number whose 4% is 72.

Solution: Let the required number be x. Then, 4% of x = 72

$\Rightarrow \frac{4}{100} \times x = 72 \Rightarrow x = \frac{100}{4} \times 72 = 1800$.

Example 5: What per cent of 25 Kg is 3.5 Kg?

Solution: Let x% of 25 Kg be 3.5 Kg.

Then, x% of 25 Kg = 3.5 Kg ⇒ x 100 × 25 = 3·5

$\Rightarrow x = \frac{x}{100} \times 25 = 3.5 \Rightarrow x = \frac{3.5 \times 100}{25} = 14$.

Hence, 3.5 Kg is 14% of 25 Kg.

## 4.2 Short-cut Methods

01 (a) I f A is x% more than that of B, then B is less than that of A by

$$\left[\frac{x}{100+x} \times 100\right]\%$$

(b) I f A is x% less than that of B, then B is more than that of A by

$$\left[\frac{x}{100-x} \times 100\right]\%$$

Explanation

Given $A=B+\frac{x}{100}B = \frac{100+x}{100}B$

$\therefore A - B = \frac{100+x}{100}B - B$

$= \left(\frac{100+x}{100} - 1\right)B = \frac{x}{100}B$

So, $\frac{A-B}{A} = \frac{\frac{x}{100}B}{\frac{100+x}{100}B} = \frac{x}{100+x}$

$\Rightarrow A - B = \left(\frac{x}{100+x} \times 100\right)\% \ of\ A.$

Therefore, B is less than that of A by

$$\left(\frac{x}{100+x} \times 100\right)\%$$

Similarly, (b) can be proved.

Example 6: If Mohan's salary is 10% more than that of Sohan, then how much per cent is Sohan's salary less than that of Mohan?

Solution: Here, x = 10.

∴ Required answer =

$\left(\frac{x}{100+x} \times 100\right)\%$

=

$\left(\frac{x}{100+x} \times 100\right)\%$

$=9\frac{1}{11}\%$

Example 7: If A's income is 40% less than B's income, then how much per cent is B's income more than A's income?

Solution: Here, x = 40

∴Required answer = =

$$\left(\frac{x}{100-x} \times 100\right)\%$$

$=\left(\frac{40}{100-40} \times 100\right)\%=66\frac{2}{3}\%$

02 If A is x% of C and B is y% of C, then A=$\frac{x}{y} \times 100\%\ of\ B$

Explanation

Given A=$\frac{x}{100}\ C \Rightarrow C = 100\frac{A}{x}$

And, B=$\frac{y}{100}\ C \Rightarrow C = 100\frac{B}{y}$

∴ C = $100\frac{A}{x} = 100\frac{B}{y} \Rightarrow A = \frac{x}{y}B$

Or,$\frac{x}{y} \times 100\%\ of\ B$

Example 8: If A is 20% of C and B is 25% of C, then what percentage is A of B?

Solution: Here, x = 20 and y = 25.

A=$\frac{x}{y} \times 100\%\ of\ B$

$=\frac{20}{25} \times 100\%\ of\ B, i.e, 80\%\ of\ B$

03 (a) I f two numbers are respectively x% and y% more than a third number, then the first number is $\left(\frac{100+x}{100+y} \times 100\right)\%$ of the second and the second number is $\left(\frac{100-y}{100-x} \times 100\right)\%$ of the first

(b) If two numbers are, respectively, x% and y% less than a third number, then the first number is $\left(\frac{100-x}{100-y} \times 100\right)\%$ % of the second

and the second number is $\left(\frac{100-y}{100-x} \times 100\right)\%$

Explanation Let A, B and C be the three numbers. Given:

$$A = C + \frac{x}{100}C = \left(\frac{100+x}{100}\right) C \Rightarrow c = A\left(\frac{100}{100+x}\right)$$

And, B=C+$\frac{y}{100}\ C = \left(\frac{100+y}{100}\right) C \Rightarrow C = B\left(\frac{100}{100+y}\right)$

∴ A$\left(\frac{100}{100+x}\right) = B\left(\frac{100}{100+y}\right)$

$$\Rightarrow A = \left(\frac{100+x}{100+y}\right) B\ Or\ \left(\frac{100+x}{100+y}\right) \times 100\%\ of\ B$$

$$and, B = \left(\frac{100+y}{100+x}\right) A\ or\ \left(\frac{100+y}{100+x}\right) \times 100\%\ of\ A.$$

Similarly, (b) can be proved

Example 9: Two numbers are respectively 20% and 50% more than a third number. What per cent is the first of the second?

Solution: Here, x = 20 and y = 50.

∴ First number $=\left(\frac{100+x}{100+y}\right) \times 100\%\ of\ the\ second$

$=\left(\frac{100+20}{100+50}\right) \times\ 100\%\ of\ the\ second$

i.e., 80% of the second.

Example 10: Two numbers are, respectively, 32% and 20% less than a third number. What per cent is the first of the second?

Solution: Here, x = 32 and y = 20.

∴ First number $=\left(\frac{100-x}{100-y}\right) \times 100\%\ of\ the\ second$

$=\left(\frac{100-32}{100-20}\right) \times\ 100\%\ of\ the\ second$

i.e., 85% of the second.

04 (a) If the price of a commodity increases by P%, then the reduction in consumption so as not to increase the expenditure is

$$\left(\frac{p}{100+p} \times 100\right)\%$$

(b) If the price of a commodity decreases by P%, then the increase in consumption so as not to decrease the expenditure is

$$\left(\frac{p}{100-p} \times 100\right)\%$$

Explanation Let the original price of the commodity be rs.100. Then, the increased price = 100 - $\frac{p}{100} \times 100$

= rs (100 + P).

Therefore, to keep the price unchanged, there should be a reduction in the consumption of the commodity by rs.P.

Decrease in rs (100 + P) = rs. P

∴ Decrease in rs100 =

$$\frac{p}{100+p} \times 100$$

∴ Required reduction in consumption is

$$\left(\frac{p}{100+p} \times 100\right)\%$$

Similarly, (b) part can be proved.

Example 11: If the price of sugar increases by 25%, find how much per cent its consumption be reduced so as not to increase the expenditure.

Solution: Reduction in consumption

$$\left(\frac{p}{100+p} \times 100\right)\%$$

=

$$\left(\frac{25}{100+25} \times 100\right)\%\ or\ 20\%$$

Example 12: If the price of a commodity decreases by 25%, find how much per cent its consumption be increased so as not to decrease the expenditure.

Solution: Increase in consumption

=

$$\left(\frac{p}{100-p}\times 100\right)\%$$

=

$\left(\frac{25}{100-25}\times 100\right)\%$ or $33\frac{1}{3}\%$

05 I f a number is changed (increased/decreased) successively by x% and y%, then net % change is given by$\left(x+y+\frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign is +ve or –ve. If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

Explanation

Let the given number be N.

If it is increased by x%, then it becomes

N+x% of N=N+$\frac{Nx}{100}=\frac{N(x+100)}{100}$

If it is further increased by y%, then it becomes

$$\frac{N(x+100)}{100}+\frac{y}{100}\times\frac{N(x+100)}{100}$$

=$\frac{N(x+100)(y+100)}{(100)^2}$

∴ Net change =$\frac{N(x+100)(y+100)}{(100)^2}$

=$\frac{N(100x+100y+xy)}{(100)^2}$

∴ % change =$N\left(x+y+\frac{xy}{100}\right)\times\frac{1}{100}\times\frac{100}{N}$

$$\left(x+y+\frac{xy}{100}\right)\%$$

Example 13: If salary of a person is first increased by 15% and thereafter decreased by 12%, what is the net change in his salary?

Solution: Here, x = 15 and y = –12.

∴ The net% change in the salary

=$\left(x+y+\frac{xy}{100}\right)\%=\left(15-12-\frac{15\times12}{100}\right)\%\ or\ 1.2\%$

Since the sign is +ve, the salary of the person increases by 1.2%

Example 14: The population of a town is decreased by 25% and 40% in two successive years. What per cent population is decreased after two years?

Solution: Here, x = – 25 and y = – 40.

∴ The net% change in population

=$\left(x+y+\frac{xy}{100}\right)\%$

=$\left(-25-40+\frac{25\times40}{100}\right)\%\ or\ -55\%$

Since the sign is –ve, there is decrease in population after two years by 55%

06 If two parameters A and B are multiplied to get a product and if A is changed (increased/ decreased) by x% and another parameter B is changed (increased/decreased) by y%, then the net % change in the product (A × B) is given

$\left(x+y+\frac{xy}{100}\right)$ 100%which represents increase or decrease in value according as the sign in +ve or –ve. If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign. \

Example 15: If the side of a square is increased by 20%, its area is increased by k%. Find the value of k.

Solution: Since side × side = area

∴ Net% change in area

$$\left(x+y+\frac{xy}{100}\right)\% = \left(20+20+\frac{20\times 20}{100}\right)\%$$

=44%

Therefore, the area is increased by 44%. Here, k = 44.

Example 16: The radius of a circle is increased by 2%. Find the percentage increase in its area.

Solution: Since π × radius × radius = area ∴ Net% change in area

$$=\left(x+y+\frac{xy}{100}\right)\% = \left(2+2+\frac{2\times 2}{100}\right)\%$$

[Here, x = 2 and y = 2]

$=4\frac{1}{25}\%$

Therefore, the percentage increase in area is $4\frac{1}{25}\%$

Example 17: The tax on a commodity is diminished by 15% and its consumption increases by 10%. Find the effect on revenue.

Solution: Since tax × consumption = revenue ∴ Net% change in revenue

$$=\left(x+y+\frac{xy}{100}\right)\% = \left(-15+10+\frac{15\times 10}{100}\right)\%$$

[Here, x = – 15 and y = 10]

= – 6.5%

∴ The revenue decreases by 6.5%

07 If the present population of a town (or value of an item) be P and the population (or value of item) changes at r% per annum, then

(a) Population (or value of item) after n years

$=p\left(1+\frac{r}{100}\right)^n$

(b) Population (or value of item) n years ago

$=\frac{p}{\left(1+\frac{r}{100}\right)^n}$

here r is +ve or –ve according as the population

(or value of item) increases or decreases.

Explanation

Population at the end of first year

=P+$\frac{r}{100}p = p\left(1+\frac{r}{100}\right)$

Now, the population at the beginning of second year

=P$\left(1+\frac{r}{100}\right)$

∴ Population at the end of second year

=P$\left(1+\frac{r}{100}\right)+\frac{r}{100}p\left(1+\frac{r}{100}\right) = p\left(1+\frac{r}{100}\right)^2$

Population at the end of n years=p$\left(1+\frac{r}{100}\right)^n$

Example 18: The population of a town increases 5% annually. If its present population is 84000, what will it be in 2 years time?

Solution: Here, P = 84000, r = 5 and n = 2.

∴ Population of the town after 2 years

=p$\left(1+\frac{r}{100}\right)^n = 84000\text{p}\left(1+\frac{5}{100}\right)^2$

=84000$\times\frac{105}{100}\times\frac{105}{100} = 92610$

Example 19: The population of a town increases at the rate of 5% annually. If the present population is 4410, what it was 2 years ago?

Solution: Here, P = 4410, r = 5 and n = 2.

∴ Population of the town 2 years ago

$$=\frac{p}{\left(1+\frac{r}{100}\right)^n}=\frac{4410}{\left(1+\frac{5}{100}\right)^2}=\frac{4410}{\frac{105}{100}\times\frac{105}{100}}=4000$$

08 If a number A is increased successively by x% followed by y% and then by z%, then the fi nal value of A will be

A$\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$

In case a given value decreases by any percentage, we will use a -ve sign before that.

Example 20: The population of a town is 144000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years?

Solution: Here, P = 144000, x = 5, y = – 10 and z = 15.

∴ Population of the town after 3 years

=A$\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$

=144000$\left(1+\frac{5}{100}\right)\left(1-\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$

=$\frac{144000\times105\times90\times115}{1001\times001\times00}$=156492

09 In an examination, the minimum pass percentage is x% If a student secures y marks and fails by z marks, then the maximum marks in the examination is $\frac{100(y+z)}{x}$

Explanation Let the maximum marks be m. Given: x% of m = y + z

$\Rightarrow \frac{x}{100} \times m = y + z \text{ or } m = \frac{100(y+z)}{x}$

Example 21: In an examination, a student must get 60% marks to pass. If a student who gets 120 marks, fails by 60 marks, fi nd the maximum marks.

Solution: Here, x = 60, y = 120 and z = 60.

∴ Maximum marks

$$= \frac{100(y+z)}{x} = \frac{100(120+60)}{60} = \frac{100 \times 180}{60} = 300$$

10 In an examination x% and y% students respectively fail in two different subjects while z% students fail in both the subjects, then the percentage of students who pass in both the subjects will be (100 – (x + y – z)) %

Explanation

Percentage of students who failed in one subject = (x – z) %

Percentage of students who failed in other subject = (y – z) %

Percentage of students who failed in both the subjects = z%

∴ Percentage of students who passed in both the subjects = [100 – [(x – z) + (y – z) + z]] % = (100 – (x + y – z))%

Example 22: In an examination, 42% students failed in Mathematics and 52% failed in Science. If 17% failed in both the subjects, find the percentage of those who passed in both the subjects.

Solution: Here, x = 42, y = 52 and z = 17.

∴ Percentage of students passing both the subjects = (100 – (x + y – z))% = (100 – (42 + 52 – 17))% or

23%.

n this article, we deal with the concept of successive percentage change. This is a problem type in Percentages and using the formula in this article, you can easily solve questions based on this concept in matter of seconds.

## 4.3 Successive Percentage Change

What is Successive Percentage Change?

The concept of successive percentage change deals with two or more percentage changes applied to quantity consecutively. In this case, the final change is not the simple addition of the two percentage changes (as the base changes after the first change).

Formula for Percentage Change:

Suppose a number N undergoes a percentage change of x % and then y%, the net change is:

New number = N × (1 + x/100) × (1 + y/100).

Now, (1 + x/100) × (1 + y/100) = 1 + x/100 + y/100 + xy/10000

If we say that x + y + xy/100 = z, then (1 + x/100) × (1 + y/100) = 1 + z/100

Here, z is the effective percentage change when a number is changed successively by two percentage changes.

Various cases for Percentage Change:

Both percentage changes are positive:

x and y are positive and net increase = (x+y+xy/100) %.

One percentage change is positive and the other is negative:

x is positive and y is negative, then net percentage change = (x-y-xy/100)%

Both percentage changes are negative:

x and y both are negative and imply a clear decrease= (-x-y+xy/100)%

Percentage Change involving three changes:

If value of an object/number P is successively changed by x%, y% and then z%, then final value.

$=P(1 \pm \frac{x}{100})(1 \pm \frac{y}{100})(1 \pm \frac{z}{100})$

Example 23: A's salary is increased by 10% and then decreased by 10%. The change in salary is

Solution:

Percentage change formula when x is positive and y is negative = {x – y – (xy/100)}%

Here, x = 10, y = 10

= {10 – 10 – (10 x 10)/100} = -1%

As negative sign shows a decrease, hence the final salary is decreased by 1%.

## 4.4 Weighted Averages and Expected Values

1. Find weighted averages, and compute and interpret expected values. Visitors to a Web site give books ratings from 1 to 5 stars. The chart shows the ratings that one book received from 10 visitors to the site. To find the average rating, you can add the numbers of stars in the ratings and then divide by the number of ratings. [4 + 1 + 3 + 5 + 3 + 5 + 2 + 5 + 5 + 4]/10 = 3.7

| Visitor Ratings |
|---|
| ★★★★ |
| ★ |
| ★★★ |
| ★★★★★ |
| ★★★ |
| ★★★★★ |
| ★★ |
| ★★★★★ |
| ★★★★★ |
| ★★★★ |

2. The average rating is 3.7 stars. You can also find the average rating by multiplying each rating by the number of times it occurs, adding these products, and then dividing by the total number of ratings.
   [1(1) + 2(1) + 3(2) + 4(2) + 5(4)]/10 = 3.7
3. Notice that some ratings occur more frequently than others, and these ratings are "weighted" more heavily in the numerator because they are multiplied by greater values. This idea can be generalized to give a definition of a weighted average. Note that the weights do not have to be based on frequencies, but can be based on other factors, such as relative importance or value.

## Summary

The key concepts learned from this unit are: -

- We have learnt percentage commodity price increase/decrease
- We have learnt successive percent changes and budget based problems.

## Keywords

- Percentage
- Commodity
- Successive percent
- Budget

## Self Assessment

1. Express 56% as a fraction:
A. (a)14/25
B. (b)18/25
C. (c)19/25
D. (d)20/25

2. Express 4% as a fraction:
A. 1/24.
B. 1/20.
C. 3/25.
D. 1/25.

3. Express 0.6% as a fraction:
A. 3/70
B. 3/125
C. 13/50
D. 3/500

4. Express 0.008%as a fraction:
A. 12/1250
B. 13/1250
C. 1/1250
D. 7/1250

5. Express 6% as a Decimal:
A. 0.06.
B. 0.28.
C. 0.002.
D. 0.004

6. Express 28% as a Decimal:
A. 0.06.

B. 0.28.

C. 0.002.

D. 0.004

7. Express 0.04% as a Decimal :

A. 0.06.

B. 0.28.

C. 0.002.

D. 0.004

8. 2 is what percent of 50 ?

A. 4%

B. 12%

C. 0.2%

D. 0.04%

9. ?% of 25 = 20125.

A. 12.5

B. 9

C. 8.5

D. 14.5

10. Which is greatest in 16 ( 2/3) %, 2/5 and 0.17 ?

A. 2/5

B. 16

C. 2/3

D. 0.17

11. An inspector rejects 0.08% of the meters as defective. How many will beexamine to project ?

A. 2400

B. 2500

C. 3000

D. 4000

12. Sixty five percent of a number is 21 less than four fifth of that number. Whatis the number ?

A. 130

B. 140

C. 150

D. 200

13. In expressing a length 810472 km as nearly as possible with three significant digits,find the percentage error

A. 0.0034

B. 0.034

C. 0.34

D. 3.4

14. if 50% of (x-y) = 30% of (x+y) then what percent of x is y?
A. 25%
B. 50%
C. 75%
D. 100%

15. The salary of a person was reduced by 10% .By what percent should hisreduced salary be raised so as to bring it at par with his original salary ?
A. (100/9)%
B. (20/9)%
C. (100/19)%
D. (100/29)%

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | D | 3. | D | 4. | C | 5. | A |
| 6. | B | 7. | D | 8. | A | 9. | C | 10. | D |
| 11. | B | 12. | B | 13. | B | 14. | A | 15. | A |

## Review Questions

1. When the price for a product was decreased by 10%, the number sold increased by 30%. What was the effect on the total revenue?
2. If the numerator of a fraction be increased by 15% and its denominator be diminished by 8% , the value of the fraction is 15/16. Find the original fraction.
3. The population of a town is 1, 76,400 . If it increases at the rate of 5% per annum, what will be its population 2 years hence? What was it 2 years ago?
4. In the new budget, the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?
5. The value of a machine depreciates at the rate of 10% per annum. If its present is Rs.1,62,000 what will be its worth after 2 years ? What was the value of the machine 2 years ago?
6. During one year, the population of town increased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year ?
7. If A earns 99/3% more than B, how much percent does B earn less then A?
8. How many kg of pure salt must be added to 30kg of 2% solution of salt and water to increase it to 10% solution?
9. In an examination, 80% of the students passed in English , 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.
10. The salary of a person was reduced by 20% .By what percent should his reduced salary be raised so as to bring it at par with his original salary ?

11. The capacity of a ground was 100000 at the end of 2012. In 2013, it increased by 10% and in 2014, it decreased by 18.18%. What was the ground's capacity at the end of 2014?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 05: Profit and Loss

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We have learnt how to calculate Cost Price, Selling Price and Profit or Gain.
- We have learnt how to calculate successive discount and marked price

## Introduction

Nowadays, transactions have become a common feature of life. When a person deals in the purchase and sale of any item, he either gains or loses some amount generally. The aim of any business is to earn profit.

Profit (P)

The amount gained by selling a product with more than its cost price.

Loss (L)

The amount that you lose after selling a product with less than its cost price.

Cost Price – This is the rate at which the commodity is bought. The amount paid for a product or commodity to purchase it is called a cost price. It can be divided into two parts:

Fixed Cost

Variable Cost

Selling Price – This is the rate at which the commodity is sold.

Profit – When the Cost Price of the commodity is less than the Selling Price.

Profit = Selling Price – Cost Price

Loss – When the Cost Price of the commodity is greater than the Selling Price.

Loss = Cost Price – Selling Price

If there is a PROFIT of x%, the calculating figures would be 100 and (100 + x).

If there is a LOSS of x%, the calculating figures would be 100 and (100 - x).

Calculating figures be Cost Price and Selling Price respectively.

The commonlyused terms in dealing with questions involving sale and purchase are:

Cost Price: The cost price of an article is the price at which an article has been purchased. It is abbreviated as C.P.

Selling Price: The selling price of an article is the price at which an article has been sold. It is abbreviated as S.P.

Profit or Gain: If the selling price of an article is more than the cost price, there is a gain or profit. Thus, Profit or Gain = S.P. - C.P.

Loss: If the cost price of an article is greater than the selling price, the seller suffers a loss. Thus, Loss = C.P. - S.P. respect to the cost price of the item.

Marked Price Formula (MPF)

Discount=Marked price- selling price

Discount percentage=(Discount/Marked price)*100

Example 1: (i) If C.P. = `235, S.P. = `240, then profit=?

(ii) If C.P. = `116, S.P. = `107, then loss =?

Solution: (i) Profit= S.P. - C.P. = 240 - 235 = rs.5.

(ii) Loss = C.P. - S.P. = 116 - 107 = rs.9.

Basic Formulae

01 Gain on rs100 is Gain per cent Gain

Gain%= $\frac{Gain \times 100}{C.P}$

Loss on rs100 Loss per cent

Loss%= $\frac{loss \times 100}{c.p}$

Another Example: A toy that cost 80 rupees is sold at a profit of 20 rupees. Find the percent or rate of

profit.

Solution: Gain/cost × 100 = % profit.

20/80 × 100 = 25%. - Answer

Example 2: The cost price of a shirt is rs.200 and selling price is rs. 250.Calculate the % ofprofit.

Solution: We have, C.P. = rs.200, S.P. = rs.250.

Profit= S.P. - C.P. = 250 - 200 = rs.50.

∴ Profit% $= \frac{profit \times 100}{C.P} = \frac{50 \times 100}{200}$ 25%

Example 3: Anu bought a necklace for rs.750 and sold it for rs.675. Find her percentage of loss.

Solution: Here, C.P. = rs.750, S.P. = rs.675

Loss = C.P. - S.P. = 750 - 675 = rs75.

$\therefore loss\% = \frac{loss \times 100}{C.P} = \frac{75 \times 100}{750} = 10\%$

02 When the selling price and gain%aregiven:

$$C.P = \left(\frac{100}{100 + Gain\%}\right) \times S.P$$

03 When the cost and gain per cent are given:

S.P=$\left(\frac{100+Gain\,\%}{100}\right)\times C.P$

Explanation

Since profit%=$\frac{profit\ \times 100}{C.P}$

$\therefore \frac{profit\ \%}{100}=\frac{S.P}{C.P}-1$

Or $\frac{S.P}{C.P}=1+\frac{profit\ \%}{100}$

$\therefore$ S.P. =$\left(\frac{100+profit\ \%}{100}\right)\ \times C.P$

and, C.P. =$\left(\frac{100}{100+profit\ \%}\right)\ \times S.P$

04 When the cost and loss per cent are given:

$$s.p=\left(\frac{100-loss\%}{100}\right)\times c.p$$

05 When the selling price and loss per cent are given:

C.P=$\left(\frac{100}{100-loss\,\%}\right)\times s.p$

Explanation

Since loss%=$\frac{loss\times 100}{C.P}$

=$\left[\frac{(C.P-S.P)\times 100}{C.P}\right]$

$\therefore \frac{loss\%}{100}=1-\frac{s.p}{c.p}$

Or $\frac{S.P}{C.P}=1-\frac{loss\%}{100}$

$\therefore$S.P=$\left(\frac{100-loss\,\%}{100}\right)\times C.P$

And, C.P=$\left(\frac{100}{100-loss\,\%}\right)\times S.P.$

Another Example: A damaged chair that cost Rs.110 was sold at a loss of 10%. Find the loss and the selling price.

Solution: Cost x percent loss = loss. 110 x 1/10 = 11, loss. Cost - loss = selling price. 110 - 11 = 99, selling price.

Example 4: Mr Sharma buys a cooler for rs.4500. For how much should he sell it to gain 8%?

Solution: We have, C.P. = rs.4500, gain%=8%

$\therefore$S.P=$\left(\frac{100+Gain\,\%}{100}\right)\times C.P$

=$\left(\frac{100+8}{100}\right)\times 4500$

=$\frac{108}{100}\times 4500=rs.4860$

Example 5: By selling a fridge for rs.7200, Pankaj loses10%. Find the cost price of the fridge.

Solution: We have, S.P. = rs.7200, loss% =10%

$\therefore$ C.P=$\left(\frac{100}{100-loss\,\%}\right)\times s.p$

=$\left(\frac{100}{100-10}\right)\times 7200$

$=\frac{100}{90} \times 7200 = rs.8000$

To find the profit and the cost when the selling price and the percent profit are given, multiply the selling price by the percent profit and subtract the result from the selling price.

Another Example: A toy is sold for Rs. 6.00 at a profit of 25% of the selling price. Separate this selling price into cost and profit.

Answer :

Selling price x % profit = profit.

Selling price = profit + cost.

6.00 x .25 = 1.50, profit.

6.00 - 1.50 = 4.50, cost

Example 6: By selling a pen for rs.99, Mohan gains $12\frac{1}{2}$%. Find out cost price of the pen.

Solution: Here, S.P. = rs.99,gain% = 12

$\therefore C.P = \left(\frac{100}{100+GAIN\%}\right) \times s.p$

$=\left(\frac{100}{100+\frac{25}{2}}\right) \times 99$

$=\left(\frac{100\times 2}{225}\right) \times 99 = rs88$

## 5.1 Some Useful short cut methods

01 If a man buys x items for `y and sells z items for `w, then the gain or loss per cent made by him is

$$\left(\frac{xw}{zy} - 1\right) \times 100\%$$

Explanation S.P. of z iterms =rs.w

S.P. of x items = rs.$\frac{w}{z}x$

Net profit= $\frac{w}{z}x - y$

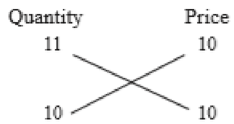
$\therefore$ %profit=$\frac{\frac{w}{z}x-y}{y} \times 100\%$

i.e,$\left(\frac{xw}{zy} - 1\right) \times 100\%$

which represents loss, if the result is negative.

Example 7: If 11 oranges are bought for `10 and sold at 10 for rs.11, what is the gain or loss %?

Solution:



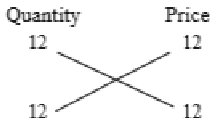

$\therefore$ %profit=$\left(\frac{xw}{zy} - 1\right) \times 100\%$

$=\left(\frac{11\times11}{10\times10}-1\right)\times100\%$

$=\frac{21}{100}\times100\%=21\%$

Example 8:A fruit seller buys apples at the rate of rs.12 per dozen and sells them at the rate of 15 for rs.12. Find out his percentage gain or loss.

Solution:




%gain or loss=$\left(\frac{xw}{zy}-1\right)\times100\%$

$=\left(\frac{12\times12}{15\times12}-1\right)\times100\%$

$$\left(\frac{144}{180}-1\right)\times100\%$$

$$\left(\frac{-36}{180}\times100\%\right)$$

Since the sign is -ve, there isa loss of 25%

02 If the cost price of m articles is equal to the selling price of n articles, then

%gain or loss=$\left(\frac{m-n}{n}\right)\times100$

[If m >n,it is % gain and, if m < n, it is % loss]

Explanation

Let, the C.P. of an article be rs.1.

∴ C.P. of m articles = `m × 1 = rs.m

S.P. of n articles = rs.m

∴ S.P. of an article =rs.$\frac{m}{n}$

∴ Profit on 1 article =rs$\left(\frac{m}{n}-1\right)$

i.e.,rs.$\left(\frac{m-n}{n}\right)$

∴%profit=$\left(\frac{m-n}{n}\right)\times100$i.e,$\left(\frac{m-n}{n}\right)\times100$

Example 9: A shopkeeper professes to sell his goods on cost price, but uses 800 gm, instead of 1 Kg. What is his gain %?

Solution: Here, cost price of 1000 gm is equal to selling price of 800 gm,

∴%gain=$\left(\frac{m-n}{n}\right)\times100$

$=\left(\frac{1000-800}{800}\right)\times100$

$=\frac{200}{800}\times100=25\%$

Example 10: If the selling price of 12 articles is equal to the cost price of 18 articles, what is the Profit %?

Solution: Here, m = 18, n = 12

∴ Profit %=$\left(\frac{m-n}{n}\right) \times 100$

=$\left(\frac{1000-800}{800}\right) \times 100$

=$\frac{200}{800} \times 100 = 25\%$

03 If an article is sold at a price S.P.1., then % gain or % loss is x and if it is sold at a price S.P2., then % gain or % loss is y. If the cost price of the article is C.P., then

$$\frac{S.p}{100+x} = \frac{S.p_2}{100+y} = \frac{c.p}{100} = \frac{S.p_1 - s.p_2}{x-y}$$

where x or y is -ve, if it indicates a loss, otherwise it is +ve.

Example 11: By selling a radio for rs. 1536, Suresh lost 20%. What per cent shall he gain or lose by selling it for rs. 2000?

Solution: Here, $S.P_1$ = 1536, x = - 20

(-ve sign indicates loss)

$S.P_2$ = RS.2000, y = ?

Using the formula:

$$\frac{s.p_1}{100+x} = \frac{s.p_2}{100+y}$$

we get,$\frac{1536}{100-20} = \frac{2000}{100+y}$

⇒100+y=$\frac{2000\times80}{1536}$=104$\frac{1}{6}$

⇒y=4$\frac{1}{6}$%

Thus, Suresh has a gain of 4$\frac{1}{6}$% by selling it for rs.2000

04 I f 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%. If 'C' pays `z for it to 'B', then the cost price for 'A' is

$$\left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

Where m or n is -ve, of it indicates a loss, otherwise it is +ve.

Example 12: Mohit sells a bicycle to Rohit at a gain of 10% and Rohit again sells it to Jyoti at a profit of 5%. If Jyoti pays rs.462 to Rohit, what is the cost price of the bicycle for Mohit?

Solution: Here, m = 10, n = 5, z = rs.462.

Using the formula,

$$C.P = \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

we get, C.P. for Mohit =$\left[\frac{100^2 \times 462}{(100+10)(100+5)}\right]$

=

$$\frac{462 \times 10000}{110 \times 105} = rs. 400$$

Example 13: 'A' sells a DVD to 'B' at a gain of 17%and 'B' again sells it to 'C' at aloss of 25%. If 'C' pays `1053 to 'B', what is the cost price of the DVD to 'A'?

Solution: We have, m = 17, n = -25, z = rs.1053.

∴ Cost price of DVD to

=$\left[\frac{100^2 z}{(100+m)(100+n)}\right]$

=$\frac{100 \times 100 \times 1053}{(100+17)(100-25)}$

=$\frac{100 \times 100 \times 1053}{117 \times 75} = rs. 1200$

05 If 'A' sells an article to 'B' at a gain/loss of m%, and 'B' sells it to 'C' at a gain/loss of n%, then the resultantProfit/loss percent is given by

$\left(m + n + \frac{mn}{100}\right)$ (1)

Where m or n is -ve, if it indicates a loss, otherwise it is +ve.

Example 14: 'A' sells a horse to 'B' at a profit of 5% and 'B' sells it: to 'C' at a profit of 10%. Find out the resultantprofit percent.

Solution: We have, m = 5 and n = 10.

∴ Resultant profit %= $\left(m + n + \frac{mn}{100}\right)$

$$\left(5 + 10 + \frac{5 \times 10}{100}\right)$$

=$\frac{31}{2}\%$ $or\ 15\frac{1}{2}$

Example 15: Manoj sells a shirt to Yogesh at a profit of 15%, and Yogesh sells it to Suresh at a loss of 10%. Find the resultantprofit or loss.

Solution: Here, m = 15, n = -10

∴ Resultant profit/loss % =$\left(m + n + \frac{mn}{100}\right)$

=$\left(15 - 10 + \frac{15 \times -10}{100}\right)$ =$\left(15 - 10 - \frac{150}{100}\right)$

=7/2% or $3\frac{1}{2}\%$

which represents profit as the sign is +ve.

06 When two different articles are sold at the same selling price, getting gain/loss of x% on the first and gain/loss of y% on the second, then the overall % gain or % loss in the transaction is given by

$$\left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

The above expression represents overall gain or loss according as its sign is +ve or -ve.

07 When two different articles are sold at the same selling price getting a gain of x% on the first and loss of x% on the second, then the overall % loss in the transaction is given by

$$\left(\frac{x}{10}\right)^2\%$$

Note that in such questions, there is always a loss.

Explanation

Let, each article be sold at rs.z.

Since gain/loss of x% is made on the first, cost price of the first article

=rs.z$\left(\frac{100}{100+x}\right)$

Also, gain/loss of y% is made on the second. Therefore, cost price of the second article

=rs. z$\left(\frac{100}{100+y}\right)$

∴ Total C.P=z$\left(\frac{100}{100+x}\right)+z\left(\frac{100}{100+y}\right)$

=z$\left[\frac{100(100+y)+100(100+x)}{(100+x)(100+y)}\right]$

Total S.P. = 2z.

∴ Overall % gain or loss $=\frac{S.P-C.P}{C.P}\times 100$

$=\frac{2z-\frac{100z\{100+x+100+y\}}{(100+x)(100+y)}}{\frac{100z[100+x+100+y\}}{(100+x)(100+y)}}\times 100$

$=\frac{2(100+x)(100+y)-100(200+x+y)}{100(200+x+y)}\times 10$

$=\frac{100x+100y+2xy}{(100+x)(100+y)\%}$

$=\left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$

Example 16: Mahesh sold two scooters, each for `24000. If he makes 20% profit on the first and 15% loss on the second, what is his gain or loss per cent in the transactions?

Solution: Here, x = 20 and y = -15.

∴ Over all gain/loss %

$=\left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$

$=\left[\frac{100(20-15)+2\times 20\times -15}{(100+20)+(100-15)}\right]\%$

$=-\frac{100}{205}\%=-\frac{20}{41}\%$

which represents loss, being a -ve expression.

Example 17: Rajesh sold two horses for `990 each; gaining 10% on the one and losing 10% on the other. Find out his total gain or loss per cent.

Solution: Here, x = 10.

∴ Overall loss % $=\left(\frac{x}{10}\right)^2\% = \left(\frac{10}{10}\right)^2\% = 1\%$

08 A merchant uses faulty measure and sells his goods at gain/loss of x%. The overall % gain/loss(g) is given by

$$\frac{100+g}{100+x} = \frac{Trye\ measure}{Faulty\ measure}$$

09 A merchant uses y% less weight/length and sells his goods at gain/loss of x%.The overall %gain/ loss is given by

$$\left[\left(\frac{y+x}{100-y}\right)\times 100\right]\%$$

## 5.2 False weight

Example 18: A dishonest shopkeeper professes to sell cloth at the cost price, but he uses faulty meter rod. His meter rod measures 95 cm only. Find his gain per cent.

Solution: Here, true measure = 100 cm.

False measure = 95 cm.

Since the shopkeeper sells the cloth at cost price, ∴ x = 0. ∴Overall gain % is given by

$$\frac{100+g}{100+x} = \frac{True\ measure}{False\ measure}$$

$\Rightarrow\frac{100+g}{100} = \frac{100}{95}$ ⇒100+g=$\frac{100\times100}{95}$

⇒g=$\frac{10000}{95} - 100 = 5\frac{5}{19}\%$

Example 19: A dishonest shopkeeper professes to sell his goods at cost price, but he uses a weight of 800 g for the Kg weight. Find out his gain per cent.

Solution: True measure = 1000 g. False measure = 800 g. Also, x = 0.

∴ Overall gain % is given by

$$\frac{100+g}{100+x} = \frac{True\ measure}{False\ measure}$$

$\Rightarrow\frac{100+g}{100} = \frac{1000}{800}$ ⇒100+g=$\frac{1000\times100}{800}$

⇒g=$\frac{1000}{8} - 100 = 25\%$

Example 20: A shopkeeper sells goods at 44% loss on cost price, but uses 30% less weight. What is his percentage profit or loss?

Solution: Here, x = -44 and y = 30.

∴ Overall gain/loss% $=\left(\frac{y+x}{100-y}\right)\times 100\%$

$=\left(\frac{30-44}{100-30}\times 100\right)\%$

$=\left(\frac{-14}{70}\times 100\right)\% = -20\%$

which represents loss being a negative expression.

10 A person buys two items for `A and sells one at a loss of l% and the other at a gain of g%. If each item was sold at the same price, then

(a) The cost price of the item sold at loss

$=\frac{A(100+\%gain)}{(100-\%loss)+(100+\%gain)}$

(b) The cost price of the item sold at gain

$$\frac{A(100+\%gain)}{(100-\%loss)+(100+\%gain)}$$

Example 21: Ramesh buys two books for rs.410. He sells one at a loss of 20% and the other at a gain of 25%. If both the books are sold at the same price, find out the cost price of two books.

Solution: Cost price of the book sold at a loss of 20%

$=\frac{410(100+25)}{(100-20)+(100+25)}$

$=\frac{410\times 125}{80+125} = rs.250$

Cost price of the book sold at a profit of 25%

$=\frac{410(100-20)}{(100-20)+(100+25)} = \frac{410\times 80}{80+125}$=rs.160.

## 5.3 Successive Discount and marked price

11 If two successive discounts on an article are m% and n% respectively, then a single discount equivalent to the two successive discounts will be:

$$\left(m+n-\frac{mn}{100}\right)\%$$

Explanation

Let, the marked price of the article be rs.100.

∴ S.P. after the first discount = rs. (100 - m) and discount at n% on rs(100-m)=rs.$\frac{(100-m)\times n}{100}$

∴ Single equivalent discount

$=\left[m+\frac{(100-m)\times n}{100}\right]\%$

$=\left(\frac{100m+100n-mn}{100}\right)\%$

$=\left(m+n-\frac{mn}{100}\right)\%$

12 If three successive discounts on an article are l%, m% and n% respectively, then a single discount equivalent to the three successive discounts will be

$$\left[l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{100^2}\right]\%$$

Explanation

Let, the marked price of the article be rs.100.

∴ S.P. after the first discount = rs.(100 - l).

Second discount at m% on rs.(100 - l)

=rs.$\left(\frac{(100-l)\times m}{100}\right)$

∴ S.P. after second the discount

=rs.(100-l)-$\left(\frac{(100-l)\times m}{100}\right)$

=rs.$\frac{100(100-l)-(100-l)m}{100}$

=rs.$\frac{(100-l)(100-m)}{100}$

Third discount at n% on rs.$\frac{(100-l)(100-m)}{100}$

=rs.$\frac{(100-l)(100-m)}{100\times 100}$

∴ S.P. after the third discount

=rs. $\frac{(100-l)(100-m)}{100} - \frac{(100-l)(100-m)n}{100\times 100}$

=rs.$\frac{(100-l)(100-m)(100-n)}{100\times 100}$

=$\left(l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{(100)^2}\right)$

∴ Single equivalent discount

=$\left(l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{(100)^2}\right)$ %

Example 22: Find a single discount equivalent to two successive discounts of 10% and 20%.

Solution: The equivalent single discount is given by

$$\left(10+20-\frac{10\times 20}{100}\right)\% i.e..28\%$$

Example 23: Find out a single discount equivalent to three successive discounts of 10%, 20% and 30%.

Solution: The equivalent single discount is given by

$$\left(10+20+30-\frac{(10\times 20+10\times 30+20\times 30)}{100}+\frac{10\times 20\times 30}{100^2}\right)\%$$

i.e$\left(60-11+\frac{6}{10}\right)\% = \frac{496}{10}\% or\ \ 49.6\%$

Example 24: Two shopkeepers sell machines at the same list price. The first allows two successive discounts of 30% and 16% and the second 20% and 26%.Which discount series is more advantageous to the buyers?

Solution: A single discount equivalent to the two successive discounts of 30% and 16% is

$$\left(30 + 16 - \frac{301 \times 16}{100}\right)\%$$

Or, $\left(46 - \frac{26}{5}\right)\%\ or\ 41\frac{1}{5}\%$

Also, a single discount equivalent to the two successive discounts of 20% and 26% is

$$\left(20 + 26 - \frac{20 \times 26}{100}\right)\%$$

Or, $\left(46 - \frac{26}{5}\right)\%\ or\ 40\frac{4}{5}\%$

Clearly, the discount series being offered by the first shopkeeper is more advantageous to the buyers.

13 A shopkeeper sells an item at rs.z after offering a discount of d% on labelled price. Had he not offered the discount, he would have earned a profit of p% on the cost price.

The cost price of each item is given by

$$C.P = \left[\frac{100^2 Z}{(100-d)(100+p)}\right]$$

Example 25:A shopkeeper sold sarees at `266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a Profit of 12% on the cost price. What was the cost price of each saree?

Solution: We have, labelled price z = rs.266, discount d = 5% and profit p = 12% Using the formula

$$C.P = \left[\frac{100^2 Z}{(100-d)(100+p)}\right]$$

we get the cost price of each saree

$$\left[\frac{100 \times 100 \times 266}{(100-5)(100+12)}\right]$$

$=\frac{100 \times 100 \times 266}{(100-5)(100+12)}$

## Summary

The key concepts learned from this unit are: -

- We have learnt how to calculate Cost Price, Selling Price and Profit or Gain.
- We have learnt how to calculate successive discount and marked price

## Keywords

- Cost Price
- Selling Price
- Profit
- Gain.

- discount price
- Marked price

## Self Assessment

1. A man buys an article for rs.27.50 and sells it for rs.28.50. His gain % is?

A. 1.

B. 2.

C. 3.

D. 4.

2. If the a radio is sold for rs 490 and sold for rs 465.50. The loss% is?

A. 2

B. 3

C. 4

D. 5

3. S.P when CP=56.25, gain=20% is ?

A. 67

B. 68

C. 67.5

D. 68

4. CP when SP = rs 40.60, gain=16% is?

A. 30

B. 35

C. 40

D. 45

5. A person incures loss for by selling a watch for rs1140 at what price should the watch be sold to earn a 5% profit ?

A. 1260

B. 3300

C. 450

D. 50

6. By selling 33 metres of cloth , one gains the selling price of 11 metres . Then the gain percent is?

A. 50

B. 55

C. 60

D. 65

7. If the cost price is 96% of sp then what is the profit %?

A. 2.32

B. 3.67

C. 4.17

D. 5.98

8. A man brought toffees at for a rupee. How many for a rupee must he sell to gain 50%?

A. 1

B. 2

C. 4

D. 5

9. A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120kg sugar at Rs.16per kg. At what rate should he sell the mixer to gain 16%?

A. 30.2

B. 20.3

C. 17.4

D. 51.8

10. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight . Find his gain percent.

(a) 3.2

(b) 4.1

(c) 1.8

(d) 5.8

11. Monika purchased a pressure cooker at 9/10th of its selling price and sold it at 8% more than its S.P .Then her gain percent is?

A. 20

B. 12

C. 17

D. 4

12. If the manufacturer gains 10%,the wholesale dealer 15% and the retailer 25%,then find the cost of production of a ,the retail price of which is Rs.1265?.

(a) 900

(b) 800

(c) 200

(d) 300

13. A An article is sold at certain price. By selling it at 2/3 of its price one losses 10%,find the gain at original price?

A. 30

B. 20

C. 35

D. 51

14. A the single discount equivalent to a series discount of 20% ,10% and 5% is ?

A. 12

B. 76.2

C. 68.4

D. 55

15. A retailer buys 40 pens at the market price of 36 pens from a wholesaler, if he sells these pens giving a discount of 1%, what is the profit %?

A. 30
B. 20
C. 15
D. 10

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | D | 3. | C | 4. | B | 5. | A |
| 6. | B | 7. | C | 8. | B | 9. | C | 10. | B |
| 11. | A | 12. | B | 13. | C | 14. | C | 15. | D |

## Review Questions

1. At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?
2. An uneducated retailer marks all its goods at 50% above the cost price and thinking that he will still make 25% profit, offers a discount of 25% on the market price. what is the actual profit on the sales?
3. A man bought a horse and a car riage for Rs 3000.he sold the horse at a gain of 20% and the carriage at a loss of 10%, thereby gaining 2% on the whole. find the cost of the horse.
4. Alfred buys an old scooter for Rs. 4700 and spends Rs. 800 on its repairs. If he sells the scooter for Rs. 5800, his gain percent is?
5. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is?
6. If selling price is doubled, the profit triples. Find the profit percent.
7. In a certain store, the profit is 320% of the cost. If the cost increases by 25% but the selling price remains constant, approximately what percentage of the selling price is the profit?
8. A vendor bought toffees at 6 for a rupee. How many for a rupee must he sell to gain 20%?
9. The cost price of three varieties of oranges namely A, B and C is Rs 20/kg, Rs 40/kg and Rs 50/kg. Find the selling price of one kg of orange in which these three varieties of oranges are mixed in the ratio of 2 : 3 : 5 such that there is a net profit of 20%?
10. A sweet seller sells 3/5th part of sweets at a profit of 10% and remaining at a loss of 5%. If the total profit is Rs 1500, then what is the total cost price of sweets?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing

3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 06: Direction Sense Test

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand Distance Related Questions
- Understand Left Right Movement

## Introduction

The questions based on directions require the candidates to identifying the direction of an individual or shadow from a set of statements. We should know of some of the important points which helps to solve the questions with ease.

## 6.1 General Directions

In general, there are four main directions i.e. North, South, East and West. Apart from these four, there are four additional directions derived from the main ones. They are called North-East, North-West, South-East and South-West. A chart is given below for reference.

## 6.2 Casting of Shadows

A lot of times questions in CAT are asked regarding casting of shadows. It is important and useful to know the below concepts to understand

If a man faces a rising sun, his shadow will always be in the west.

If a man faces north, his shadow will be on his right during sunrise and on his left during sunset.

Similarly, if a man faces south, his shadow will be on his left during sunrise and on his right during sunset.

In mid-noon, no shadows are seen as the sun's rays are vertically downwards.

## 6.3 Distance Related Questions

In most of the direction and distance questions, it is required to calculate a certain parameter from the question statement. The questions can be related to the total distance walked, shortest path, the distance between two entities, etc.

While solving the distance related question, one must be thorough with the Pythagorean Theorem to be able to solve most of the questions. The Pythagorean Theorem is used to calculate the shortest path traveled, the minimum distance between two points, etc. A couple of examples and cat questions are given below to illustrate the direction and distance questions better.

## 6.4 Left Right Movement

A person facing north, on taking left will face towards west and on taking the right turn towards east.

A person facing west, on taking left will face towards south and on taking right turn towards north.

A person facing east, on taking left will face towards north and on taking the right turn towards south.

A person facing south, on taking left will face towards east and on taking the right turn towards west.

Example 1: Airplanes A, B, C and D started flight towards east. After flying 125 kms planes A and D flew towards right while planes B and C flew towards left. After 115 km, planes B and C flew towards their left while planes A and D also turned towards their left. In which directions are the airplanes A, B, D, C respectively flying now?

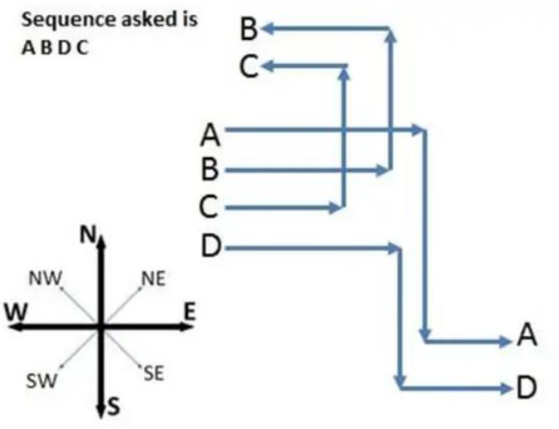
1. North, South, East, West
2. East, West, West, East
3. East, West, East, West
4. South, North, North, South

Let us look at A and D first. After flying for 125 kms, A and D would still be in east direction. Now , they take a right turn which makes their direction as south. From south, if you take a left, you will again come in the east direction. So planes A and D are in east direction.

Now, let us look at planes B and C. After flying for 125 kms, B and C would still be in east direction.

From east direction, they take a left turn which means now they are in north. From north, they again take a left which makes them land in the west.

So correct answer would be: East, west, East, West (Option C)



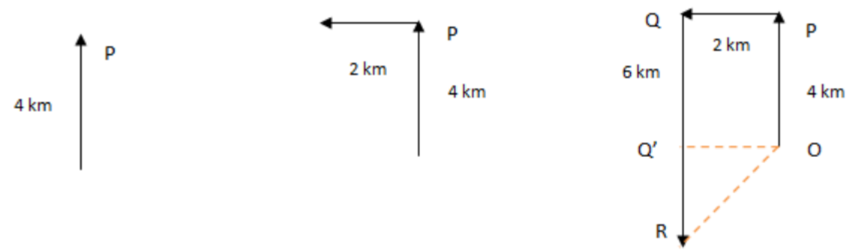

Example 2: A person starts walking towards North for about 4 km and reaches a point P. From point P, he takes a left turn and walks for 2 km to reach a point EXAMPLE:- From Q, he again takes a left turn and walks for 6 km and reaches a point R. How far and in which direction is the point R from the starting point?

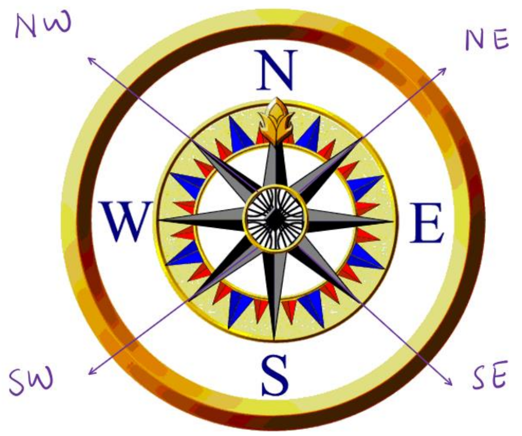
Solution: Let us first draw a rough sketch of the above reasoning direction problem.

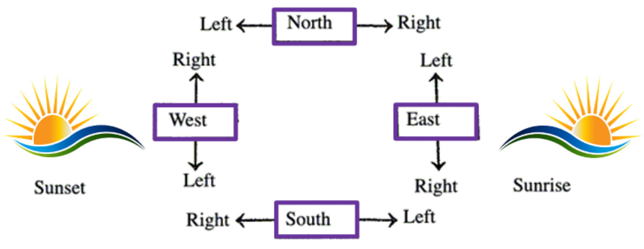
Joining the starting point O to the endpoint R and drawing a perpendicular on QR at Q′.

In a right angled triangle OQ′R, We have OQ′ = PQ = 2 km, and Q′R = QR – QQ′ = 6 – 4 = 2 km.

So, $OR^2$ = 22 + 22 = 4 + 4 = 8. or, OR = √8 = 2√2 km. R is in the South-West direction from the starting point O.

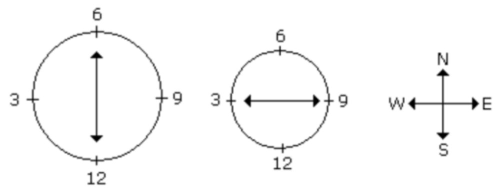





Main directions

Example 3:- Rahul put his timepiece on the table in such a way that at 6 P.M. hour hand points to North. In which direction the minute hand will point at 9.15 P.M. ?




At 9.15 P.M., the minute hand will point towards west.

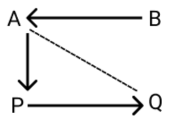
Example 4:- Town A is to the West of Town B. Town P is to the South of Town A. Town Q is to the East of Town P. Then Town Q is towards which direction of Town A?

(a) South

(b) South West

(c) South East

(d) North




Solution- (c) South East

Example 5:- Siva starting from his house, goes 5 km in the East, then he turns to his left and goes 4 km.

Finally, he turns to his left and goes 5 km. Now how far is he from his house and in what direction?

1. If North-east becomes West and South-east becomes North then what will West becomes?

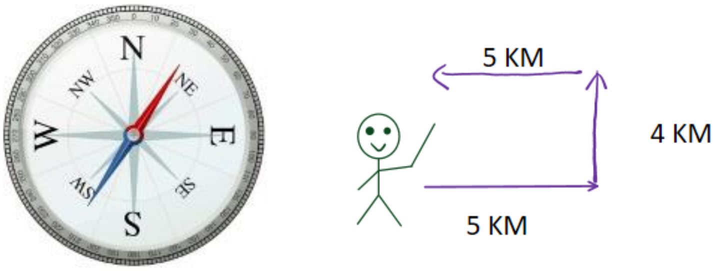
A. South-east

B. North-east

C. South

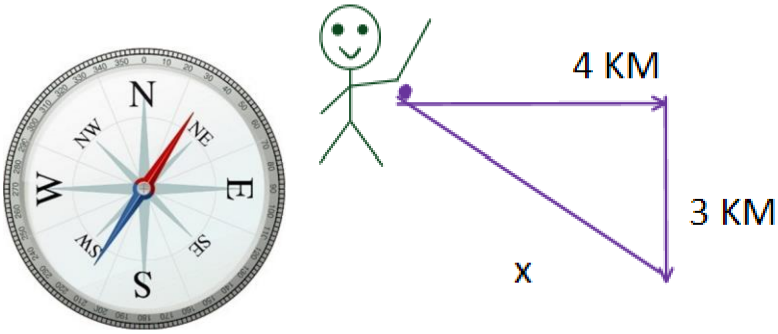
D. North-west

E. None of these




Suresh starting from his house, goes 4 km in the East, then he turns to his right and goes 3 km.

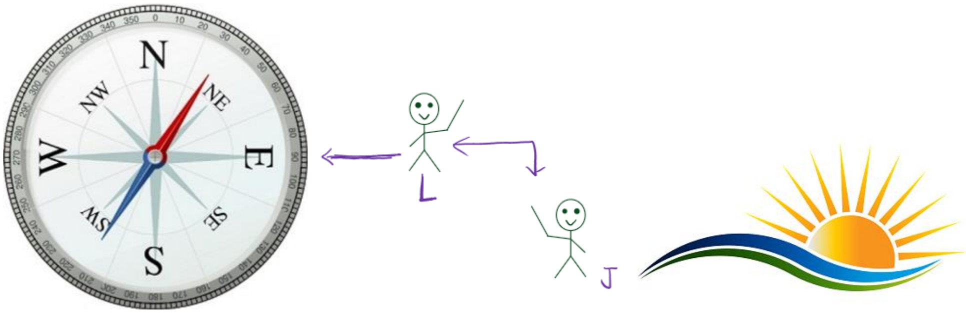
What minimum distance will be covered by him to come back to his house?




$X^2=4^2+3^2=16+9=25$

$X=5.$

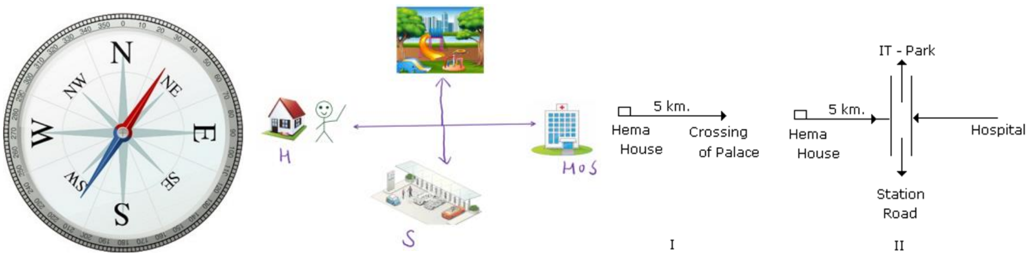
Example 6:- One morning after sunrise Juhi while going to school met Lalli at Boring road crossing. Lalli's shadow was exactly to the right of Juhi. If they were face to face, which direction was Juhi facing?




Hema starting from her house walked 5 km to reach the crossing of Palace.

In which direction she was going, a road opposite to this direction goes to Hospital.

The road to the right goes to station. If the road which goes to station is just opposite to the road which IT-Park, then in which direction to Hema is the road which goes to IT-Park?



It is clear that the road which goes to IT-Park is left to Hema.

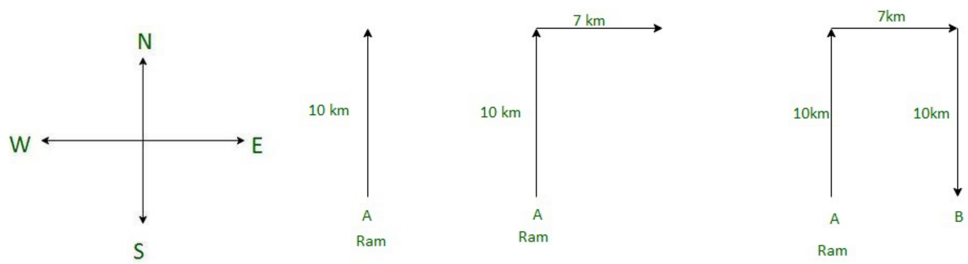
Example 7 – One morning after sunrise, Mahesh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?

Solution –

The sun rises in the East (E) in the morning. As the shadow of Mahesh falls to his right.

So he must be facing South. Hence, the answer is the south.

Example 8- Ram starts from a point A walks 10 km north, then turns right and walks for 7 km, then turns right again and walks for another 10 km. And reaches point B. How far is Ram from the starting point?




Example 9:

One evening, two friends Riya and Priya were talking to each other, with their backs towards each other, sitting in a park. If Riya's shadow was exactly to the left of her, then which direction was Priya facing?

Solution:

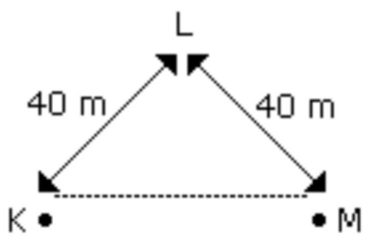
Riya's shadow fell to her left i.e. towards East (as it was evening). So, Riya was facing "South". As Priya had her back towards Riya, hence, Priya was facing "North". North is the correct answer.

Example 10:

K is 40 m South-West of L. If M is 40 m South-East of L, then M is in which direction of K?

Solution:

M is in the East of K as shown in the following diagram. East is the correct answer.




Example 11:- A man walks 6 km towards the north, then turns towards his left and walks for 4 km. He again turns left and walks for 6 km. At this point he turns to his right and walks for 6 km. How many km and in what direction is he from the starting point?

A. 10 km, West

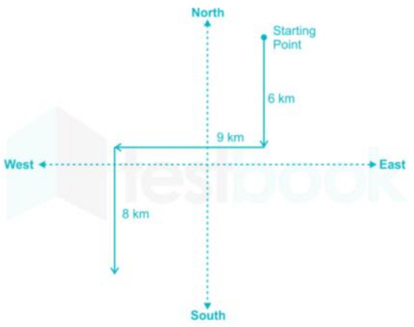
B. 6 km, South

C. 4 km, South

D. 8 km, West

E. 10 km, East

Solution:- A. 10 km, West




EXAMPLE 12:- If North-east becomes West and South-east becomes North then what will West

becomes?

A. South-east

B. North-east

C. South

D. North-west

E. None of these

EXAMPLE 13:-While facing East you turn to your left and walk 10 yards; then turn to your left

and walk 10 yards: and now you turn 45o towards your right and go straight to

cover 25 yards. Now, in which direction are you from your starting point ?

A. North-East

B. South-West

C. East

D. North-West

E. None of these

Solution:- D. North-West

Let be the initial position of the man. He initially faced East and then turned his left in the direction of North and walked 10 yards. Then, turned his left in the direction of West and walked 10 yards.

Now, he turned 45° to his right and walked 50 yards straight in the same direction. Now, the direction of the man with respect to his starting point is North - West.

EXAMPLE 14:-A direction pole was situated on the crossing. Due to an accident the pole turned

in such a manner that the pointer which was showing East, started showing South.

One traveler went to the wrong direction thinking it to be West. In what direction

actually he was travelling ?

A. South

B. East

C. West

D. North

E. None of these

Solution:- D. (North We can see that according to graph Due to an accident, the pointer showing East, started showing South. It means, the pole has been rotated through 90° clockwise. So, when she was travelling towards West, actually it was North.)

EXAMPLE 15:-My friend is facing east. She turned 120°in the clockwise direction and then 165o

in the anti-clockwise direction. Which direction is she facing now?

A. East.

B. North-east.

C. North.

D. South-west.

E. None of these

Answer:- B. North-east.

EXAMPLE 16:-Five friends are playing a game. Nitin is facing South. He follows instructions

given by his friends.

(i) When Kimi touch him, Nitin walks 50 metre.

(ii) When Neha touch him, Nitin turns right and walks 40 metre.

(iii) When Annu touch him, Nitin turns left and walks 80 metre.

(iv) When Punit touch him, Nitin walks 70 metre in the direction opposite to

which he faces.

If Kimi, Annu, Punit and Neha touch him in that order, how far does Nitin reach

from his starting point?

A. 60 metre.

B. 72 metre.

C. 80 metre.

D. 75 metre.

E. None of these

Solution:- B. 72 metre.

EXAMPLE 17:- Read the situations given below to answer the questions:

Nine cars P, Q, R, S, T, U, V, W and X are parked such that:

(1) R is 4 km east of Q.

(2) P is 2 km north of Q.

(3) W is 4 km south of P.

(4) V is 2 km west of W.

(5) S is 6 km east of V.

(6) U is 4 km north of V.

(7) X is parked in the middle of Q and R.

(8) T is parked in the middle of W and S.

What is the distance between W and S?

A. 6 km.

B. 4 km.

C. 3 km.

D. 2 km.

E. 5 km

Solution:- B. 4 km.

EXAMPLE 18:-What is the direction of Q with respect to W?

A. East

B. West

C. South

D. North

E. Canno

Solution:- D. North

EXAMPLE 19:- What is the distance between U and W?

A. 20 km

B. √28 km

C. √20 km

D. 3 4/5 km

E. Cannot be determined

Solution:- C. √20 km

EXAMPLE 20:- Which car is parked between V and T?

A. S

B. Q

C. W

D. T

E. Ut be determined

Solution:- C. W

EXAMPLE 21:- A road network has parallel and perpendicular roads running north-south or eastwest only. Junctions/ Intersections on this road network are marked as A, B, C, D.... All junctions are at exactly half a kilometer distance from each other. The following is known about junctions A, B, C, H and X.

'A' is east of 'B' and west of 'C', 'H' is southwest of 'C' and southeast of B. 'B'

is southeast of 'X'. Which junctions are the farthest south and the farthest east?

A. H, B

B. H, C

C. C, H

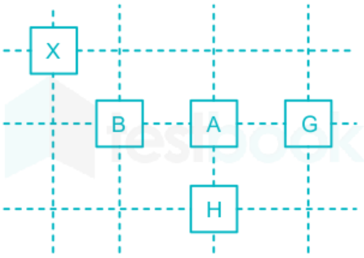
D. B, H

E. B, A
Solution:- B. H, C

A road network has parallel and perpendicular roads running north-south or eastwest only. Junctions/ Intersections on this road network are marked as A, B, C, D.... All junctions are at exactly half a kilometer distance from each other. The following is known about junctions A, B, C, H and X.

'A' is east of 'B' and west of 'C', 'H' is southwest of 'C' and southeast of B. 'B'

is southeast of 'X'. Which junctions are the farthest south and the farthest east? Hence farthest south is H and Farthest East is C. Hence option 2 is correct answer.




DIRECTIONS ( EXAMPLE 22:- ) : Ashini, Bindiya and Sanika are standing at the corners of an equilateral triangle drawn In an open farm. Study the questions given below which are based on the following diagram and answer them by marking the appropriate choice from the choices.

From the positions shown in the figure,if Ashini, Bindiya and Sanika run along the sides in clockwise direction and stop after covering 1 12 sides, then which of the following statement is true?

A. Bindiya is to the north of Sanika

B. Ashini is to the south-west of Bindiya

C. Ashini is to the east of Sanika

D. Bindiya is to the north-west of Sanika

E. Sanika is to the north-east of Ashini

Solution:- B. Ashini is to the south-west of Bindiya

From the positions mentioned in the figure of above question, if all of them run in the anticlockwise direction covering two sides and then stop, then which of the following statements is correct?

A. Bindiya is to the west of Sanika

B. Ashini is to the south of Bindiya

C. Ashini is to the south-west of Sanika

D. Bindiya is to the north-west of Ashini

E. Bindiya is to the south of Sanika

Solution:- D. Bindiya is to the north-west of Ashini

DIRECTIONS( EXAMPLE 23:- ) : Read the following information to answer the questions given below it.

Pradeep starts walking from his college towards his house. He starts from the front gate of his College and walks 5 km, then turns left and walks 2 km, then turns left again and walks 4 km, then he turns to his right and walks 3 km,then turns left and walks 1 km and then turns to his left again and walks 4 km, then turns to his right and walks 10 km and finally turns right and walks 3 km and thus reaches the front gate of his house.

If Pradeep's house is facing south, in which direction did he start walking?

A. West

B. North

C. South

D. East

E. None of these

Solution:- D. East

Pradeep's house is in what direction with respect to his college If Pradeep's house is facing south?

A. Northwest

B. Southwest

C. South

D. Cannot be determined

E. None of these

Solution:- A. Northwest

DIRECTIONS ( EXAMPLE 24:-) : Read the following information carefully and answer the questions given below it:

(A) P α Q means Q is to the right of P at a distance of one metre.

(B) P β Q means Q is to the North of P at a distance of one metre.

(C) P λ Q means Q is to the left of P at a distance of one metre.

(D) P η Q means Q is to the South of P at a distance of one metre.

(E) In each of the following questions all persons face South.

If A η B λ L β K, then K is in which direction with respect to A?

A. South

B. East

C. North

D. West

E. None of these

Solution:- B. East

If G α L η R α M then M is in which direction with respect to L?

A. North-east

B. North-west

C. South-east

D. South-west

E. None of these

Solution:- D. South-west

If A α B λ C β D, then D is in which direction with respect to A?

A. North

B. South

C. East

D. North-east

E. None of these

Solution:- A. North

Pran and Khan start from their office and walks in opposite direction, each traveling 10 km. Pran then turns left and walks 10 km. While Khan turns right and walks 10 km. How far they are now from each other?

A. 0 km

B. 5 km

C. 10 km

D. 20 km

E. None of these

Solution:- D. 20 km

Facing to the South Mr. Jha turns in certain ways, which of the following turns will not lead him to the same side?

A. left, left, right, left, left, right

B. left, left, left, right, right, right

C. left, right, left, right, left, right

D. right, left, right, left, right, left

E. None of these

Solution:- A. left, left, right, left, left, right

One morning after sunrise, Vikram and Shailaish were standing in a town with their backs towards each other. Vikram's shadow fell exactly towards left and side. Which direction was Shailaish facing ?

A. East

B. West

C. North

D. South

E. North south

Solution:- D. South

## Summary

The key concepts learned from this unit are: -

- We have learnt how to calculate Distance Related Questions
- We have learnt how to calculate Left Right Movement.

## Keywords

- Distance
- Directions

## Self Assessment

1. If North-east becomes West and South-east becomes North then what will West becomes?
A. South-east
B. North-east
C. South
D. North-west

2. A man walks 6 km towards the north, then turns towards his left and walks for 4 km. He again turns left and walks for 6 km. At this point he turns to his right and walks for 6 km. How many km and in what direction is he from the starting point?
A. 10 km, West
B. 6 km, South
C. 4 km, South
D. 8 km, West

3. A river flows west to east and on the way turns left and goes in a semi-circle round a hillock and then turns left at right angles. In which direction is the river finally flowing?
A. West
B. East
C. North
D. South

4. While facing East you turn to your left and walk 10 yards; then turn to your left and walk 10 yards: and now you turn 45o towards your right and go straight to cover 25 yards. Now, in which direction are you from your starting point ?
A. North-East
B. South-West
C. East
D. North-West

5. A direction pole was situated on the crossing. Due to an accident the pole turned in such a manner that the pointer which was showing East, started showing South. One traveler went to the wrong direction thinking it to be West. In what direction actually he was travelling ?
A. South
B. East
C. West
D. North

6. Five friends are playing a game. Nitin is facing South. He follows instructions given by his friends.
(i) When Kimi touch him, Nitin walks 50 metre.
(ii) When Neha touch him, Nitin turns right and walks 40 metre.
(iii) When Annu touch him, Nitin turns left and walks 80 metre.
(iv) When Punit touch him, Nitin walks 70 metre in the direction opposite to which he faces.
If Kimi, Annu, Punit and Neha touch him in that order, how far does Nitin reach from his starting point?
A. 60 metre.
B. 72 metre.
C. 80 metre.
D. 75 metre.

7. Pradeep starts walking from his college towards his house. He starts from the front gate of his College and walks 5 km, then turns left and walks 2 km, then turns left again and walks 4 km, then he turns to his right and walks 3 km,then turns left and walks 1 km and then turns to his left again and walks 4 km, then turns to his right and walks 10 km and finally turns right and walks 3 km and thus reaches the front gate of his house.

If Pradeep's house is facing south, in which direction did he start walking?

A. West
B. North
C. South
D. East

8. I am facing East. Turning to the right I go 20 m, then turning to the left I go 20 m and turning to the right I go 20 m, then again turning to the right I go 40 m and then again I go 40 m to the right. In which direction am I from my original position?

A. North
B. West
C. South
D. East

9. Bhairav walked 30 ft towards North, then took a left turn and walked 15 ft. He again took a left turn and walked 30 ft. How far and in which direction is Bhairav from the starting point?

A. 15 ft to the West
B. 45 ft to the South
C. 30 ft to the East
D. 15 ft to the North

10. Seema started early in the morning on the road towards the Sun. After some time she turned to her left. Again after some time she turned to her right. After moving some distance she again turned to her right and began to move. At this time, in what direction was she moving?

A. South
B. North-West
C. North-East
D. East

11. At dusk, Rohit started walking facing the sun. After a while, he met his friend and both turned to their left. They halted for a while and started moving by turning again to their right. Finally Rohit waved 'good bye' to his friend and took a left turn at a corner. At which direction is Rohit moving now?

A. South
B. West
C. North
D. East

12. Sumi ran a distance of 40 m towards South. She then turned to the right and ran for about 15 m, turned right again and ran 50 m. Turning to right then ran for 15 m. Finally she turned to the left an angle of 45° and ran. In which direction was she running finally?

A. South-East
B. South-West
C. North-East
D. North-West

13. A man starting from his home moves 4 km towards East, then he turns right and moves 3 km. Now what will be the minimum distance covered by him to come back to his home?

A. 4
B. 5
C. 12
D. 13

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | A | 3. | B | 4. | D | 5. | A |
| 6. | B | 7. | D | 8. | B | 9. | D | 10. | A |
| 11. | A | 12. | D | 13. | B | | | | |

## Review Questions

1. Satish starts from his house and takes two right turns and then one left turn. Now he is moving towards south. In which direction Satish started from his house?
2. A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now?
3. Sumi ran a distance of 40 m towards South. She then turned to the right and ran for about 15 m, turned right again and ran 50 m. Turning to right then ran for 15 m. Finally she turned to the left an angle of 45° and ran. In which direction was she running finally?
4. A child is looking for his father. He went 90 metres in the east before turning to his right. He went 20 metres before turning to his right again to look for his father at his uncle's place 30 metres from this point. His father was not there. From there, he went 100 metres to his north before meeting his father in a street. How far did the son meet his father from the starting point?
5. Deepa moved a distance of 75 metres towards the north. She then turned to the left and walking for about 25 metres, turned left again and walked 80 metres. Finally, she turned to the right at an angle of 45°. In which direction was she moving finally?
6. From his house, Rahul went 25 km to north. Then he turned towards west and covered 15 km. Then he turned south and covered 10 km. Finally, turning to east, he covered 15 km. In which direction is he from his house?
7. Rasik walked 20 m towards north. Then he turned right and walks 30 m. Then he turns right and walks 35 m. Then he turns left and walks 15 m. Finally he turns left and walks 15 m. In which direction and how many metres is he from the starting position?
8. One morning after sunrise, Suresh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?
9. One morning Udai and Vishal were talking to each other face to face at a crossing. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?
10. Ranuka started walking from her house, she first walked for 3 km towards west, then she turned towards north and moved 4 km in that direction. How far is renuka from her house?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 07: Blood Relation

CONTENTS



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand how find Family or Blood Relationship.
- We will understand how find Relationships Involving the Term '-in-law' General
- We will understand how find Relationships Involving Father-in-law, Mother-in-law, Son-in-law and Daughter-in-law
- We will understand how find Relationships Involving Brother-in-law and Sister-in-law
- We will understand how find Relationships Half Sibling and Step Relations.

## Introduction

Blood Relations is the most studied topic of Logical Reasoning and is one of the few topics which has found its importance in almost every entrance exam. This topic tests the analytical skills of the students and how one can approach the solution of logical problems with the help of diagrams instead of calculations. The diagrams play an important role here. The right diagrams come from the right analysis. The following is the list of exams in which the appearance of questions from blood relations are frequent. Any relation in the world which either by birth or by marriage is called a Blood Relation. Relation as we know is the connection between any two things or between any two persons. So when speaking of blood relation, it means the connection between two people by blood or basically by birth.

Example: Any relation by birth will be mother, father, son, daughter, etc. and any relation by marriage will be father-in-law, mother-in-law, etc.

## 7.1 Types of Questions on Blood Relation

Over the years, the standard and type of questions which are being asked from the blood relation topic have seen a slight turn. Initially, the questions used to be less complex and statement or dialogue-based, but with the increased competition, the variety of questions being asked have also changed.

Being one of the most common concepts from which questions are asked in exams, given below are the different ways in which the blood relation questions may be asked in the competitive exams, for the assistance of candidates:

**Dialogue/ Conversation Based** – Based on Dialogue or conversation. In such questions, one person describes his/her relation with another person (this may or may not be related to the person with whom the conversation is being made).

**Based on Puzzles** – To make the questions complex, blood relation questions are also being asked in the form of a puzzle. A piece of brief information about multiple people being interrelated is given and sub-questions based on the same may be asked.

**Coding-Decoding** – The relationship between two people may be denoted using symbols. This has become a common method of asking blood relation questions in competitive exams, nowadays.

Keep in mind that from a person's name, we cannot judge the gender of that person. The name doesn't always show the gender beyond a reasonable doubt.

A tree as we know has roots firmly grounded and then it has its stem which branches out and gets leaves in it.

Basic Terms used in Blood Relations

Parents: Mother and father.

Children: Son or Daughter

Siblings: Brother or Sister( of the same parents)

Spouse: Husband or wife.

Aunt: Aunt means father's sister, mother's sister, father's brother's wife, or mother's brother's wife.

Uncle: Uncle means father's brother, Mother's brother, Father's sister's husband, mother's sister's husband.

Niece: Brother's and sister's daughter.

Nephew: Brother's and sister's son.

Cousin: Children of aunt and uncle.

Father-in-law: Father of the spouse.

Mother-in-law: Mother of Spouse.

Sister-in-law: Sister of the spouse, wife of brother

Co-sister: Wife of spouse's brother.

Brother-in-law: Brother of the spouse, husband of the sister

Co- brother: husband of spouse's sister.

Maternal: Relations or family members who are from the mother's side.

Paternal: Relations or family members who are from the father's side.

Note:

I. Any relation of Mother's side is called 'Maternal'.

II. Any relation of Father's side is called 'Paternal'.

For example: Pointing towards a boy Veena said "He is the son of only son of my grandfather". How is that boy related to Veena?

Solution: He is the son of only son of my grandfather – Veen's Father's son

Therefore, that boy is a Veena's brother.

The same is the case with a family tree, wherein we can imagine our ancestors as the roots and then their children and grandchildren and so on act as branches and leaves.

Family Tree a pictorial or visual representation of our lineage. With the help of a family tree, it not only gives us a better understanding of our lineage but also helps us understand our relationship with different people who have common ancestors.

There also are a few things which need to be kept in mind while solving the blood relation questions. Given below are few such important pointers:

You cannot assume the gender of the person based on the name

If the statement says X is the son of Y, the gender of Y cannot be determined unless mentioned in the question

In puzzle based questions, a web of relations can be formed, so do not solve such questions in a haste

These questions are scoring and easy to solve, so do not panic if the question seems lengthy

In case of coding-decoding blood relation, use a pictorial description to solve the question. This will make the symbols and relation more clear

To ace the reasoning section, candidates can visit the 3 Sutras to Prepare Reasoning Ability at the linked article.

Let us now move on to solving a few sample questions to get a better understanding of the concept.

## 7.2 Family Or Blood Relations Test

Tips on finding a relationship between two members of a family:

Step 1: The first and foremost step is to choose the two persons among whom the relationship is to be established.

Step 2: Now once you have chosen the two persons, pin-point the intermediate relationship between two persons, i.e. such relationship through which long drawn relationship can be established between the required persons.

Step 3: In the end, conclude the relationship between two required persons.

Make sure that you represent the gender of a female with a "□" sign and that of a male with a "⊕

**Definition and Concept**

Family or Blood Relationship means persons connected by relations like – father-mother, sondaughter, brother-sister, grandfather-grandmother, uncle-aunty, nephew-niece, brother-in-lawsister-in-law etc. The list can go on and on adding members from father's side and mother's side etc.

Questions in Test of Reasoning on Family /Blood Relationship are about the relationship of a particular person with another person of the family, based on the chain of relationships between other members of that family.

The questions depict relationships among the various members of a family in a roundabout chain. The candidate is expected to find the relation of two particular persons mentioned in the question. An example of a question on Blood Relationship is given below to understand the concept in a better way:

Example 1: Introducing Neeta, Anil said, 'She is wife of my mother's only son.' How is Neeta related to Anil?

(1) Mother

(2) Wife

(3) Sister

(4) Daughter-in-law

(5) None of these

Solution: Neeta is the wife of Anil's mother's only son, who is Anil himself. Hence, answer is Neeta is Anil's wife. i.e. (2) Wife.

Example 2: 'Ram' is the father of 'Kusha' but 'Kusha' is not his son. 'Mala' is the daughter of 'Kusha'.'Shalaka' is the spouse of 'Ram'. 'Gopal' is the brother of 'Kusha'. 'Hari' is the son of 'Gopal'. 'Meena' is the spouse of 'Gopal'. 'Ganpat' is the father of 'Meena'. Who is the grand daughter of 'Ram'?

(1) Hari

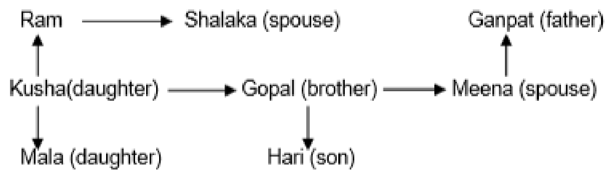
(2) Mala

(3) Meena

(4) Shalaka

(5) None of these

Solution:




'Mala' is the daughter of 'Kusha' and 'Ram' is the father of 'Kusha'. So, 'Mala' is the granddaughter of 'Ram'. Hence, answer is (2) Mala.

Family/Blood Relation Tests are an exercise to test the candidate's ability to comprehend and come to the crux of an issue from complex, lengthy and unclear data.

On a lighter note, this topic of Family/Blood Relations should be of interest to the candidates who are fans of Hindi Cinema, as the nature of the questions on Family/Blood Relations are of the type 'Hum Aapke Hai Kaun

## 7.3 Family/Blood Relations Described General

Family/Blood Relations tests largely depend on the candidate's knowledge of family relations. Various family relationships are described below to help the candidates to understand the relationships better and to attempt the questions based on them with confidence.

### Some Common Terms

Meaning of some terms often used in questions on family relationship are given below:

a) Parent – Mother or father

b) Child – Son or daughter (even if an adult)

c) Sibling – Brother or sister (Including half brother and half sister - one parent in common)

d) Spouse – Husband or wife

Basic Relationships Aunt, Uncle, Niece and Nephew

Most English speakers use "uncle" for any of four relationships: father's brother, mother's brother, father's sister's husband, or mother's sister's husband.

Again, "aunt" in English could mean father's sister, mother's sister, father's brother's wife, or mother's brother's wife.

Brother's or sister's son is called nephew. Brother's or sister's daughter is called niece.

Children of aunt or uncle are called cousins.

## 7.4 Relationships Involving the Term '-in-law' General

Any relationship term ending with -in-law indicates that the relationship is by marriage and not by blood. In other words, -in-law will be a blood relative of the spouse. 13 In-law relationship terms are always written with hyphens. And the plural is formed on the part before the "-in-law"; for example, "brothers-in-law" and not "brother-in-laws". The only exception is the general term "in-laws", which is always plural.

## 7.5 Father-in-law, Mother-in-law, Son-in-law and Daughter-in-law

Father-in-law is the father of spouse; mother-in-law is the mother of spouse. If parents get divorced and remarry, their new spouses are called stepparents, not mother-in-law and father-in-law.

The husband of daughter is son-in-law; the wife of son is daughter-in-law. If spouse has children from a previous marriage, those are called stepchildren, not sons-in-law or daughters-in-law. The person is their stepfather or stepmother, not their father-in-law or mother-in-law.

## 7.6 Brother-in-law and Sister-in-law

Brother-in-law" and "Sister-in-law" each have two or three meanings as follows:

a) Sister-in-law could be

i) The sister of spouse, or

ii) The wife of brother, or

iii) The wife of spouse's brother.

b) Similarly, Brother-in-law could be

i) The brother of spouse, or

ii) The husband of sister, or

iii) The husband of spouse's sister

### Relationships Involving the Terms 'Grand' and 'Great'

The relationships of the second generation are prefixed with the word Grand. For example, for a person, the first generation below him/her would be that of his/her child/children. The next/second generation would be the children of the children who would be called Grand Children of that person. The next/ third generation children would be called Great Grand Children of that person. This also applies to Niece and Nephew. For example, Son of nephew of a person is called Grand Nephew and so on

Similarly, for a person, the first generation above him would be that of his/her parents (Father/ Mother). The next/second generation above him/her would be the parents of the parents who would be called Grand Parents/ Grand Father/ Grand Mother of that person. The next/ third generation parents would be called Great Grand Parents/ Great Grand Father/ Great Grand Mother of that person.

This also applies to the collateral relationships. For example, Son of nephew of a person is called Grand Nephew; Brother of Grand Father is called Grand Uncle and so on.

The fourth-generation relationships are called Great Great Grand. For example, Son of Great Grand Son is Great Great Grand Son.

## 7.7 Half Sibling and Step Relations

Questions on Half Sibling and Step Relations are not very common in Bank exams. The information given below is only for very discerning candidates.

A half sibling (half brother or half sister) is a sibling with one shared biological parent.

When a parent remarries, the new spouse is the stepfather or stepmother of any children from the previous marriage. The children from a previous marriage are stepsons and stepdaughters. One is called stepbrother or stepsister if they have no parents in common but their parents have married each other. There are two ways Martha could have a stepsister:

a) If Martha's mother marries second time, and her new husband (Martha's new stepfather) already has a daughter from a previous marriage, that daughter is Martha's stepsister because one of her parents is married to one of Martha's parents.

b) If Martha's father marries second time, and his new wife already has a daughter, that daughter is again Martha's stepsister.

A similar rule gives the two ways for stepbrother.

### Summary of Some Common Relationships

Summary of some common Relationships is given below in tabular forms:

| Relation | Commonly Used Terms |
|---|---|
| Grandfather's or Grandmother's only son | Father |
| Grandfather's or Grandmother's only daughter-in-law | Mother |
| Father's father or Mother's | Grandfather |
| Father's Mother or Mother's | Grandmother |
| Father's brother or Mother's | Uncle |
| Father's sister or Mother's | Aunt |
| Son's wife | Daughter-in-law |
| Daughter's husband | Son-in-law |
| Husband's or wife's sister | Sister-in-law |
| Husband's or wife's brother | Brother-in-law |
| Brother's wife | Sister-in-law |
| Brother's or sister's son | Nephew |
| Brother's or sister's daughter | Niece |
| Uncle's or aunt's son or daughter | Cousin |
| Sister's husband | Brother-in-law |
| Brother's wife | Sister-in-law |
| Grand son's or grand daughter's daughter | Grand Grand Daughter |

| | |
|---|---|
| Grand son's or grand daughter's son | Great Grand Son |

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | Common Ancestor (Husband-Wife) | Child (son or daughter) | Grandchild | Great Grandchild | 2 Great Granchild |
| 1 | Child (son or daughter) | Sibling (brother or sister) | Nephew or Niece | Grand Nephew or Niece | Great Grand Nephew or Niece |
| 2 | Grandchild | Nephew or Niece | First Cousin | | |
| 3 | Great Grandchild | Grand Nephew or Niece | | Second Cousin | |
| 4 | 2 Great Grandchild | Great Grand Nephew or Niece | | | Third Cousin |

## Types of Questions on Blood Relations

Questions on Blood Relations are of the following types:

a) Mixed-Up Relationship Descriptions.

b) Relationships Riddle.

c) Coded Relations

## Type I - Mixed-Up Relationship Descriptions

Concept and Example

In questions of Mixed-Up Relationship Descriptions, a cluttered and roundabout description of relationships is given. The candidate is required to decipher the whole chain of relations and identify the direct/ actual relationship between the concerned persons.

A solved example of the Mixed-Up Relationship Descriptions is given below to understand the concept and questions based on it.

Example: Pointing to a gentleman, Dinesh said "His only brother is the father of my daughter's father." How is the gentleman related to Dinesh?

(1) Uncle

(2) Grandfather

(3) Father

(4) Brother- in-law

(5) None of these

Solution: The gentleman's only brother is the father of Dinesh (Dinesh daughter's father is Dinesh himself.). Gentleman is brother of Dinesh's father. Gentleman is Dinesh's uncle. Hence, answer is (1) Uncle.

**Basic Skills and Tips for Solving Questions on Mixed-Up Relationship Descriptions**

To make the chain of relationships clear, where necessary a rough sketch of family tree may be prepared in pencil on question paper on the basis of descriptions given in the question. The person's of same generation may be placed on same horizontal level and that of different generation one below the other. It may sometimes be necessary to draw two different diagrams and then put them together based on the link provided.

The relationship may be shown by drawing lines / arrows. Short forms as follows may be used to indicate the nature of relationships:

| | | | |
|---|---|---|---|
| g – Gentleman/ Male | s – Sister | u – Uncle | snl – Son-in-law |
| l – Lady/ Female | b – Brother | a – Aunt | dl – Daughter-in-law |
| sp -Spouse | sn – Son | cb – Cousin Brother | pu – Paternal Uncle |
| ch – Child | d – Daughter | cs – Cousin Sister | mu – Maternal Uncle |
| h – Husband | gf – Grandfather | nf – Nephew | |
| w - Wife | gm – Grandmother | nc – Niece | |
| m – Mother | gs – Grandson | bl – Brother-in-law | |
| f – Father | gd – Granddaughter | sl – Sister-in-law | |

Use of Small letters is suggested to indicate the nature of relationships to avoid confusion of Capital alphabets used in the questions like 'A is mother of C', 'D is sister of B's husband'

Sometimes even re-writing the given information using the short forms helps in reducing the confusion. The candidates may also, where possible try to correlate the given relationships with their own kith and kin. This works wonderfully in understanding the relationship.

It is observed that the names given in the questions are sometimes deceptive as to the gender. Candidates are advised to follow the instructions given in the question ignoring the name of the gender as at times it is deceptive. e.g. Kamal, Milan, Preetam, Kiran, Jasbir, Jasprit and etc.

Quite often descriptions of superfluous (unnecessary/ redundant) are given. It is thus better to first identify relationship between which two persons is exactly required to be found out in the question. And then proceed to track the relationship based on the descriptions connected to them. 35 Again properly understanding the relationship between which two persons is exactly required to be found out in the question is important as the relationship between A and B would be different than between B and A. For example, if Ravi and Mala are brother and sister, Ravi is related to Mala as brother, whereas Mala is related to Ravi as sister.

Also remember that terms like 'only son' only means that the person do not have other son, but it does not mean that the person doesn't have daughter/s. However, when it is said a person does not have any brothers and sisters, it can be safely concluded that he is the only child of his father/ mother.

Quite often a candidate depending upon whether he is male or female presumes that a person whose sex is not explicitly referred to in the relationships is of the same sex as that of his/her. Such

bias about the sex of the persons referred to the relationships should be avoided and the candidate should go strictly by the description given in the question.

Often the relationships are described in a round about way. Some examples of such descriptions and their actual/direct meaning is given below:

a) Only son of my grand father – My Father

b) Only son of my grandmother – My Father

c) Only daughter of my grandfather – My Mother

d) Only daughter of my grandmother – My Mother

e) Sister of my mother – My Aunt

f) Son/Daughter of my husband – My son / daughter

g) Son/Daughter of my wife – My son / daughter

h) Only daughter of my grandfather’s only son – My Sister

i) Grandmother of my father’s only son – My Grandmother

j) Father of my daughter’s father – My Father k) Father of my son’s father – My Father

l) My son’s sister – My daughter

m) Daughter-in-law of grandmother of my father’s only son – My Mother

n) A is the father of B but B is not the son of A – B is daughter of A

After the answer is found, it is good practice to quickly check back the answers with the relevant information given in the question.

Example:- Pinky, who is Victor's daughter, say to Lucy, "Your Mother Rosy is the younger sister of my Father, who is the third child of Joseph." How is Joseph related to Lucy?

a) Father - In - Law

b) Father

c) Maternal Uncle

d) Grand Father

Solution

(Option D)

Example. Mohan is the Son of Arun's Father's sister. Prakash is the son of Reva, who is the mother of Vikas

and Grandmother of Arun. Pranab is the father of Neela and the grandfather of Mohan. Reva is the wife of Pranab. How is the wife of Vikas related to the neela?

a) Sister

b) Sister - In - Law

c) Niece

d) None of The Above

Solution

(Option B)

Example: T, S and R are three brothers. T's son Q is married to K and they have one child Rahul blessed to them. M the son of S is married to H and this couple is blessed with a daughter Madhvi. R has a

daughter N who is married to P. This couple has one daughter Karuna born to them. How is Madhvi

related to S?

a) Daughter

b) Niece

c) Grand Daughter

d) None of The Above

Solution

(Option C)

Example:- Arti and Saurabh are the children of Mr and Mrs Shah. Ritu and Shakti are the children of Mr and

Mrs. Mehra. Saurabh and Ritu are married to each other and two daughter Mukti and Shruti are born to them. Shakti is married to Rina and two children Subhash and Reshma are born to them. How is Arti

related to Shruti?

a) Mother

b) Mother - in - Law

c) Sister

d) Aunt

Solution

(Option D)

## Summary

The key concepts learned from this unit are: -

- We have learnt how find Family or Blood Relationship.
- We have learnt how find Relationships Involving the Term '-in-law' General
- We have learnt how find Relationships Involving Father-in-law, Mother-in-law, Son-in-law and Daughter-in-law
- We have learnt how find Relationships Involving Brother-in-law and Sister-in-law
- We have learnt how find Relationships Half Sibling and Step Relations.

## Keywords

- Blood Relationship
- Relationships
- Coding-Decoding

## Self Assessment

1. Anil, introducing a girl in a party, said, she is the wife of the 12, grandson of my mother. How is Anil related to the girl?

A. Father
B. Grandfather
C. Husband
D. Father-in-law

2. A man said to a woman, ―Your mother's husband's sister is my aunt.‖ How is the woman related to the man?

A. Granddaughter
B. Daughter
C. Sister
D. Aunt

3. Introducing Rajesh, Neha said, ―His brother's father is the only son of my grandfather‖. How Neha is related to Rajesh?

A. Sister
B. Daughter
C. Mother

D. Niece

4. Vinod is the brother of Bhaskar. Manohar is the sister of Vinod. Biswal is the brother of Preetam and Preetam is the daughter of Bhaskar. Who is the uncle of Biswal?
A. Bhaskar
B. Manohar
C. Vinod
D. Insufficient data

5. A man said to a woman, ―Your brother's only sister is my mother.‖ What is the relation of the woman with the maternal grandmother of that man?
A. Mother
B. Sister
C. Niece
D. Daughter

6. Pointing to a photograph, a man said, ―I have no brother or sister but that man's father is my father's son.‖ Whose photograph was it?

A. His own
B. His son's
C. His father's
D. His nephew's

7. Pointing to a photograph, a lady tells Pramod, ―I am the only daughter of this lady and her son is your maternal uncle,‖ How is the speaker related to Pramod's father?
A. Sister-in-law
B. Wife
C. Neither (a) nor (b)
D. Aunt

8. A is the brother of B. A is the brother of C. To find what is the relation between B and C. What minimum information from the following is necessary?
(i) Gender of C (ii) Gender of B
A. Only (i)
B. Only(ii)
C. Either (i) or (ii)
D. both (i) and (ii)

9. Looking at a portrait of a man, Harsh said, "His mother is the wife of my father's son. Brothers and sisters I have none. "At whose portrait was Harsh looking?
A. His son
B. His cousin
C. His uncle
D. His nephew

10. A, B, C, D, E, F and G are members of a family consisting of 4 adults and 3 children, two of whom, F and G are girls. A and D are brothers and A is a doctor. E is an engineer married to one of the brothers and has two children. B is married to D and G is their child. Who is C?
A. G's brother
B. F's father
C. E's father
D. A's son

11. In a family of 5, P is the father of R. S is Q's son. S has R as sister. Therefore, if U has P as brother, then the relationship between Q and U is as follows.
A. Q is U's daughter
B. U is Q's wife
C. Q is the sister-in-law of U
D. Q is U's brother-in-law

12. A, B and C are sisters. D is the brother of E and E is the daughter of B. How is A related to D?

A. Sister

B. Cousin

C. Niece

D. Aunt

13. F is the brother of A. C is the daughter of A. K is the sister of F. G is the brother of C. Who is the uncle of G?

A. A

B. C

C. F

D. K

14. P is the brother of Q and R. S is the R's mother. T is P's father. Which of the following statements cannot be definitely true?

A. T is Q's father

B. S is P's mother

C. T is S's husband

D. S is T's son

15. A party consisted of a man, his wife, his three sons and their wives and three children in each son's family. How many were there in the party?

A. 24

B. 22

C. 13

D. 17

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | C | 3. | A | 4. | C | 5. | D |
| 6. | B | 7. | B | 8. | D | 9. | A | 10. | D |
| 11. | C | 12. | D | 13. | C | 14. | D | 15. | D |

## Review Questions

1. Pointing to Ajay, Radha said, "His father is the only son of my grandfather". How is Radha Related to Ajay?
2. Lalita said to Tina, "You are the daughter-in-law of the grandmother of my father's only son."
3. Pointing to a photograph, Amar said, "I have no brother or sister but that man's father is my father's son." Whose photograph was it? How is Lalita related to Tina?
4. Looking at the portrait of a man, Ashok said, 'His mother is the wife of my father's son. Brothers and sisters I have none'. At whose portrait was Ashok looking?
5. Ahmad said to Saira, 'Your only brother's son is my wife's brother'. How is Saira related to the Ahmad's wife?
6. Pointing to a gentleman, Abdul said, "His only brother is the father of my daughter's father". How is the gentleman related to Abdul?"

7. Pointing to a man in a photograph, Malati tells, “His brother’s father is the only son of my grandfather.” How is Malati related to the man in the photograph?
8. Pointing to Dharmendra, Hema said, “He is the son of my father’s only son.” How is Dharmendra’s mother related to Hema?
9. Mr. ‘Ashok’ meets Mr.’Babu’. ‘Babu’ is the father of a son ‘Dharmendra’ and a daughter ‘Chandrika’. ‘Shalini’ is the mother of ‘Ashok’ ‘Dharmendra’ is married has one son. ‘Shalini’ is the daughter-in-law of ‘Babu’. How is ‘Ashok’ related to ‘Babu’.
10. Pointing to a photograph, a lady tells Bhushan, “I am the only daughter of this lady and her son is your maternal uncle.” How is the speaker related to Bhushan’s father?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 08: Number, Ranking and Time Sequence

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand different types of questions on Number Test.
- We will understand different types of questions on Ranking Test.
- We will understand different types of questions on Time Sequence Test.

## Introduction

Number-Ranking-Time Sequence section has all the different types of questions on Number Test, Ranking Test, and Time Sequence Test. In this type of questions, generally, you are given a long series of numbers. The candidate is required to find out how many times a number satisfying the conditions, specified in the question, occurs.

1 Minute = 60 seconds

1 Hour = 60 minutes

1 Day = 24 hours

1 Week = 7 days

1 Month = 4 weeks

1 Year = 12 months

1 Ordinary year = 365 days

1 Leap year = 3 66 days

1 Century =100 years.

| Month Name | Total number of days |
|---|---|
| January | 31 |
| February | 28 |
| March | 31 |
| April | 30 |
| May | 31 |
| June | 30 |
| July | 31 |
| August | 31 |
| September | 30 |
| October | 31 |
| November | 30 |
| December | 31. |

Other facts to be remembered

A day is the period of the earth's revolution on its axis.

A 'Solar year' is the time taken the earth to travel round the sun. It is equal to 365 days, 5 hours, 48 minutes and 47- seconds nearly.

A 'Lunar month' is the time taken the moon to travel round the earth. It is equal to nearly 28 days.

Leap Year

If the number of a given year is divisible by 4, it is a leap year. Hence, the years like 1996,2008,2012 are leap years. But years like 1997,1991, 2005,2007 are not divisible by 4 and therefore, such years are not leap years.

In a leap year, February has 29 days.

A leap year has 52 weeks and 2 days.

## 8.1 Number Test

In this type of question, generally, a set group or series of numerals is given and the candidate is asked to trace out numerals following certain given conditions or lying at specifically mentioned positions after shuffling according to a certain given pattern. In these types of questions, a number, a set of numbers, series of digit is given and the candidate is asked to trace out digit following certain given conditions. In these sort of problems, a number, an arrangement of numbers, arrangement of digit is given and the applicant is requested that follow out digit taking after certain given conditions or lying at particular said positions in the wake of rearranging as indicated by a specific given example.

Example1. How many such 5s are there in the following number sequence each of which is immediately preceded by 3 or 4 but not immediately followed by 8 or 9?

3 5 9 5 4 5 5 3 5 8 4 5 6 7 3 5 7 5 5 4 5 2 3 5 1 0

(A) None

(B) Three

(C) Four

(D) Five

(E) None of these

Solution:

As you know, a number that comes after a given number is said to follow it while the one which comes before the given number precedes it.

Thus, the number satisfying the given conditions may be marked as follows:

3 5 9 5 4 5 3 5 8 4 6 7 3 7 5 5 4 2 3 1 0

Clearly, there are five such 5s. Hence, the answer is (d).

Example2. What will be last digit of the 3rd number from top when the numbers given below are arranged in descending order after reversing the position of the digits within each number?

517 325 639 841 792

(a) 2

(b) 5

(c) 7

(d) 3

Sol. (d) The given numbers are:

517 325 639 841 792

After reversing, the numbers becomes as follows: 715 523 936 148 297

When arranged in descending order the numbers become as follows:

936 715 523 297 148;

Now, the third number from top is 523. Hence, the last digit of 523 is 3.

∴Option (d) is correct.

Example 3 How many 5s are there in the following number sequence which are immediately preceded by 7 and immediately followed by 6?

Terms : 7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 6 4 3 2 5 6 7 8

(a) 1

(b) 2

(c) 3

(d) 4

Solution. (a) Here,7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 6 4 3 2 5 6 7 8

Preceded by 7 and followed by 6 So, there is only one such 5.

738 429 156 273 894

Example: Which of the following will be the second digit of the third number from the top when they are arranged in descending order, after the first digit in each number is changed to its next higher digit?

A) 2

B) 3

C) 5

D) 7

E) 9

Answer:

Option A

Explanation:

The new numbers formed are : 838, 529,256,373,994.

These, in descending order, are : 994, 838, 529, 373,256.

The third number from the top is 529, and its second digit is 2.

The digits of each of the following five numbers are written in reverse order and five new number are obtained

513 726 492 865 149

Example: Which of the following will be the middle digit of the third number from the top when the new numbers are arranged in descending order?

A) 1

B) 2

C) 6

D) 9

E) None of these

Answer:

Option C

Explanation:

The new numbers, arranged in descending order are 941 627 568 315 294 .

The third number from the top is 568 and its middle digit is 6.

Example: In the following sequence of instructions, 1 stands for Run, 2 stands for Stop, 3 stands for Go, 4 stands for Sit and 5 stands for Wait. If the sequence were continued, which instruction will come next

4 4 5 4 5 3 4 5 3 1 4 5 3 1 2 4 5 4 5 3 4 5 3

A) Wait

B) Sit

C) Go

D) Stop

E) Run

Answer:

Option E

Explanation:

The given sequence may be analysed as under:

4 / 45 / 453 / 4531 / 45312 / 45 / 453 /453

Following the above sequence, the next number is 1 which stands for 'Run'

## 8.2 Ranking Test

In this type of questions, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. However, sometimes this question is put in the form of a puzzle of interchanging seats by two persons. In these sort of questions, generally the rank of a person both from the top and from the base are said and the aggregate number of persons is asked. In any case, in some cases this grouping is placed as a riddle of interchanging seats by two persons. In these types of questions, generally the ranks of a person both from the top and from the bottom are mentioned and the total number of persons is asked. When we are doing number and ranking we know basically two type of symbols first is greater than(>)and the other one is less than(<).To solve these questions we only know the three things:

Symbol of Greater Than and Less than means if A>B IMPLIES THAT (A is greater than B)or B < A(B is smaller than A). These are for the same person:

TOTAL={(TOP+BOTTOM)-1}

TOTAL={(LEFT+RIGHT)-1}

In numbering and ranking arrangement questions, position/rank of a person from left-right/top-bottom of a row/class is to be determined or rank/position is given & total no. of persons is to be calculated. You may also be asked to determine, using data given, which floor which person lives on.

Note:

1) Read the statement line by line and apply the cases as explained below.

2) Position can be from either sides of row and rank is always from top or bottom of the row

**One of the type of Number Ranking.**

1) Total number of persons = {(sum of positions of same person from both sides i.e. left and right side) – 1}

OR

2) Position of a person from opposite side = {(Total no. of persons – Position of same person from given side) +1}

Example:- In a row of persons, position of A from left side of the row is 27th and position of A from right side of the row is 34th. Find total no. of persons in the row?

Solution:

Total no. of students = (Position of A from left + Position of A from right) -1

$\Rightarrow$Total no. of students = (27 + 34) – 1 = 61 – 1 = 60.

Example 4 : A class of boys stands in a single line. One boy is nineteenth in order from both the ends. How many boys are there in the class?

(a) 27

(b) 37

(c) 38

(d) 39

Solution. (b) Clearly, number of boys in the row = (18 + 18 + 1) = 37

The question can be solved by formula also, total number of boys in the row = 19 + 19 - 1 = 37

Formula Total number of persons in a row or class = (Rank of a person from upper end or left end) + (Rank of that person from lower or right end) - 1

Example:- Rohan ranks seventh from the top and twenty-sixth from the bottom in a class. How many students are there in the class?

(A) 31

(B) 32

(C) 33

(D) 34

Solution:

Clearly, the whole class consists of:

(i) 6 student who have ranks higher than Rohan;

(ii) Rohan; and

(iii) 25 students who have ranks lower than Rohan, i.e, (6+1+25) = 32 students. Hence, the answer is (b).

In such problems, the ranks of a person both from the top from the bottom are given and on the basis of this the total number of persons is asked. Sometimes question is twisted also and position of a particular person is asked.

Example 5. Karishma ranks 10th from the top and 15th from the bottom in an examination. Find the total number of students in Karishma's class.

(a) 35

(b) 31

(c) 28

(d) 30

Sol. (d) As per the question; the class has

(i) 15 students higher than Karishma

(ii) 14 students lower than Karishma

(iii) Karishma

∴Total number of students = 15 + 14+1=30 Hence, option (d) is correct.

Example 6f:- In a row of trees, one tree is fifth from either end of the row. How many trees are there in the row ?

Option:

A. 8

B. 9

C. 10

D. 11

Answer: B . 9

Justification: Clearly, number of trees in the row = ( 4 + 1 + 4 ) = 9.

## 8.3 Time Sequence Test

Example.1. Satish remembers that his brother's birthday is after fifteenth but before eighteenth of February whereas his sister Kajal remembers that her brother's birthday is after sixteenth but below nineteenth of February. On which day in February is Satish's brother's birthday?

(A) 16th

(B) 17th

(C) 18th

(D) 19th

(E) None of these

Solution:

According to Satish, the brother's birthday is on one of the days among 16th and 17th February.

According to Kajal, the brother's birthday is on one of the days among 17th and 18th February.

Clearly, Satish's brother's birthday is on the day common to both the above groups, i.e., 17th February.

Example7. Neena returned house after 3 days earlier than the time she had told her mother. Neena's sister Veena reached five days later than the day Neena was supposed to return. If Neena returned on Thursday, on what day did Veena return?

(a) Friday

(b) Saturday

(c) Wednesday

(d) Sunday

Sol. (a) Neena returned home on Thursday. Neena was supposed to return 3 days later, i.e., on Sunday.

Veena returned five days later from Sunday, i.e., on Friday.

∴ Option (a) is the correct option.

Example 8:- If the seventh day of a month is three days sooner than Friday, what day will it be on the nineteenth day of the month.
Solution:
Given that seventh day of a month is three days sooner than Friday. So, the seventh day of the month is three days earlier than Friday, which is Tuesday. Therefore, the fourteenth day is also Tuesday. Hence, nineteenth day is Sunday.

Example 9. Vandana remembers that her father's birthday is between 13th and 16th of June. Whereas her brother remembers that their Father's birthday is between 14th and 18th of June. On which day is their Fathers birthday?

(a) 14th June

(b) 16th June

(c) 15th June

(d) 18th June

Sol. (c) According to Vandana her father's birthday is on one of the days among 14th and 15th June. Vandana's brother, the father's birthday is one of the days among 15th 16th and 17th June.

It is obvious that the father's birthday is on the day to both the above groups. The common day is 15th. Hence, the father's birthday falls on 15th June. 1 Option (c) is the correct option.

Example 10:- How many such 6's are there in the following number sequence each of which is immediately preceded by 3 or 4 but not immediately followed by 8 or 9?

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

Solution:

Given number sequence is

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

A number which comes after a given number is said to follow it while the one which precedes the given number goes before it.

Thus, the numbers satisfying the given conditions may be marked(with red colour) as follows:

3 6 9 6 4 6 6 3 6 8 4 6 5 7 3 6 7 6 6 4 6 2 3 6 1 0

Therefore, there are five such 6's.

Example 11:- How many pairs of successive numbers have a difference of 3 each? If the series:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 4 2 1 4 2 6 3.

Solution:

Given series is:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 2 1 4 2 6 3.

Here,

7 – 4 = 3

4 – 1 = 3

8 – 5 = 3

5 – 2 = 3

4 – 1 = 3

6 – 3 = 3

So, the pairs of successive numbers having a difference of 3 can be shown below:

7 4 1 2 2 8 4 1 2 1 8 5 5 2 2 1 7 1 2 1 4 2 6 3

Therefore, there are six such pairs.

## Summary

The key concepts learned from this unit are: -

- We have learnt how to solve different types of questions on Number Test.
- We have learnt how to solve different types of questions on Ranking Test.
- We have learnt how to solve different types of questions on Time Sequence Test.

## Keywords

- Series of numbers.
- Number sequence
- Number Test.
- Ranking Test.
- Time Sequence Test.

## Self Assessment

1. How many 3's are there in the following sequence which is neither preceded by nor immediately followed by 9?

9 3 6 6 3 9 5 9 3 7 8 9 1 6 3 9 6 3 9

A. One
B. Two
C. Three
D. None of these

2. Count each 7 which is not immediately preceded by 5 but is immediately followed by either 2 or 3. How many such 7's are there?
5 7 2 6 5 7 3 8 3 7 3 2 5 7 2 7 3 4 8 2 6 7 8
A. 2
B. 3
C. 4
D. 5

3. How many 7's are there in the following series which are preceded by 6 which is not preceded by 8?
8 7 6 7 8 6 7 5 6 7 9 7 6 1 6 7 7 6 8 8 6 9 7 6 8 7

A. Nil
B. One
C. Two
D. None of these

4. In a queue, Amrita is 10th from the front while Mukul is 25th from behind and Mamta is just in the middle of the two. If there were 50 persons in the queue, what position does Mamta occupy from the front?
A. 20th
B. 19th
C. 18th
D. 17th

5. Manisha ranked sixteenth from the top and twenty-ninth from the bottom among those who passed an examination. Six boys did not participate in the competition and five failed in it. How many boys were there in the class?
A. 40
B. 44
C. 50
D. 55

6. Some boys are sitting in a row. P is sitting fourteenth from the left and Q is seventh from the right. If there are four boys between P and Q, how many boys are there in the row?
A. 25
B. 23
C. 21
D. 19

7. A bus for Delhi leaves every thirty minutes from a bus stand. An enquiry clerk told a passenger that the bus had already left ten minutes ago and the next bus will leave at 9.35 a.m. At what time did the enquiry clerk give this information to the passenger?

A. 9.10 a.m.
B. 8.55 a.m.
C. 9.08 p.m.
D. 9.15 a.m.

8. Sangeeta remembers that her father's birthday was certainly after eighth but before thirteenth of December. Her sister Natasha remembers that their father's birthday was definitely after ninth but before fourteenth of December. On which date of December was their father's birthday? (Bank P.O. 1998)

A. 10th
B. 11th
C. 12th
D. Data inadequate

9. The train for Lucknow leaves every two and a half hours from New Delhi Railway Station. An announcement was made at the station that the train for Lucknow had left 40 minutes ago and the next train will leave at 18.00 hrs. At what time was the announcement made?

A. 15.30 hrs
B. 15.50 hrs
C. 16.10 hrs
D. 17.10 hrs

10. In the series given below , count the number of 9's , each of which is not immediately preceded by 5 but is immediately followed by either 2 or 3. How many such 9's are there ?

1 9 3 2 1 7 4 2 6 9 7 4 6 1 3 2 8 7 4 1 3 8 3 2 5 6 7 4 3 9 5 8 2 0 1 8 7 4 6 3

A. one
B. three
C. five
D. six

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | A | 3. | D | 4. | C | 5. | D |
| 6. | A | 7. | D | 8. | D | 9. | C | 10. | B |

## Review Questions

1. Pointing to Ajay, Radha said, "His father is the only son of my grandfather". How is Radha Related to Ajay?
2. How many 4's are there preceded by 7 but not followed by 3? 5 9 3 2 1 7 4 2 6 9 7 4 6 1 3 2 8 7 4 1 3 8 3 2 5 6 7 4 3 9 5 8 2 0 1 8 7 4 6 3.
3. Rohan ranks 7th from the top and 26th from the bottom in the class. How many students are there in the class?
4. Manik is 14 th from the right end in the row of 40 students . What is his position from the left end ?
5. In a row of boys facing the north , A is 16th from the left end and C is 16 th from the right end . B , who is 4th to the right of A , is 5th to the left of C, in a row. How many boys are there in a row?
6. In the following series of numbers, find out how many times, 1, 3 and 7 have appeared together, 7 being in the middle and 1 and 3 on either side of 7 ?

   2 9 7 3 17 3 7 7 1 3 3 17 3 8 5 7 1 3 7 7 1 7 3 9 0 6

7. If the numbers from 5 to 85 which are exactly divisible by 5 are arranged in descending order, which would come at the eleventh place from the bottom ?
8. In a row of students, Anil is 7th from left, while Sunil is 18th from right. Both of them interchanged their positions such that Anil becomes 21st from left. What will be the total number of students in the class?
9. Mohan is taller than Shyam but shorter than Ramesh. Ramesh is taller than Rajat but shorter than Gautam. If Shyam is taller than Rajat, then who is the shortest among all?
10. Sunita leaves her house at 20 min to seven in the morning, reaches Vineeta's house in 25 min, they finish their breakfast in another 15 min and leave for their office which takes another 35 min. At what time, did they leave Vineeta's house to reach their office?

## Further Readings


1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 09: Ratio and Proportion

CONTENTS



## Objectives

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand how to comparison of two quantities
- We will understand how to find Proportion
- We will understand how to Compare in a given mixture
- We will understand how to find Fraction
- We will understand how to find Percent in a given mixture
- We will understand how to find Equivalent ratios
- We will understand how to find Mean Proportional

## Introduction

A ratio is a comparison of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantities may be of same kind or different kinds. For example, when we consider the ratio of the weight 45 Kg of a bag of rice to the weight 29 Kg of a bag of sugar, we are considering the quantities of same kind but when we talk of allotting 2 cricket bats to 5 sportsmen, we are considering quantities of different kinds. Normally, we consider the ratio between quantities of the same kind.

If a and b are two numbers, the ratio of a to b is a/b or a ÷ b and is denoted by a:b. The two quantities that are being compared are called terms. The fi rst is called antecedent and the second term is called consequent.

For example, the ratio 3:5 represents 3/5 with antecedent 3 and consequent 5.

1. A ratio is a number in order to find the ratio of two quantities and they must be expressed in the same units.

2. A ratio does not change, if both of its terms are multiplied or divided by the same number. Thus, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

A ratio is a comparison of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantities may be of same kind or different kinds. For example, when we consider the ratio of the weight 45 Kg of a bag of rice to the weight 29 Kg of a bag of sugar, we are considering the quantities of same kind but when we talk of allotting 2 cricket bats to 5 sportsmen, we are considering quantities of different kinds. Normally, we consider the ratio between quantities of the same kind.

If a and b are two numbers, the ratio of a to b is a/b or a ÷ b and is denoted by a:b. The two quantities that are being compared are called terms. The fi rst is called antecedent and the second term is called consequent. For example, the ratio 3:5 represents 3 5 with antecedent 3 and consequent 5.

## 9.1 Ratios and Proportions

At a certain college, there are 500 students who are under 25 years of age and 1500 students who are 25 years old or older. The ratio of students under 25 to students over 25 is 500 to 1500, which is also written 500:1500 or as a fraction $\frac{500}{1500}$. We know that as fractions $\frac{500}{1500} = \frac{1}{3}$. A ratio is a division relationship between two numbers. The ratio of a to b, written as a:b, is another meaning for the division or fraction $\frac{a}{b}$.

**Equivalent ratios:** We have already noticed that the ratios $\frac{500}{1500} = \frac{1}{3}$ are equal as fractions. Notice too that $500 \cdot 3 = 1 \cdot 1500$. Two ratios are equal if and only if their cross products are equal. $\frac{a}{b} = \frac{c}{d}$ if and only if $a \cdot d = c \cdot b$. Observe how the cross products are constructed. The numerator on the left side times the denominator on the right is equal to the numerator on the right times the denominator on the left.

**Proportions:** An equation of two ratios is called a proportion. $\frac{500}{1500} = \frac{1}{3}$ is a proportion, and $\frac{64}{360} = \frac{16}{90}$ is a proportion. The equality of two ratios is called proportion. If a/b = c/d , then a, b, c and d are said to be in proportion and we write a:b::c:d. This is read as "a is to b as c is to d".

A proportion involves 4 numbers. We can use cross-multiplication and some algebra to solve a proportion equation for an unknown.

Example: Find t so that this proportion is true.

Set the cross products equal and solve the equation.

$$\frac{140}{210} = \frac{8}{t}$$
$$140t = 8 \cdot 210$$
$$140t = 1680$$
$$t = \frac{1680}{140}$$
$$t = 12$$

**Proportions can be used to solve many word problems.**

Example 2: How far does a train going 68 miles per hour travel in 3 and a half hours?

$$\frac{68\ \text{miles}}{1\ \text{hour}} = \frac{x\ \text{miles}}{3.5\ \text{hours}}$$
$$68 \cdot 3.5 = x \cdot 1$$
$$x = 238\ \text{miles}$$

Example: 200 is 80% of what number?

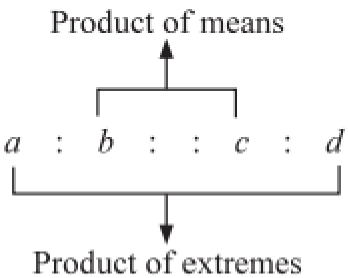
Let b represent the unknown number which is the base in the percent relationship and set up the proportion. Cross-multiply and solve. It helps to reduce the ratio first.

$$\frac{200}{b} = \frac{80}{100}$$
$$\frac{200}{b} = \frac{4}{5}$$
$$4b = 200 \cdot 5$$
$$b = \frac{1000}{4} = 250$$

Exercises:

For example, since 3/4 = 6/8 , we write 3:4::6:8 and say 3, 4, 6 and 8 are in proportion. Each term of the ratio a/b and c/d is called a proportional. a, b, c and d are respectively the fi rst, second, third and fourth proportionals. Here, a, d are known as extremes and b, c are known as means.

1. If four quantities are in proportion, then Product of Means = Product of Extremes For example, in the proportion a:b::c:d, we have, bc = ad




From this relation, we see that if any three of the four quantities are given, then the fourth quantity can be determined.

**Fourth proportional**: If a:b::c:x, then x is called the fourth proportional of a, b, c.

We have a/b = c / x or x = b*c/x Thus, fourth proportional of a, b, c is b * c/ a

2:5::4:x or, 2/ 5 = 4/x

$\therefore$ x = $\frac{5 \, x \, 4}{2}$ = 10

**Third Proportional:** If a:b::b:x, then x is called the third proportional of a, b.

We have , a/b = b/ x or $b^2/2$ Thus, third proportional of a, b is $b^2$/a

Example: Find a third proportional to the numbers 2.5, 1.5.

Solution: Let x be the third proportional, then

2·5:1·5::1·5:x or $\frac{2.5}{1.5} = \frac{1.5}{x}$ $\therefore$ $x = \frac{1.5 \, X \, 1.5}{2.5}$ = 0.9

**Mean Proportional:** If a:x::x:b, then x is called the mean or second proportional of a, b. We have, a x = b x or, $x^2$ = ab or, x = $\sqrt{ab}$ $\therefore$ Mean proportional of a and b is $\sqrt{ab}$ . We also say that, a, x, b are in continued proportion.

Example: Find the mean proportional between 48 and 12.

solution :- let x be the mean proportional . Then

48:x::x:12 or $\frac{48}{x} = \frac{x}{12}$ or $x^2$ = 576 or x = 24

$\frac{a+b}{b} = \frac{c+d}{d}$ ( componends)

1) $\frac{a+b}{b} = \frac{c+d}{d}$ ( componends)

2) $\frac{a-b}{b} = \frac{c-d}{d}$ (dividendo)

3) $\frac{a \mp b}{a-b} = \frac{c-d}{c-d}$ (compound and dividend)

4) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

Example: Find a third proportional to the numbers 2.5, 1.5.

Solution: Let x be the third proportional, then

2·5:1·5::1·5:x or $\frac{2.5}{1.5} = \frac{1.5}{x}$ $\therefore$ $x = \frac{1.5 \, X \, 1.5}{2.5}$ = 0.9

**Mean Proportional:** If a:x::x:b, then x is called the mean or second proportional of a, b. We have, a x = b x or, $x^2$ = ab or, x = $\sqrt{ab}$ $\therefore$ Mean proportional of a and b is $\sqrt{ab}$ . We also say that, a, x, b are in continued proportion.

Example: The sum of two numbers is c and their quotient is $\frac{p}{q}$. Find the numbers.

Solution: Let the numbers be x, y.

Given: x + y = c …(1)

And $\frac{x}{y} = \frac{p}{q}$ …(2)

$\therefore \frac{x}{x+y} = \frac{p}{p+q}$

$= \frac{x}{c} = \frac{p}{p+q}$ [ using (1) ]

= x = $\frac{pc}{p+q}$

**SHORTCUT – METHOD**

a) If two numbers are in the ratio of a:b and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b}$ and $\frac{bx}{a+b}$, respectively.

Or

If in a mixture of x litres, two liquids A and B are in the ratio of a:b, then the quantities of liquids A and B in the mixture will be $\frac{ax}{a+b}$ litres and bx a $\frac{bx}{a+b}$ litres, respectively.

b) If three numbers are in the ratio of a:b:c and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b+c}$, $\frac{bx}{a+b+c}$ and $\frac{cx}{a+b+c}$ respectively

## SOLUTION:

Let the three numbers in the ratio a:b:c be A, B and C.

Then, A = ka, B = kb, C = kc and, A + B + C = ka + kb + kc = x

⇒ k(a + b + c) = x

⇒ $k = \frac{x}{a+b+c}$

$A = ka = \frac{ax}{a+b+c}$

$B = kb = \frac{bx}{a+b+c}$

$C = kc = .\frac{cx}{a+b+c}$

Example: Two numbers are in the ratio of 4:5 and the sum of these numbers is 27. Find the two numbers. Solution: Here, a = 4, b = 5 and x = 27.

∴ the first number $= \frac{ax}{a+b} = \frac{4\ x\ 27}{4+5} = 12$

And , the Second number $= \frac{bx}{a+b} = \frac{5\ x\ 27}{4+5} = 15$

Example: Three numbers are in the ratio of 3:4:8 and the sum of these numbers is 975. Find the three numbers. Solution: Here, a = 3, b = 4, c = 8 and x = 975.

∴ the first number $= \frac{ax}{a+b+c} = \frac{3x\ 975}{3+4+8} = 195$

And , the Second number $= \frac{bx}{a+b+c} = \frac{4x\ 975}{3+4+8} = 260$

and, the third number $= \frac{cx}{a+b+c} = \frac{8\ x\ 975}{3+4+8} = 520$

2) If two numbers are in the ratio of a:b and difference between these numbers is x, then these numbers will be

a) $\frac{ax}{a-b}$ and $\frac{bx}{a-b}$, respectively ( where a>b)

b) $\frac{ax}{b-a}$ and $\frac{bx}{b-a}$, respectively ( where a<b)

**Solution**

Let the two numbers be ak and bk. Let a > b. Given: ak - bk = x

⇒ (a - b)k = x or, $k = \frac{ax}{b-a}$

Therefore, the two numbers are $= \frac{ax}{b-a} = \frac{bx}{b-a}$

Example 10: If A:B = 3:4 and B:C = 8:9, find A:B:C.

Solution: Here, $n_1 = 3$, $n_2 = 8$, $d_1 = 4$ and $d_2 = 9$.

∴ A:B:C = $(n_1 \times n_2):(d_1 \times n_2):(d_1 \times d_2)$

= (3 × 8):(4 × 8):(4 × 9)

= 24:32:36 or, 6:8:9.

Example 11: If A:B = 2:3, B:C = 4:5 and C:D = 6:7, find A:D.

Solution: Here, $n_1 = 2$, $n_2 = 4$, $n_3 = 6$, $d_1 = 3$, $d_2 = 5$ and $d_3 = 7$.

∴ A:B:C:D = $(n_1 \times n_2 \times n_3):(d_1 \times n_2 \times n_3):(d_1 \times d_2{:}n_3):(d_1 \times d_2 \times d_3)$

= (2 × 4 × 6) :(3 × 4 × 6):(3 × 5 × 6) :(3 × 5 × 7)

= 48:72:90:105 or, 16:24:30:35.

Thus, A:D = 16:35.

4) (a) The ratio between two numbers is a:b. If x is added to each of these numbers, the ratio becomes c:d. The two numbers are given as:

$\frac{ax(c-d)}{ad-bc}$ and $\frac{bx(c-d)}{ad-bc}$

**Solution :-**

Let two number be ak and bk.

Given : $\frac{ak+x}{bk+x} = \frac{c}{d} \rightarrow$ akd + dx = cbk + cx

= k(ad-bc) = x(c-d)

= k = $\frac{x(c-d)}{ad-bc}$

Therefore , the two number are $\frac{ax(c-d)}{ad-bc}$ and $\frac{bx(c-d)}{ad-bc}$

(b) The ratio between two numbers is a:b. If x is subtracted from each of these numbers, the ratio becomes c:d.

The two numbers are given as: $\frac{ax(d-c)}{ad-bc}$ and $\frac{bx(d-c)}{ad-bc}$

**Solution:**

Let two number be ak and bk.

Given : $\frac{ak+x}{bk+x} = \frac{c}{d} \rightarrow$ akd -xd = bck -xc

= k(ad-bc) = x(d-c)

= k = $\frac{x(d-c)}{ad-bc}$

Therefore, the two number are $\frac{ax(d-c)}{ad-bc}$ and $\frac{bx(d-c)}{ad-bc}$

Example 12: Given two numbers which are in the ratio of 3:4. If 8 is added to each of them, their ratio is changed to 5:6. Find the two numbers.

Solution: We have, a:b = 3:4, c:d = 5:6 and x = 8.

The first number $\frac{ax(c-d)}{ad-bc}$

= $\frac{3\ x\ 8\ x\ (5-6)}{(3\ x\ 6\ -4x\ 5)}$ = 12

The second number $\frac{bx(c-d)}{ad-bc}$

= $\frac{5\ x\ 8\ x\ (5-6)}{(3\ x\ 6\ -4x\ 5)}$ = 16

Example 13: The ratio of two numbers is 5:9. If each number is decreased by 5, the ratio becomes 5:11. Find the numbers.

Solution: We have, a:b = 5:9, c:d = 5:11 and x = 5

The first number $\frac{ax(c-d)}{ad-bc}$

$= \frac{5\,x\,5\,x\,(11-15)}{(5x11\ -9x\ 5)} = 16$

The second number $\frac{bx(c-d)}{ad-bc}$

$= \frac{9\,x\,5\,x\,(11-15)}{(5x11\ -9x\ 5)} = 27$

5) (a) If the ratio of two numbers is a:b, then the numbers that should be added to each of the numbers in order to make this ratio c:d is given by $\frac{ad-bc}{c-d}$.

**Solution :-**

Let the required number be x.

Given: $\frac{a+x}{b+x} = \frac{c}{d}$

= ad + xd = bc + xc

=x(d-c = bc – ad

Or , x = $\frac{ad-bc}{c-d}$

(b) If the ratio of two numbers is a:b, then the number that should be subtracted from each of the numbers in order to make this ratio c:d is given by $\frac{bc-ad}{c-d}$

**Solution :-**

Let the required number be x.

Given : $\frac{a-x}{b-x} = \frac{c}{d}$

= ad - xd = bc - xc

=x(c-d) = bc – ad

Or , x =$\frac{bc-ad}{c-d}$

Example: Find the number that must be subtracted from the terms of the ratio 5:6 to make it equal to 2:3.

Solution: We have, a:b = 5:6 and c:d = 2:3.

∴ the required number = $\frac{bc-ad}{c-d}$

$= \frac{6\,x2\ -5\,x3}{2-3} = 3$

Example 15: Find the number that must be added to the terms of the ratio 11:29 to make it equal to 11:20.

Solution: We have, a:b = 11:29 and c:d = 11:20.

$\therefore \text{The required number} = \frac{ad-bc}{c-d}$

$= \frac{11\,x\,20\ -29\,x\,11}{11-20} = 11$

6) There are four numbers a, b, c and d.

(i) The number that should be subtracted from each of these numbers so that the remaining numbers may be proportional is given by

$$\frac{ad - bc}{(a+d)-(b-c)}$$

**Solution**

Let x be subtracted from each of the numbers.

The remainders are a - x, b - x, c - x and d - x.

Given: $\frac{a-x}{b-x} = \frac{c-x}{d-x}$

⇒ (a - x) (d - x) = (b - x) (c - x)

⇒ ad - x(a + d) + $x^2$ = bc - x(b + c) + $x^2$

⇒ (b + c)x - (a + d)x = bc – ad

∴ x = $\frac{bc - ad}{(b+c)-(a+d)}$ or $\frac{ad - bc}{(a+d)-(b+c)}$

(ii) The number that should be added to each of these numbers so that the new numbers ma be proportional is given by

x = $\frac{bc - ad}{(a+d)-(b+c)}$

**Solution**

Let x be added to each of the numbers. The new numbers are a + x, b + x, c + x and d + x.

Given : $\frac{a+x}{b+x} = \frac{c+x}{d+x}$

⇒ (a + x) (d + x) = (b + x) (c + x)

⇒ ad + x(a + d) + $x^2$ = bc + x(b + c) + $x^2$

⇒ (a + d)x - (b + c)x = bc - ad.

∴ x = $\frac{bc - ad}{(a+d)-(b+c)}$

Example 16: Find the number subtracted from each of the numbers 54, 71, 75 and 99 leaves the remainders which are proportional. Solution: We have, a = 54, b = 71, c = 75 and d = 99

The required number = ∴ x = $\frac{ad-bc}{(a+d)-(b+c)}$

= $\frac{54 \times 99 - 71 \times 75}{(54+99)-(71+75)}$ = 3

7) The incomes of two persons are in the ratio of a:b and their expenditures are in the ratio of c:d. If the saving of each person be `S, then their incomes are given by

₹$\frac{aS(d-c)}{ad-bc}$ and ₹$\frac{bS(d-c)}{ad-bc}$

and their expenditures are given by

₹$\frac{cS(d-a)}{ad-bc}$ and ₹$\frac{aS(d-a)}{ad-bc}$

**Solution**

Let their incomes be ₹ ak and ₹ bk, respectively. Since each person saves ₹ S,

∴ Expenditure of fi rst person = ₹(ak - S)

and, expenditure of second person = ₹(bk - S).

Given $\frac{ak-S}{bk-S} = \frac{c}{d}$

= akd - Sd = bkc - Sc

⇒ k(ad - bc) = (d - c)S or, k = $\frac{(d-c)}{ad-bc}$

Therefore, the incomes of two persons are

$\frac{a(d-c)S}{ad-bc}$ and ₹$\frac{b(d-c)S}{ad-bc}$

And their expenditure are

Ak-S and bk-s

i .e $\frac{a(d-c)S}{ad-bc}$-S and ₹$\frac{b(d-c)S}{ad-bc}$-S

$\frac{aS(d-c)}{ad-bc}$ and $\frac{dS(d-a)}{ad-bc}$

Example 17: Annual income of A and B is in the ratio of 5:4 and their annual expenses bear a ratio of 4:3. If each of them saves `500 at the end of the year, then find their annual income.

Solution: We have, a:b = 5:4, c:d = 4:3 and S = 500.

Annual income of A = $\frac{aS(d-c)}{ad-bc}$

$= \frac{5\ x\ 500\ x\ (3-4)}{(5x3-4x4)}$

= ₹2500

And annual income of B = $\frac{dS(d-c)S}{ad-bc}$

$= \frac{4\ x500\ x(3-4)}{(5x3-4x4)}$

= ₹2000

Example 18: The incomes of Mohan and Sohan are in the ratio 7:2 and their expenditures are in the ratio 4:1. If each saves `1000, find their expenditures.

Solution: We have, a:b = 7:2, c:d = 4:1 and S = 1000.

∴ Mohan's expenditure = $\frac{cS(b-a)}{ad-bc}$

$= \frac{4\ x\ 1000\ x\ (2-7)}{(7\ x\ 1-2x4)}$

= ₹2000

Sohan's expenditure = $\frac{dS(b-a)}{ad-bc}$

$= \frac{1\ x\ 1000x(2-7)}{(7x1-2x4)}$

₹5000

8) (a) If in a mixture of x litres of two liquids A and B, the ratio of liquids A and B is a:b, then the quantity of liquid B to be added in order to make this ratio.

C:d is $\frac{x(ad-bc)}{c(a+b)}$

Quantity of liquid A in the mixture = $\frac{ax}{a+b}$ . Quantity of liquid B in the mixture = $\frac{bx}{a+b}$

Let/litres of liquid B to be added in order to make this ratio as c:d

Then $\frac{ax}{a+b} : \frac{bx}{a+b}$ + l = c:d

Or $\frac{ax}{a+b} : \frac{bx+l(a+b)}{a+b}$ = c:d

Or, : $\frac{ax}{bx+l(a+b}$ = C/d or axd = bcx +cl(a+b)

L=$\frac{x(ad-bc)}{(a+b)c}$

(b) In a mixture of two liquids A and B, the ratio of liquids A and B is a:b. If on adding x litres of liquid B to the mixture, the ratio of A to B becomes a:c, then in the beginning the quantity of liquid A in the mixture was ax/c-b− litres and that of liquid B was bx /c b − litres.

**Solution**

Let the quality of mixture be M liters

Let the quantity of mixture be M litres.

Then, the quantity of liquid A = $\frac{aM}{a+b}$ litres

and the quantity of liquid B = $\frac{bM}{a+b}$ litres.

If x litres of liquid B is added, then

$\frac{aM}{a+b}:\frac{bM}{a+b}$ + x= a:c

Or, $\frac{aM}{a+b}$ : $\frac{bM+x(a+b)}{a+b}$ = a:c

or $\frac{aM}{bM+x(a+b)} = \frac{a}{c}$ or cM = bM +x(a+b)

Or M = $\frac{x(a+b)}{c-b}$

∴ quantity of liquid A = $\frac{ax(a+b)}{(c-b)(a+b)}$

= $\frac{ax}{c-b}$ litres

And, quantity of liquid B = $\frac{bx(a+b)}{(c-b)(a+b)}$

= $\frac{bx}{c-b}$ litres

Example 19: 729 ml of a mixture contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture containing milk and water in the ratio of 7:3. Solution: Here, x = 729, a:b = 7:2 and c:d = 7:3. ∴ The quantity of water to be added

= $\frac{x(ad-bc)}{c(a+b)}$

= $\frac{729\ x\ (7x\ 3-2\ x7)}{7(7+2)}$

= 81 ml

Example 20: A mixture contains alcohol and water in the ratio of 6:1. On adding 8 litres of water, the ratio of alcohol to water becomes 6:5. Find the quantity of water in the mixture.

Solution: We have, a:b = 6:1, a:c = 6:5 and x = 8. ∴ The quantity of water in the mixture

= $\frac{bx}{c-b} = \frac{1\ x\ 8}{5-1}$

= 2 litres

9)When two ingredients A and B of quantities $q_1$ and $q_2$ and cost price/unit $c_1$ and $c_2$ are mixed to get a mixture c having cost price/unit cm, then

a) $\frac{q_1}{q_2} = \frac{c_2-c_m}{c_m-c_1}$

b) $c_m = \frac{c_1 x q_1 + c_2 x q_2}{q_1+q_2}$

Example 21: In what ratio the two kinds of tea must be mixed together, one at ₹9 per Kg and another at ₹15 per Kg, so that mixture may cost ₹10.2 per Kg?

Solution: We have, $c_1$ = 9, $c_2$ = 15, cm = 10.2

∴ $\frac{q_1}{q_2} = \frac{c_2-c_m}{c_m-c_1} = \frac{15-10.2}{10.2-9}$

$=\frac{4.8}{1.2}$

$=\frac{4}{1}$

Thus, the two kinds of tea are mixed in the ratio 4:1

Example 22: In a mixture of two types of oils $O_1$ and $O_2$, the ratio $O_1:O_2$ is 3:2. If the cost of oil $O_1$ is ₹4 per litre and that of $O_2$ is ₹9 per litre, then find the cost per litre of the resulting mixture. Solution: We have, $q_1 = q_2 = 2$, $c_1 = 4$ and $c_2 = 9$. ∴ The cost of resulting mixture

$= \frac{c_1 x q_1 + c_2 x q_2}{q_1 + q_2} = \frac{4x3+9\,x2}{3+2}$

$= \frac{30}{5}$

= ₹ 6

(a) If a mixture contains two ingredients A and B in the ratio a:b, then

percentage of A in the mixture = $\frac{a}{a+b}$x100% and percentage of B in the mixture $\frac{b}{a+b}\times$ 100%

(b) If two mixtures $M_1$ and $M_2$ contain ingredients A and B in the ratios a:b and c:d, respectively,

then a third mixture $M_3$ obtained by mixing $M_1$ and $M_2$ in the ratio x:y will contain

$\left[\frac{\frac{a*x}{a+b}+\frac{c*y}{c+d}}{x+y}\right]$ x 100% ingredient A , and

$\left[100\% - \left\{\frac{\frac{ax}{a+b}+\frac{cy}{c+d}}{x+y}\right\}\right]$

Or $\left[\frac{\frac{bx}{a+b}+\frac{dy}{c+d}}{x+y}\right]$ x 100% ingredient B

Example 23: Two alloys contain silver and copper in the ratio 3:1 and 5:3. In what ratio the two alloys should be added together to get a new alloy having silver and copper in the ratio of 2:1? Solution: We have, a:b = 3:1, c:d = 5:3 Let the two alloys be mixed in the ratio x:y. Then, percentage quantity of sliver in the new alloy

$= \left[\frac{\frac{a*x}{a+b}+\frac{c*y}{c+d}}{x+y}\right]$ x 100%

$= \left[\frac{\frac{3x}{4}+\frac{5y}{8}}{x+y}\right]$ x 100%

$= \frac{6x+5y}{8(x+y)}$ x 100% ..(1)

Since, the ratio of silver and copper in the new alloys is 2:1.

∴ percentage quantity of sliver in the new alloy

$= \frac{2}{2+1}$ x 100% $= \frac{200}{3}$% ….(2)

From (1) and (2), we get

$= \frac{6x+5y}{(x+y)} = \frac{2}{3}$ or 18x + 15 y = 16x + 16 y

or, 2x = y or, x:y = 1:2. Hence, the two alloys should be mixed in the ratio 1:2.

## Summary

The key concepts learned from this unit are: -

- We have learnt how to solve to comparison of two quantities
- We have learnt how to solve Proportion
- We have learnt how to compare in a given mixture
- We have learnt how to solve and find Fraction

- We have learnt how to solve Equivalent ratios
- We have learnt how to solve Mean Proportional

## Keywords

- Ratio
- Proportion
- Comparison
- Fraction
- Percent
- Equivalent ratios
- Mean Proportional

## Self Assessment

1. Divide $200 in the ratio 3 : 2. What is the result?
A. $140 : $60
B. $80 : $120
C. $300 : $200
D. $120 : $80

2. The ratio of men to women at a party was 6 : 7, respectively.
There were 28 women. Calculate the number of people at the party.
A. 24
B. 28
C. 52
D. 104

3. $x is divided among three boys, Ryan, Keith and Andrew, in the ratio 2 : 3 : 7, respectively.
If Andrew gets $15 more than Keith, what is the value of x ?
A. $27
B. $45
C. $57
D. $180

4. A sum of J$300 is divided in the ratio 2 : 3.
Calculate the amount of the LARGER share.
A. J$120
B. J$150
C. J$180
D. J$240

5. A metal is made from copper, zinc and lead in the ratio 13 : 6 : 1. The mass of the zinc is 90 kg. Calculate the mass of the metal.
A. 195 kg
B. 210 kg
C. 285 kg
D. 300 kg

6. The ratio of Rs 8 to 80 paise is
A. 1 : 10
B. 10 : 1
C. 1: 1
D. 100 : 1

7. The length and breadth of a steel tape are 10m and 2.4cm, respectively. The ratio of the

length to the breadth is

A. 5 : 1.2

B. 25: 6

C. 625: 6

D. 1250: 3

8. Find the missing number in the box in the following proportion:: 8 :: 12 : 32

A. 2

B. 3

C. 4

D. 5

9. State whether the given statement are true or false: 12 : 18 = 28 : 56

10. Fill in the blanks:

If two ratios are ______, then they are in proportion.

11. The first, second and fourth terms of a proportion are 16, 24 and 54 respectively. Then the third term is:

A. 36;

B. 28;

C. 48;

D. 32

12. If x, y and z are in proportion, then:

A. x : y : : z : x;

B. x : y : : y : z;

C. x : y : : z : y;

D. x : z : : y : z

13. Length and width of a field are in the ratio 5 : 3. If the width of the field is 42 m then its length is:

A. 100 m;

B. 80 m;

C. 50 m;

D. 70 m

14. The value of m, if 3, 18, m, 42 are in proportion is:

A. 6;

B. 54;

C. 7;

D. none of these

15. The ratio of 1 hour to 300 seconds is:

A. 1 : 12;

B. 12 : 1;

C. 1 : 5;

D. 5 : 1

16. State whether the given statements are true or false: 25 persons : 130 persons = 15kg : 78kg

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | C | 3. | B | 4. | C | 5. | D |
| 6. | D | 7. | B | 8. | D | 9. | B | 10. | Equal/Same |
| 11. | A | 12. | B | 13. | D | 14. | C | 15. | B |
| 16. | A | | | | | | | | |

## Review Questions

1. You jog 3.6 miles in 30 minutes. At that rate, how long will it take you to jog 4.8 miles?

2. You earn $33 in 8 hours. At that rate, how much would you earn in 5 hours?

3. An airplane flies 105 miles in ½ hour. How far can it fly in 1 ¼ hours at the same rate of speed?

4. What is the cost of six filters if eight filters cost $39.92?

5. If one gallon of paint covers 825 sq. ft., how much paint is needed to cover 2640 sq. ft.?

6. A map scale designates 1″ = 50 miles. If the distance between two towns on the map is 2.75 inches, how many miles must you drive to go from the first town to the second?

7. Bob is taking his son to look at colleges. The first college they plan to visit is 150 miles from their home. In the first hour they drive at a rate of 60 mph. If they want to reach their destination in 2 ½ hours, what speed must they average for the remainder of their trip?

8. Four employees can wash 20 service vehicles in 5 hours. How long would it take 5 employees to wash the same number of vehicles?

9. These two figures are similar. Use a proportion to find the length of side n.

20 m 12 m 30 m n

Set up a proportion to solve each word problem. (You may use the back to show your work)

10. Zach can read 7 pages of a book in 5 minutes. At this rate, how long will it take him to read the entire 175 page book? (Convert your final answer to hours and minutes)

11. Shannon's bicycle travels 50 feet for every 3 pedal turns. How many pedal turns are needed to travel one mile (1 mile = 5,280 feet)?

12. A cookie recipe for 5 dozen cookies calls for 4 cups of flour. How much flour is needed to make 90 cookies?

13. A man can make 12 birdhouses in 8 hours. How many hours will it take him to make 30 birdhouses?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 10: Alligation or Mixture

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand what is an Alligation.
- We will understand what are Rule of Alligation.

## Introduction

Alligation literally means 'linking'. It is a rule to find:

(a) The ratio in which two or more ingredients at their respective prices should be mixed to give a mixture at a given price.

(b) The mean or average price of a mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given. Here, cost price of a unit quantity of a mixture is called the 'mean price'.

## 10.1 Alligation Rule

**1. Alligation**

Alligation is the rule which enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a specified price.

**2. Mean Price**

Mean price is the cost price of a unit quantity of the mixture

**3. Rule of Alligation**

If two ingredients are mixed, then

$$\left(\frac{Quantity\ of\ cheaper}{Quantity\ of\ dearer}\right) = \left(\frac{C.P.of\ dearer\ -\ Mean\ Price}{Mean\ price - C.P\ of\ cheaper}\right)$$

The above formula can be represented with the help of the following diagram which is easier to understand.

=> (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - c)

4. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid $=\left[x\left(1-\frac{\cdot}{y}\right)^{n}\right]$units.

Suppose ₹d per unit be the price of first ingredient (superior quality) mixed with another ingredient (cheaper quality) of price `c per unit to form a mixture whose mean price is `m per unit, then the two ingredients must be mixed in the ratio:

i.e., the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price. The above rule may be represented schematically as under:

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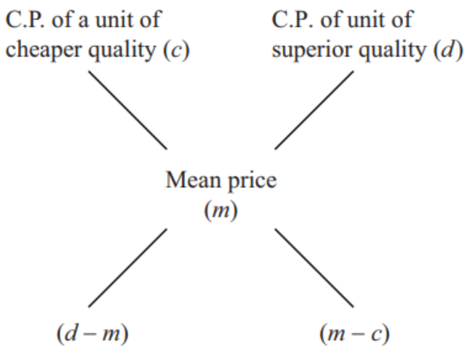
i.e., the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price. The above rule may be represented schematically as under:

$\frac{Quantity\ of\ cheaper}{Quantity\ of\ superior} = \frac{C.p.\ superior - Mean\ price}{Mean\ price - C.P.\ of\ chapter}$

i.e., the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price. The above rule may be represented schematically as under:




$\frac{Quantity\ of\ cheapter\ quality}{Quantity\ of\ superior\ quality} = \frac{d-m}{m-c}$

Explanation:- Suppose, x Kg of cheaper quality is mixed with y Kg of superior quality.

Price of cheaper ingredient = ₹cx

Price of superior ingredient = ₹ dy

∴ Price of mixture = ₹$\left(\frac{cx+dy}{x+y}\right)$

and quantity of mixture = (x + y) Kg.

∴ $\frac{CX+dy}{x+y}$ = m

cx + dy = mx + my

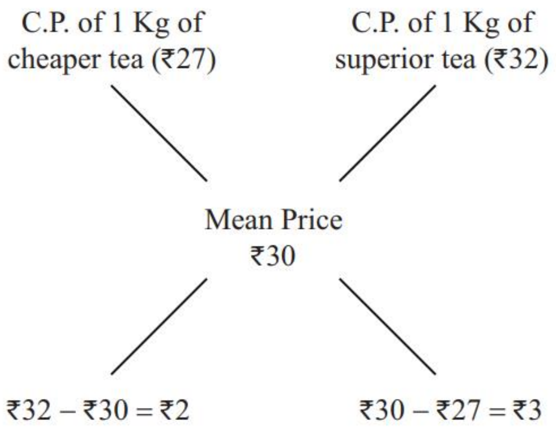
⇒ dy – my = mx – cx

⇒ y (d – m ) = x(m – c )

⇒ $\frac{x}{y} = \frac{d-m}{m-c}$

Example:- 1: In what ratio two varieties of tea, one costing ₹ 27 per Kg and the other costing ₹ 32 per Kg, should be blended to produce a blended variety of tea worth ₹ 30 per Kg. How much should be the quantity of second variety of tea, if the first variety is 60 Kg.

Solution:




The required ratio of the two varieties of tea is 2:3, i.e.,

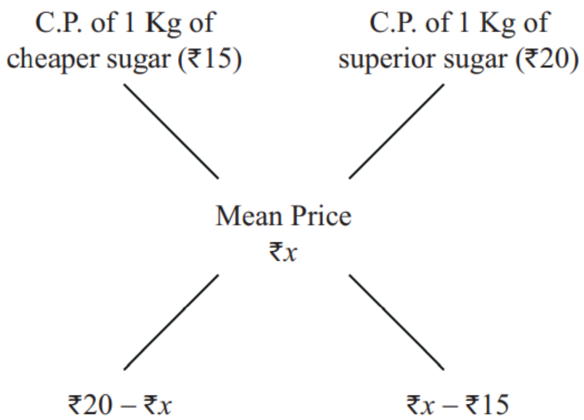
$\frac{Quantity\ of\ cheaper\ tea}{Quantity\ of\ superior\ tea} = \frac{2}{3}$

∴ Quantity of superior tea = $\frac{60\ X\ 3}{2}$ = 90 kg

Thus, the second variety of tea is 90 Kg.

Example:- 2: Sugar at ₹ 15 per Kg is mixed with sugar at ₹ 20 per Kg in the ratio 2:3. Find the per Kg price of the mixture.

Solution: Let, the mean price of the mixture be ₹ ₹ x




$\frac{Quantity\ of\ cheaper\ sugar}{Quantity\ of\ dearer\ sugar} = \frac{20 - x}{x - 15}$

⇒ 60 – 3x = 2x – 30

⇒ 5x = 90or, x = 18.

Thus, the per Kg price of the mixture is `18.

01) A vessel, full of wine, contains a litres of it of which b litres are withdrawn. The vessel is then filled with water. Next, b litres of the mixture are withdrawn and, again the vessel is filled with water. This process is repeated n times. Then

$$\frac{wine\ left\ in\ the\ vessel\ after\ nth\ operations}{original\ quantity\ of\ wine\ in\ the\ vessal}$$

$= \left(\frac{a-b}{a}\right)^n$

Explanation:-

Amount of wine after the first operation

= a-b = $\left(1-\frac{b}{a}\right)$ X a

Ratio of wine and water after the fi rst operation is (a – b): b.

∴ In b litres of mixture withdrawn in the second operation, amount of wine withdrawn

= $\frac{a-b}{(a-b)+b}$ X b = (a-b)$\frac{b}{a}$

∴ Amount of wine left after the second operation

= (a-b) –(a-b) x $\frac{b}{a}$ =$\left(1-\frac{b}{a}\right)^2$ a

In general , quantity of wine left after nth operation =$\left(1-\frac{b}{a}\right)^n$ a

∴ $\frac{wine\ left\ after\ nth\ operation}{orginal\ quantity\ of\ wine}$ = =$\left(1-\frac{b}{a}\right)^n$

Example 3: Two salt solutions have concentrations 15% and 40%. In what ratio should they be mixed to get a solution of concentration 30%?

Solution:

So, they should be mixed in the ratio 2: 3.

Example 4:- X and Y two alloys are made by mixing aluminum and magnesium metals in the ratio of 8: 5 and 9:16 respectively. If equal amounts of alloys are melted to form a new alloy Z, what will be the ratio of aluminum and magnesium in Z?

a) 317:333

b) 316:319

c) 314:333

d) 313:317

Answer: a

Solution:

The ratio of aluminum and magnesium in Z = [8/13+9/25] : [5/13+16/25]

= 317/325: 333/325

= 317:333

Example 5:- A cistern contains 100 liters of water. 10 liters of water is taken out of it and replaced by the same quantity of soda. This process is repeated one more time. After that 20 liters of the solution is replaced by the same quantity of whiskey. Find the proportion of soda, water, and whiskey in the final mixture?

a) 19: 71: 25

b) 38: 72: 50

c) 19: 81: 25

d) 29: 71: 25

Answer: c

Solution:

Given

A cistern contains 100 liters of water.

Calculation

Initially

Water = 100 lit

After 1st replacement

Water: Soda = 90: 10 = 9: 1

After 2nd replacement

Water = 90 – (10 × 9/10) = 81 lit

Soda = 10 – (10 × 1/10) + 10 = 19 lit

After 3rd replacement

Water = 81 – (20 × 81/100) = 64.8 lit

Soda = 19 – (20 × 19/100) = 15.2 lit

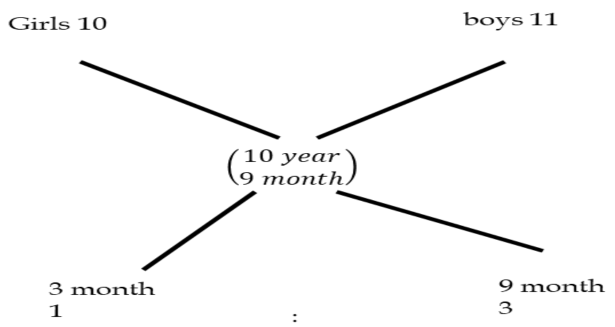
Whiskey = 20 lit

∴Required ratio = 15.2: 64.8: 20

⇒19: 81: 25

Example 6:- There are 400 students in a Mathematics class. Average age of girls is 10 years and average age of boys is 11 years. Find the number of girls in the class, if the average age of all the students is 10 years 9 months.

Solutions:-




So, number of girls =1/4×400=100

Example 7:- A vessel contains 125 litres of wine. 25 litres of wine was taken out of the vessel and replaced by water. Then, 25 litres of mixture was withdrawn and again replaced by water. The operation was repeated for the third time. How much wine is now left in the vessel? Solution: Amount of wine left in the vessel

$= \left(1 - \frac{25}{125}\right)^3$ X 125

$= \frac{100\ X\ 100\ X\ 100\ X\ 125}{125\ X\ 125\ X\ 125}$

= 64 Liters

There are n vessels of equal size filled with mixtures of liquids A and B in the ratio $a_1$:$b_1$:$a_2$, $b_2$...,$a_n$:$b_n$, respectively. If the contents of all the

vessels are poured into a single large vessel, then

$$\frac{Quantity\ of\ liquid\ A}{Quantity\ of\ liquid\ B} = \frac{\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\cdots\frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\cdots\frac{b_n}{a_n+b_n}\right)}$$

Explanation:- Let the capacity of each vessel be c litres. Amount of liquid A in different vessels

$= \frac{a_1 c}{a_1+b_1}, \frac{a_2 c}{a_2+b_2}, \frac{a_3 c}{a_3+b_3}, \dots \frac{a_n c}{a_n+b_n}$

Amount of liquid B in different vessels

$= \frac{b_1 c}{a_1+b_1}, \frac{b_2 c}{a_2+b_2}, \frac{b_3 c}{a_3+b_3}, \dots \frac{b_n c}{a_n+b_n}$

So, in the resulting mixture, amount of liquid A

$= \left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2}, + \cdots \frac{a_n}{a_n+b_n}\right)$ x c

Amount of liquid B

$= \left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2}, + \cdots \frac{b_n}{a_n+b_n}\right)$ x c

$\frac{Quantity\ of\ liquid\ A}{Quantity\ of\ liquid\ B} = \frac{\left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2} + \cdots \frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2} + \cdots \frac{b_n}{a_n+b_n}\right)}$

Example 8:- How many kilograms of sugar of Rs.5.4 per kg should be mixed with 10 kg of sugar of Rs.4.5 per kg, such that there may be gain of 20% by selling the mixture at Rs.5.94 per kg.

a) 10 kg

b) 12 kg

c) 15 kg

d) 8 kg

Answer: A

Solution:

Let, the amount of rice of Rs.5.4 per kg = x kg

According to the question,

x × 5.4 + 4.5 × 10 = 5.94 × (10 + x) ÷ 120 × 100

=5.4x + 45 = 4.95 × (10 + x)

=5.4x + 45 = 49.5 + 4.95x

=5.4x – 4.95x

= 49.5 – 45

=0.45x = 4.5

=x = 10 kg

Example 9**:-** Three equal glasses are filled with mixture of milk and water. The proportion of milk and water in each glass is as follows: In the first glass as 3:1, in the second glass as 5:3 and in the third as 9:7. The contents of the three glasses are emptied into a single vessel. What is the proportion of milk and water in it?

Solution:

$\frac{Quantity\ of\ milk}{Quantity\ of\ water} = \frac{\frac{3}{3+1} + \frac{5}{5+3} + \frac{9}{9+7}}{\frac{1}{3+1} + \frac{3}{5+3} + \frac{7}{9+7}}$

$= \frac{31/16}{17/16}$

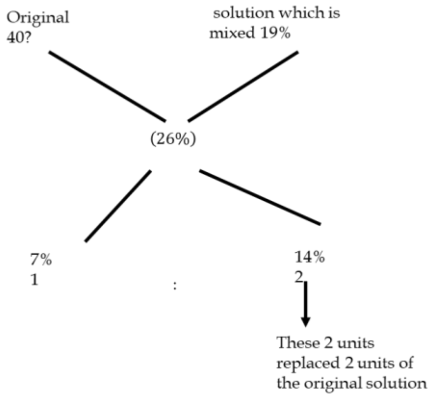
= 31:17

There are n vessels of sizes $c_1$, $c_2$, , $c_n$ filled with mixtures of liquids A and B in the ratio $a_1 : b_1$, $a_2 : b_2$, , $a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{Quantity\ of\ milk}{Quantity\ of\ water} = \frac{\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\cdots+\frac{a_n}{a_n+b_n}}{\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\cdots+\frac{b_n}{a_n+b_n}}$$

Example 10:- Some part of a sugar solution which contains 40% sugar is replaced with another sugar solution which contains 19% sugar. Part of sugar in the new mixture became 26% what fraction of the original sugar solution was replaced with another sugar solution?

Solutions:




Total number of units =1+ 2=3

2 units of original solution, out of 3, were replaced.

=2/3of the original solution was replaced.

Example 11:- Three glasses of sizes 3 litres, 4 litres and 5 litres, contain mixture of milk and water in the ratio 2:3, 3:7 and 4:11, respectively. The contents of all the three glasses are poured into a single vessel. Find out ratio of milk to water in the resulting mixture

Solution : $\frac{quantity\ of\ milk}{quantity\ of\ water}$

$$= \frac{\left(\frac{2X\,3}{2+3}+\frac{3X4}{3X7}+\frac{4X5}{4+11}\right)}{\left(\frac{3X3}{2+3}+\frac{7X4}{3+7}+\frac{11X5}{4+11}\right)}$$

$$= \frac{\frac{6}{5}+\frac{12}{10}+\frac{20}{15}}{\frac{9}{5}+\frac{28}{10}+\frac{55}{15}}$$

= 56:124

OR 14: 31

Example 12:- A jar had 330-liter mixture of Milk and water in the respective ratio of 2 : 1.60 liters of this mixture is taken out and 'x' liter each of milk and water is added to the jar (remaining mixture). The respective ratio between milk and water was 8 :5 respectively. What was the total quantity of both milk and water added to the jar?

a) 90 liter

b) 170 liter

c) 160 liter

d) 120 liter

Answer: d

Solution:

Given

Total mixture = 330

liter Ratio of milk and water = 2 : 1

Calculation

Mixture left in jar after 60 liters taken out = 330 – 60 = 270 liters

Milk = (2/3) × 270 = 180 liter

Water = (1/3) × 270 = 90 liter

Now according to question

(180 + x)/ (90 + x) = 8/5

⇒900 + 5x = 720 + 8x

⇒3x = 180

⇒x = 60

∴Quantity that was added = 2 × 60 = 120 liter.

Example 13:- A container contains two liquids A and B in the ratio of 3 :2. When 8 L of liquid B is added to X L of the mixture, the ratio of liquid A to liquid B becomes 1 :1. What is the value of X?

a) 50 L

b) 45 L

c) 40 L

d) None of these

Ans: c)

Sol:

Given, Ratio of A and B in the mixture 3 : 2

Volume of B added = 8 L

Final ratio of A and B in the mixture = 1 : 1

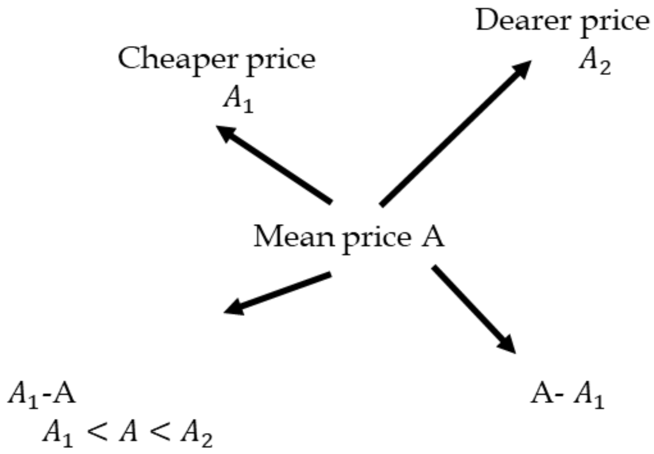
Formula used,

To find the ratio in which two Quantities are mixed to formed a new mixture is as shown below.

Where A1, A2 and A are the cheaper price, Dearer price and Mean price, respectively. Then ratio of cheaper Quantity to Dearer quantity is calculated as (n1: n2)

n1: n2 = (A2 – A) / (A - A1)

Also, the Mean value should lie between Cheaper price and Dearer price




applying Alligation concept

Proportion B in the mixture = 2/5 (cheaper)

Pure liquid B = 1 (dearer)

Proportion of B in final mixture = 1/2 (mean)

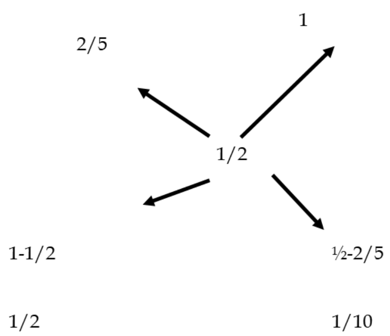
Using Alligation concept Ratio of cheaper to dearer = 1/2 : 1/10 = 5 : 1

1R refers to pure liquid of B = 8 L

∵ 1R = 8 L

∴ 5R = 40 L

Therefore, X = 40 L




Example 14:- In what proportion a milkman must mixed water with milk, so that after selling the mixture at half the cost of milk, gains 40%

a) 9 : 5

b) 5 : 9

c) 4 : 5

d) 3 : 5

Ans: a

Sol:

Given,

Cost of mixture = 1/2 × cost of milk

Percentage profit = 40%

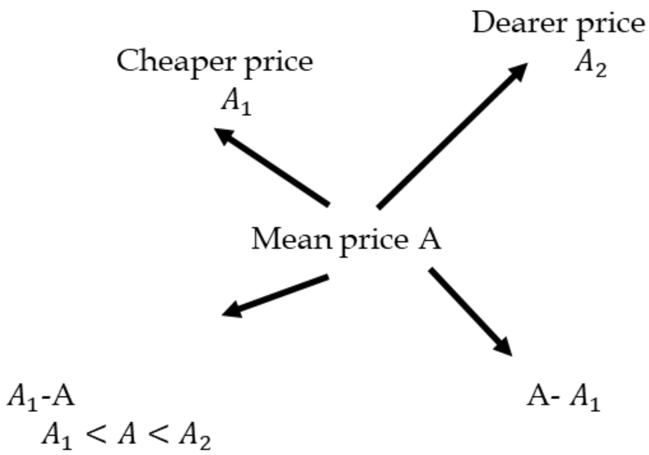
Formula used,

To find the ratio in which two Quantities are mixed to formed a new mixture is as shown below.

Where A1, A2 and A are the cheaper price, Dearer price and Mean price, respectively. Then ratio of cheaper Quantity to Dearer quantity is calculated as

(n1: n2)

n1: n2 = (A2 – A) / (A - A1) Also the Mean value should lie between Cheaper price and Dearer price




Let cost of milk = Rs. X per litre

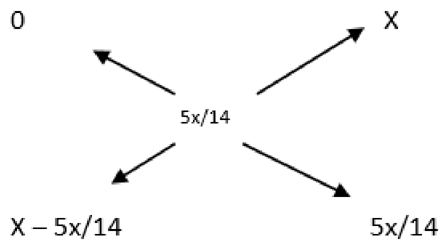
∴Selling price of mixture of milk and water = Rs. X/2 per litre(dearer)

Cost of water =Rs. 0 per litre(cheaper)

Percentage profit after selling the mixture = 40%

∴ Cost price of the mixture = X/2 × 100/140 = 5X/14 (mean)

Applying Alligation rule:-




Ratio of cheaper to dearer = 9X/14 : 5X/14 = 9 : 5

Therefore, ratio of water and milk = 9 : 5

Example 15:- Ratio of Milk and water in Vessel A is 7:4 and same mixture in the ratio of 5:3, 22 liters mixture from vessel A taken out and poured in vessel B new ratio of milk to water is 27:16. If new quantity of mixture in vessel B is equal to initial quantity of mixture in vessel A, then find quantity of Milk after 8 liters of mixture has been taken out from Vessel A?

A) 200/29 liters

B) 546 / 11 liters

C) 120/13 liters

D) 220/19 liters

Answer: b

Let Ratio of Milk and water in Vessel A is 7x and 4y

Let ratio of milk and water in vessel B is 5y and 3y

Now according to question.

(5y + 22 * 7 / 11) / (3y + 22 * 4 / 11) = 27 / 16

(5y + 14)/ (3y + 8) = 27 / 16

80y + 224 = 81y + 216

Y = 8

New quantity of mixture in vessel B

= > (8 * 5 + 14) + (8 * 3 + 8)

= > 54 + 32

= > 86

Therefore, initial quantity of vessel A = 86 liters

Quantity of milk remaining in Vessel A

86 * 7 / 11 – 8 * 7 / 11

= > 546 / 11 litres.

Example 16:- A mixture contains 40% milk and another mixture contains milk and water in the ratio of 1: 5. How many litres of the latters must be mixed with 6 litres of the former so that the resulting mixture may contains milk and water in the ratio of 1: 2.

a) 10 L

b) 15 L

c) 18 L

d) 20 L

Ans: b

Sol: Given,

Ratio of milk and water in 1st mixture = 40 ∶ 60 or 2 ∶ 3

Ratio of milk and water in 2nd mixture = 1 ∶ 5

Quantity of 1st mixture = 6 L

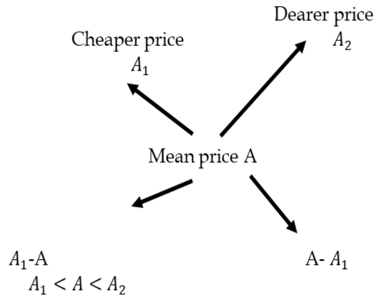
Ratio of milk and water in final mixture = 1 ∶ 2

Formula used,

To find the ratio in which two Quantities are mixed to formed a new mixture is as shown below.

Where A1, A2 and A are the cheaper price, Dearer price and Mean price, respectively.

Then ratio of cheaper Quantity to Dearer quantity is calculated as (n1∶ n2) n1∶ n2 = (A2 – A) / (A - A1) Also the Mean value should lie between Cheaper price and Dearer price

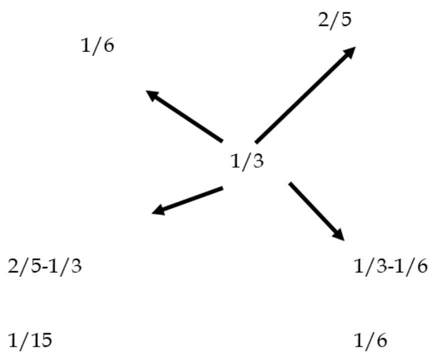



Proportion of milk in 1st mixture = 2/5 (Dearer value)

Proportion of milk in 2nd mixture = 1/6 (Cheaper value)

Proportion of milk in final mixture = 1/3 (Mean value)

Applying Alligation rule,




Cheaper : Dearer = 1/15 : 1/6 = 2 : 5

2R refers to former quantity of mixture

3R refers to latter quantity of mixture

∵ 2R = 6 L ∴ 5R = 15 L

Therefore, Quantity of latter mixture = 15 L .

Example 17:- Three containers of equal capacity are containing a mixture of wine and water in the ratio of 2 :1, 5:4 and 3 : 4 respectively. These three containers are emptied into a 4th container. What is the approximate percentage of wine in 4th container?

a) 50%

b) 60%

c) 45%

d) 55%

Ans: d

Sol:

Given,

Ratio of wine and water in 1st container = 2 : 1 Ratio of wine and water in 2nd container = 5 : 4 Ratio of wine and water in 33rd container = 3 : 4

1/6 2/5

1/3

2/5 – 1/3 1/3 – 1/6

1/15 1/6

Proportion of wine in 1st container = 2/3 and proportion of water = 1/3

Proportion of wine in 2nd container = 5/9 and proportion of water = 4/9

Proportion of wine in 3rd container = 3/7 and proportion of water = 4/7

Required ratio of wine and water in 4th container is

(2/3 + 5/9 + 3/7) :(1/3 + 4/9 + 4/7)

104 : 85

Percentage of wine = 104/189 × 100 = 55%

## Summary

The key concepts learned from this unit are: -

- We have learnt what is an Alligation.
- We will understand what are Rule of Alligation.

## Keywords

- Alligation.
- Mean price.
- Mean
- Average
- Ratio

## Self Assessment

1. Two varieties of sugar are mixed in the proportion of 4:7 and the mixture is sold at Rs. 39 per kg at a profit of 30%. If the second variety of sugar cost Rs. 5.5 more than that of first variety of sugar. Find the cost price of the first variety of sugar.

A. 31 Rs/kg
B. 35.5 Rs/kg
C. 37.5 Rs/kg
D. None of these

2. How many kg of salt at 42 paise per kg must a man mix with 25 kg of salt at 24 paise per kg, so that he may, on selling the mixture at 40 paise per kg. gain 25% on the outlay?

A. 20
B. 30
C. 40
D. 50

3. The ratio of the quantities of acid and water in a mixture is 1:3. If 5 litres of acid is further added to the mixture, the new ratio becomes 1:2. The quantity of new mixture in litres is

A. 32
B. 40
C. 42
D. 45

4. A barrel contains a mixture of wine and water in the ratio 3: 1. How much fraction of the mixture must be drawn off and substituted by water so that the ratio of wine and water in the resultant mixture in the barrel becomes 1: 1?

A. 1/4
B. 1/3

C. 2/3
D. 3/2

5. A mixture contains spirit and water in the ratio 3 : 2. If it contains 3 litres more spirit than water, the quantity of spirit in the mixture is
   A. 10 litres
   B. 12 litres
   C. 8 litres
   D. 9 litres

6. There are 81 litres pure milk in a container. One-third of milk is replaced by water in the container. Again one-third of mixture is extracted and equal amount of water is added. What is the ratio of milk to water in the new mixture?
   A. 1: 2
   B. 1: 1
   C. 2: 1
   D. 4: 5

7. Three vessles whose capacities are 3:2:1 are completely filled with milk mixed with water. The ratio of milk and water in the mixture of vessels are 5:2, 4:1 and 4:1 respectively. Taking 1/3 of first, 1/2 of second and 1/7 of third mixtures, a new mixture, kept in a new vessel is prepared. The percentage of water in the new mixture is
   A. 32
   B. 28
   C. 30
   D. 24

8. Three containers whose volumes are in the ratio of 2:3:4 are full of mixture of spirit and water. In the 1st container the ratio of spirit and water is 4:1 in 2nd container the ratio is 11:4 and in the 3rd container ratio is 7:3. All the three mixtures are mixed in a big container. The ratio of spirit and water in the resultant mixture is:
   A. 4:9
   B. 9:5
   C. 11:4
   D. 5:10

9. Bottles 1 contains a mixture of milk and water in 7:2 ratio and Bottle 2 contains a mixture of milk and water in 9:4 ratio. In what ratio of volumes should the liquids in Bottle 1 and Bottle 2 be combined to obtain a mixture of milk and water in 3:1 ratio?
   A. 27:14
   B. 27:13
   C. 27:16
   D. 27:18

10. A milk vendor sells 10 litres of milk from a can containing 40 litres of pure milk to the 1st customer. He then adds 10 litres of water to the milk can. He again sells 10 litres of mixture to the 2nd customer and then adds 10 litres of water to the can. Again, he sells 10 litres of mixture to the 3rd customer and then adds 10 litres of water to the can and so on. What amount of pure milk will the 5th customer receive?
   A. (510/128) Litres
   B. (505/128) Litres
   C. (410/128) Litres
   D. (405/128) Litres

11. Two liquids A and B are mixed in the ratio of 1 : 4 and the mixture is sold at Rs.46 per litre at a profit of 15%. If the liquid A costs Rs. 10 less than the cost of B. cost of B is (In Rs.)
   A. 40
   B. 42
   C. 50
   D. 32

12. A can contains a mixture of two liquids A and B in the ratio of 7: 9. When X litres of mixture is removed and is replaced by B. The ratio of A and B becomes 21: 43. Find X is how much percent of the initial mixture.

A. 25%
B. 20%
C. 35%
D. 30%

13. There is 128 L of pure acid in a container. 1/4th of acid is removed and replaced by water. This process is repeated further 2 more times. Find the difference of acid and water in the final mixture.

A. 40 L
B. 20 L
C. 30 L
D. Can't be determined

14. Two containers A and B contains milk and water in the ratio of 2 : X and 4 : 1. These two containers are mixed in the ratio of 3 : 1, making mixture half milk and half water. Find X

A. 5
B. 7
C. 3
D. None of these

15. A milkman bought 20 L of milk and mixed 4 L water in it. If the cost of mixture becomes Rs. 30 per litre. What is the cost of milk per litre?

A. Rs. 40
B. Rs. 38
C. Rs. 36
D. Rs. 35

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | D | 2. | A | 3. | D | 4. | B | 5. | B |
| 6. | D | 7. | D | 8. | C | 9. | B | 10. | D |
| 11. | B | 12. | A | 13. | B | 14. | C | 15. | C |

## Review Questions

1. Two containers A and B contains milk and water in the ratio of 2 : X and 4 : 1. These two containers are mixed in the ratio of 3 : 1, making mixture half milk and half water. Find X.
2. There is 128 L of pure acid in a container. 1/4th of acid is removed and replaced by water. This process is repeated further 2 more times. Find the difference of acid and water in the final mixture.
3. A can contains a mixture of two liquids A and B in the ratio of 7: 9. When X litres of mixture is removed and is replaced by B. The ratio of A and B becomes 21: 43. Find X is how much percent of the initial mixture.
4. A Jar 'A', 140 litre milk was mixed with 40 litre water. Some of this mixture was taken out from Jar 'A' and put in Jar 'B'. If before the operation, there was 17 litres of milk in Jar 'B', and afterwards the resultant ratio between milk and water in Jar 'B' was 19:3 respectively, what was the amount of mixture that was taken out from Jar 'A'? (in litre)
5. In Jar A, 180 litre milk was mix with 36 litre water. Some of this mixture was taken out from Jar A and put it in Jar B. If after adding 6 litres of water in the mixture, the respective

ratio between milk and water in Jar B was 5:2 respectively, what was the amount of mixture that was taken out from Jar A? (in litres)

6. A vessel contains a mixture of grape, pineapple and banana juices in the respective ratio of 4: 6: 5. 15 litres of this mixture is taken out and 8 litres of grape juice and 2 litres of pineapple juice is added to the vessel. If the resultant quantity of grape juice is 10 litres less than the resultant quantity of pineapple juice, what was the initial quantity of mixture in the vessel?
7. Suresh is having three brands of whisky with him White Dog, Renders Pride, MAT49 having ratio of alcohol to water 2:3, 5:7, 6:11 respectively. If he mixes equal quantity of each brand. Find the ratio of water to alcohol in the resultant mixture?
8. Container P contains V litres of pure water. Container Q contains V litres of pure milk. X litres are taken out from container P and poured into container Q. Now, X litres are taken from container Q and poured into container P. Which of the following statement is true?

I. % of milk in container P = % of water in Container Q

II. % of water in container P = % of milk in Container Q

III. % of milk in container Q < % of water in container P

IV. % of milk in container P > % of water in container Q

9. In what ratio must a person mix three kinds of wheat costing him 1.20, 1.44 and 1.74 per kg, so that mixture may worth 1.41 per kg?
10. Two varieties of sugar are mixed in the proportion of 4:7 and the mixture is sold at Rs. 39 per kg at a profit of 30%. If the second variety of sugar cost Rs. 5.5 more than that of first variety of sugar. Find the cost price of the first variety of sugar.

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 11: Problem on Ages and Numbers

**CONTENTS**



## Objectives

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand how to calculate age some years ago
- We will understand how to calculate present age
- We will understand how to calculate age some years hence

## Introduction

Problems based on ages are generally asked in most of the competitive examinations. To solve these problems, the knowledge of linear equations is essential. In such problems, there may be three situations:

(i) Age some years ago

**(ii)** Present age

**(iii)** Age some years hence

Two of these situations are given and it is required to find the third. The relation between the age of two persons may also be given. Simple linear equations are framed and their solutions are obtained. Sometimes, short cut methods given below are also helpful in solving such problems.

## 11.1 Shortcut Method

1. If the age of A, t years ago, was n times the age of B and at present A's age is $n_2$ times that of B, then

A's present age $= \left(\frac{n_1-1}{n_1-n_2}\right) n_2 t$ Years

$n_2t$ years and, B's present age $= \left(\frac{n_1-1}{n_1-n_2}\right) t$ years

**Explanation:-**

Let the present age of B be x years. Then, the present age of A = $n_2x$ years

Given, t years ago,

$n_1(x - t) = n_2x - t$ or, $(n_1 - n_2)x = (n1 - 1)t$

or X = $\left(\frac{n_1-1}{n_1-n_2}\right)t$

Therefore, B's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$t years

And A's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$ $n_2$t years.

**Example 1:** The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, then what is father's present age?

**Solution:** The father's present age

= $\left(\frac{n_1-1}{n_1-n_2}\right)$ $n_2t$ [Here, $n_1$ = 7, $n_2$ = 4 and t = 5]

= $\left(\frac{7-1}{7-4}\right)$ 4 x 5 = $\frac{6 \, x \, 4 \, x \, 5}{3}$

= 40 years

2) The present age of A is $n_1$ times the present age of B. If t years hence, the age of A would be $n_2$ times that of B, then

And A's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$ $n_2$t years.

And And B's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$ t years.

2) The present age of A is $n_1$ times the present age of B. If t years hence, the age of A would be $n_2$ times that of B, then

And A's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$ $n_2$t years.

And And B's present age = $\left(\frac{n_1-1}{n_1-n_2}\right)$ t years.

**Explanation**

Let the present age of B be x years.

Then, the present age of A = $n_1$x

Given, t years hence,

($n_1$x + t) = $n_2$ (x + t)

or, ($n_1$ - $n_2$)x = ($n_2$ - 1)t

Or $\left(\frac{n_1-1}{n_1-n_2}\right)$ t years.

Therefore B's present age = $\left(\frac{n_2-1}{n_1-n_2}\right)$ $n_1$t years.

And A's present age = $\left(\frac{n_2-1}{n_1-n_2}\right)$ $n_1$t years.

**Example 2:** The age of Mr Gupta is four times the age of his son. After ten years, the age of Mr Gupta will be only twice the age of his son. Find the present age of Mr Gupta's son.

**Solution:** The present age of Mr Gupta's son

= $\left(\frac{n_2-1}{n_1-n_2}\right)$ t years.

= $\left(\frac{2-1}{4-2}\right)$ 10

[Here, $n_1$ = 4, $n_2$ = 2 and t = 10]

= 5 years.

3) The age of A, $t_1$ years ago, was $n_1$ times the age of B. If t 2 years hence A's age would be n2 times that of B, then

A' s present age = $\frac{n_1(t_1+t_2)(n_2-1)}{n_1-n_2} + t_1$ years

B' s present age = $\frac{t_2(n_2+1)+t_1(n_1-1)}{n_1-n_2}$ years

**Explanation:-**Let A's present age = x years and B's present age = y years.

Given: $x - t_1 = n_1 (y - t_1)$ and $x + t_2 = n_2 (y + t_2)$ i.e., $x - n_1 y = (1 - n_1) t_1$ (1)

and, $x - n_2 y = (-1 + n_2) t_2$ (2)

Solving (1) and (2), we get

$X= \frac{n_1(t_1+t_2)(n_2-1)}{n_1-n_2} + t_1$

And ,$y = \frac{t_2(n_2+1)+t_1(n_1-1)}{n_1-n_2}$

**Example 3:** 10 years ago Anu's mother was 4 times older than her daughter. After 10 years, the mother will be twice older than her daughter. Find the present age of Anu is:

**Solution:** Present age of Anu

$= \frac{t_1(n_2-1)+t_1(n_1-1)}{n_1-n_2}$ years

[Here, $n_1 = 4$, $n_2 = 2$, $t_1 = 10$ and $t_2 = 10$]

$= \frac{10(2-1)+10(4-1)}{4-2}$

$= \frac{10+30}{2}$

= 20 years

4) The sum of present ages of A and B is S years. If, t years ago, the age of A was n times the age of B, then

Present age of A $=\frac{Sn-t(n-1)}{n-1}$ years

And present age of B = $\frac{S-t(n-1)}{n-1}$ years

**Explanation**

Let the present ages of A and B be x and y years

respectively.

Given: x + y = S (1)

and, x - t = n (y - t)

or, x - ny = (1 - n)t (2)

Solving (1) and (2), we get

$X = \frac{Sn-t(n-1)}{n-1}$

$Y = \frac{S-t(n-1)}{n-1}$ years

**Example 4:** The sum of the ages of A and B is 42 years. 3 years back, the age of A was 5 times the age of B. Find the difference between the present ages of A and B.

**Solution:** Here, S = 42, n = 5 and t = 3

∴ Present age of A

$= \frac{Sn-t(n-1)}{n-1}$

$= \frac{42 \times 5-3(5-1)}{5+1}$

$= \frac{198}{6}$

= 33 years

And, present age of B

$= \frac{42+t(n+1)}{n+1}$

$= \frac{42+3(5-1)}{5+1}$

$= \frac{54}{6}$

= 9 years

∴ Difference between the present ages of A and B = 33 - 9 = 24 years.

6) The sum of present ages of A and B is S years. If, t years hence, the age of A would be n times the age of B, then

Present age of A $= \frac{Sn-t(n-1)}{n-1}$ years

And present age of B = $\frac{S-t(n-1)}{n-1}$ years

**Explanation:-**

Let the present ages of A and B be x and y years,

respectively

Given: x + y = S (1)

and, x + t = n ( y + t)

or, x - ny = t (n - 1) (2)

Solving (1) and (2), we get

X = $\frac{Sn-t(n-1)}{n-1}$ years

Y = $\frac{S-t(n-1)}{n-1}$

**Example 5:** The sum of the ages of a son and father is 56 years. After four years, the age of the father will be three times that of his son. Find their respective ages.

**Solution:** The age of the father

$= \frac{S_n+t(n-1)}{n+1}$

$= \frac{56x\ 3+4(3-1)}{3+1}$

[Here, S = 56, t = 4 and n = 3]

$= \frac{176}{4}$

= 44 years

6) If the ratio of the present ages of A and B is a:b and t years hence, it will be c:d, then

A' s present age = $\frac{at(c-d)}{ad-bc}$

And, B's present age = $\frac{bt(c-d)}{ad-bc}$

**Example 6:** The ratio of the age of father and son at present is 6:1. After 5 years, the ratio will become 7:2. Find the present age of the son.

**Solution:** The present age of the son = $\frac{bt(c-d)}{ad-bc}$

[Here, a = 6, b = 1, c = 7, d = 2 and t = 5]

$= \frac{1\ x\ 5(7-2)}{6x2-1x7}$

= 5 years

**Example 7**: 6 years ago Mahesh was twice as old as Suresh. If the ratio of their present ages is 9:5 then, what is the difference between their present ages?

**Solution:** Present age of Mahesh

$=\frac{-at(c-d)}{ad-bc}$

$= \frac{-9x6(2-1)}{1x9-5x2}$

[Here, a = 9, b = 5, c = 2, d = 1 and t = 6 ]

= 54 years

Present age of Suresh

$= \frac{bt(c-d)}{ad-bc}$

$= \frac{-5x6(2-1)}{1x\,9-5x2}$

= 30 years

Difference of their ages = 54 – 30 = 24 years

Example1. 10, years ago, Mohan was thrice as old as Ram was but 10 years hence, he will be only twice as old as Ram. Find Mohan's present age.

(a) 60 years

(b) 80 years

**(c) 70 years**

(d) 76 years

Ans: (c) Let, Mohan's present age be x years and Ram's present age be y years.

Then, according to the fi rst condition,

x − 10 = 3(y − 10)

or, x − 3y = −20 …(1)

Now, Mohan's age after 10 years

= (x + 10) years Ram's age after 10 years

= (y + 10)

∴ (x + 10) = 2 (y + 10)

or, x − 2y = 10 …      (2)

Solving (1) and (2), we get

x = 70 and, y = 30

∴ Mohan's age = 70 years and Ram's age = 30 years.

Example 2. The ages of Ram and Shyam differ by 16 years. 6 years ago, Mohan's age was thrice as that of Ram's, fi nd their present ages.

**(a) 14 years, 30 years**

(b) 12 years, 28 years

(c) 16 years, 34 years

(d) 18 years, 38 years

Ans: (a) Let, Ram's age = x years So, Mohan's age = (x + 16) years Also, 3(x − 6) = x + 16 − 6 or, x = 14

∴ Ram's age = 14 years and, Mohan's age = 14 + 16 = 30 years.

Example 3. 15 years hence, Rohit will be just four times as old as he was 15 years ago. How old is Rohit at present?

(a) 20

**(b) 25**

(c) 30

(d) 35

Ans: (b) Let, the present age of Rohit be x years Then, given: x + 15 = 4(x − 15) ⇒ x = 25.

Example 4. A man's age is 125% of what it was 10 years ago, but 83 $\frac{1}{3}$% of what it will be after ten 10 years. What is his present age?

(a) 45 years

**(b) 50 years**

(c) 55 years

(d) 60 years

Ans: (b) Let, the present age be x years.

Then, 125% of (x − 10) = x

and, 83 $\frac{1}{3}$ % of (x + 10) = x

∴ 125 % of (x − 10) = 83 $\frac{1}{3}$ % of (x + 10)

or, $\frac{5}{4}$ (x − 10) = $\frac{5}{6}$ (x + 10)

or, $\frac{5}{4}x - \frac{5}{6}x = \frac{50}{6} + \frac{50}{4}$

or, $\frac{5x}{12} = \frac{250}{12}$ $or, x = 50 years.$

Example 5. If twice the son's age be added to the father's age, the sum is 70 years and if twice the father's age is added to the son's age, the sum is 95 years. Then father's age is:

**(a) 40 years**

(b) 35 years

(c) 42 years

(d) 45 years

Ans: (a) Let, son's age ( in years ) = x and father's age (in years) = y Given: 2x + y = 70 and, x + 2y = 95 Solving for y, we get y = 40.

Example 6. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today? What is the age of the child?

(a) 3 years

(b) 5 years

**(c) 2 years**

(d) 1 year

Ans: (c) Present age of 5 members = 5 × 17 + 3 × 5 = 100 years Also, present ages of 5 members + Age of the baby = 6 × 17 = 102 years ∴ Age of the baby = 102 − 100 = 2 years.

Example 7. The ratio of A's and B's ages is 4:5. If the difference between the present age of A and B 5 years hence is 3, then what is the sum of present ages of A and B?

(a) 68 years

**(b) 72 years**

(c) 76 years

(d) 64 years

Ans: (b) Given: $\frac{A}{B}$= 4 5or, B = $\frac{5}{4}$ A

and, B − (A + 5) = 3 or, B = A + 8

∴ $\frac{5}{4}$ A = A + 8

or, $A\left(\frac{5}{4} - 1\right) = 8$

∴ A = 32 years and, B = $\frac{5}{4}$ × 32 = 40 years

∴ A + B = 40 + 32 = 72 years.

Example 8. The ages of A and B are in the ratio 6:5 and sum of their ages is 44 years. The ratio of their ages after 8 years will be:

(a) 4:5

(b) 3:4

(c) 3:7

**(d) 8:7**

Ans: (d) Let, present ages (in years) of A and B respectively, are 6x and 5x. Given: 6x + 5x = 44 ⇒ x = 4

Ratio of ages after 8 years will be 6x + 8:5x + 8 or, 32:28 or, 8:7.

Example 9. One year ago the ratio between Samir and Ashok's age was 4:3. One year hence the ratio of their ages will be 5:4. What is the sum of their present ages in years?

(a) 12 years

(b) 15 years

**(c) 16 years**

(d) Cannot be determined

Ans: (c) Let, one year ago

Samir's age be 4x years

and, Ashok's age be 3x years

Present age of Samir = (4x + 1) years

Present age of Ashok = (3x + 1) years

One year hence

Samir's age = (4x + 2) years

Ashok's age = (3x + 2) years

According to question, $\frac{4x+2}{3x+2}=\frac{5}{4}$ ⇒16x + 8 = 15x + 10

or, x = 2.

∴ Sum of their present ages = 4x + 1 + 3x + 1

= 7x + 2

= 7 × 2 + 2 = 16 years.

Example 10. Ratio of Ashok's age to Pradeep's age is 4:3. Ashok will be 26 years old after 6 years. How old is Pradeep now?

(a) 18 years

(b) 21 years

**(c) 15 years**

(d) 24 years

Ans: (c) Let, the present ages of Ashok and Pradeep be 4x and 3x

So that $4x + 6 = 26 \Rightarrow x = 5$

$\therefore$ Present age of Pradeep is $3x = 3 \times 5$, i.e., 15 years

Example 11. Jayesh is as much younger to Anil as he is older to Prashant. If the sum of the ages of Anil and Prashant is 48 years, what is the age of Jayesh?

(a) 20 years

**(b) 24 years**

(c) 30 years

(d) Cannot be determined

Ans: (b) A – J = J – P

2J = A + P

2J = 48

= J 24 years

Example 12. 5 years ago, Mr Sohanlal was thrice as old as his son and 10 years hence he will be twice as old as his son. Mr Sohanlal's present age (in years) is:

(a) 35

(b) 45

**(c) 50**

(d) 55

Ans: (c) Let, Mr Sohanlal's age (in years) = x and his son's age = y Then, x - 5 = 3 (y - 5) i.e., x - 3y + 10 = 0 and, x + 10 = 2 ( y + 10) i.e., x - 2y - 10 = 0 Solving the two equations, we get x = 50, y = 20.

Example 13. Three times the present age of a father is equal to eight times the present age of his son. 8 years hence the father will be twice as old as his son at that time. What are their present ages? (a) 35, 15

**(b) 32, 12**

(c) 40, 15

(d) 27, 8

Ans: (b) 3F = 8S ⇒ F 8/3S

F + 8 = 2 (S + 8)

F + 8 = 2S + 16

$\frac{8}{3}$S +8= 2S +16

8S + 24 = 6S + 48

2S = 24

S= 12

F =8/3 ×S

=8/3 × 12⇒ 32

Example 14. The sum of the ages of a father and son is 45 years. 5 years ago, the product of their ages was four times the father's age at that time. The present age of the father is:

(a) 39 years

**(b) 36 years**

(c) 25 years

(d) None of these

Ans: (b) Let, father's present age = x years Then, son's present age = (45 - x) years Given: (x - 5)(45 - x - 5) = 4(x - 5) or, $x^2$ - 41x + 180 = 0 or, (x - 36)(x - 5) = 0 ∴ x = 36 years.

Example 15. One year ago, a father was four times as old as his son. In 6 years time his age exceeds twice his son's age by 9 years. Ratio of their ages is:

(a) 13:4

(b) 12:5

**(c) 11:3**

(d) 9:2

Ans: (c) Let, the present ages of father and son be x and y years, respectively Then, (x - 1) = 4(y - 1) or, 4y - x = 3 …(1)

and, (x + 6) - 2(y + 6) = 9 or, -2y + x = 15 …(2)

Solving (1) and (2), we get, x = 33, y = 9

∴ Ratio of their ages = 33:9 = 11:3.

Example 16. The ages of A, B and C together is 185 years. B is twice as old as A and C is 17 years older than A. Then, the respective ages of A, B and C are:

(a) 40, 86 and 59 years

**(b) 42, 84 and 59 years**

(c) 40, 80 and 65 years

(d) None of these

Ans: (b) Let, A's age be x years B's age be 2x years C's age = (x + 17) years According to the question, x + 2x + (x + 17) = 185 ∴ 4x = 185 - 17 = 168

∴ x = 42

∴ A's age = 42 years B's age = 84 years C's age = 42 + 17 = 59 years.

Example 17. A father's age is three times the sum of the ages of his two children, but 20 years hence his age will

be equal to the sum of their ages. Then, the father's age is:

**(a) 30 years**

(b) 40 years

(c) 35 years

(d) 45 years

Ans: (a) Let, the present age of father be x years and the present age of son be y years. ∴ x = 3y … (1)

Also, (x + 20) = (y + 20 + 20) … (2)

Solving (1) and (2), we get x = 30 years.

EXERCISE 2

Example 1. Eighteen years ago, the ratio of A's age to B's age was 8:13. Their present ratios are 5:7. What is the present age of A?

(a) 70 years

**(b) 50 years**

(c) 40 years

(d) 60 years

Ans: (b) Let A → age of A (Present)

B → age of B (Present)

$$\frac{A-18}{B-18} = {}^{8}\!/_{13}$$

$$\frac{\frac{5}{7}\,B-18}{B-18} = {}^{8}\!/_{13}$$

$$\frac{5B-126}{7B-126} = {}^{8}\!/_{13}$$

65B – 1638 = 56B – 1008

9B = 630

B=70

A=$\frac{5}{7} \times B$

=$\frac{5}{7} \times 70 = 50 year.$

Example 2. The average age of 30 students of a class is 14 years 4 months. After admission of 5 new students in the class the average becomes 13 years 9 months. The youngest one of the fi ve new students is 9 years 11 months old. The average age of the remaining 4 new students is

(a) 12 years 4 months

(b) 11 years 2 months

**(c) 10 years 4 months**

(d) 13 years 6 months

Ans: (c) Let a be the age of youngest student.

According to the question,

$$30\left(14\frac{1}{3}\right) + 9\frac{11}{2} + 4a = 35 \times 13 + \frac{35 \times 3}{4}$$

∴a=10 $^{1}/_{3}$

∴= 10 years 4 months.

Example 3. The sum of the ages of two brothers, having a difference of 8 years between them, will double after 10 years. What is the ratio of the age of the younger brother to that of the elder brother?

(a) 8: 9

(b) 10: 13

(c) 7: 11

**(d) 3: 7**

Ans: (d) 1st brother → A (present age)

2nd brother → B (present age)

Given ⇒A-B=8year (1)

Sum of their age will double after 10 years

∴ A+10+B+10=2(A+B)

A+B=20 (2) From (1) and (2) ⇒ $A = 14$ $B = 6$ ∴ $\frac{B}{A} = \frac{6}{14} = {}^{3}/_{7}$

Example 4. After replacing an old member by a new member, it was found that the average age of fi ve members of a club is same as it was 3 years ago. The difference between the ages of the replaced and the new members is:

(a) 2 years

(b) 4 years

(c) 8 years

**(d) 15 years**

Ans: (d) Increase in ages of five members in 3 years = (3 × 5) years = 15 years

Since the average age remains same, therefore, required difference = 15 years

Example 5. The ratio of the present ages of A and B is 7:9. 6 years ago the ratio of $\frac{1}{3}$ of A's age at that time and $\frac{1}{3}$of B's ageat that time was 1:2. What will be the ratio of A's to B's age 6 years from now?

(a) 4:5

(b) 14:15

**(c) 6:7**

(d) 18:25

(e) 22:25

Ans: (c) Let, the present age of A be 7x years and that of B be 9x years.

Now, 6 years ago,$\frac{3(7x-6)}{3(9x-6)} = \frac{1}{2}$

or, 42x - 36= 27x - 18 or, 15x = 18

∴x = $\frac{6}{5}$ years

Ratio after 6 years

$$\frac{\frac{7 \times 6}{5} + 6}{9 \times \frac{6}{5} + 6} = \frac{42 + 30}{54 + 30} = \frac{72}{84} = 6:7$$

∴ Required ratio = 6:7

Example 6. The present age of Romila is 1 4 that of her father. After 6 years her father's age will be twice the age of Kapil. If Kapil celebrated fi fth birthday 8 years ago, what is Romila's present age?

(a) 7 years

(b) 7.5 years

**(c) 8 years**

(d) 8.5 years

(e) None of these

Ans: (c) Kapil's present age = (8 + 5) = 13 years

Kapil's age after 6 years =13 + 6 = 19 years

Now, Romila's father's age = 2 × Kapil's age = 2 × 19 = 36 years

Father's present age = 38 − 6 = 32 years

Romila's present age =$\frac{1}{4}$ × father's present age =× = $\frac{1}{4}$ ×32 8years

Example 7. The average age of women and child workers in factory was 15 years. The average age of all the 16 children was 8 years and the average age of women workers was 22 years. If 10 women workers were married, then the number of unmarried women workers is:

(a) 16

(b) 12

(c) 8

**(d) 6**

Ans: (d) Let, unmarried women workers are x, then as per question,$\frac{16\times8+22\times(10+x)}{16+10+x} = 15$

⇒ 128 + 220 + 22x = 390 + 15x ⇒ 7x = 42

∴ x = 6

Example 8. The age of a father is three times of that of his son. After 5 years, the double of father's age will be fi ve times the age of son. The present age of father and son is:

(a) 30 years, 10 years

(b) 36 years, 12 years

(c) 42 years, 14 years

**(d) 45 years, 15 years**

Ans: (d) Let, present age of son is x years and then present age of father is 3x years then, $5(x + 5) = 2(3x + 5) \Rightarrow 5x + 25 = 6x + 10$

$\therefore x = 15$ years Present age of father = 45 years.

Example 9. The sum of the ages of 4 members of a family, 5 years ago, was 94 years. Today, when the daughter has been married off and replaced by a daughter-in-law, the sum of their ages is 92. Assuming that there has been no other change in the family structure and all the people are alive, what is the difference between the age of the daughter and that of the daughter-in-law?

**(a) 22 years**

(b) 11 years

(c) 25 years

(d) 19 years

(e) 15 years

Ans: (a) There are four members in a family. Five years ago the sum of ages of the family members = 94 years Now, sum of present ages of family members = $94 + 5 \times 4 = 114$ years $\therefore$ Daughter is replaced by daughter-in-law. Thus, sum of family member's ages becomes 92 years. $\therefore$ Difference = $114 - 92 = 22$ years

Example 10. The ratio between the present age of Manisha and Deepali is 5:x. Manisha is 9 years younger than Parineeta. Parineeta's age after 9 years will be 33 years. The difference between Deepali's and Manisha's age is same as the present age of Parineeta. What will come in place of x?

(a) 23

(b) 39

(c) 15

**(d) None of these**

Ans: (d) Given Parineeta's age after 9 years = 33 years

$\therefore$ Parineeta's present age = $33 - 9 = 24$ years

$\therefore$ Manisha's present age = $24 - 9$

= 15 years

$\therefore$ Deepali's present age = $15 + 24$

= 39 years Hence, ratio between Manisha and Deepali = 15:39 = 5:13

$\therefore x = 13$

Example 11. The ratio between the present ages of Ram and Rakesh is 6:11. 4 years ago, the ratio of their ages was 1:2. What will be Rakesh's age after five

years?

(a) 45 years

(b) 29 years

(c) 49 years

(d) Cannot be determined

Ans: (c) Let, the age of Ram = x and, Rakesh = y, then, $\frac{x}{y}$

$= \frac{6}{11}$

$\therefore$ x $= \frac{6y}{11}$

According to the question,

$$\frac{x-4}{y-4} = \frac{1}{2}$$

2x - 8 = y - 4

$$2 \times \frac{6y}{11} - 8 = y - 4$$

$$\frac{12y}{11} - y = -4 + 8$$

$$\frac{y}{11} = 4$$

Y=44 years

$\therefore$ Age of Rakesh after 5 years = 44 + 5 = 49 years

Example 12. The ratio between the present ages of Ram Rohan and Raj is 3:4:5. If the average of their present ages is 28 years then what will be the sum of the ages of Ram and Rohan together after 5 years?

(a) 45 years

(b) 55 years

(c) 52 years

**(d) 59 years**

Ans: (d) Let, the ages of Ram, Rohan and Raj be 3x, 4x and 5x respectively.

Then,

$$\frac{3x + 4x + 5x}{3} = 28$$

⇒ 4x = 28

⇒ $x = \frac{28}{4} = 7 years$

So, the present ages of Ram and Rohan together

= 3x + 4x = 7x = 7 × 7 = 49 years

Hence, the sum of the ages of Ram and Rohan together after 5 years

= 49 + 5 × 2 = 49 + 10 = 59 years

Example 13. In a family, mother's age is twice that of daughter's age. Father is 10 years older than mother. Brother is 20 years younger than his mother and 5 years older than his sister. What is the age of the father?

(a) 62 years

**(b) 60 years**

(c) 58 years

(d) 55 years

Ans: (b) Let, the age of the daughter be x. Then, age of brother = x + 5 years Therefore, age of mother = 2x years ∴ 2x - 20 = x + 5 ⇒ 2x - x = 5 + 20 ⇒ x = 25 years. Age of mother = 2x = 2 × 25 = 50 years Age of father = 50 + 10 = 60 years

## Summary

The key concepts learned from this unit are: -

- We have learnt how to calculate age some years ago.
- We have learnt how to calculate present age.
- We have learnt how to calculate age some years hence.

## Keywords

- Age.
- Present age.
- Ratio of ages.

## Self Assessment

Q1.10, years ago, Mohan was thrice as old as Ram was but 10 years hence, he will be only twice as old as Ram. Find Mohan's present age.

A. 60 years
B. 80 years
C. 70 years

D. 76 years

Q2. The ages of Ram and Shyam differ by 16 years. 6 years ago, Mohan's age was thrice as that of Ram's, find their present ages.

A. 14 years, 30 years
B. 12 years, 28 years
C. 16 years, 34 years
D. 18 years, 38 years

Q3. In a family, mother's age is twice that of daughter's age. Father is 10 years older than mother. Brother is 20 years younger than his mother and 5 years older than his sister. What is the age of the father?

A. 62 years
B. 60 years
C. 58 years
D. 55 years

Q4. The ratio between the present ages of Ram Rohan and Raj is 3:4:5. If the average of their present ages is 28 years then what will be the sum of the ages of Ram and Rohan together after 5 years?

A. 45 years
B. 55 years
C. 52 years
D. 59 years

Q5. The age of a father is three times of that of his son. After 5 years, the double of father's age will be five times the age of son. The present age of father and son is:

A. 30 years, 10 years
B. 36 years, 12 years
C. 42 years, 14 years
D. 45 years, 15 years

Q6. The ratio between the present age of Manisha and Deepali is 5:x. Manisha is 9 years younger than Parineeta. Parineeta's age after 9 years will be 33 years. The difference between Deepali's and Manisha's age is same as the present age of Parineeta. What will come in place of x?

A. 23
B. 39
C. 15
D. None of these

Q7. After replacing an old member by a new member, it was found that the average age of fi ve members of a club is same as it was 3 years ago. The difference between the ages of the replaced and the new members is:

A. 2 years
B. 4 years
C. 8 years

D. 15 years

Q8.One year ago, a father was four times as old as his son. In 6 years time his age exceeds twice his son's age by 9 years. Ratio of their ages is:

A. 13:4

B. 12:5

C. 11:3

D. 9:2

Q9.The sum of the ages of a father and son is 45 years. 5 years ago, the product of their ages was four times the father's age at that time. The present age of the father is:

A. 39 years

B. 36 years

C. 25 years

D. None of these

Q10. Three times the present age of a father is equal to eight times the present age of his son. 8 years hence the father will be twice as old as his son at that time. What are their present ages?

A. 35, 15

B. 32, 12

C. 40, 15

D. 27, 8

Q11. The ratio of A's and B's ages is 4:5. If the difference between the present age of A and B 5 years hence is 3, then what is the sum of present ages of A and B?

A. 68 years

B. 72 years

C. 76 years

D. 64 years

Q12. The sum of the ages of A and B is 42 years. 3 years back, the age of A was 5 times the age of B. Find the difference between the present ages of A and B.

A. 24 years

B. 36 years

C. 72 years

D. 60 years

Q13. The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, then what is father's present age?

A. 24 years

B. 40 years

C. 72 years

D. 60 years

Q14. 10 years ago Anu's mother was 4 times older than her daughter. After 10 years, the mother will be twice older than her daughter. Find the present age of Anu is:

A. 24 years

B. 40 years

C. 20 years

D. 60 years

Q15. 6 years ago Mahesh was twice as old as Suresh. If the ratio of their present ages is 9:5 then, what is the difference between their present ages?

A. 24 years

B. 40 years

C. 20 years

D. 60 years

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | C | 2. | A | 3. | B | 4. | D | 5. | D |
| 6. | D | 7. | D | 8. | C | 9. | B | 10. | B |
| 11. | B | 12. | A | 13. | B | 14. | C | 15. | A |

## Review Questions

1. Two Ratio of the ages of Tania and Rakesh is 9:10. 10 years ago, the ratio of their ages was 4:5. What is the present age of Rakesh?
2. The ratio of the ages of a father and a son at present is 5:2. 4 years hence, the ratio of the ages of the son and his mother will be 1:2. What is the ratio of the present ages of the father and the mother?
3. The ratio of the ages of Anubha and her mother is 1:2. After 6 years the ratio of their ages will be 11:20. 9 years before, what was the ratio of their ages?
4. The ratio of the present ages of Swati and Trupti is 4:5. 6 years hence the ratio of their ages will be 6:7. What is the difference between their ages?
5. Radha's present age is three years less than twice her age 12 years ago. Also the ratio between Raj's persent age and Radha's present age in 4:9. What will be Raj's age after 5 years?
6. The ratio of the age of Tina and Rakesh is 9:10. 10 years ago the ratio of their ages was 4:5. What is the present age of Rakesh?
7. In a family, mother's age is twice that of daughter's age. Father is 10 years older than mother. Brother is 20 years younger than his mother and 5 years older than his sister. What is the age of the father?
8. The sum of the ages of two brothers, having a difference of 8 years between them, will double after 10 years. What is the ratio of the age of the younger brother to that of the elder brother?

9. After replacing an old member by a new member, it was found that the average age of fi ve members of a club is same as it was 3 years ago. The difference between the ages of the replaced and the new members is?
10. The age of a father is three times of that of his son. After 5 years, the double of father's age will be five times the age of son. The present age of father and son is?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle

Dr. Nitin K. Mishra, Lovely Professional University

# Unit 12: Permutation and Combination, Probability

CONTENTS



## Objectives

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- Understand what are the basics of counting.
- Understand the use of product rule in the basics of counting.
- Understand the use of Sum rule in the basics of counting.
- Understand the use of the Subtraction rule in the basics of counting.
- Understand the use of tree diagram in the basics of counting.
- Understand the Division rule in the basics of counting.
- Understand how to find the total number of functions in the basics of counting.

## Introduction

Suppose that a password on a computer system consists of six, seven, or eight characters. Each of these characters must be a digit or a letter of the alphabet. Each password must contain at least 1 one digit. How many such passwords are there? The techniques needed to answer this question and a wide variety of other counting problems will be introduced in this section.

Counting problems arise throughout mathematics and computer science. For example, we must count the successful outcomes of experiments and all the possible outcomes of these experiments to determine the probabilities of discrete events. We need to count the number of operations used by an algorithm to study its time complexity.

We will introduce the basic techniques of counting in this section. These methods serve as the foundation for almost all counting techniques. We first present two basic counting principles, the product rule and the sum rule. Then we will show how they can be used to solve many different counting problems. The product rule applies when a procedure is made up of separate tasks.

## 12.1 The Product Rule

Suppose that a procedure can be broken down into a sequence of two tasks. If there are n1 ways to do the first task and for each of these ways of doing the first task, there are n2 ways to do the second task, then there are n1n2 ways to do the procedure. Example s 1–10 show how the product rule is used.

Example 1

A new company with just two employees, Sanchez and Patel, rents a floor of a building with12 offices. How many ways are there to assign different offices to these two employees?

Solution: The procedure of assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the officer assigned to Sanchez, which can be done in 11 ways. By the product rule, there are $12 \cdot 11 = 132$ ways to assign offices to these two employees.

Example 2

The chairs of an auditorium are to be labelled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labelled differently?

Solution: The procedure of labelling a chair consists of two tasks, namely, assigning to the seat one of the 26 uppercase English letters, and then assigning to it one of the 100 possible integers.

The product rule shows that there are $26 \cdot 100 = 2600$ different ways that a chair can be labelled. Therefore, the largest number of chairs that can be labelled differently is 2600.

Example 3

There are 32 microcomputers in the computer centre. Each microcomputer has 24 ports. How

many different ports to a microcomputer in the centre are there?

Solution: The procedure of choosing a port consists of two tasks, first picking a microcomputer and then picking a port on this microcomputer. Because there are 32 ways to choose the micro-computer and 24 ways to choose the port no matter which microcomputer has been selected, the product rule shows that there are $32 \cdot 24 = 768$ ports.

An extended version of the product rule is often useful. Suppose that a procedure is carried out by performing the tasks $T_1, T_2, \ldots, T_m$ in sequence. If each task $T_i$, $i = 1, 2, \ldots, n$, can be done in ni ways, regardless of how the previous tasks were done, then there are $n_1 \cdot n_2 \cdot \cdots \cdot$ nm ways to carry out the procedure. This version of the product rule can be proved by mathematical induction from the product rule for two tasks.

Example 4

How many different bit strings of length seven are there?

Solution:

Each of the seven bits can be chosen in two ways because each bit is either 0 or 1. Therefore, the product rule shows there are a total of $2^7 = 128$ different bit strings of length seven.

Example 5

How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

----  ---- ---              ----------

26 choices          10 choices

for each               for each

letter                     digit

Solution:

There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of 26 · 26 · 26 · 10 · 10 · 10 = 17,576,000 possible license plates.

Example 6 Counting Functions How many functions are there from a set with m elements to a set with

n elements?

Solution:

A function corresponds to a choice of one of the n elements in the codomain for each of the elements in the domain. Hence, by the product rule, there are $n \cdot n \cdot \cdots \cdot n = n^m$ functions from a set with m elements to one with n elements. For example, there are $5^3 = 125$ different functions from a set with three elements to a set with five elements.

Example 7

Counting One-to-One Functions How many one-to-one functions are there from a set with

m elements to one with n elements?

Solution:

First, note that when m>n there are no one-to-one functions from a set with m

elements to a set with n elements to a set with n elements.

Now let $m \leq n$. Suppose the elements in the domain are $a_1, a_2, \ldots, a_m$. There are n ways to choose the value of the function at a1. Because the function is one-to-one, the value of the function at $a_2$ can be picked in $n - 1$ way (because the value used for $a_1$ cannot be used again).

In general, the value of the function at $a_k$ can be chosen in $n - k + 1$ ways. By the product rule, there are $n(n - 1)(n - 2) \cdots (n - m + 1)$ one-to-one functions from a set with m elements to one with n elements. For example, there are $5 \cdot 4 \cdot 3 = 60$ one-to-one functions from a set with three elements to a set with five elements.

## 12.2 More Complex Counting Problems

Many counting problems cannot be solved using just the sum rule or just the product rule. However, many complicated counting problems can be solved using both of these rules in combination. We begin by counting the number of variable names in the programming language BASIC. (In the exercises, we consider the number of variable names in JAVA.) Then we will count the number of valid passwords subject to a particular set of restrictions.

Example 15

In a version of the computer language BASIC, the name of a variable is a string of one or two alphanumeric characters, where uppercase and lowercase letters are not distinguished. (An alphanumeric character is either one of the 26 English letters or one of the 10 digits.) Moreover, a variable name must begin with a letter and must be different from the five strings of two characters that are reserved for programming use. How many different variable names are there in this version of BASIC?

Solution:

Let V equal the number of different variable names in this version of BASIC. Let $v_1$be the number of these that are one character long and $v_2$ be the number of these that are two characters long. Then by the sum rule, V = $v_1$ + $v_2$. Note that $v_1$= 26, because a one-character variable name must be a letter. Furthermore, by the product rule, there are 26 · 36 strings of length two that begin with a letter and end with an alphanumeric character. However, five of these are excluded, so $v_2$ = 26 · 36 − 5 = 931. Hence, there are V = $v_1$ + $v_2$ = 26 + 931 = 957 different names for variables in this version of BASIC.

Example 16

Each user on a computer system has a password, which is six to eight characters long, where

each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Solution:

Let P be the total number of possible passwords, and let$p_6$, $p_7$, and $p_8$, denote the number of possible passwords of length 6, 7, and 8, respectively. By the sum rule,

P =$P_6$+ $P_7 + P_8$ We will now find $P_6$+ $P_7 an\ d + P_8$ and $P_8$. Finding $P_6$ directly is difficult. To find $P_6$ it is easier to find the number of strings of uppercase letters and digits that are six characters long, including those with no digits, and subtract from this the number of strings with no digits. By the product rule, the number of strings of six characters is 366, and the number of strings with no digits is 266. Hence,

$P_6$ = $36^6$ − $26^6$ = 2,176,782,336 − 308,915,776 = 1,867,866,560. Similarly, we have $P_7$= $36^7$ − $26^7$ = 78,364,164,096 − 8,031,810,176 = 70,332,353,920 and $P_8$= $36^8$ − $26^8$ = 2,821,109,907,456 − 208,827,064,576 = 2,612,282,842,880.

Consequently, P = $P_6$+ $P_7 + P_8$ = 2,684,483,063,360.

Example 17

Counting Internet Addresses On the Internet, which is made up of interconnected physical networks of computers, each computer (or more precisely, each network connection of a computer) is assigned an Internet address. In Version 4 of the Internet Protocol (IPv4), now in use,

Bit number 0 1 2 3 4 8 16 24 31

| | | | | | | |
|---|---|---|---|---|---|---|
| Class A | 0 | netid | | | | hostid |
| Class B | 1 | 0 | netid | | | hostid |
| Class C | 1 | 1 | 0 | netid | | hostid |
| Class D | 1 | 1 | 1 | 0 | Multicast address | |
| Class E | 1 | 1 | 1 | 1 | 0 | address |

. an address is a string of 32 bits. It begins with a network number (netid ). The netid is followed by a host number (hostid ), which identifies a computer as a member of a particular network. Three forms of addresses are used, with different numbers of bits used for netids and hostids. Class A addresses, used for the largest networks, consist of 0, followed by a 7-bit netid and a 24-bit hostid. Class B addresses, used for medium-sized networks, consist of 10, followed by a 14-bit netid and a 16-bit hostid. Class C addresses, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid. There are several restrictions on addresses because of special uses: 1111111 is not available as the netid of a Class A network, and the hostids consisting of all 0s and all 1s are not available for use in any network. A computer on the Internet has either a Class A, a Class B, or a Class C address. (Besides Class A, B, and C addresses, there are also Class D addresses, reserved for use in multicasting when multiple computers are addressed at a single time, consisting of 1110 followed by 28 bits, and Class E addresses, reserved for future use, consisting of 11110 followed by 27 bits. Neither Class D nor Class E addresses are assigned as the IPv4 address of a computer on the Internet.)

Figure 1 illustrates the IPv4 addressing. (Limitations on the number of Class A and Class B netids have made IPv4 addressing inadequate; IPv6, a new version of IP, uses 128-bit addresses to solve this problem.)

How many different IPv4 addresses are available for computers on the Internet?

Solution:

Let x be the number of available addresses for computers on the Internet, and let and$x_A$, $x_B$, $x_c$ denote the number of Class A, Class B, and Class C addresses available, respectively.

By the sum rule, x =$x_A$+ $x_B$ + $x_c$

To find $x_A$, note that there are 27 − 1 = 127 Class A netids, recalling that the netid 1111111 is unavailable. For each netid, there are 224 − 2 = 16,777,214 hostids, recalling that the hostids consisting of all 0s and all 1s are unavailable. Consequently, xA = 127 ·16,777,214 = 2,130,706,178.

To find $x_A$ and $x_c$, note that there are 214 = 16,384 Class B netids and 221 = 2,097,152 Class C netids. For each Class B netid, there are 216 − 2 = 65,534 hostids, and for each Class C netid, there are 28 − 2 = 254 hostids, recalling that in each network the hostids consisting of all 0s and all 1s are unavailable. Consequently, xB = 1,073,709,056 and xC = 532,676,608. We conclude that the total number of IPv4 addresses available is x = $x_A + x_B + x_C$= 2,130,706,178 + 1,073,709,056 + 532,676,608 = 3,737,091,842.

Example 18

How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Solution:

We can construct a bit string of length eight that either starts with a 1 bit or ends with the two bits 00, by constructing a bit string of length eight beginning with a 1 bit or by constructing a bit string of length eight that ends with the two bits 00. We can construct a bit string of length eight that begins with a 1 in $2^7$ = 128 ways. This follows by the product rule because the first bit can be chosen in only one way and each of the other seven bits can be chosen in two ways. Similarly, we can construct a bit string of length eight ending with the two bits 00, in $2^6$ = 64 ways. This follows by the product rule because each of the first six bits can be chosen in two ways and the last

two bits can be chosen in only one way.

| | | | |
|---|---|---|---|
| 1 | $2^7$=128 way | | |
| 0 | $2^6$ =64 ways | 0 | 0 |
| 1<br>0 | $2^5$=32 ways | | 0 |

Some of the ways to construct a bit string of length eight starting with a 1 are the same as the ways to construct a bit string of length eight that ends with the two bits 00. There are 25 = 32 ways to construct such a string. This follows by the product rule because the first bit can be chosen in only one way, each of the seconds through the sixth bits can be chosen in two ways, and the last two bits can be chosen in one way. Consequently, the number of bit strings of length eight that begin with a 1 or end with a 00, which equals the number of ways to construct a bit string of length eight that begins with a 1 or that ends with 00, equals 128 + 64 − 32 = 160. We present an example that illustrates how the formulation of the principle of inclusion-exclusion can be used to solve counting problems.

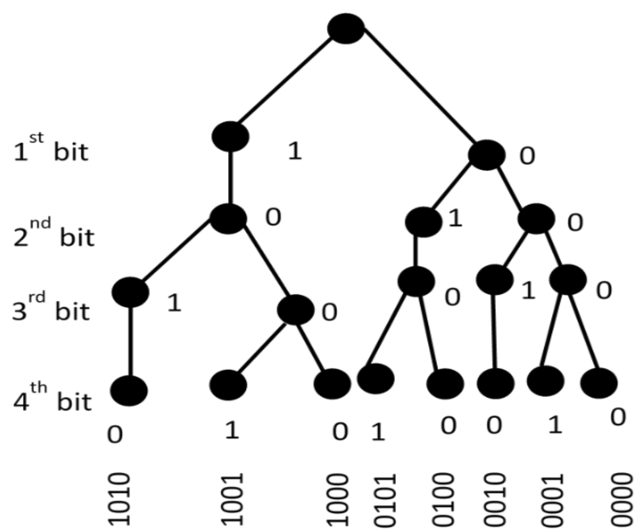
Example 19

A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

Solution:

To find the number of these applicants who majored neither in computer science nor in business, we can subtract the number of students who majored either in computer science or in business (or both) from the total number of applicants. Let A1 be the set of students who majored in computer science and A2 the set of students who majored in business. Then $A_1 \cup A_2$ is the set of students who majored in computer science or business (or both), and $A_1 \cap A_2$ is the set of students who majored both in computer science and in business. By the subtraction rule, the number of students who majored either in computer science or in business (or both) equals

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$ = 220 + 147 − 51 = 316. We conclude that 350 − 316 = 34 of the applicants majored neither in computer science nor in business. ▲ The subtraction rule, or the principle of inclusion-exclusion, can be generalized to find the number of ways to do one of n different tasks or, equivalently, to find the number of elements in the union of n sets, whenever n is a positive integer. We will study the inclusion-exclusion principle and some of its many applications in Chapter 8.




## 12.3 Tree Diagrams

**FIGURE 2 Bit strings of length four without consecutive 1's**

Counting problems can be solved using tree diagrams. A tree consists of a root, several branches leaving the root, and possible additional branches leaving the endpoints of other branches. To use trees in counting, we use a branch to represent each possible choice. We represent the possible outcomes by the leaves, which are the endpoints of branches not having other branches starting at them.

Note that when a tree diagram is used to solve a counting problem, the number of choices

of which branch to follow to reach a leaf can vary.

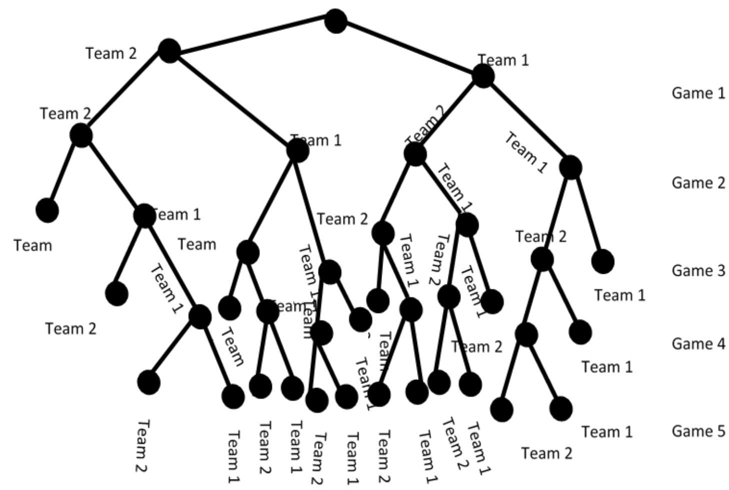



FIGURE 4. Best three games out of five playsoff

Example 21

How many bit strings of length four do not have two consecutive 1s?

Solution:

The tree diagram in Figure 2 displays all bit strings of length four without two consecutive 1s. We see that there are eight-bit strings of length four without two consecutive 1s.

Example 22

A playoff between two teams consists of at most five games. The first team that wins three games wins the playoff. In how many different ways can the playoff occur?

Solution:

The tree diagram in Figure 3 displays all the ways the playoff can proceed, with the winner of each game shown. We see that there are 20 different ways for the playoff to occur.

Example 23

Suppose that "I Love New Jersey" T-shirts come in five different sizes: S, M, L, XL, and XXL.

Further suppose that each size comes in four colours, white, red, green, and black, except for XL, which comes only in red, green, and black, and XXL, which comes only in green and black. How many different shirts do a souvenir shop have to stock to have at least one of each available size and colour of the T-shirt?

Solution:

The tree diagram in Figure 4 displays all possible size and colour pairs. It follows that the souvenir shop owner needs to stock 17 different T-shirts.

## 12.4 Permutations and Combinations

Introduction Many counting problems can be solved by finding the number of ways to arrange a specified number of distinct elements of a set of a particular size, where the order of these elements matters. Many other counting problems can be solved by finding the number of ways to select a particular number of elements from a set of a particular size, where the order of the elements selected does not matter. For example, in how many ways can we select three students from a group of five students to stand in line for a picture? How many different committees of three students can be formed from a group of four students? In this section, we will develop methods to answer questions such as these. Permutations We begin by solving the first question posed in the introduction to this section, as well as related questions.

EXAMPLE 1

In how many ways can we select three students from a group of five students to stand in line for a picture? In how many ways can we arrange all five of these students in a line for a picture? Solution: First, note that the order in which we select the student's matters. There are five ways to select the first student to stand at the start of the line. Once this student has been selected, there are four ways to select the second student in the line. After the first and second students have been selected, there are three ways to select the third student in the line. By the product rule, there are 5 · 4 · 3 = 60 ways to select three students from a group of five students to stand in line for a picture. To arrange all five students in a line for a picture, we select the first student in five ways, the second in four ways, the third in three ways, the fourth in two ways, and the fifth in one way. Consequently, there are 5 · 4 · 3 · 2 · 1 = 120 ways to arrange all five students in a line for a picture.

The notion of permutation is used with several slightly different meanings, all related to the act of permuting (rearranging) objects or values. Informally, a permutation of a set of objects is an arrangement of those objects in a particular order. When doing permutation, the order of the items is important. For example, permutations of three items a, b, c are ab, ba, ac, ca, bc and cb. The number of permutations of n things taking r at a time is denoted by ${}^{n}P_r$ and the expression is

${}^{n}P_r = \frac{n!}{(n+r)!}$

Some other formulae on permutations are as follows:

The number of permutations of n different things taken r at a time is given by

${}^{n}P_r = n^r$

The number of permutations of n things taken all together, when x of the things is alike of one kind, y of the things is alike of one kind and z are rest of the things that are alike, is given by $nP_r = \frac{n!}{x!y!z!}$

3. The number of ways of arranging n distinct objects along a round table is given by (n-1)!

4. The number of ways of arranging n persons along a round table so that no person has the same two neighbors is given by $\frac{1}{2}(n-1)!$

EXAMPLE 2

Let S = {1, 2, 3}. The ordered arrangement 3, 1, 2 is a permutation of S. The ordered arrangement 3, 2 is a 2-permutation of S. The number of r-permutations of a set with n elements is denoted by P (n, r). We can find P (n, r) using the product rule.

EXAMPLE 3

Let S = {a, b, c}. The 2-permutations of S are the ordered arrangements a, b; a, c; b, a; b, c; c, a; and c, b. Consequently, there are six 2-permutations of this set with three elements. There are always six 2-permutations of a set with three elements. There are three ways to choose the first element of the arrangement. There are two ways to choose the second element of the arrangement because it must be different from the first element. Hence, by the product rule, we see that P (3, 2) = 3 · 2 = 6. the first element. By the product rule, it follows that P (3, 2) = 3 · 2 = 6.

We now use the product rule to find a formula for P (n, r) whenever n and r are positive integers with 1 ≤ r ≤ n.

THEOREM 1

If n is a positive integer and r is an integer with $1 \le r \le n$, then there are $P(n, r) = n(n-1)(n-2)\cdots(n-r+1)$ r-permutations of a set with n distinct elements. Proof: We will use the product rule to prove that this formula is correct. The first element of the permutation can be chosen in n ways because there are n elements in the set. There are $n - 1$ ways to choose the second element of the permutation because there are $n - 1$ elements left in the set after using the element picked for the first position. Similarly, there are $n - 2$ ways to choose the third element, and so on, until there are exactly $n - (r - 1) = n - r + 1$ ways to choose the $r^{th}$ element. Consequently, by the product rule, there are $n(n-1)(n-2)\cdots(n-r+1)$ r-permutations of the set. Note that $P(n, 0) = 1$ whenever n is a non-negative integer because there is exactly one way to order zero elements. That is, there is exactly one list with no elements in it, namely the empty list. We now state a useful corollary of Theorem 1. COROLLARY 1

If n and r are integers with $0 \le r \le n$, then $P(n, r) = \frac{n!}{(n-r)!}$. Proof: When n and r are integers with $1 \le r \le n$, by Theorem 1 we have $P(n, r) = n(n-1)(n-2)\cdots = \frac{n!}{(n-r)!}$. Because $\frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$ whenever n is a non-negative integer, we see that the formula $P(n, r) = \frac{n!}{(n-r)!}$. also holds when $r = 0$.

By Theorem 1 we know that if n is a positive integer, then $P(n, n) = n!$. We will illustrate this result with some examples.

EXAMPLE 4

How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Solution:

Because it matters which person wins which prize, the number of ways to pick the three prize winners is the number of ordered selections of three elements from a set of 100 elements, that is, the number of 3-permutations of a set of 100 elements. Consequently, the answer is $P(100, 3) = 100 \cdot 99 \cdot 98 = 970{,}200$.

EXAMPLE 5

Suppose that there are eight runners in a race. The winner receives a gold medal, the second-place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties?

Solution:

The number of different ways to award the medals is the number of 3-permutations of a set with eight elements. Hence, there are $P(8, 3) = 8 \cdot 7 \cdot 6 = 336$ possible ways to award the medals.

EXAMPLE 6

Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?

Solution:

The number of possible paths between the cities is the number of permutations of seven elements because the first city is determined, but the remaining seven can be ordered arbitrarily. Consequently, there are $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ ways for the saleswoman to choose her tour. If, for instance, the saleswoman wishes to find the path between the cities with minimum distance, and she computes the total distance for each possible path, she must consider a total of 5040 paths!

EXAMPLE 7

How many permutations of the letters ABCDEFGH contain the string ABC?

Solution:

Because the letters ABC must occur as a block, we can find the answer by finding the number of permutations of six objects, namely, the block ABC and the individual letters D, E, F, G, and H. Because these six objects can occur in any order, there are 6! = 720 permutations of the letters ABCDEFGH in which ABC occurs as a block.

Combinations We now turn our attention to counting unordered selections of objects. We begin by solving a question posed in the introduction to this section of the chapter.

EXAMPLE 8

How many different committees of three students can be formed from a group of four students? Solution:

To answer this question, we need only find the number of subsets with three elements from the set containing the four students. We see that there are four such subsets, one for each of the four students because choosing three students is the same as choosing one of the four students to leave out of the group. This means that there are four ways to choose the three students for the committee, where the order in which these students are chosen does not matter.

A combination is a way of selecting several things out of a larger group, where (unlike permutations) order does not matter. For example, combinations of three items a, b, c are ab, ac and by.

Number of combinations of n things taking r at a time is denoted by ${}^{n}C_r$ and expression is

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

Some other formulae on combinations are as follows:

The number of ways in which a + b things can be divided into two groups containing a and b things, respectively, is

$${}^{a,\,b}C_b = \frac{(a+b)!}{a!b!}$$

The number of ways in which a selection can be made out of a + b + c things of which a are alike of one kind, b is alike of another kind and remaining c are different is given by (a+1) (b+1)$2^c - 1$

The number of ways of arranging n distinct objects along a round table is given by

(n-1)!

The number of ways of arranging n persons along a round table so that no person has the same two neighbors is given by

$$\frac{1}{2}(n-1)!$$

Partitions

A combination is nothing but portioning a set into two subsets, one containing k objects and the other containing the remaining n − k objects. Hence, in general the set S = {1, 2, …, n} can be partitioned into r subsets.

Consider the following iterative situation leading to partition of S. We first select a subset of $n_1$elements from S. Having chosen the first subset, we select second subset containing $n_2$elements from the remaining n − $n_1$ elements and so on until no elements remain. This procedure yields a partition of S into r subsets, with the p[th] subset containing exactly $n_p$elements.

Now we wish to count the number of such partitions. We know that there are $\frac{n!}{n!(n-n_1)!}$ ways to form the first subset

Hence, we can formulate that there are $\frac{(n-n_1-...-n_{p-1})!}{n_p!(n-n_1-...-n_p)!}$ ways to form the the subset.

Using the counting principle, the total number of partitions is then given by $\frac{n!}{n_1!n_2!...n_r!}$

This expression is called a multinomial coefficient.

## 12.5 Introduction to Probability

The word probability or chance is very frequently used in day-to-day life. For example, we generally say, 'He may come today' or 'probably it may rain tomorrow' or 'most probably he will get through the examination'. All these phrases involve an element of uncertainty and probability is a concept which measures these uncertainties. The probability when defined in simplest way is the chance of occurring of a certain event when expressed quantitatively, i.e., probability is a quantitative measure of the certainty. The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of predictions for future. The following remarks may be important for learning this chapter on probability.

1. Die: A die is a small cube used in games of chance. On its 6 faces dots are marked as . .. ... : : : . :

Plural of die is dice. The outcome of throwing (or tossing) a die is the number of dots on its uppermost face. An ace on a die means 1 dot. 2. Cards: A pack (or deck) of playing cards has 52 cards, divided into 4 suits:

Spades (♠)

Clubs (♣)

Hearts (♥)

Diamonds (♦)

Each suit has 13 cards, nine cards numbered 2 to 10, an Ace (bDdk), a King (ckn'kkg), Queen (csxe) and a Jack or Knave (xqyke). Spades and Clubs are blackfaced cards while Hearts and Diamonds are red-faced cards. The Aces, Kings, Queens and Jacks are called face cards and other cards are called number cards. The Kings, Queens and Jacks are called court cards.

3. The number of combinations of n objects taken r at a time (r ≤ n) is denoted by C(n, r) or n$C_r$ and is defined as

$$n_{C_r}=\frac{n!}{r!\,(n-r)!}=\frac{n(n-1)(n-2)\ldots to\ r\ factors}{1.2.3\ldots r.}$$

Illustration 1:$5_{C_3}=\frac{5.4.3}{1.2.3}=10 n_{C_0}=1\ and\ n_{C_n}=1$

If r>$\frac{n}{2}$, then it is better to simplify $n_{C_r}\ as\ n_{C_{n-r}}=$

Illustration 2:$52_{C_{50}}=52_{C_{52-50}}=52_{C_2}=\frac{52.51}{2.1}$

=26.51=1326

When, r > n, $n_{C_r}$= 0.

Some Important terms and concepts Random Experiment or Trial: The performance of an experiment is called a trial. An experiment is characterised by the property that its observations under a given set of circumstances do not always lead to the same observed outcome but rather to the different outcomes. If in an experiment all the possible outcomes are known in advance and none of the outcomes can be predicted with certainity, then such an experiment is called a random experiment. For example, tossing a coin or throwing a die are random experiments.

Event: The possible outcomes of a trial are called events. Events are generally denoted by capital letters A, B, C and so on.

Illustration 3:

When a coin is tossed the outcome of getting a head or a tail is an event.

When a die is thrown the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event. Sample Space: The set of all possible outcomes of an experiment is called a sample space. We generally denote it by S.

Illustration 4:

When a coin is tossed, S = {H, T} where H = head, T = tail.

When a die is thrown, S = {1, 2, 3, 4, 5, 6}.

When 2 coins are tossed simultaneously, S = {HH, HT, TH, TT} Equally Likely Events: Events are said to be equally likely if there is no reason to expect any one in preference to other. Thus, equally likely events mean outcome is as likely to occur as any other outcome.

Illustration 5:

In throwing a die, all the 6 faces (1, 2, 3, 4, 5, 6) are equally likely to occur. Simple and compound Events In the case of simple events we consider the probability of happening or non-happening of single events.

Illustration 6: We might be interested in finding out the probability of drawing an ace from a pack of cards. In the case of compound events we consider the joint occurrence of two or more events.

Illustration 7: If from a bag, containing 8 red and 5 green balls, two successive draws of 2 balls are made, we shall be finding out the probability of getting 2 red balls in the fi rst draw and 2 green balls in the second draw. We are thus dealing with a compound event. Exhaustive Events: It is the total number of all possible outcomes of any trial.

Illustration 8:

When a coin is tossed, either head or tail may turn up and therefore, there are two exhaustive cases.

There are six exhaustive cases or events in throwing a die.

If two dice are thrown simultaneously, the possible outcomes are:

(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)

(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)

(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)

(1, 4) (2, 4) (3, 4) (4, 4) (5, 4) (6, 4)

(1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)

(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6)

Thus, in this case, there are $36(=6^2)$ ordered pairs. Hence, the number of exhaustive cases in the simultaneous throw of two dice is 36.

Three dice are thrown, the number of exhaustive cases is $6^3$, i.e., 216

algebra of Events

If A and B are two events associated with sample space S, then

$A \cup B$ is the event that either A or B or both occur.

$A \cap B$ is the event that A and B both occur simultaneously.

A is the event that A does not occur.

(iv) AB ∩ is an event of non-occurrence of both A and B, i.e., none of the events A and B occurs.

Illustration 9: In a single throw of a die, let A be the event of getting an even number and B be the event of getting a number greater than 2. Then,

A = {1, 3, 5}, B = {3, 4, 5, 6} ∴ A ∪ B = {1, 3, 4, 5, 6}

A ∪ B is the event of getting an odd number or a number greater than 2. A ∩ B = {3, 5}. A ∩ B is the event of getting an odd number greater than 2.

A = {2, 4, 6} [Those elements of S which are not in A.] A is the event of not getting an odd number, i.e., getting an even number.

$\bar{B}$ = {1, 2}.

$\bar{B}$ is the event of not getting a number greater that 2, i.e., getting a number less than or equal to 2.

$\bar{A} \cap \bar{B}$= {2}. AB ∩ is the event of neither getting an odd number nor a number greater than 2.

Mutually Exclusive Events

In an experiment, if the occurrence of an event precludes or rules out the happening of all the other events in the same experiment.

Illustration 10:

When a coin is tossed either head or tail will appear. Head and tail cannot appear simultaneously. Therefore, occurrence of a head or a tail are two mutually exclusive events.

(ii) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others in the same trial, is ruled out.

Illustration 11:

(i) If the random experiment is 'a die is thrown' and A, B are the events, A: the number is less than 3; B: the number is more than 4, then A = {1, 2}, B = {5, 6}. A ∩ B = ϕ, thus A and B are mutually exclusive events.

(ii) If the random experiment is 'a card is drawn from a well-shuffled pack of cards' and A, B are the events A: the card is Black; B: the card is an ace. Since a black card can be an ace, A ∩ B ≠ ϕ, thus A and B are not mutually exclusive events.

Mutually Exclusive and Exhaustive Events

Events $E_1$, $E_2$, ..., $E_n$ are called mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n$ = S, i.e., $\bigcup_{t=1}^{n} E_t = s$ and $E_i \cap E_j$ = ϕ for all i ≠ j.

For example, in a single throw of a die, let A be the event of getting an even number and B be event of getting odd numbers, then

A = {2, 4, 6}, B = {1, 3, 5}

A ∩ B = ϕ, A ∪ B = {1, 2, 3, 4, 5, 6} = S

∴ A and B are mutually exclusive and exhaustive events.

Illustration 12: Two dice are thrown and the sum of the numbers which come up on the dice noted. Let us consider the following events:

A: 'the sum is even'

B: 'the sum is a multiple of 3'

C: 'the sum is less than 4'

D: 'the sum is greater than 11'

Which pairs of these events are mutually exclusive? Solution: There are 6 × 6 = 36 elements in the sample space (Refer to Example 2).

A is the event "the sum is even". It means we have to consider those ordered pairs (x, y) in which (x + y) is even. Thus, A = [(1, 1), (2, 2) (1, 3), (1, 5), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6) (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)].

Similarly, B = [(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)]

C = [(1, 1), (2, 1), (1, 2)] D = [(6, 6)].

We find that A ∩ B = [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)] ≠ ϕ Thus, A and B are not mutually exclusive.

Similarly, A ∩ C ≠ ϕ, A ∩ D ≠ ϕ, B ∩ C ≠ ϕ, B ∩ D ≠ ϕ, C ∩ D = ϕ. Thus, C and D are mutually exclusive.

Probability of an Event

The probability of an event is defined in the following two ways:

Mathematical (or a priori) definition

Statistical (or empirical) definition. Mathematical Definition of Probability: Probability of an event A, denoted as P(A), is defined as

$$P(A)=\frac{Number\ of\ cases\ favourable\ to\ A}{Numbers\ of\ possible\ outcomes}$$

Thus, if an event A can happen in m ways and fails (does not happen) in n ways and each of m + n ways is equally likely to occur then the probability of happening of the event A (also called success of A) is given by

$$P(A) = \frac{m}{m+n}$$

and that the probability of non-occurrence of the A (also called its failure) is given by

$$P\ (\text{not } A) \text{ or } P(\bar{A}) = \frac{n}{m+n}$$

If the probability of the happening of a certain event is denoted by p and that of not happening by q, then

$$p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1$$

Here, p, q are non-negative and cannot exceed unity, i.e., $0 \le p \le 1$ and $0 \le q \le 1$. When p = 1, then the event is certain to occur. When p = 0, then the event is impossible. For example, the probability of throwing eight with a single die is zero.

Probability as defined above is sometimes called Priori Probability, i.e., it is determined before hand, that is, before the actual trials are made.

Illustration 13: A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up 'tails'?

Solution: The coin can come up either "heads" (H) or "tails" (T). Thus, the set S of all possible outcomes is S = {H, T}

$$\therefore P\ (T) = \frac{1}{2}$$

Illustration 14: What is the probability of getting an even number in a single throw of a die?

Solution: Clearly, a die can fall with any of its faces upper most. The number on each of the faces is, therefore, a possible outcome. Thus, there are total 6 outcomes. Since there are 3 even numbers on the die, namely, 2, 4 and 6,

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$$

Illustration 15: What is the probability of drawing a 'king' from a well-shuffl ed deck of 52 cards?

Solution: Well-shuffl ed ensures equally-likely outcomes. There are 4 kings in a deck. Thus, P(a king) $=\frac{4}{52}=\frac{1}{13}$

odds of an Event

Suppose, there are m outcomes favourable to a certain event and n outcomes unfavourable to the event in a sample space, then odds in favour of the event

$=\frac{Number\ of\ favourable\ outcomes}{Number\ of un\ favourable\ outcome}=\frac{m}{n}$

and odds against the event

$=\frac{Number\ of\ unfavourable\ outcomes}{Number\ of\ favourable\ outcome}=\frac{n}{m}$

If odds in favour of an event A are a:b, then the probability of happening of event

of happening of event A = P(A) = $\frac{a}{a+b}$ and probability of not happening of event p($\bar{A}$) = $\frac{b}{a+b}$

If odds against happening of an event A are a:b, then probability of happening of event A = P(A) =$\frac{b}{a+b}$

and probability of not happening of eventA =p($\bar{A}$) = $\frac{b}{a+b}$

Illustration 16: What are the odds in favour of getting a '3' in a throw of a die? What are the odds against getting a '3'?

Solution: There is only one outcome favourable to the event "getting" a 3, the other fi ve outcomes, namely, 1, 2, 4, 5, 6 are unfavourable. Thus, Odds in favour of getting a '3'

$=\frac{Number\ of\ favourable\ outcomes}{Number\ of un\ favourable\ outcome}=\frac{1}{5}\ or\ 1\ to\ 5$

Odd against getting a '3'

$$\frac{Number\ of\ favourable\ outcomes}{Number\ of un\ favourable\ outcome}=\frac{1}{5}\ or\ 1\ to\ 5$$

Illustration 17: If the odds in favour of an event are 4 to 5, fi nd the probability that it will occur. Solution: The odds in favour of the event are $\frac{4}{5}\ thus,$

$$\frac{P(A)}{1-P(A)}=\frac{4}{5}\ i.e..,4[1-P(A)]=5P(A),$$

$$i.e..,P(A)=\frac{4}{9}$$

The probability that it will occur =$\frac{4}{9}$

fundamental theorems on ProbabIlIty

Theorem 1: In a random experiment, if S is the sample space and E is an event, then

P(E) ≥ 0

P(ϕ) = 0

(iii) P(S) = 1. Remarks: It follows from above results that,

probability of occurrence of an event is always nonnegative;

probability of occurrence of an impossible event is 0;

probability of occurrence of a sure event is 1.

Theorem 2: If E and F are mutually exclusive events, then,

P (E ∩ F) = 0 and,

(ii) P (E ∪ F) = P(E) + P(F).

Theorem 3: If E and F are two mutually exclusive and exhaustive events, then P(E) + P(F) = 1.

Theorem 4: Let E be any event and be its complementary event, then = 1 - P($\bar{E}$). Theorem 5: For any two events E and F, P(E - F) = P(E) - P(E ∩ F).

Theorem 6: (Addition Theorem). For any two events E and F,

P(E ∪ F) = P(E) + P(F) - P(E ∩ F)

Notes

1. We may express the above results as P (E or F) = P (E) + (F) - P (E and F)

2. If E and F are mutually exclusive, then P(E ∩ F) = 0 and so P(E ∪ F) = P(E) + P(F).

Theorem 7: If $E_1$ and $E_2$be two events such that $E_1 \subseteq E_2$, then prove that P($E_1$) ≤ P($E_2$). Theorem 8: If E is an event associated with a random experiment, then 0 ≤ P (E) ≤ 1.

Theorem 9: For any three events E, F, G P(E ∪ F ∪ G) = P(E) + P(F) + P(G) - P(E ∩ F) - P(F ∩ G) - P(E ∩ G) + P(E ∩ F ∩ G)

Illustration 18: A card is drawn at random from a wellshuffl ed pack of 52 cards. Find the probability of getting (i) a jack or a queen or a king, (ii) a two of heart or diamond.

Solution:

(i) In a pack of 52 cards, we have: 4 jacks, 4 queens and 4 kings.

Now, clearly a jack and a queen and a king are mutually exclusive events.

Also ,P(a jack) =$\frac{4_{C_1}}{52_{C_1}} = \frac{4}{52} = \frac{1}{13}$

P(a queen) =$\frac{4_{C_1}}{52_{C_1}} = \frac{4}{52} = \frac{1}{13}$

P(a king) $\frac{4_{C_1}}{52_{C_1}} = \frac{4}{52} = \frac{1}{13}$

∴ By the addition theorem of Probability, P(a jack or a queen or a king) = P(a jack) + P(a queen) + P(a king)

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}$$

(ii) P(two of heart or two of diamond) = P(two of heart) + P(two of diamond)

=$\frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}$

Illustration 19: Find the probability of getting a sum of 7 or 11 in a simultaneous throw of two dice.

Solution: When two dice are thrown we have observed that there are 36 possible outcomes. Now, we can have a sum of 7 as 1 + 6 = 7, 2 + 5 = 7, 3 + 4 = 7, 4 + 3 = 7, 5 + 2 = 7, 6 + 1 = 7

Thus, the six favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

∴ P(a sum of 7) =$\frac{6}{36} = \frac{1}{6}$

Again, the favourable cases of getting a sum of 11 are (5, 6), (6, 5)

∴ P(a sum of 11) =$\frac{2}{36} = \frac{1}{18}$

Since the events of getting ‘a sum of 7’ or ‘a sum of 11’ are mutually exclusive:

∴ P(a sum of 7 or 11) = P(a sum of 7) + P(a sum of 11)

$=\frac{1}{6}+\frac{1}{18}=\frac{4}{18}=\frac{2}{9}$

Illustration 20: From a well-shuffl ed pack of 52 cards, a card is drawn at random, fi nd the probability that it is either a heart or a queen.

Solution: A: Getting a heart card B: Getting a queen card

P(A) $=\frac{13}{52}, P(B)=\frac{4}{52}$, P(A ∩ B) $=\frac{1}{52}$

Required probability = P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

$=\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$

Independent Events

Two event A and B are said to be independent if the occurrence (or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 21: In the simultaneous throw of 2 coins, 'getting a head' on fi rst coin and 'getting a tail on the second coin are independent events.

Illustration 22: When a card is drawn from a pack of well-shuffl ed cards and replaced before the second card is drawn, the result of second draw is independent of fi rst draw. We now state, without proof, the theorem which gives the probabilities of simultaneous occurrence of the independent events.

Theorem 10: If A and B are two independent events, then P(A and B) = P(A). P(B)

Illustration 23: Two dice are thrown. Find dthe probability of getting an odd number on the one die and a multiple of three on the other. Solution: Since the events of 'getting an odd number' on one die and the event of getting a multiple of three on the other are independent events,

P(A and B) = P (A) × P(B) (1)

Now, P(A) = P(an odd number ) $=\frac{3}{6}=\frac{1}{2}$ = [There are 3 odd numbers 1, 3, 5] and P(B) = P(a multiple of 3)$=\frac{2}{6}=\frac{1}{3}$ [Multiples of 3 are 3 and 6]

∴ From (1), required probability $=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

Illustration 24: Arun and Tarun appear for an interview for 2 vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that

only 1 of them will be selected,

none of them be selected. Solution: Let A: Arun is s selected B: Tarun is selected.

Then, P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{5}$Clearly

'A' and 'not B' are independent also 'not A' and 'not B' are independent, 'B' and 'not A' are independent. (i) P (only 1 of them will be selected)

= P (A and not B or B and not A) = P (A) P (not B) + P (B) P (not A)

$=\frac{1}{3}\left(1-\frac{1}{5}\right)+\frac{1}{5}\left(1-\frac{1}{3}\right)$

$=\frac{1}{3}\times\frac{4}{5}+\frac{1}{5}\times\frac{2}{3}=\frac{4}{15}+\frac{2}{15}$

$=\frac{6}{15}=\frac{2}{5}$

P (onle 1 of them be selected)

= P (not A and not B) = P(not A) × P(not B)

$=\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)$

$=\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$

## 12.6 Bayes' Rule

P(A | B) = P(A ∩ B) / P(B)

• Allows us to express P(A | B) in terms of P(B | A), which may be easier to work with.

Example: Machine 1 makes 30% of all parts in a factory, and Machine 2 makes the rest. 2% of all parts made by Machine 1 are defective, and 3% of all parts made by Machine 2 are defective. Suppose we find a part that is defective. What is the probability that it came from Machine 1?

Example 2

In Orange County, 51% of the adults are males. (It doesn't take too much advanced mathematics to deduce that the other 49% are females.) One adult is randomly selected for a survey involving credit card usage.

a. Find the prior probability that the selected person is a male.

b. It is later learned that the selected survey subject was smoking a cigar. Also, 9.5% of males smoke cigars, whereas 1.7% of females smoke cigars (based on data from the Substance Abuse and Mental Health Services Administration). Use this additional information to find the probability that the selected subject is a male.

Solution

Let's use the following notation:

M = male

$\bar{M}$= female (or not male)

C = cigar smoker

$\bar{C}$ = not a cigar smoker.

a. Before using the information given in part b, we know only that 51% of the adults

in Orange County are males, so the probability of randomly selecting an adult and

getting a male is given by P(M) = 0.51.

b. Based on the additional given information, we have the following:

P($\bar{M}$) = 0.51 because 51% of the adults are males

P(M) = 0.49 because 49% of the adults are females (not males)

P(C|M) = 0.095 because 9.5% of the males smoke cigars (That is, the probability of getting someone who smokes cigars, given that the person is a male, is 0.095.)

Let's now apply Bayes' theorem by using the preceding formula with M in place

of A, and C in place of B. We get the following result:

$$P(M \mid C) = \frac{P(M)\cdot P(C|M)}{[P(M)\cdot P(C|M)]+[P(\overline{M})\cdot P(C|\overline{M})]}$$

$$= \frac{0.51\cdot 0.095}{[0.51\cdot 0.095]+[0.49\cdot 0.017]}$$

$$= 0.85329341$$

$$= 0.853 \text{ (rounded)}$$

Before we knew that the survey subject smoked a cigar, there is a 0.51 probability that the survey subject is male (because 51% of the adults in Orange County are males). However, after learning that the subject smoked a cigar, we revised the probability to 0.853. There is a 0.853 probability that the cigar−smoking respondent is a male. This makes sense, because the likelihood of a male increases dramatically with the additional information that the subject smokes cigars (because so many more males smoke cigars than females).

## Summary

The key concepts learned from this unit are: -

- We have learned what are the basics of counting.
- We have learned how to use the product rule.
- We have learned the use of the product rule in the basics of counting.
- We have learned the use of the Sum rule in the basics of counting
- We have learned the use of the Subtraction rule in the basics of counting.
- We have learned the use of the product rule, the Sum Rule and the Subtraction rule in the basics of counting.
- We have learned the use of a tree diagram in the basics of counting.
- We have learned the Division rule in the basics of counting.
- We have learned how to find the total number of functions in the basics of counting
- We have learned how to count the different possible ways of combination.
- We have learned some counting related to the deck of cards with different examples
- We have learned what a pigeonhole principle is.
- We have learned with different examples how to count the different possible ways of combination.

## Keywords

- basics of counting.
- Product rule
- Sum rule
- Subtraction rule
- tree diagram
- Division rule
- Combination.
- Deck of cards
- Pigeonhole principle.

## Self Assessment

1. There are four bus lines between A and B and three bus lines between B and C , In how many ways can a man travel by bus from A to c by way of B ?

A. 12
B. 13
C. 14
D. 15

2. Suppose repetitions are not permitted. How many three- digit numbers can be formed from the six digits 2,3,5,6,7 and 9

A. 120

B. 140

C. 150

D. 200

3. Find the number of ways that a party of seven persons can arrange themselves: in a row of seven chairs.

A. 15!

B. 17!

C. 20!

D. 22!

4. The number of distinct permutations that can be formed from all the letters of each word : RADAR is

A. 30

B. 40

C. 50

D. 60

5. In how many ways can four mathematics books, three history books ,three chemistry books. And two sociology books be arranged on a shelf so that all books of the same subject are together?

A. 350

B. 450

C. 472

D. 879

6. If p(n,2)= 72, then n=

A. 9

B. 8

C. 7

D. 5

7. In how many ways are find examinations be scheduled in a week so that no two examinations are scheduled on the same day considering Sunday as a holidays?

A. 15

B. 12

C. 10

D. 6

8. In a certain programming language ,variables should be of length three and should be made up of two letters followed by a digit or of length two made up of a letter followed by a digit. How many possible variables ?

A. 250
B. 260
C. 720
D. 200

9. Six boys and 6 girls are to be seated in a row , how many ways can they be seated if All boys are to be seated togethere and all girls are to be seated together.

A. 6!x6!
B. 6!x2
C. 6!x6!x2
D. 6!x6!x2x2

10. How many ways can be letter in word MISSISSIPPI can be arranged ?

A. $\frac{11!}{4!4!}$
B. $\frac{11!}{4!2!}$
C. $\frac{10!}{4!4!2!}$
D. $\frac{11!}{4!4!2!}$

11. In how many ways can letters a, b, c, d, m and n be arranged if M and n always appear together

A. 10!x3
B. 5!x2
C. 5!x5
D. 10!x4

12. If repetitions are not permitted ,how many four digit number can be formed from digit 1,2,3,7,8, and 5. Total number of less than 5000.

A. 100
B. 180
C. 280
D. 380

13. A word that reads the same when read in forward or backward is called as palindrome.How many seven -letters palindromes can be formed from english alphabets?

A. 26^2
B. 25^4
C. 26^4
D. 26^5

14. Suppose repetitions are not permitted. How many of these numbers are less than 400?

A. 30

B. 40

C. 50

D. 60

15. Find the number of ways that a party of seven persons can arrange themselves: around a circular table.

A. (n)!

B. (n-1)!

C. (n-2)!

D. (n-3)!

*16*. A woman has 11 close friends. In how many ways can she invite five of them to dinner?

A. 456;

B. 462;

C. 450;

D. 451;

17. A man has 11 close friends. In how many ways if two of the friends are married and will not attend separately?

A. 210;

B. 222;

C. 223;

D. 234;

18. A person has 11 close friends. In how many ways if two of them are not on speaking terms and will not attend together?

A. 456

B. 786

C. 897

D. 252

19. A woman has 11 close friends of whom six are also women. In how many ways can she invite three or more to a party?

A. $2^{11} - 1 - \binom{11}{2} - \binom{11}{2} = 1981$

B. $2^{9} - 1 - \binom{17}{2} - \binom{17}{2} = 1980$

C. $2^7 - 1 - \binom{15}{2} - \binom{15}{2} = 1982$

D. $2^5 - 1 - \binom{10}{2} - \binom{10}{2} = 1983$

20. A woman has 11 close friends of whom six are also women. In how many ways can she invite three or more of them if she wants the same number of men as women (including herself)?

A. 325
B. 360
C. 400
D. 480

21. A student is to answer 10 out of 13 questions on an exam. How many choices has he?

A. 567
B. 456
C. 786
D. 286

22. A student is to answer 10 out of 13 questions on an exam. How many if he must answer the first two questions?

A. 897
B. 890
C. 165
D. 987

23. A student is to answer 10 out of 13 questions on an exam. How many if he must answer the first or second question but not both?

A. 675
B. 876
C. 110
D. 654

24. A student is to answer 10 out of 13 questions on an exam. How many if he must answer exactly three out of the first five question?

A. 876
B. 80

C. 87
D. 90

25. A student is to answer 10 out of 13 questions on an exam. How many if he must answer at least three of the five questions?

A. 897
B. 276
C. 234
D. 543

26. The minimum number of students needed to guarantee that five of them belong to the same class (freshman, sophomore, junior, senior)

A. Here the n=3 classes are the pigeonholes and k+1=7 so k=8. Thus, among any kn+1=14 student (pigeons), five of them belong to the same class.
B. Here the n=4 classes are the pigeonholes and k+1=5 so k=4. Thus, among any kn+1=17 student (pigeons), five of them belong to the same class.
C. Here the n=6 classes are the pigeonholes and k+1=50 so k=41. Thus, among any kn+1=107 student (pigeons), five of them belong to the same class.
D. Here the n=24 classes are the pigeonholes and k+1=15 so k=4. Thus, among any kn+1=17 student (pigeons), five of them belong to the same class.

27. Let L be a list (not necessarily in alphabetical order) of the 26 letters in the english alphabet (which consists of 5 vowels,A,E,I,O,U and 21 consonants) then L has a sublist consisting ……………

A. of four or more consecutive consonants.
B. of five or more consecutive consonants
C. of six or more consecutive consonants
D. of eight or more consecutive consonants

28. Let L be a list (not necessarily in alphabetical order) of the 26 letters in the english alphabet (which consists of 5 vowels,A,E,I,O,U and 21 consonants). Assuming L begins with a vowel , say A, then L has a sublist consisting …………….

A. of four or more consecutive consonants.
B. of five or more consecutive consonants
C. of six or more consecutive consonants

D. of eight or more consecutive consonants

29. The minimum number n of integers to be selected from S = {1, 2,..., 9}so that the sum of two of the n integers is even is…………

A. 3
B. 4
C. 5
D. 6

30. The minimum number n of integers to be selected from S = {1, 2,..., 9}so that the difference of two of the n integers is 5 is

A. 5
B. 6
C. 7
D. 8

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | A | 3. | B | 4. | A | 5. | C |
| 6. | A | 7. | D | 8. | B | 9. | C | 10. | D |
| 11. | B | 12. | B | 13. | C | 14. | B | 15. | B |
| 16. | B | 17. | A | 18. | D | 19. | A | 20. | A |
| 21. | D | 22. | C | 23. | C | 24. | B | 25. | B |
| 26. | B | 27. | A | 28. | B | 29. | A | 30. | B |

## Review Questions

1. How many words can be formed out of the letters of the word "UNITED" taking all the letters at a time considering no letter is to be repeated?

Solution: The word "UNITED" consists of 6 different letters. Hence the required number of permutations

$= {}^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Therefore, the required number of ways is

$({}^8C_2 \times^4 C_2)+ ({}^8C_1 \times^4 C_3)+({}^4C_4)=\left(\frac{8\times7}{2} \times \frac{4\times3!}{4}\right) + (8 \times 4) + (1)$

=168+33+1=201

2. From a group of 8 men and 4 women, 4 persons are to be selected to form a committee so that at least 2 women are there on the committee. In how many ways can it be done?

Solution: We may have (2 men and 2 women) or (1 man and 3 women) or (0 man and 4 women).

3. How many words can be formed from the letters of the word "COMMUTE" taking all the letters at a time considering no letter is to be repeated and the first and last letter of the words are "M"?

Solution: The word "COMMUTE" has two M's. The first and the last letter should be M. The remaining letters are C, O, U, T, E. So, the number of permutations is

${}^{5}P_{5} = 5 \times 4 \times 3 \times 2 \times 1 = 120$

4. In how many ways 6 prizes can be given to 10 students when a student may receive at most two prizes?

Solution: The first prize can be given in 10 ways as any student can receive it. Also, since a student may receive at most two prizes, the second prize can be given in 10 ways. Now, the third prize can be given in 9 ways since a student cannot get more than two prizes. Similarly, the fourth prize can be given in 9 ways. The fifth and sixth prize can be given in 8 ways each. Hence, the total number of ways in which the prizes can be given to 10 students when no student can get more than two prizes = $10 \times 10 \times 9 \times 9 \times 8 \times 8 = 518400$

5. How many natural numbers less than 1000 can be formed with the digits 1, 2, 3, 4 and 5, considering repetition of digits is not allowed?

Solution: The numbers can be of 1, 2 or 3 digits. The numbers can be formed using the digits 1, 2, 3, 4, 5. So the total number of natural numbers less than 1000 using 1, 2, 3, 4, 5 is

$=5 + 5 \times 4 + 5 \times 4 \times 3 = 5 + 20 + 60 = 85$

6. In how many ways can 5 beads of different colors form a necklace?

Solution: Here, we are to form circular permutations of 5 things taken all at a time. This can be done in (5-1)! ways. But in case of a necklace, clockwise and anti-clockwise, permutations are same. Hence, the required number of ways is

$$\frac{1}{2}(5-1)! = \frac{1}{2}4! = \frac{1}{2} \times 24 = 12$$

7. Mary has 4 bananas, 6 oranges and 5 apples. How many selections of fruits can be made by selecting at least one of them?

Solution: Five apples can be selected in 6 different ways, that is, we can choose 1, 2, 3, 4, 5 or none of the apple. Similarly, 4 bananas can be selected in 5 different ways and 6 oranges can be selected in 7 different ways. Therefore, Number of ways = 6×5×7 = 210

This also includes the case in which none of the fruits is selected. Rejecting this case, we get required number of ways = 210 − 1 = 209.

8. In how many ways can the word "APOCALYPSE"be arranged such that all the vowels come together?

Solution: The word "APOCALYPSE" has 10 letters. Now, all the vowels are supposed to come together, we consider all the vowels "A, O, A, E" as one. Hence, number of ways this can be arranged in

(7! /2!) (since 4 vowels are treated as 1, the number of arrangements become 7! and because "P" is repeated twice). Also, the vowels can be arranged in 4! number of ways within themselves but the vowel "A" is repeated twice. Hence, the total number of arrangements of the word "APOCALYPSE" such that all the vowels come together

$\frac{7!}{2!} \times \frac{4!}{2!} = 7! \times 3! = 5040 \times 6 = 30240$

9. $If\ ^{n-1}P_3 : nP_4 = 1:6, then\ what\ ia\ the\ value\ of\ n?$

Solution: We have

$^{n-1}P_3 : {^nP_4} = 1:6$

$\Rightarrow \frac{(n-1)!}{(n-1-3)!} \times \frac{(n-4)!}{(n1)!} = \frac{1}{6}$

$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{(n)!} = \frac{1}{6}$

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{6}$

$\Rightarrow \frac{(n-1)!}{n.(n-1)!} = \frac{1}{6}$

$\Rightarrow n=6$

10. How many triangles can be formed by joining the

Vertices of an octagon?

Solution: Total number of vertices in an octagon = 8. We know that a triangle is formed by joining three vertices. Hence, this can be done in $^8C_3$ number of ways. Thus, the total number of triangles that can be formed is

$^8C_3 = \frac{8!}{3! \times 5!} = \frac{8 \times 7 \times 6}{3 \times 2} = 56.$

11. Record plant location (either 1, 2, 3) of next maintenance call for spinning machine repair:

Sample space =

12. Toss 2 fair coins:

Sample space =

13. An engineering design firm is up for a Nissan contract and a Ford contract:

Sample space =

14. Plant 1 has 22 machines, plant 2 has 60 machines, plant 3 has 18 machines. Probability of next repair being from plant 3, denoted:

P(3) =

15. Event A = 'next repair from odd-numbered plant'

P(A) =

16. Event B = 'get at least one contract'

P(B) =

17. A manufacturer (Acme) of automobile lamps categorizes the lamps produced as "Good", "Satisfactory", or "Unsatisfactory" for both *intensity* and *useful life*. For 200 lamps, consider the table:

*Useful Life*

| *Intensity* | Good | Satis | Unsatis | Total |
|---|---|---|---|---|
| Good | 100 | 25 | 5 | |
| Satis | 35 | 10 | 5 | |
| Unsatis | 10 | 8 | 2 | |
| Total | | | | 200 |

Select a lamp at random. What is the probability that it will be rated "Good" in *intensity*?

A =

P(A) =

What is the probability that it will be rated "Good" in *useful life*?

B =

P(B) =

What is the probability that it will be rated "Good" in *intensity* and *useful life*?

What is the probability that it will be rated "Good" in *useful life* or *intensity*?

18. Suppose we toss 10 coins. Define event A = {obtain at least 2 "heads"}.

What is P(A)?

18. What is the probability that the lamp is Good in *intensity* given that the lamp is Good in *useful life*?

19. What is the probability that the lamp is Good in *intensity* given that the lamp is Unsatisfactory in *useful life*?

20. An auto-parts store purchases 30% of its lamps from the Acme manufacturer. Estimate the probability that a randomly chosen lamp in the store is from Acme and is "Unsatisfactory" in *intensity*.

21. When a computer goes down, there is a 75% chance that it is due to an overload and a 15% chance that it is due to a software problem. There is a 15% chance that *neither* an overload nor a software problem is the cause. For a random computer malfunction, what is the probability that both an overload and a software problem are liable?

22. 80% of accidents at a foundry involve human error and 40% involve equipment malfunction. 35% involve both problems. If an accident involves an equipment malfunction, what is the probability that there was also human error?

23. Acme example: A = {Good in *intensity*}, B = {Good in *useful life*}.

Are A and B independent?

24. Roll 1 die. Let A = {even number} and B = {less than 4}. Are A and B independent?

25. If two events A and B are mutually exclusive (and P(A) > 0 and P(B) > 0), then they cannot be independent. Why?

26. Four electrical components are connected in series. The reliability of each component is 0.90. If the components are independent, what is the probability that the circuit works when the switch is thrown?

27. Baye's Theorem

| | Positive Test Result (Pregnancy is indicated) | Negative Test Result (Pregnancy is not indicated) |
|---|---|---|
| Subject is Pregnant | 80 | 5 |
| Subject is Not Pregnant | 3 | 11 |

1. a. If one of the 99 test subjects is randomly selected, what is the probability of getting a subject who is pregnant?

b. A test subject is randomly selected and is given a pregnancy test. What is the probability of getting a subject who is pregnant, given that the test result is positive?

Ans: a. 85/99 or 0.859

b. 80/83 or 0.964

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

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# Unit 13: Logical Venn Diagram and Set Theory, Syllogism

**CONTENTS**



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand how to show the relationships among things or finite groups of things.
- We will understand what are sets
- We will understand what are different types of sets
- We will understand what are operations on a sets
- We will understand what is Syllogism.

## Introduction

The concept of set is fundamental in all branches of mathematics. Sets are the most basic tools of mathematics which are extensively used in developing the foundations of relations and functions, logic theory, sequences and series, geometry, probability theory, etc. In fact, these days most of the concepts and results in mathematics are expressed in the set theoretic language.

The modern theory of sets was developed by the German mathematician Georg Cantor (1845–1918AD). In this chapter, we will study some basic definitions and operations involving sets. We will also discuss the applications of sets.

## 11.1 Venn Diagrams

The diagrams drawn to represent sets are called Venn— Euler diagrams or simply Venn diagrams. In Venn diagrams, the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle. If a set A is a subset of B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B we draw two intersecting circles. Two disjoint sets are represented by two non-intersecting circles.

## 11.2 Set

We observe that in nature, varieties of objects occur in groups. These groups are given different names such as, a collection of books, a bunch of keys, a herd of cattle, an aggregate of points, etc., depending on the characteristic of objects they represent. In literal sense, all these works have the same meaning. (i.e., a group or a collection). In mathematical language, we call this collection of objects, a set. From the above examples, it can be seen that each collection has a well-defined property (characteristic) of its own.

Thus, a set is a well-defined collection of objects. When we say well defined, we mean that the objects follow a given rule or rules. With the help of this rule, we will be able to say whether any given object belongs to this set or not. For example, if we say that we have a collection of short students in a class, this collection is not a set as 'short students', is not well defined. However, if we say that we have a collection of students whose height is less than 5 feet, then it represents a set.

It is not necessary that a set may consist of same type of objects, For example, a book, a cup and a plate lying on a table may also form a set, their common property being that they form a collection of objects lying on the table.

Example 1: Some other examples of sets are:

(i)The set of numbers 1, 3, 5, 7, 9, 14.

(ii)The set of vowels in the alphabets of English.

(iii)The set of rivers in India.

(iv)The set of all planets.

(v)The set of points on a circle.

(vi)The set of mathematics books in your library.

(vii)The set of even positive integers (i.e., 2, 4, 6, 8, ...).

(viii)The set of multiples of 4 (i.e., 4, 8, 12, ...).

(ix)The set of factors of 12. (i.e., 1, 2, 3, 4, 6, 12).

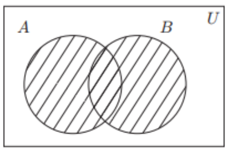
(x)The set of integers less than zero (i.e., -1, -2, -3, ...).

Notations

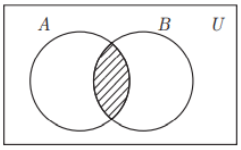
Sets are usually denoted by capital letters A, B, C, etc., and their elements by small letters a, b, c, etc. Let, A be any set of objects and let 'a' be a member of A, then we write a ∈ A and read it as 'a belongs to A' or 'a is an element of A' or 'a is a member of A'. If a is not an object of A, then we write a ∉ A and read it as 'a does not belong to A' or 'a is not an element of A'.

## 11.3 Operation on Sets

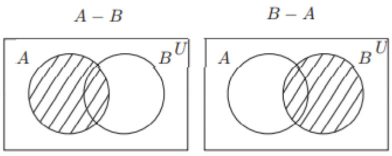
1. Union of sets: A U B



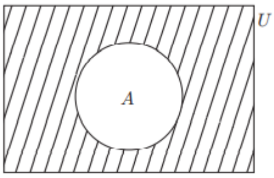

2. Intersection of sets: A ∩ B




3. Difference of sets: A − B or B – A




4. Complement of a set: A′




## 11.4 Venn Diagram with Two Attributes

In this section, we discuss the Venn diagram with two attributes and give the generalized formulae. Figure 1 shows Venn diagram representation of two sets.

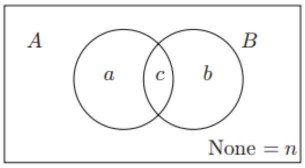
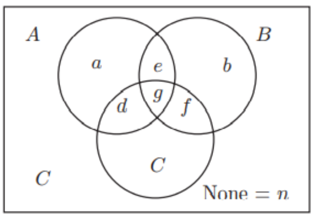



Figure 1 | Venn diagram representation of two sets A and B

1. n (A) = a+c
2. n (B) = b+ c
3. Things belonging to exactly one attribute = +a b
4. n (A ∩B) = c
5. Things belonging to none of the attributes = n
6. n A( ) ∪B a = + b c

## 11.5 Venn Diagram with Three Attributes

In this section, we discuss the Venn diagram with two attributes and give the generalized formulae. Figure 2 shows Venn diagram representation of three sets.\




1. n (A) = +a e + +g d

2. n (B ) = +b f + +e g
3. n (C ) = c +d +g+ f
4. Things belonging to exactly one attribute = a + b + c
5. n (A ∩B) = e+ g
6. n (B ∩C) = g+ f
7. n (A ∩ C ) = d+ g
8. n (A ∩ B ∩ C )= g
9. Things belonging to none of the attributes = n

## 11.6 Representation of Sets

There are two ways of expressing a set. These are

1. Tabular form or roster form.

2. Set-builder form or rule method.

Tabular Form or Roster Form

In this method, we list all the members of the set separating them by means of commas and enclosing them in curly brackets {}.

Example 2: Let, A be the set consisting of the numbers 1, 3, 4 and 5, then we write A = {1, 3, 4, 5}.

**NOTES:-** The order of writing the elements of a set is immaterial. For example, {1, 3, 5}, {3, 1, 5}, {5, 3, 1} all denote the same set. • An element of a set is not written more than once. Thus, the set {1, 5, 1, 3, 4, 1, 4, 5} must be written as {1, 3, 4, 5}.

## 11.7 Set Builder Form or rule Method

In this method, instead of listing all elements of a set, we write the set by some special property or properties satisfied by all its elements and write it as A = {x : P (x)} or, A = {x | x has the property P (x)} and read it as "A is the set of all elements x such that x has the property P". The symbol ':' or '|' stands for 'such that'.

Example 3: Let, A be the set consisting of the elements 2, 3, 4, 5, 6, 7, 8, 9, 10. Then, the set A can be written as A = {x : 2 ≤ x ≤ 10 and x e N}.

## 11.8 Finite and Infinite Sets

### Finite Set

A set having no element or a definite number of elements is called a finite set. Thus, in a finite set, either there is nothing to be counted or the number of elements can be counted, one by one, with the counting process coming to an end.

Example 4: Each of the following sets is a finite set:

(i) A = the set of prime numbers less than 10

= {2, 3, 5, 7};

(ii) B = the set of vowels in English alphabets

= {a, e, i, o, u};

(iii) C = {x | x is divisor of 50}.

cardinal number of a Finite Set

The number of distinct elements in a finite set S is called the cardinal number of S and is denoted by n(S).

Example 5: If A = {2, 4, 6, 8} then n(A) = 4.

**Infinite Set**

A set having unlimited number of elements is called an infinite set. Thus, in an infinite set, if the elements are counted one by one, the counting process never comes to an end.

Example 6: Each of the following sets is an infinite set:

(i) the set of all natural numbers = {1, 2, 3, 4, ...).

(ii) the set of all prime numbers = {2, 3, 5, 7, ...).

(iii) the set of all points on a given line.

(iv) the set of all lines in a given plane.

(v) {x | x ε R and 0 < x < 1}.

## 11.9 Empty Set or Null Set

The set which contains no element is called the empty set or the null set or void set.

The symbol for the empty set or the null set is f. Thus, f = {}, since there is no element in the empty set.

The empty set is a finite set .Since any object x which is not equal to itself does not exist, the set A = {x : x ≠ x} is the empty set ∅.

A set which is not empty, i.e., which has at least one element is called a non-empty set or a non-void set.

Example 7:

(i) The set of natural numbers less than 1 is an empty set.

(ii) the set of odd numbers divisible by 2 is a null set.

(iii) {x | x ∈ Z and x2 = 2} = f, because there is no

integer whose square is 2.

(iv) {x | x ∈ R and x2 = -1} = f, because the square of

a real number is never negative.

(v) {x | x ∈ N, 4 < x < 5} is the empty set.

(vi) {x | x ∈ Z, - 1 < x < 0} is the null set.

The empty set should not be confused with the set {0}. It is the set containing one element, namely 0.

**Singleton**

A set containing only one element is called a singleton.

Example 8:

(i) The set {0} is a singleton since it has only one element 0.

(ii) The set of even prime numbers is the set {2} which is a singleton.

(iii) $\{x \mid x \text{ is an integer and } -1 < x < 1\} = \{0\}$ is a singleton.

### Equal set

Two sets A and B are said to be equal if they have the same elements and we write A = B. Thus, A = B if every element of A is an element of B and every element of B is an element of A.

In symbols, $A = B$ iff $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$. To indicate that two sets A and B are not equal, we will write $A \neq B$.

**Example 9:**

(i) If $A = \{2, 3, 4\}$ and $B = \{x \mid 1 < x < 5, x \in N\}$ then A = B.

(ii) If A = the set of letters in the word 'WOLF' and B = the set of letters in the word 'FOLLOW' then A= B as each = {W, O, L, F}, remembering that in a set the repetition of elements is meaningless and order of elements is immaterial.

### Equivalent Sets

Two finite sets A and B are said to be equivalent if they have the same number of elements, i.e., if we can find a one -to-one correspondence between the elements of the two sets.

The symbol ' ~ ' is used to denote equivalence. Thus, A ~ B is read as "A is equivalent to B". Two finite sets A and B are equivalent if n(A) = n(B), i.e., if they have the same cardinal number.

Equivalent sets have the same number of elements, not necessarily the same elements. The elements in two equivalent sets may or may not be the same. Thus, equal sets are always equivalent but equivalent sets may or may not be equal.

**Example 10:**

(i) If A = {1, 2, 3} and B = {2, 4, 6) then A ~ B.

(ii) If A = {a, b, c, d} and B = {p, q, r, s} then A ~ B.

(iii) If A = {3, 5, 7, 9} and B = {9, 7, 5, 3} then A ~ B.

Also, since A and B have same elements, ∴ A = B.

### Subset of set

If A and B are any two sets, then B is called a subset of A if every element of B is also an element of A.

Symbolically, we write it as $B \subseteq A$ or $A \supseteq B$

(i) $B \subseteq A$ is read as B is contained in A or B is a subset of.

(ii) $A \supseteq B$ is read as A contains B or A is super set of B.

**Example 11:**

(i) The set A = {2, 4, 6} is a subset of B = {1, 2, 3, 4, 5, 6}, since each number 2, 4 and 6 belonging to

A, also belongs to B.

(ii) The set A = {1, 3, 5} is not a subset of B = {1, 2, 3, 4} since $5 \in A$ but $5 \notin B$.

(iii) The set of real numbers is a subset of the set of complex numbers. The set of rational numbers is a subset of the set of real numbers. The set of integers is a subset of the set of rational numbers. Finally, the set of natural numbers is a subset of the set of integers. Symbolically, $N \subseteq Z \subseteq Q \subseteq R \subseteq C$

### Proper Subsets of a Set

A set B is said to be a proper subset of the set A if every element of set B is an element of A whereas every element of A is not an element of B.

We write it as B ⊂ A and read it as "B is a proper subset of A".

Thus, B is a proper subset of A if every element of B is an element of A and there is at least one element in A which is not in B.

Example 12:

(i) If A = {1, 2, 5} and B = {1, 2, 3, 4, 5}. Then A is a proper subset of B.

(ii) The set N of all natural numbers is a proper subset of the set Z of all integers because every natural number is an integer, i.e., N ⊂ Z but every integer need not be a natural number, i.e., N ≠ Z.

**Power set**

Elements of a set can also be some sets. Such sets are called set of sets. For example, the set {f, {1}, {2}, {3, 4}} is a set whose elements are the sets f, {1}, {2}, {3, 4}.

The set of all the subsets of a given set A is called the power set of A and is denoted by P(A).

Example 13:

(i) If A = {a}, then P(A) = {f, A}.

(ii) If B = {2, 5}, then P(B) = {f, {2}, {5}, B}.

(iii) If S = {a, b, c}, then P(S) = {f, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, S}.

**Comparable sets**

If two sets A and B are such that either A ⊂ B or B⊂ A, then A and B are said to be comparable sets. If neither A ⊂ B nor B ⊂ A, then A and B are said to be non-comparable sets.

Example 14:

(i) If A = {1, 3, 5} and B = {1, 2, 3, 4, 5}, then A and B are comparable sets because A ⊂ B.

(ii) If A = B, then A and B are comparable sets.

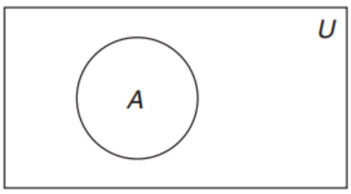
**Universal sets:-**

If in any discussion on set theory, all the given sets are subsets of a set U, then the set U is called the universal set. Example 15: Let, A = {2, 4, 6}, B = {1, 3, 5}, C = {3, 5, 7, 11}, D = {2, 4, 8, 16} and U = {1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 16} be the given sets. Here the sets A, B, C, D are subsets of the set U. Hence U can be taken as the universal set.

## 11.10 Venn Diagram

In order to visualize and illustrate any property or theorem relating to universal sets, their subsets and certain operations on sets, Venn, a British mathematician developed what are called Venn diagrams. He represented a universal set by interior of a rectangle and other sets or subsets by interiors of circles

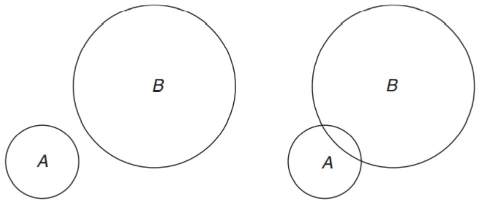
Examples of certain relationships Between Sets by Venn diagrams 1. If U be a set of letters of English alphabets and A, a set of vowels, then A ⊂ U. This relationship is illustrated by Fig. (a).




If A ⊂ B and A ≠ B, then A and B can be represented by either of the diagrams [Figure (b) andFigure (c)].

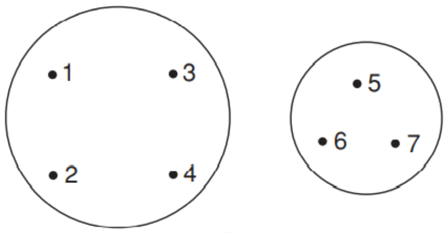

(*b*) (*c*)

3. If the sets A and B are not comparable, then neither of A or B is a subset of the other. This fact can be represented by either of the diagrams [Figure (d) and Figure (e)].




(d) (c)

4. If A = {1, 2, 3, 4} and B = {5, 6, 7}, then A and B are disjoint. These can be illustrated by Venn diagram given in Fig. (f).



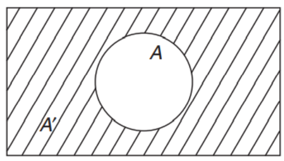

(f)

**Complement of sets :-**

Let, A ⊂ U (i.e., A is a proper subset of universal set U). Evidently, U consists of all the elements of A together with some elements which are not in A. Let us now constitute another set consisting of all the elements of U not in A. Naturally, it will form another proper subset of U. We call this subset the complement of the subset A in U and denote it by A′ or by Ac i.e., Ac = {x : x ∈ U, x ∉ A}.

Thus, the complement of a given set is a set which contains all those members of the universal set that do not belong to the given set.

Example of A′ by Venn diagram Let, A be a subset of the universal set U. The shaded area in fi gure below represents the set A′ which consists of those elements of U which are not in A.




Example 16:

(i) If the universal set is {a, b, c, d} and A = {a, b, d} then A′ = {c}.

(ii) If the universal set U = {1, 2, 3, 4, 5, 6} and A = {2, 4, 6}, then A’ = {1, 3, 5}.

(iii) If U = N and A = O (the set of odd natural numbers), then A′= E (the set of even natural numbers).

(iv) If U = I, A = N,then A′ = {0, - 1, - 2, - 3 ...}.

(v) If U = {1, 2, 3, 4}, A = {1, 2, 3, 4}, then A′ = f.

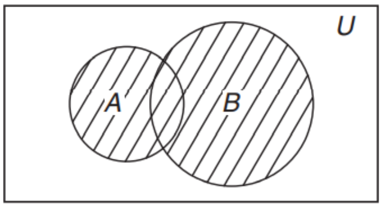
**(a) Union of Sets**

Let A and B be two given sets. Then the union of Aand B is the set of all those elements which belong to either A or B or both. The union of A and B is denoted by A ∪ B and is read as A union B. The symbol ∪ stands for union. It is evident that union is 'either, or' idea. Symbolically,A ∪ B = {x : either x ∈ A or x ∈ B}.

NOTES

the union set contains all the elements of A and B, except that the common elements of both A and B are exhibited only once. N

**Example** of A ∪ B by Venn diagram Let A and B be any two sets contained in a universal set U. Then A ∪ B is indicated by the shaded area in the figure below.




Example 17:

(i) Let, A = {1, 2, 3, 4}, B = {2, 3, 6, 7, 9}, then, A ∪ B = {1, 2, 3, 4, 6, 7, 9}.

(ii) If A = O (set of odd natural numbers), B = E (set of even natural numbers), then A ∪ B = N.

(iii) If A is the set of rational numbers and B the set of irrational numbers, then A ∪ B = R.

(iv) If A = {x : $x^2$ = 4, x ∈ I} = {2, -2}, B = {y : $y^2$ =9, y ∈ I} = {3, -3}, thenA ∪ B = {-3, -2, 2, 3}.

(v) If A = {x : 1 < x < 5, x ∈ N} = {2, 3, 4}, B = {y : 3< y < 7, y ∈ N} = {4, 5, 6}, then A ∪ B = {2, 3, 4, 5, 6}.

NOTES

From the definition of the union of two sets A and B, it is clear that

- x ∈ A ∪ B ⇔ x ∈ A or x ∈ B
- x ∉ A ∪ B ⇔ x ∉ A and x ∉ B
- A ⊆ A ∪ B and B ⊆ A ∪ B.

## (b) Intersection of Sets

Let A and B be two given sets. Then the intersection of A and B is the set of elements which belong to both A and B. In other words, the intersection of A and B is the set of common members of A and B. The intersection of A and B is denoted by A ∩ Band is read as A intersection B. The symbol ∩ stands for intersection. It is evident that intersection is an 'and' idea.

Symbolically,

A ∩ B = {x : x ∈ A and x ∈ B}.

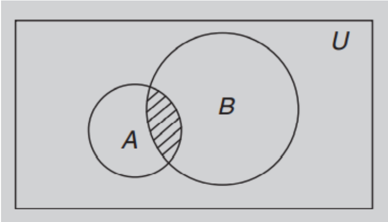
NOTES :-

From the defi nition of the intersection of two sets Aand B, it is clear that

- x ∈ A ∩ B ⇔ x ∈ A and x ∈ B
- x ∉ A ∩ B ⇔ x ∉ A or x ∉ B
- A ∩ B ⊆ A and A ∩ B ⊆ B.

Let A and B be any two sets contained in the universal set U. Then A ∩ B is indicated by the shaded area, as shown in the figure below.

NOTES




Example 18:

(i) If A = {1, 2, 3, 6, 9, 18},and B = {1, 2, 3, 4, 6, 8, 12, 24},then A ∩ B = {1, 2, 3, 6}.

(ii) If A is the set of odd natural numbers and B is the set of even natural numbers, then A ∩ B = f.

[Intersection of two disjoint sets is empty set]

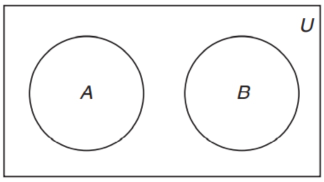
(iii) If A and B are sets of points on two distinct concentric circles, then A ∩ B = f.

(iv) If A = {x : 1 < x < 6, x ∈ N } = {2, 3, 4, 5}, B = {y : 2 < y < 9, y ∈ N} = {3, 4, 5, 6, 7, 8}

then, A ∩ B = {3, 4, 5}.

### Disjoint Sets

If A ∩ B = f, then A and B are said to be disjoint sets. For example, let A = {2, 4, 6, 8} and B = {1, 3, 5, 7}. Then, A and B are disjoint sets because there is no element which is common to both A and B. The disjoint sets can be represented by Venn diagram as shown in the figure below.




### Difference of Sets

Let A and B be two given sets. The difference of sets A and B is the set of elements which are in A but not in B. It is written as A - B and read as A difference B.

Symbolically,

A - B = {x : x ∈ A and x ∉ B}.

Similarly, B - A = {x : x ∈ B and x ∉ A}.

Caution: In general, A - B ≠ B - A.

Example 19:

(i) If A = {1, 2, 3, 4}, B = {2, 4, 7, 9}, then

A - B = {1, 3} and B - A = {7, 9}.

Hence A - B ≠ B - A.

(ii) If A = {12, 15, 17, 20, 21},

B = {12, 14, 16, 18, 21}

and C = {15, 17, 18, 22}, then

A - B = {15, 17, 20}

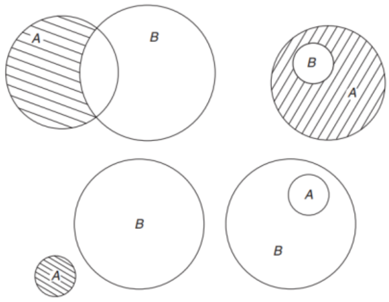
B - C = {12, 14, 16, 21}

C - A = {18, 22}

B - A = {14, 16, 18}

A - A = f.

**Example** of A − B by Venn Diagrams In the four cases shown by the diagrams below, A - Bis given by shaded area.




### Applications of Sets

1. If a set S has only a finite number of elements, we denote by n(S) the number of elements of S.

Example 20: If U = {1, 2, 3, 4, 5}, then n(U) = 5.

2. For any two sets A and B, with finite number of elements, we have the following formula:

n(A ∪ B) = n(A) + n(B) - n(A ∩ B).

3. If A and B are disjoint sets, thenn(A ∪ B) = n(A) + n(B).

Example 21: X and Y are two sets such that

n(X) = 17, n(Y) = 23, n(X ∪ Y) = 38,

find n(X ∩ Y).

**Solution:** n(X) = 17, n(Y) = 23, n(X ∪ Y) = 38,

n(X ∩ Y) = ?

Now, n (X ∪ Y) = n (X) + n (Y) - n (X ∩ Y).

Then, 38 = 17 + 23 - n(X ∩ Y)

⇒ n(X ∩ Y) = 17 + 23 - 38 = 2.

### Ordered Pair

Let A and B be two non-empty sets. If a ∈ A and b ∈ B, an element of the form (a, b) is called an ordered pair, where 'a' is regarded as 'the first element' and 'b' as the second element. It is evident from the definition that

1. (a, b) ≠ (b, a)

2. (a, b) = (c, d) if and only if a = c and b = d.

### Equality of two ordered pairs.

Two ordered pairs (a, b) and (c, d) are said to be equal if and only if a = cand b = d. The ordered pairs (2, 4) and (2, 4) are equal while the ordered pairs (2, 4) and (4, 2) are different. The distinction between the set {2, 4} and the ordered pair (2, 4) must be noted carefully. We have {2, 4} = {4, 2} but (2, 4) ≠ (4, 2).

### CARTESIAN Product Of sets

Let, A and B be two non-empty sets. The cartesian product of A and B is denoted by A × B (read as 'A cross B') and is defined as the set of all ordered pairs (a, b), where a ε A and b ε B. Symbolically,A × B = {(a, b) : a ε A and b ε B}

Example 22: Suppose, A = {2, 4, 6} and B = {x, y}Then,

A × B = {(2, x), (4, x), (6, x), (2, y), (4, y), (6, y)}

B × A = {(x, 2), (x, 4), (x, 6), (y, 2), (y, 4), (y, 6)}

Thus, we note that if A ≠ B, then A × B ≠ B × A.

Example 23: Let, A = {1, 2, 3} and B = f. Then, A × B = f, as there will be no ordered pair belonging

to A × B. Thus, we note that A × B = f if A or B or both of A and B are empty sets.

Example 24: Let, n(A) represents the number of elements in set A. In Example 22, we can see that n(A) = 3, n(B) = 2 and n(A × B) = 6. Thus, we note that n(A × B = n(A) × n (B).In other works, if a set A has m elements and a set Bh as n elements, then A × B has mn elements. Further, it may be noted that n(A × B) = n(B × A). This implies that A × B and B × A are equivalent sets.

Example 25: If there are three sets A, B, C and a∈ A, b ∈ B, c ∈ C, we form an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the cartesian product of the sets A, B, C. That is, × B × C = {(a, b, c) : a ∈ A, b ∈ B, c ∈ C}.

## 11.11 SYLLOGISMS: Deductive Reasoning

**Remember:** There is a difference between asserting that a premise is untrue, and asserting that the logic of the argument is faulty. "All dogs can fly. Fido is a dog. Fido can fly." That is a perfectly valid argument in terms of logic, but this flawless logic is based on an untrue premise. If a person accepts the major and minor premises of an argument, the conclusion follows undeniably by the sheer force of reason. If in an argument, the logic reaches a conclusion that seems absurd, it behooves you to analyze each sentence separately (to see if each premise is true without exception) and then to analyze the structure of the argument (to see if the reasoning of the argument itself is valid). Also be on the lookout for "equivocation," the use of two different meanings of one word during the process of an argument.

**Directions:** Decide whether the following syllogisms are valid in format (as opposed to using

true premises). If one is invalid, either explain why, or illustrate using circles.

**Invalid example:** All snakes are cold-blooded.

All snails are cold-blooded.

All snails are snakes.

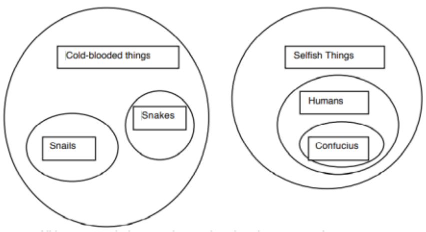
**INVALID:** Snakes and snails could overlap as categories, but they both could also premises are true, but if we accept these be in the largest circle without overlapping.

**VALID.** We might not agree that the individual premises are true, but if we accept these premises, the conclusion does logically follow.

**Valid example:** All humans are selfish.

Confucius is a human.

Confucius is selfish.




1)All human societies are doomed to deteriorate over time.

America is a human society.

America is doomed to deteriorate over time.

2)No philosophers are evil.

All Greeks are philosophers.

No Greeks are evil.

3) All women are potential mothers.

Betty is a potential mother.

Betty is a woman.

4. All students are eligible for student government.

No teachers are eligible for student government.

No teachers are students.

5. All barbiturates are drugs.

Marijuana is not a barbiturate.

Marijuana is not a drug.

6. Pizza is a substance made of cardboard.

All substances made of cardboard are good to eat.

Pizza is a substance that is good to eat.

Example 26

**Statements Some cameras are radios**

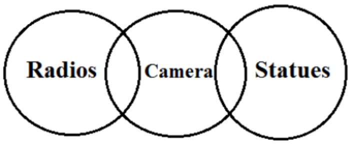
Some statues are cameras.

**Conclusions** I. Some radios are statues.

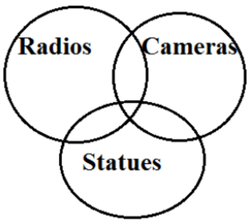
II. No radio is statue.

**Solution.** Either "some radios are statues" or "no radio is statue", as I and E-type propositions form a complementary pair.

We can draw all possible cases as given below




b) Hence using both diagrammatically representation, we can conclude either 'some radios are statues' or 'no radio is statue.' Hence, atleast, one of the conclusions must be true.




Special Cases

Facts

Combination (Conclusion) Conclusion

| | |
|---|---|
| A + O | Either I or II follows |
| E + I | Either I or II follows |
| I + O | Either I or II follows |

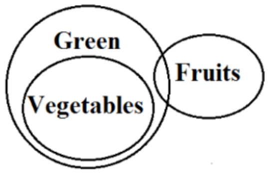
Example 27. Statements All vegetables are green.

Some greens are fruits.

**Conclusions** I. Some fruits are vegetables.

II. No fruit is vegetable.

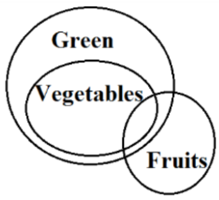
**Solution.** Here, Conclusion I is particular affirmative and Conclusion II is universal negative proposition. Hence, either Conclusion I or Conclusion II follows.




(b)

Conclusion If we follow venn diagram (a),then we can say no fruit is vegetable Conclusion II but if we follow venn diagram

(b), then we can say some fruits are vegetables. (Conclusion I) Here either venn diagram (a) or venn diagram (b) is possible. Hence, Conclusion I or Conclusion II must be followed. Minimal Possibilities We can represent statements by keeping in mind one conclusions. If we follow that our two conclusions belong to special case, then either one of them is true.




Example 28.

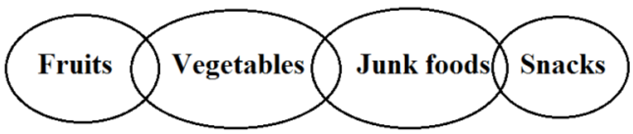
**Statements** Some fruits are vegetables.

Some vegetables are junk food.

**Conclusions** I. Some junk foods are vegetables.

II. Some junk food are fruits.

**Solution.**




It is clear the above diagram that only Conclusion I follows.

Possibility.

Possibility is a concept of inconsistency for an event which is not yet verified but if true would explain certain facts or phenomena. In other words capability of existing or happening or being true known as possibility

| Condition | Possibility |
|---|---|
| Given facts Imaginary facts | Cannot be determined Can be determined |

It is so simple to understand through the above table that possibility exists where no definite relation occurs between the objects and definite relation between the objects eliminate existence of any possibility. In simple way given with an example which will also clear the term possibility.

Example 29

**Statements** Some birds are trees.

Some trees are hens.

**Conclusions** I. Some birds being hens is a possibility.

II. All trees being hens is a possibility.

**Solution.** In Conclusion I, before deciding the possibility between birds and hens, we must notice the relation between both, we find that there is no relation between birds and hens, so possibility favours the condition and the Conclusion I is true for possibility and in Conclusion II we must notice the relation between trees and hens. We find that both have some type of relation between them so the possibility of 'All' between trees and hens is true. Hence, both the Conclusions I and II follows.

| Given Exclusive Proportion | Desired Proportions | Possibility |
|---|---|---|
| All | All | No |
| Some | Some | No |
| No | No | No |
| No | Some not | No |
| Some | All | Yes |
| No proper relations | Some All | Yes |

Note Improper relation between two objects favours the possibility (In above example Conclusion I)

Special Cases of Exclusive Proposition

| If the statement is of | Conversion | Example | Meaningful Conversion |
|---|---|---|---|
| Much, more, many, very, a few, most,<br>almost<br>Atleast<br>Definitely<br>Only<br>1% to 99% | Some<br>Some<br>No use<br>Some | Most A are B.<br>A few X are Y.<br>Atleast some A are B.<br>Some A are definitely<br>B.<br>Some X are definitely<br>not Y.<br>Only A are B.<br>38% A are B. | Some A are B.<br>Some X or Y.<br>Some A are B.<br>Some A are B.<br>Some X are not Y.<br>All B are A.<br>Some A are B.<br>Some X are Y. |

| | | | |
|---|---|---|---|
| | | 98% X are Y. | |

## Summary

The key concepts learned from this unit are: -

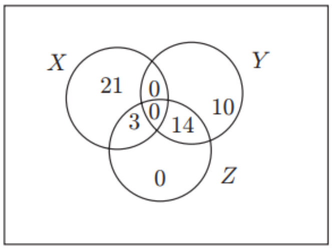
- We have learnt how to show the relationships among things or finite groups of things.
- We have learnt what are sets
- We have learnt what are different types of sets
- We have learnt what are operations on a sets
- We have learnt what is Syllogism.

## Keywords

- Logical Venn diagram
- set theory
- Syllogism

## Self Assessment

Direction (Q1−Q5): Solve the examples on the basis of the following diagram:




Note: X represents even numbers, Y represents odd numbers and Z represents prime numbers in the sample.

1. How many even numbers are there which are not prime?

A. 21

B. 22

C. 24

D. 45

2. How many odd numbers are there which are prime?

A. 21

B. 14

C. 25

D. 49

3. How many even numbers are prime?

A. 1

B. 12

C. 3

D. 4

4. How many numbers are present which are not prime?

A. 31
B. 12
C. 32
D. 42

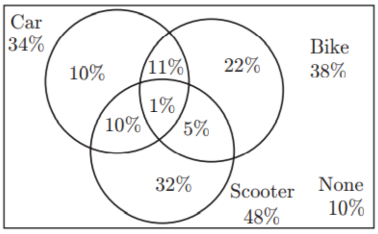
5. How many total odd numbers are present?

A. 30
B. 12
C. 33
D. 24

Direction (Q6—Q10): In a survey conducted for a society, 10% of the houses have just car, 22% have

just bike, 32% have just scooter, 1% have a car, bike and scooter, and 10% have none of the three

things. So, 40 houses do not have any vehicle.




Say x is (car ∩ bike), y is (car ∩ scooter) and z is (bike ∩ scooter). Therefore,

31 = 10+ x+ 1

38= 22+x +1+z

48= 32 +y +1+z

Solving the above equations for x, y and z, we get

X = 10%

Y=10%

Z=5%

Also, total houses surveyed = $\frac{100 \ x \ 40}{10}$ = 400

6. How many houses have bikes only?

A. 35
B. 10
C. 33
D. 88

7. How many houses have exactly two vehicles? Solution: Total houses with more than one vehicle = 25%

A. 36
B. 96
C. 39

D. 80

8. How many houses have only cars?

A. 30
B. 12
C. 39
D. 96

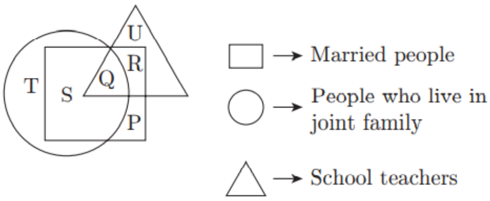
9. How many houses have only one vehicle?

A. 35
B. 100
C. 256
D. 240

10. How many houses do not have a scooter?

A. 168
B. 102
C. 340
D. 812

Direction (Q11 – Q13): The following Venn diagram depicts people living in a building who are married, who live in joint family and who are school teacher. Study the chard carefully and answer the following.




11. By which letter, the married teachers who live in joint family are represented?

A. U
B. R
C. P
D. Q

12. By which letter, the married people who live in joint family but not are school teachers are represented?

A. S
B. U
C. R
D. P

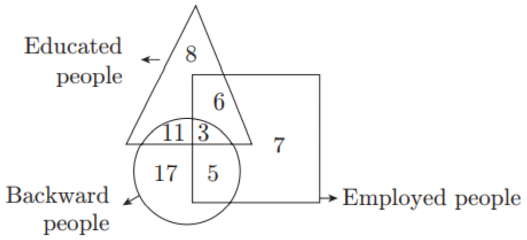
13. The people who live in joint family but are neither married nor teachers are represented by which letter?

A. S
B. U

C. R

D. T

Direction (Q14 – Q15): The following Venn diagram depicts people who are educated, backward and employed. Study the diagram carefully and answer the following.




14. How many educated people are employed?

A. 9

B. 10

C. 12

D. 15

15. How many backward people are educated?

A. 32

B. 11

C. 14

D. 50.

16. Statements:

Some ropes are walls.

Some walls are sticks.

All sticks are chairs.

All chairs are tables.

**Conclusions:**

I. Some tables are walls.

II. Some chairs are ropes.

III. Some sticks are ropes.

A. None follows

B. Only I follows

C. Only II follows

D. Only III follows

17. Statements:

Some pens are knives.

All knives are pins.

Some pins are needles.

All needles are chains.

**Conclusions:**

I. Some chains are pins.

II. Some needles are knives.

III. Some pins are pens.

A. Only I follows
B. Only II follows
C. Only III follows
D. None of these

## Answer for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | B | 3. | C | 4. | A | 5. | D |
| 6. | D | 7. | B | 8. | D | 9. | C | 10. | A |
| 11. | D | 12. | A | 13. | D | 14. | A | 15. | C |
| 16. | B | 17. | D | | | | | | |

## Review Questions

1. Specify the set A by listing its elements, where

A = { whole numbers less than 100 divisible by 16 }.

b Specify the set B by giving a written description of its elements, where

B = { 0, 1, 4, 9, 16, 25 }.

c Does the following sentence specify a set?

C = { whole numbers close to 50 }.

C = { whole numbers close to 50 }.

2. Use dots to help list each set, and state whether it is finite or infinite. i A = { even numbers between 10 000 and 20 000 } ii B = { whole numbers that are multiples of 3 }
3. If the set S in each part is finite, write down | S |. i S = { primes } ii S = { even primes } iii S = { even primes greater than 5 } iv S = { whole numbers less than 100 }
4. Let F be the set of fractions in simplest form between 0 and 1 that can be written with a single-digit denominator. Find F and | F |.
5. Rewrite in set notation:
   i All squares are rectangles.
   ii Not all rectangles are rhombuses.
6. Rewrite in an English sentence using the words ‘all’ or ‘not all’:
   i { whole number multiples of 6 } □ { even whole numbers }.
   ii { square whole numbers } □ { even whole numbers }.
7. Rewrite the statements in part (6) in an English sentence using the words ‘if …, then’. d Given the sets A = { 0, 1, 4, 5 } and B = { 1, 4 }:
   i Draw a Venn diagram of A and B using the universal set U = { 0, 1, 2, … , 8 }.

ii Graph A on the number line.

8. Let the universal set be E = {whole numbers less than 20 }, and let A = { squares less than 20 } B = { even numbers less than 20 } C = { odd squares less than 20 }
   i) Draw A and C on a Venn diagram, and place the numbers in the correct regions.
   ii) Draw B and C on a Venn diagram, and place the numbers in the correct regions.
   iii) Shade $A \cap B$ on a Venn diagram, and place the numbers in the correct regions.
   iv) Shade $A \cup B$ on a Venn diagram, and place the numbers in the correct regions.
9. Draw a Venn diagram of two sets S and T b Given that $|S| = 15$, $|T| = 20$, $|S \cup T| = 25$ and $|E| = 50$, insert the number of elements into each of the four regions. c Hence find $|S \cap T|$ and $|S \cap T^c|$.
10. Twenty-four people go on holidays. If 15 go swimming, 12 go fishing, and 6 do neither, how many go swimming and fishing? Draw a Venn diagram and fill in the number of people in all four regions.
11. In a certain school, there are 180 pupils in Year 7. One hundred and ten pupils study French, 88 study German and 65 study Indonesian. Forty pupils study both French and German, 38 study German and German only. Find the number of pupils who study: a all three languages b Indonesian only c none of the languages d at least one language e either one ot two of the three languages

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

# Unit 14: Data interpretation

CONTENTS



## Objective

The key concepts explained in this unit, including area a learner is expected to learn and master after going through the unit are: -

- We will understand simplest way to represent data.
- We will understand how to show a particular data line graph
- We will understand how to visualize the presentation of categorical with bar diagram.
- We will understand how to use table to represent data.
- We will understand how to use pie chart to represent data.

## Introduction

A line graph (or line chart) is a graph which displays information as a series of data points connected by straight line segments. A line graph shows a particular data that change at equal intervals of time.

Line graph is the simplest way to represent data. Single set or multiple sets of data can be shown in a graph. Figure 1 shows a line graph representing average daily temperature for two states, A and B.

Usually, we can analyze the following things using line graphs:

1. Increase in profit in absolute terms or in percentage terms
2. Average annual growth rate.
3. Average profit
4. Capability utilization.

   Capacity utilization =

   $capacity\ utilization\ = \frac{total\ production}{total\ capacity}$ X 100

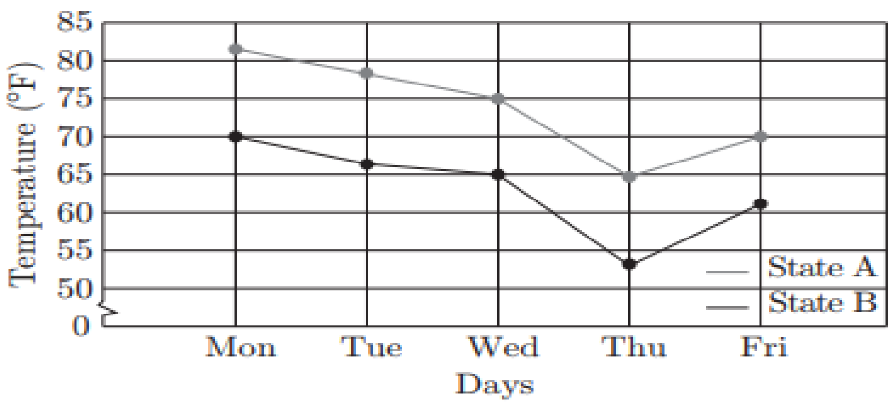



**Figure 1** | Line graph representing average daily temperature for two states.

Example 1:- The total capacity of a Hyundai i10 plant is 150 cars per day. In the month of April 2009, the plant manufactured at the rate of 120 cars per day. Find the capacity utilization in the month of April.

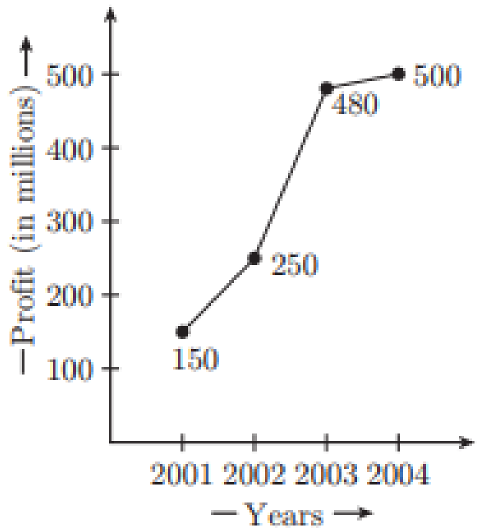
Solution : We know that capacity utilization is given by

Capacity utilization = $\frac{Total\ production}{total\ capacity}$ x 100

= $\frac{120}{150}$ x 100

= 80%

Example 2: Direction for Q2 and Q3: Consider the following graph:




Balance sheet of XYZ Corporation

2. What was the percent increase in profit in 2001—02 and 2003—04?

Solution: Percent increase in profit in 2001—2002

= $\frac{250-150}{150}$ x 100

= 66.67%

Present increase in profit in 2003-04

$= \frac{500-480}{480} \times 100$

= 4. 16 %

3. What was the average annual growth rate for XYZ Corporation?

Solution: Average annual growth rate

$= \frac{increase\ in\ profit\ for\ duration}{Base\ years\ profit} \times \frac{100}{number\ of\ years}$

$= \frac{500-150}{150} \times \frac{100}{3}$

$= \frac{350}{150}\, x\, \frac{100}{3}$

= 66.67%

Example 3:- Direction for Q4 and Q5: Consider the following graph

The above graph is showing consumption of metals versus plastics in the given years (2006—11) for car manufacturing.

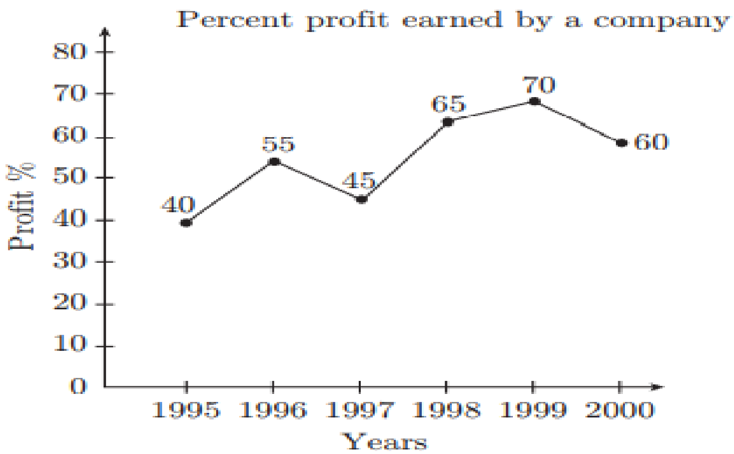
4. Which item (and for which year) shows the highest percentage change in consumption over the previous year?

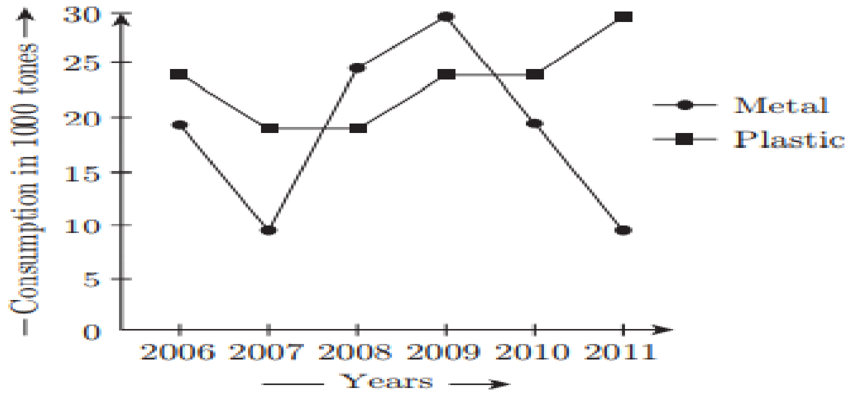
Solution: Percentage change in consumption is highest for metals in 2008. 5.

5. Find out the number of years for which the consumption of metals was less than the consumption of plastic over the given time period.

Solution: For 2006, 2007, 2010 and 2011, the consumption of metals was less than plastics. Hence, for four years the consumption of metals was less than plastics.

Example 4:- Direction for Q6—Q8: Consider the following graph:






6. If the expenditures in 1996 and 1999 are equal, then what is the approximate ratio of the income in 1996 and 1999, respectively? Solution: Let the expenditure in 1996 be x.

Also, let the incomes in 1996 and 1999 be a and b, respectively.

Then, for the year 1996, we have

$\frac{a-x}{x}$ x 100

55

a = $\frac{155\,x}{100}$

$\frac{b-x}{x}$ x 100 = 70

b = $\frac{170\,x}{100}$

From equ (1) and (2) we get

$\frac{a}{b} = \frac{(155\,x/100)}{(170\,x/100)}$

$= \frac{155}{170} \approx 9$

7. What is the average profit earned for the given years?

Solution: Average profit for the given years

$= \frac{40+55+45+65+70+60}{6}$

$= \frac{335}{6}$

= 55.83

8. If the income in 1998 was ₹264 crores, what was the expenditure in 1998?
Solution : Let the expenditure is 1998 be ₹ x crores.
$\frac{264\,-x}{x}$ x 100
= 65
X = $\frac{264x\;100}{165}$
160
Expenditure in 1998 = ₹ 160 $crores$.

## 14.1 Bar Diagram

A bar diagram or bar graph is a chart with rectangular bars in which the lengths of bars are in proportion to the values of the entities they represent.

A vertical bar chart is called a column bar chart (Fig. 1a) and a horizontal bar chart is called a row bar chart (Fig. 1b).

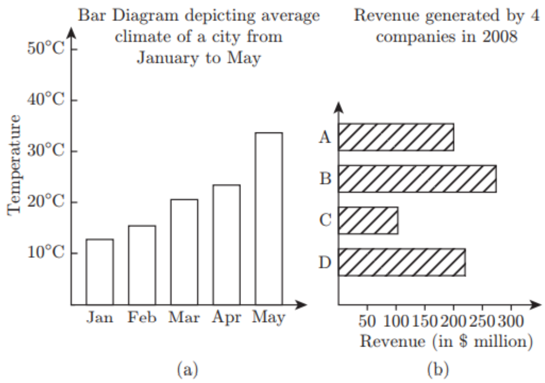



**Figure 1** | Bar diagram: (a) Column bar chart and (b) row bar chart.

Bar diagrams are often used to provide visual presentation of categorical data and are considered very reliable. Bar graphs can also be used for comparison of complex data. Also, a bar chart is very useful for displaying discrete data values.

Multiple sets of interrelated data can be represented using a multiple bar diagram. It is used to compare more than one quantity. The technique of a simple bar diagram is used to draw the diagram. However, different quantities are represented using different shapes, sizes, colours, patterns, etc. Figure 2 shows a regular multiple bar diagram.

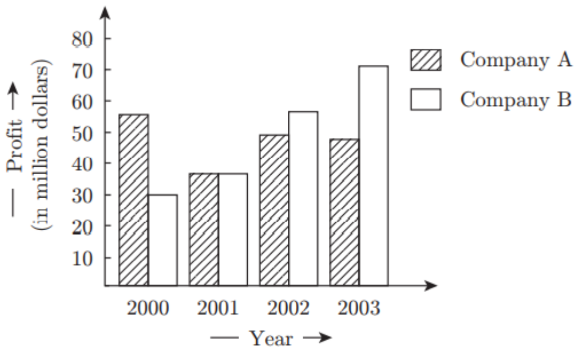



**Figure 2** | Multiple bar diagram.

A compound bar diagram is used to combine or compare two or more types of information in one chart. The bars are stacked on top of one another. However, different quantities are represented using different shapes, sizes, colours, patterns, etc. Figure 3 shows a regular compound bar diagram

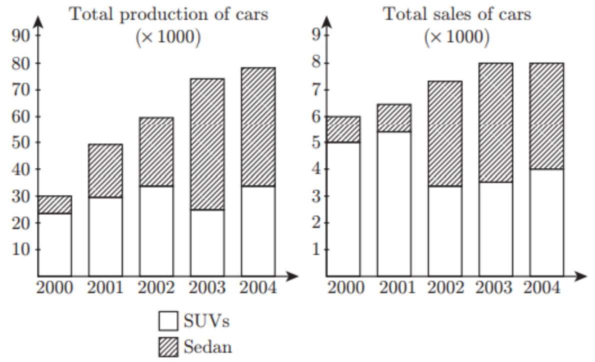



**Figure 3** | Compound bar diagram.

A bar chart may be drawn on a percentage basis. To obtain a percentage bar diagram, bars of length equal to 100 for each class are drawn and subdivided in the proportion of the percentage of their component. Figure 4 shows a regular percentage bar diagram

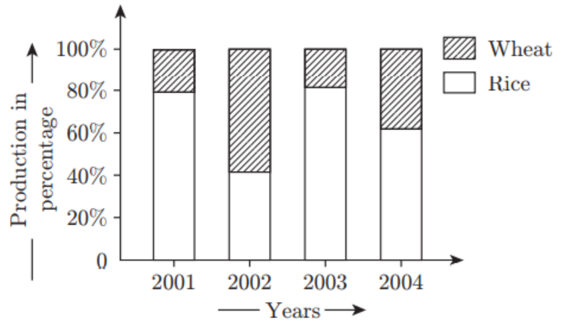
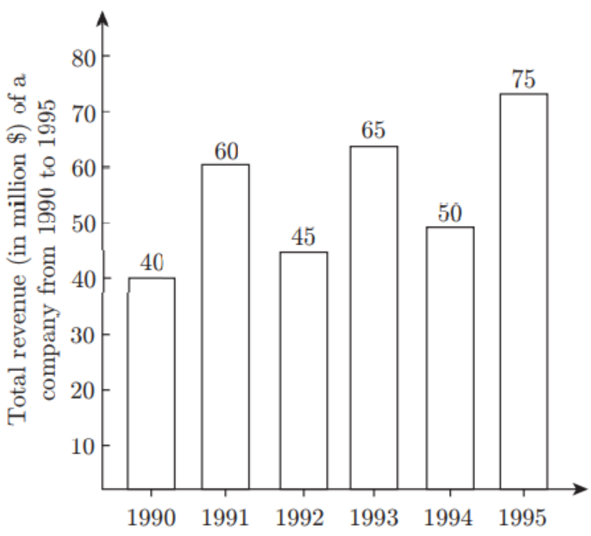



**Figure 4** | Percentage bar diagram.

Example 5:- Direction for Q1-Q4: Study the following bar diagram carefully and answer the questions.



Which pair of years had the same average revenue as that of 1990 and 1995? Solution: Total revenue in 1990 = $ 40 million Total revenue in 1995 = $ 75 million

Average revenue in 1990 and 1995 = $\frac{40+75}{2}$ $ 57.5 million

Total revenue in 1993 = $ 65 million

Total revenue in 1994 = $ 50 million Average revenue for 1993 and 1994 = $\frac{65+50}{2}$ $ 57.5 million Hence, average revenue of 1993 and 1994 was same as 1990 and 1995.

2. In which year(s), the percentage increase in production was maximum from the previous year? Solution: Percentage increase from 1990 to 1991 is

$\frac{60-40}{40}$ x 100

$\frac{20}{40}$ x 100

= 50%

Also, percentage increase from 1994 to 1995 is

$\frac{75-50}{50}$ x 100 = 50%

Thus, percentage increase was maximum in 1991 and 1995.

3. . What was the percentage drop in the revenue from 1991 to 1992?

Solution: Total revenue in 1991 = $ 60 million Total revenue in 1992 = $ 45 million

Percentage drop was

$\frac{60-45}{60}$ x 100

= $\frac{15}{60}$ x 100

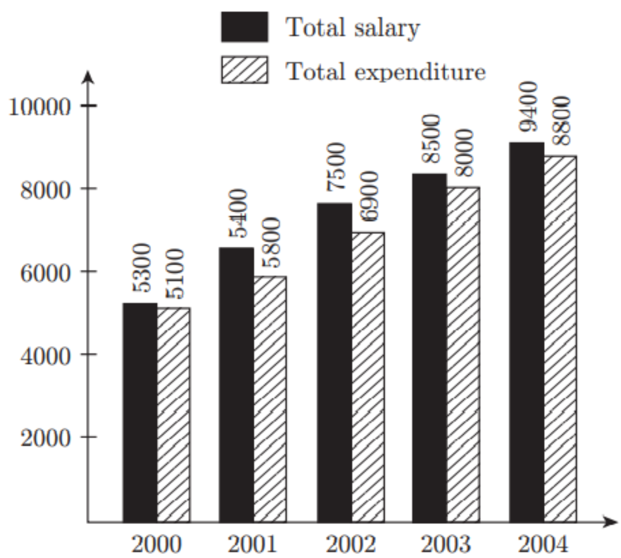
= 25%

4. What was the difference between revenue in 1995 and 1990?

Solution: Total revenue in 1995 = $ 75 million Total revenue in 1990 = $ 40 million

Difference in revenue = 75 - 40 = $ 30 million

Example 6:- Direction for Q5-Q7: Study the following bar diagram carefully and answer the questions.



5. What was the percentage increase in the "Total Salary" in 2002 as compared to 2000?

Solution: Total salary in 2002 = 7500

Total salary in 2000 = 5300

Percentage increase was $\frac{7500-5300}{7500}$ x 100 = 5.88%

6. If profit = total salary - expenditure, then in 2003 what percentage of total salary was the profit made?

Solution: Total salary in 2003 = 8500

Total expenditure in 2003 = 8000

Total profit = 500

Profit % = $\frac{500}{8500}$ x 100 = 5. 88%

7. . By what amount had the expenditure increased over the period 2000-2004?

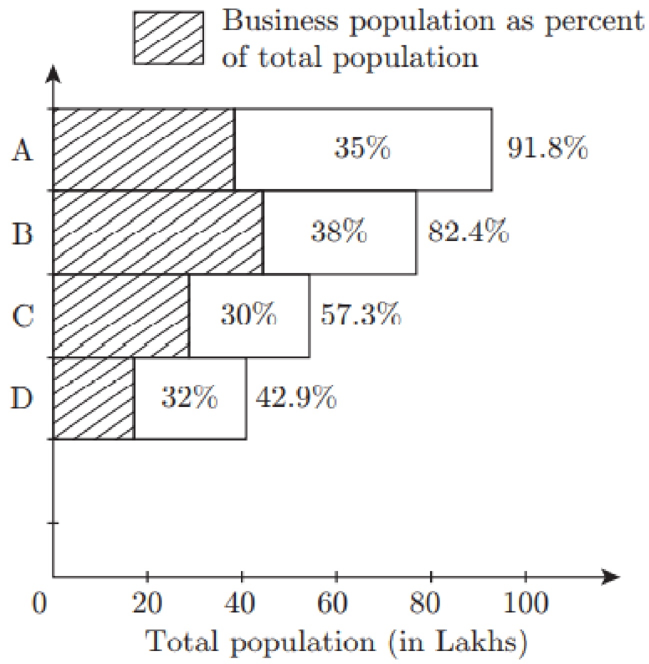
Solution: Total expenditure in 2000 = ₹5100

Total expenditure in 2004 = ₹8800

Total increase in expenditure from 2000-2004 was

8800 − 5100 = ₹3700

Example 7:- Direction for Q8-Q10: Study the following bar diagram carefully and answer the questions.




8. What is total business population of A?

Solution: Total population of A = 91.8 lakh

Total business population = 35%

$\frac{35}{100}$ x 9.18 lakh = 32.13 lakh

9. What place has the highest business population?

Solution: Business population of A = $\frac{35}{100}$ x 9.18 lakh = 32.13 lakh

Business population of B = $\frac{38}{100}$ x 8.24 lakh = 31.31 lakh

Business population of C = $\frac{30}{100}$ x 57.3 lakh = 17.19 lakh

Business population of D = = $\frac{32}{100}$ x 42.9 lakh = 13.728 lakh

Hence, A has the highest business population.

10. What is the difference between business populations of C and D?

Solution: Business population of C = $\frac{30}{100}$ x 57.3 lakh = 17.19 lakh

Business population of D = = $\frac{32}{100}$ x 49.9 lakh = 13.728 lakh

Difference between business population of C and D = 17.19 - 13.728 = 3.462 lakh

## 14.2 Cubes and Dices

A cuboid is any three-dimensional figure with length, breadth and height. If all the three sides of a cuboid are same, then the figure is called a cube. Figure 1 shows a Rubik's cube which is a cube made of 27 smaller cubes.

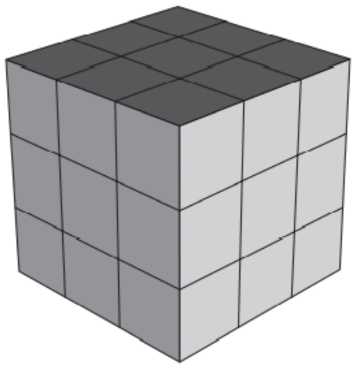


Figure 1 | Rubik's cube

In a cube:

1. There are 8 corners/vertices.

2. There are 6 faces, all equal in area.

3. There are 12 edges, all equal in area.

4. It should be noted that

Number of edges Number of vertices Number of face = + − 2

Some important tips to solve questions based on cubes are as follows:

If we paint a cube of the dimension nxnxn in any one colour and cut it to get $n^3$ cubelets, then the number of cubes with only one face painted = $(n-2)^2$ x 6

2. If we paint a cube of the dimension nxnxn in any one colour and cut it to get $n^3$ cubelets, then the number of cubes with two faces painted = $(n-2)^2$ x 12

3. If we paint a cube of the dimensions nxnxn in any one colour and cut it to get $n^3$ cubelets, then the number of cubes with three faces painted = $(n-2)^3$

## 14.3 Dices:-

Dices are small cubical structures with numbers 1 to 6 (or dots) marked on each face.

When sum of number of opposite pair of faces is same, then the dice is called a symmetric dice. Hence, for a symmetrical dice, the faces opposite to each other will be 1→6, 2→5, 3→4. Figure 2 shows symmetrical dices.

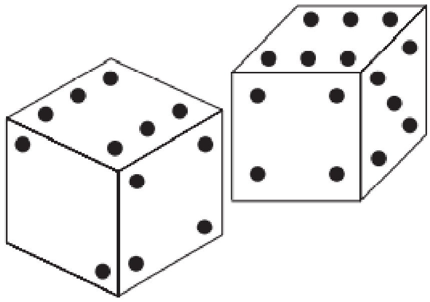


Figure 2 | Symmetric dices.

When sum of numbers on opposite pair of faces is different, then the dice is called an asymmetric dice. Figure 3 shows asymmetrical dices.

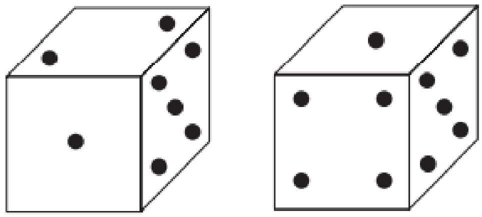


Figure 3 | Asymmetric dices.

Solved Example :-

Example 8: Direction for Q1—Q5: If a cube is cut into $n^3$ identical cubelets using minimum number of cuts, after painting all faces of cubes with white colour, then answer the following question

1. What is the maximum number of cuts required?

   Solution: There are a minimum of $(n - 1)$ equidistant cuts parallel to each of three faces which are joining the corner. Hence, total number of cuts required is $3(n - 1)$.

2. How many cubelets will have exactly two faces painted?

   Solution: Cubes with exactly two faces painted = $12(n - 2)$, where n is the number of parts into which side is divided.

3. How many cubelets will have exactly one face painted?

   Solution: Cubes with exactly one face pained = $6(n - 2)^2$ , where n is the number of parts into which side is divided.

4. How many cubelets will have at most two faces painted?

   Solution: Total number of cubelets with at most two faces painted = Total number of cubes —

   Number of cubes with three faces painted=Total number of cubes = $n^3$

   Number of cubes with three faces painted = 8

   Hence, total number of cubelets with at most two faces painted = $n^3 - 8$

5. How many cubelets will have at least one face painted?

   Solution: Total number of cubelets with at least one face painted = Total number of cubes

   Number of cubes with no-face painted.

   Number of cubes with no face painted = $(n-2)^3$

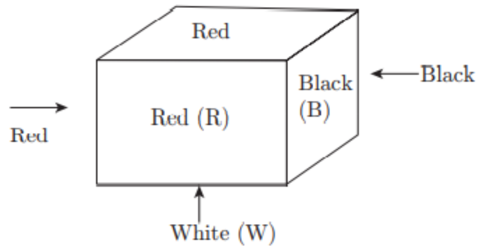
   Hence, total number of cubelets with at least one face painted = $n^3 - (n-2)^3$

Three adjacent faces of one cube are painted in red, one adjacent pair of faces is painted in black and remaining faces are painted in white.

The cube is then cut into 216 identical cubelets.

Example 9: Direction for Q6—Q10: A cube is cut into 216 cubelets = 6 x 6 x 6

We can draw the pattern of painting as shown in the following figure:




Now, let us analyze the colour combination.

Corners:

RRR = 1, BBR= 1

RRB = 2, BBW=1

RRW = 1, RWB = 2

Edges:

RR = 3, RW = 2

RB= 3, BW = 2

BB = 1

Faces:

R = 3 B = 2 W = 1

6. How many cubelets have all the three colour on them?

   Solution: The small pieces will be formed from the corners. The cubelets having all three colours (red, black and white) are 2.

7. How many cubelets have exactly two colour on them?

   Solution: The cubelets with exactly two colour found at corners = 5

   Total edges having different colour on either side = 8

   Therefore, total cubelets = 8(6-2)+5 = 37

8. How many cubelets have exactly one colour on them?

   Solution: The cubelets having exactly one colour at corner (RRR) = 1

   At the three edges each of RR categories = 3(6-2) = 12

   At one edge of BB category =1(6-2) = 4

   At middle of each of the six faces = 6(6-2)2 = 96

   Hence, total cubelets with exactly one colour = 1 + 12 + 4+ 96 = 113

9. How many cubelets do not have red colour on them?

Solution: The number of cubelets with no red colour = Total cubes – (Total cubelets from one red surface + Total cubelets from two red surface + Total cubelets from three red surfaces)= + 216 – (36 + 30 + 25 ) = 125

10. How many cubelets have black or white colour on them but not red colour on them?

    Solution: The number of cubelets with no red colour = 125

    The number of cubelets with black or white but not red = 125 – ( 6 – 2)$^3$ = 125 -64 = 61

## 14.4 A table

A table is used to represent data in rows and columns.

Tables are arguably the most common way of arranging a given data. Evidently, tables are used almost everywhere, in print media, books, presentation slides, etc.
Tables differ significantly in variety, structure, flexibility, notation, representation and use.

Tables are advantageous to us as they can arrange a large amount of data easily and without any complexity.

However, locating pattern or visual observations are difficult.

Example 10: Direction for Q1—Q5: The following table shows sales of cars of five companies over the years (in thousand).

| Company | Years | | | | |
|---|---|---|---|---|---|
| | 2000 | 2001 | 2002 | 2003 | 2004 |
| Honda | 20 | 21 | 50 | 35 | 75 |
| Hyundai | 29 | 31 | 23 | 46 | 42 |
| Maruti | 31 | 29 | 27 | 22 | 16 |
| Tata | 33 | 14 | 33 | 37 | 48 |
| Skoda | 15 | 17 | 32 | 39 | 47 |
| Total | 128 | 112 | 165 | 179 | 228 |

Consider the table and answer the following questions.

1. For which company did the amount of sales of cars increase continuously over the years?

   Solution: For Skoda, we can see that the sales of each year are always greater than the previous year.

2. For which company did the amount of sales of cars decrease continuously over the years?

   Solution: For Maruti, we can see that the sales of each year are always lesser than the previous year.

3. In 2001, which company had the biggest increase in sales from 2000?

   Solution: Three companies had more sales in 2001 than 2000.

   Percentage increase for Honda = $\frac{21-20}{20}$ x 100 = 5%

   Percentage increase for Hyundai = $\frac{31-29}{29}$ x 100= 6.9%

   Percentage increase for Skoda = $\frac{17-15}{15}$ x 100= 13.33%

   Hence, Skoda had the biggest percentage increase in sales of 2001.

4. In which year did Maruti have the largest percentage decrease in sales? Solution:

   Percentage decrease in sales in 2001 = $\frac{31-29}{20}$ x 100 = 6.45%

   Percentage decrease in sales in 2002 = $\frac{29-27}{27}$ x 100 = 18.52%

   Percentage decrease in sales in 2003 = $\frac{22-16}{16}$ x 100 =18.52%

Percentage decrease in sales in 2004 = $\frac{22-16}{16}$ x 100 = 27.27%

Hence, in 2004, the decrease in sales for Maruti was the highest

5. Which company sold the most number of cars in the five years?

Solution: Total sales of Honda = 20 + 21 + 50 + 35 + 75 = 201000

Total sales of Hyundai = 29 + 31 +23 + 46 + 42 = 171000

Total sales of Maruti = 31 + 29 + 27 + 22 + 16 = 125000

Total sales of Tata = 33 + 14 + 33 + 37 + 48 = 165000

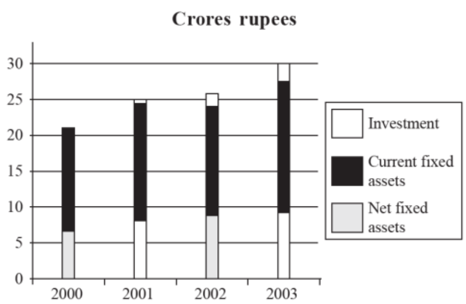
Total sales of Skoda = 15 + 17 + 32 + 39 + 47 = 150000

Hence, Honda had the highest sales from 2000 to 2004.

Example 11: Direction for Q6—Q10: The following table shows total revenue of five companies over the years (in $ millions).

## 14.5 Cumulative Bar Graph

In a cumulative bar graph, the length of the bar is divided proportionately among various quantities represented in the graph. Thus, it may be conveniently used for making comparisons, Example is as follows




## 14.6 Pie Charts

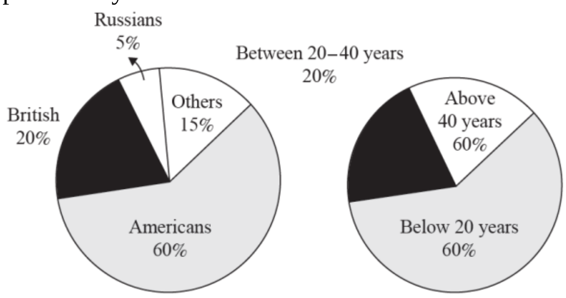
In a pie chart, the given data is distributed over a circle. Each part of the data makes a certain central angle.

For example, if from all the questions asked in a Bank PO exam, 25% are on data interpretation, then central angle made by this tern

= (25/100)X360 °= 90°

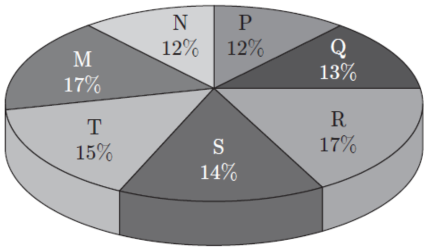
Pie charts are useful for representing percentages or proportions of variou s elements with respect to the total quantity. They also represent shares of various parts of a particular quantity.

Example **:** The following pie charts describe the characteristics of foreign tourists visiting India in a particular year.

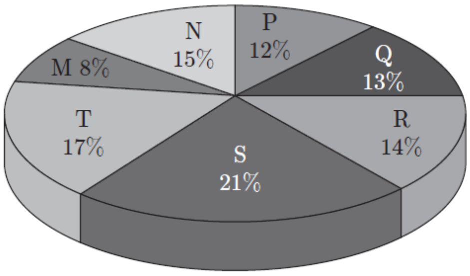



Distribution of students at graduate and post graduate levels in seven institutes (M, N, P, Q, R, S and T).

Total number of students of graduate level = 27300.




Total number of students of post graduate level = 24700.




1. How many students of institutes of M and S are studying at graduate level?

Solution: Students of institute M at graduate level =
(17/100)×27300 = 4641
Students of institute S at graduate level =
(14/100)×27300 =3822
= 14/100×27300= 3822
Total number of students at graduate level in institutes
M and S = 4641 + 3822 = 8463.

2. What is the total number of students studying at postgraduate level from institutes N and P?

Solution: Students studying at postgraduate level from N= (15/100)×24700 = 3705.
Students studying at postgraduate level from P = (12/100)×24700= 2964.
Required number = 3705 + 2964 = 6669.

3. What is the ratio between the number of students studying at postgraduate and graduate levels from institute S?

Solution: Number of postgraduate students from institute S= (21/100)×24700 = 5187
Number of graduate students from institute S =(14/100)×27300 = 3822
Required ratio= (5187/3822) = 19/14.

## Summary

The key concepts learned from this unit are: -

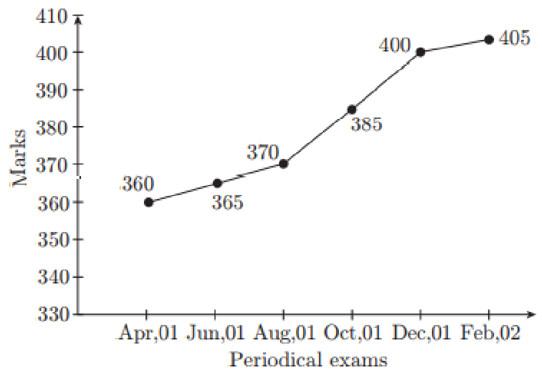
- We have learnt how t We will understand how to show a particular data line graph
- We have learnt how to visualize the presentation of categorical with bar diagram.
- We have learnt how to use table to represent data.
- We have learnt how to use pie chart to represent data.

## Keywords

- Bar Diagram
- Line graph
- Dices
- Pie Diagram

## Self Assessment

Direction for Q1 and Q3: Consider the following graph:




The above graph shows marks obtained by a student in six periodicals held in every two months during the year in the session 2001 – 2002. In addition, the maximum total marks in each periodical exam are 50

1. What is the percentage of marks obtained by the student in the periodical exams of August, 01 and October, 01 taken together?

A. 75.5%

B. 72%

C. 30%

D. 45%

2. What are the average marks obtained by the student in all the periodical exams during the last session?

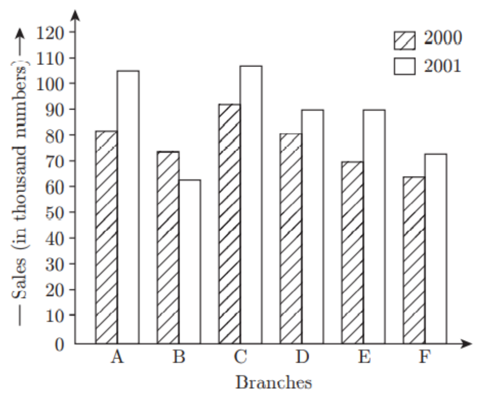
A. 380.83

B. 340

C. 370

D. 289

Direction for Q3-Q7: Study the following bar diagram carefully and answer the questions.




3. What is the ratio of the total sales of branch B for both years to the total sales of branch D for both years?

A. Required ratio = $\frac{140}{180} = \frac{7}{9}$

B. Required ratio = $\frac{140}{180} = \frac{7}{10}$

C. Required ratio = $\frac{143}{180} = \frac{7}{9}$

D. Required ratio = $\frac{140}{190} = \frac{7}{9}$

4. What is the total sale of branches A, C and E together for both the years (in thousands)?
   Solution: Total sales of branches A, C and E for both the years (in thousands)

A. 660

B. 560

C. 760

D. 860

5. What percent of the average sales of branches A, B and C in 2001 is the average sales of branches A, C and F in 2000?

A. 77.5%

B. 87.5%

C. 88.5%

D. 98.5%

6. Total sale of branch F for both the years is what percent of the total sales of branch C for both the years?

A. 70.5%

B. 12%

C. 72%

D. 73.17%

7. What is the total number of red faces?

A. 120

B. 720

C. 620

D. 320

9. How many cubes have two adjacent blue faces each?

A. 2 cubes

B. 62 cubes

C. 64 cubes

D. 60 cubes

10. How many cubes have only one red face each?

A. only one face red.

B. only two face red.

C. only three face red.

D. only four face red.

11. Which two colors have the same number of faces?

A. First 64 cubes are such that each of whose two faces are green and second 64 cubes are such that each of whose two faces are blue. Therefore, green and blue colour have the same number of faces.
B. First 64 cubes are such that each of whose two faces are red and second 64 cubes are such that each of whose two faces are blue. Therefore, red and blue colour have the same number of faces.
C. First 64 cubes are such that each of whose two faces are green and second 64 cubes are such that each of whose two faces are red. Therefore, green and red colour have the same number of faces.
D. First 46 cubes are such that each of whose two faces are green and second 64 cubes are such that each of whose two faces are blue. Therefore, green and blue colour have the same number of faces.

Consider the table and answer the following questions.

| Company | Years | | | |
|---|---|---|---|---|
| | 2000 | 2001 | 2002 | 2003 |
| A | 20 | 17 | 8 | 18 |
| B | 10 | 12 | 16 | 20 |
| C | 17 | 15 | 4 | 2 |
| D | 6 | 16 | 3 | 9 |
| E | 18 | 30 | 10 | 11 |

12. What was the total revenue of company D from 2000 to 2003?
A. $ 34 million
B. $ 44 million
C. $ 54 million
D. $ 64 million

13. What was the revenue of all the companies in the year 2002?
A. $ 11 million
B. $ 21 million
C. $ 31 million
D. $ 41 million

14. From 2000 to 2003, in which company was there a continuous increase in revenue?
A. Company A had a continuous increase in revenue from 2000 to 2003.
B. Company B had a continuous increase in revenue from 2000 to 2003.
C. Company C had a continuous increase in revenue from 2000 to 2003.
D. Company D had a continuous increase in revenue from 2000 to 2003.

15. In which two years did company B and D have the same amount of revenue?
A. in 1999 and 2001.
B. in 2001 and 2002
C. in 2004 and 2005
D. in 2000 and 2001

16. What was the percentage decrease in total revenue from 2001 to 2003?

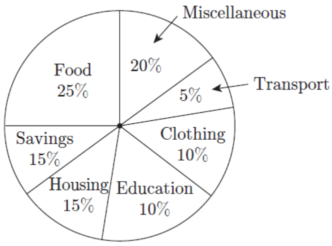
A. 33.3%

B. 53.3%

C. 63.3%

D. 73.3%

Direction for Q9—Q12: The following pie chart shows the percentage distribution of money spent by a family in August, 2013.




Consider the figure and answer the following questions.

17. If the total amount spent was Rs 120000, then what was the total amount spent on "housing"?

A. Rs 18000

B. Rs 21000

C. Rs 15000

D. Rs 24500

18. What is the ratio of the total amount of money spent on clothing to that on food?

A. 5 : 2

B. 2 : 5

C. 3 : 4

D. 4 : 3

19. How much of the total money was used for savings?

A. 36°

B. 48°

C. 52°

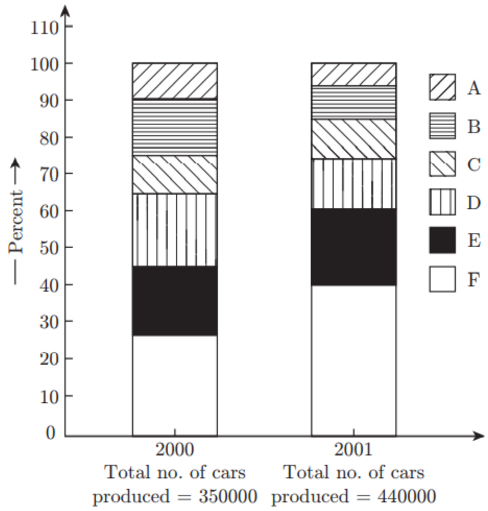
D. 54°

## Answers for Self Assessment

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | A | 2. | A | 3. | A | 4. | B | 5. | B |
| 6. | D | 7. | D | 8. | C | 9. | A | 10. | A |
| 11. | A | 12. | D | 13. | B | 14. | D | 15. | A |
| 16. | A | 17. | B | 18. | D | 19. | B | | |

## Review Questions

1. Direction for Q1-Q5: Study the following bar diagram carefully and answer the questions.




1. What is the difference in the number of E-type cars produced in 2000 and that produced in 2001?

Solution: Total number of E-type cars produced in 2001

= $\frac{(60-40)x\ 440000}{100}$ = 880000

Total number of E-type cars produced in 2000 = $\frac{(45-25)x\ 440000}{100}$ = 880000

Required difference = 88000 −77000 = 11000

2. If the percentage of production of F-type cars in 2001 were the same as that in 2000, then the number of F-type cars produced in 2001 would have been?

Solution: We are given that the percentage production of F-type cars in 2001 = percentage production of F-type cars in 2000 = 25%

Then, number of F-type cars produced in 2001 = 25% of 4,40,000 = 110000

3. Total number of cars of model B, E and F manufactured in 2000 is?

Solution: In 2000, total number of cars produced = 350000

a) F = (25 − 0)% of 350000 = $\frac{25}{100}$ x 350000 = 875000

b) E = (45 − 25)% of 350000 = $\frac{20}{100}$ x 350000 = 70000

c) D = (65 − 45)% of 350000 = $\frac{20}{100}$ x 350000 = 70000

d) c = (75 − 65)% of 350000 = $\frac{10}{100}$ x 350000 = 35000

e) B = (90 − 45)% of 350000 = $\frac{15}{100}$ x 350000 = 52500

f) A = (100 − 90)% of 350000 = $\frac{10}{100}$ x 350000 = 35000

In 2001, total number of cars produced = 44000

F = (40-0)% of 440000 = $\frac{40}{100}$ x 440000 = 176000

E = (60-40)% of 440000 = $\frac{20}{100}$ x 440000 = 88000

D= (75-60)% of 440000 = $\frac{15}{100}$ x 440000 = 66000

C= (85-75)% of 440000 = $\frac{10}{100}$ x 440000 = 44000

B = (95-85)% of 440000 = $\frac{10}{100}$ x 440000 = 44000

A= (100-95)% of 440000 = $\frac{5}{100}$ x 440000 = 22000

Total number of cars of model B, E and F manufactured in 2000 = (52500 + 70000 + 87500) = 210000

4. If 85% of the C-type cars produced in each year were sold by the company, how many C-type cars remain unsold?

Solution: Number of C-type cars which remain unsold in 2000 = $\frac{15}{100}$ X 35000 = 5250

Number of C-type cars which remained unsold in 2001 = $\frac{15}{100}$ ×44000 = 6600

Therefore, total number of C-type cars which

remained unsold = 5250 + 6600 = 11850

5. For which model was the percentage rise/fall in production from 2000 to 2001 minimum?
Solution: The percentage change in production from 2000 to 2001 for various models is:

F = $\left[\frac{(176000-87500)}{105000} x\ 100\right]$ = 101.14% rise

E = $\left[\frac{(88000-70000)}{70000} x\ 100\right]$ = 25.71% rise

D = $\left[\frac{(70000-66000)}{105000} x\ 100\right]$ = 5.71% fall

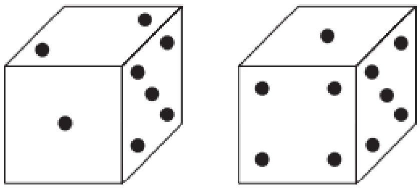
C = $\left[\frac{(44000-35000)}{35000} x\ 100\right]$ = 25.14% rise

B = $\left[\frac{(52000-44000)}{105000} x\ 100\right]$ = 16.19% fall

A = $\left[\frac{(35000-22000)}{35000} x\ 100\right]$ = 37.14% fall

Therefore, minimum change is observed in model D.

6. Observe the dots on a dice (1—6 dots) in the following figures. How many dots are contained on the face opposite to that containing 4 dots?



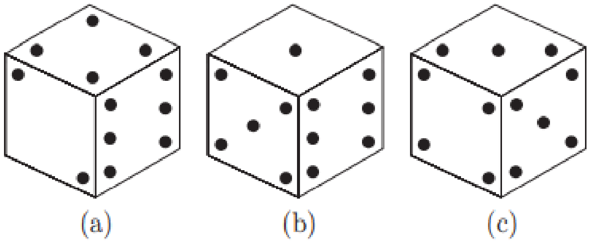
(a) (b)

Solution: We can see from Figs. (a) and (b) that the side of the dice with 5 dots remains stationary.

Hence, when we shift the dice such that we move the side with 1 dot, we observe that one side has 2 dots and the other side has 4 dots.

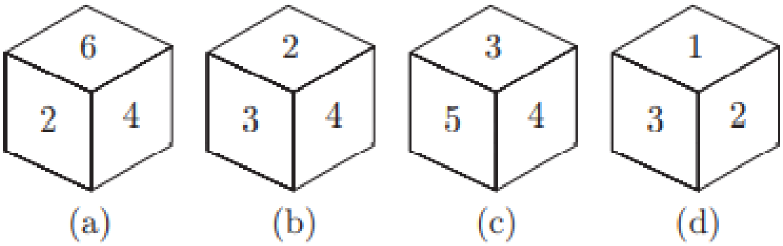
Since these two faces are opposite to each other, 2 dots are contained on the face opposite to that containing 4 dots.

7. Three different positions of a dice are shown in the following figures. How many dots lie opposite 2 dots?



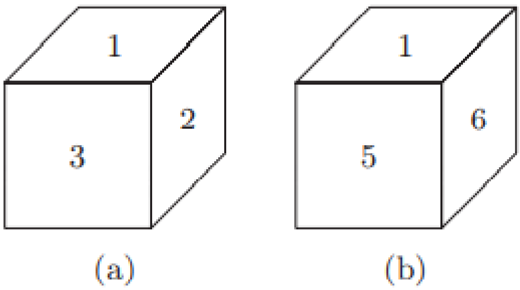
Solution: From Figs. (b) and (c), we conclude that 1, 6, 3 and 4 dots lie adjacent to 5 dots. Therefore, 2 dots must lie opposite 5 dots. Conversely, 5 dots must lie opposite 2 dots.

8. A dice is thrown four times and its four different positions are shown in the following figures. What is the number on the face opposite the face showing 2?




Solution: From Figs. (a), (b) and (d), we conclude that 6, 4, 3 and 1 lie adjacent to 2. Hence, 5 must lie opposite 2

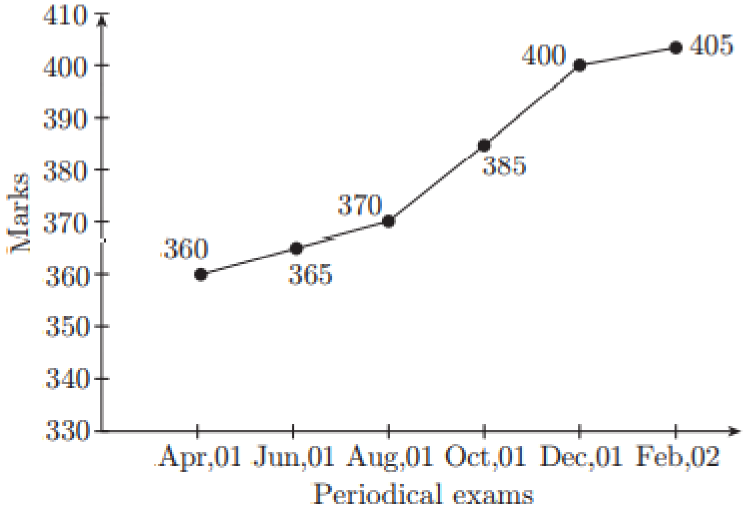
9. Two positions of a cube with its surfaces numbered are shown in the following figures. When the surface 4 touches the bottom, which surface will be on the top?




Solution: In these 2 positions, one common face with number 1 is in the same position. Also, we can see that 2, 3, 5 and 6 are adjacent to 1. Thus, 1 and 4 are opposite.

Therefore, when surface 4 touches the bottom, 1 will be on the top.

10. Consider the following graph:




The above graph shows marks obtained by a student in six periodicals held in every two months during the year in the session 2001—2002. In addition, the maximum total marks in each periodical exam are 50

What is the percentage of marks obtained by the student in the periodical exams of August, 01 and October, 01 taken together?

## Further Readings

1. Quantitative Aptitude For Competitive Examinations By Dr. R S Aggarwal, S Chand Publishing
2. A Modern Approach To Verbal & Non-Verbal Reasoning By Dr. R S Aggarwal, S Chand Publishing
3. Magical Book On Quicker Maths By M Tyra, Banking Service Chronicle
4. Analytical Reasoning By M.K. Pandey, Banking Service Chronicle

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